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DE from non-local effective gravity modifications

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Outline

Part I: theory

- Motivations and context: the DE problem
- Local and non-local GR modifications

Part II: phenomenology

- Spherical solutions (no vDVZ discontinuity)
- Cosmology: background & perturbations

Modify GR in the infrared to get a DE model
with same # of parameters as Λ CDM:

$$m \longleftrightarrow \Lambda$$

Local modifications: massive gravity

Fierz-Pauli: ghost-free at the linear level

dRGT: ghost-free at the nonlinear level

no flat FRW, instabilities

need for a reference, nondynamical metric

bigravity: extra massive spin2,
general covariant and still ghost-free

good background

perturbations under investigations

Non-local modifications: general covariance without extra degrees of freedom

degravitation: $\left(1 - \frac{m^2}{\square}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$ (Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

$$\square_r^{-1} f(x) \equiv \int d^4 y G_r(x, y) f(y)$$

retarded Green function
to ensure **causality**

Cannot be obtained from a variational principle:

$$\begin{aligned} \frac{\delta}{\delta\phi(x)} \int dx' \phi(x') (\square_{\mathbf{r}}^{-1} \phi)(x') &= \frac{\delta}{\delta\phi(x)} \int dx' dx'' \phi(x') G_{\mathbf{r}}(x', x'') \phi(x'') \\ &= \int dx' [G_{\mathbf{r}}(x, x') + G_{\mathbf{r}}(x', x)] \phi(x') \end{aligned}$$



no fundamental lagrangian: effective theory
(ghost problem cannot be addressed)

Still, non-local terms arise naturally in several contexts:

- when integrating out extra degrees of freedom (other fields or short-wavelength modes)
- in QFT, computing effective equations for in-in matrix elements (associated to classical observables)

possible implementations:

first try:

$$G_{\mu\nu} - m^2 (\square_r^{-1} G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

$$\left(S_{\mu\nu} = S_{\mu\nu}^T + (\nabla_\mu V_\nu + \nabla_\nu V_\mu) \quad \nabla^\mu S_{\mu\nu}^T = 0 \right)$$

does not work:

in general, taking the transverse part of tensors brings instabilities:

$$S_{\mu\nu}^T \leftarrow S_0^0, S_i^i, V_0$$

two ways out :

$$\text{I: } G_{\mu\nu} - m^2 (g_{\mu\nu} \square_r^{-1} R)^T = 8\pi G T_{\mu\nu}$$

$$(g_{\mu\nu} S)^T \leftarrow S$$

$$\text{II: } G_{\mu\nu} - m^2 K_{\mu\nu}^{NL} = 8\pi G T_{\mu\nu}$$

with

$$\nabla^\mu K_{\mu\nu}^{NL} \equiv 0$$

trick: “derive” it from an “action”, e.g.:

$$S_{\text{NL}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - m^2 R \frac{1}{\square_r^2} R \right] \quad \square \rightarrow \square_r$$

I and II are equivalent at linear level

Part II

Spherical solutions

Absence of vDVZ discontinuity and of a strong coupling regime

- write the eqs of motion of the non-local theory in spherical symmetry

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

- for $mr \ll 1$: low-mass expansion
- for $r \gg r_s$: Newtonian limit (perturbation over Minkowski)
- match the solutions for $r_s \ll r \ll m^{-1}$ (this fixes all coefficients)

For both models **I** and **II** one finds:

$$\text{for } r_s \ll r \ll m^{-1}: \quad A(r) \simeq 1 - \frac{r_S}{r} \left(1 + \frac{m^2 r^2}{6} \right)$$

the limit $m \rightarrow 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3} \frac{r_S}{r} \left(1 - \frac{r_S}{12 m^4 r^5} \right)$$

vDVZ discontinuity

breakdown of linearity below
 $r_V = (r_S / m^4)^{1/5}$

Both **I** and **II** are ok with solar system constraints

Cosmology: background

trade nonlocal for local terms

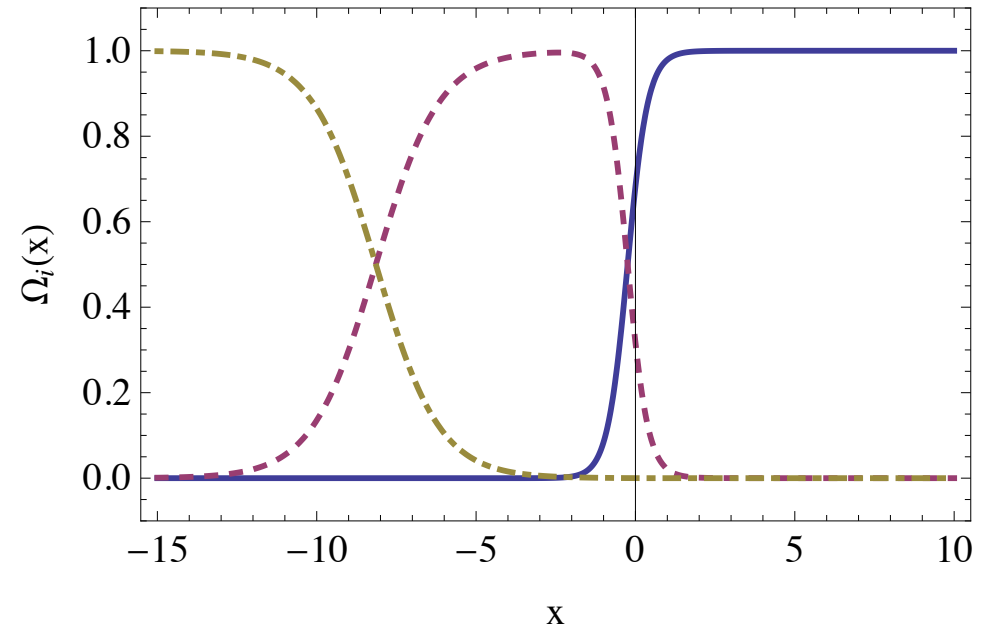
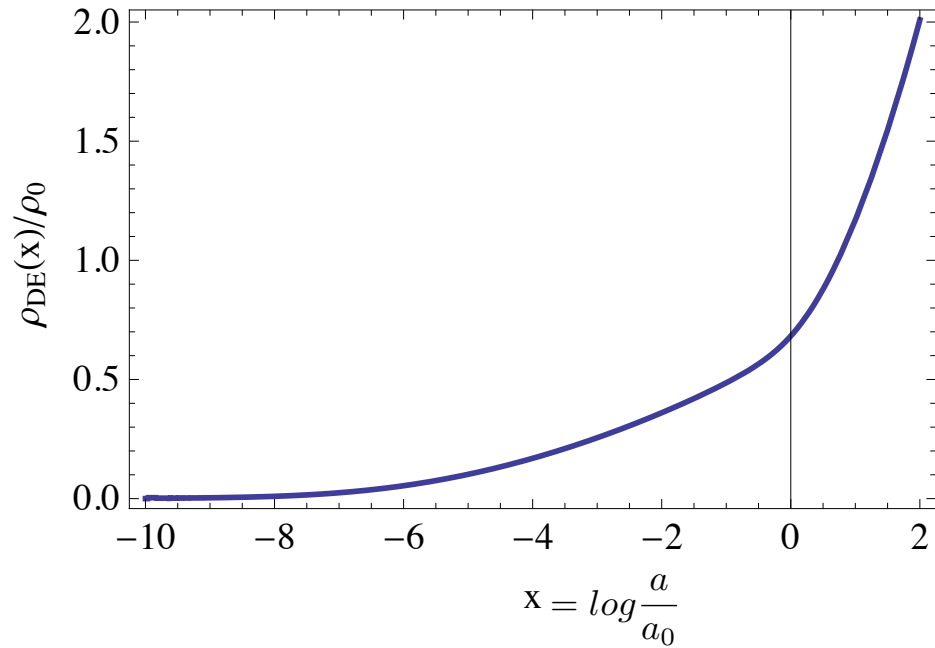
$$\square_r^{-1} R \rightarrow U : \square_r U = R, R = 0 \Rightarrow U = 0$$

and run numerical evolution

starting from deep in radiation era $(R = 0)$

adjust the only parameter m in order to have $\rho + \rho_{DE} = \rho_0$ today,
if possible

(as is done in Λ CDM, after all)



$$\gamma \equiv \frac{m^2}{9H_0^2} = \begin{array}{ll} 0.0504 & \text{(I)} \\ 0.0891 & \text{(II)} \end{array}$$

The effective DE is sourced by R and starts growing only during MD

DE equation of state

$$\dot{\rho}_{\text{DE}} + 3(1 + w_{\text{DE}})H\rho_{\text{DE}} = 0$$

$$w_{DE}^0 = \begin{array}{ll} -1.04 & \text{(I)} \\ -1.14 & \text{(II)} \end{array}$$

phantom



a general feature of DE
models sourced by R:

one unavoidably has $\dot{\rho}_{DE} > 0$ and $\rho_{DE} > 0$ at some point

Stability:

strictly speaking, one never has

$$R = 0$$

so one should check that small deviations from the natural initial conditions for the auxiliary local fields

$$U_{in} = 0$$

do not spoil the good behaviour.

Indeed one always finds:

$$\delta U \simeq e^{\alpha x}, \alpha < 0$$

Perturbations

are a much more stringent test

e.g.: Deser-Woodward model $Rf(\square^{-1}R)$

ruled out by **structure formation** at 8σ level.

Perturbations equations for models **I** and **II** have been studied in the fluid approximation:

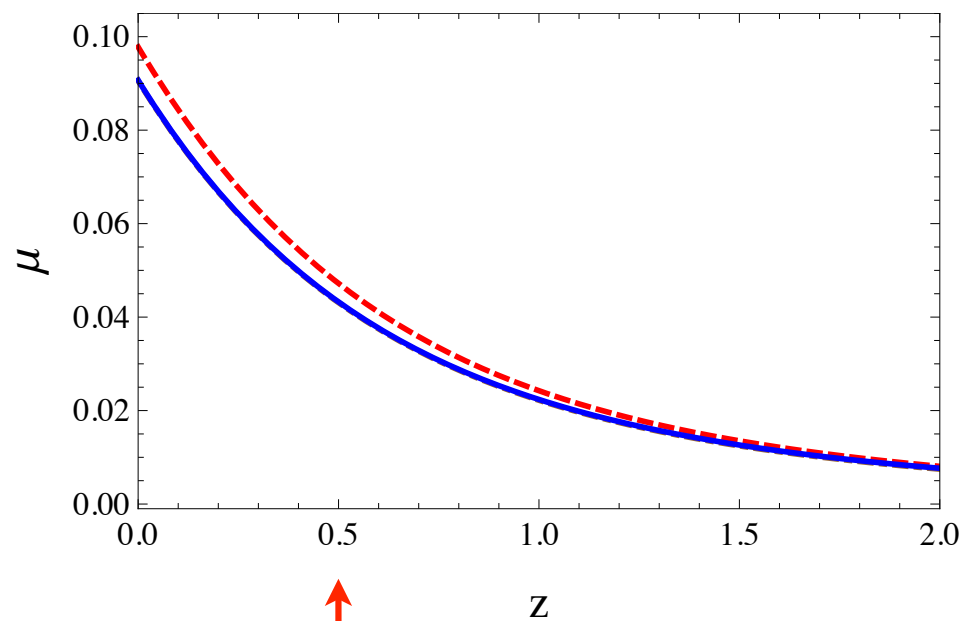
**both are well compatible with observations,
and differ from Λ CDM at a few % level**

see also: NESSERIS, TSUJIKAWA: 1402.4613

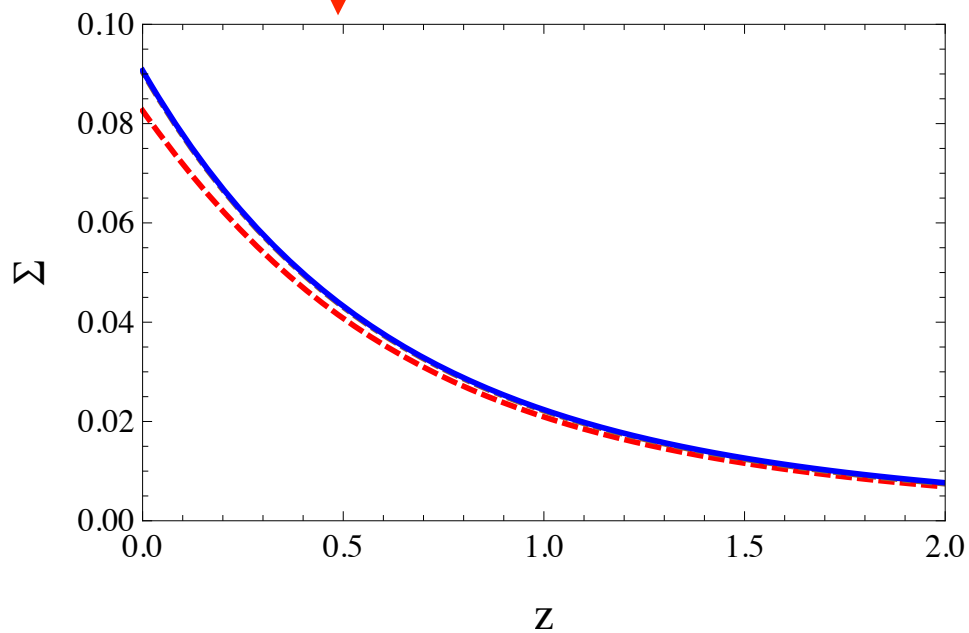
BARREIRA, LI, HELLWING, BAUGH, PASCOLI, in preparation

$$\Psi = [1 + \mu(a; k)] \Psi_{\text{GR}}$$

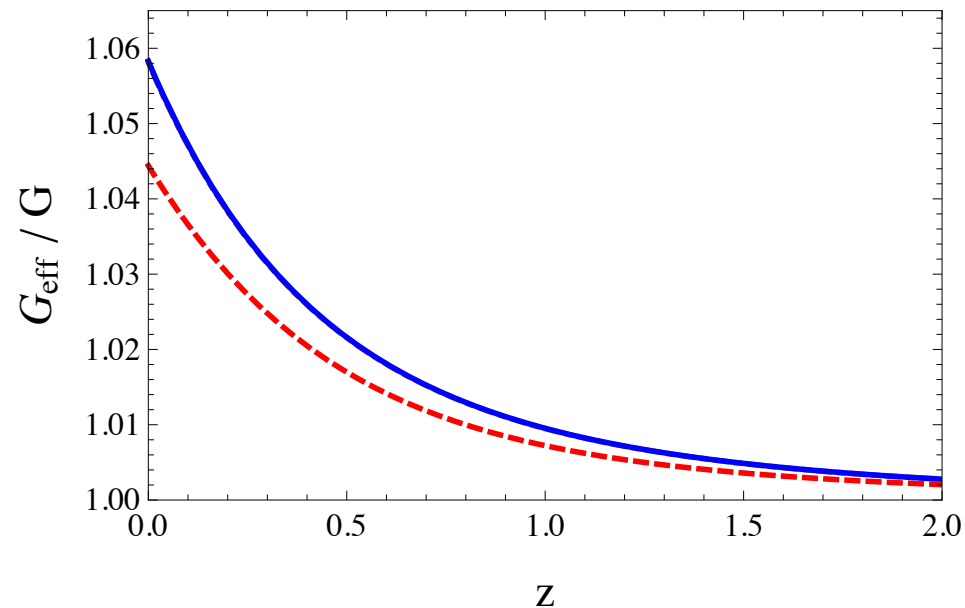
$$\Psi - \Phi = [1 + \Sigma(a; k)] (\Psi - \Phi)_{\text{GR}}$$



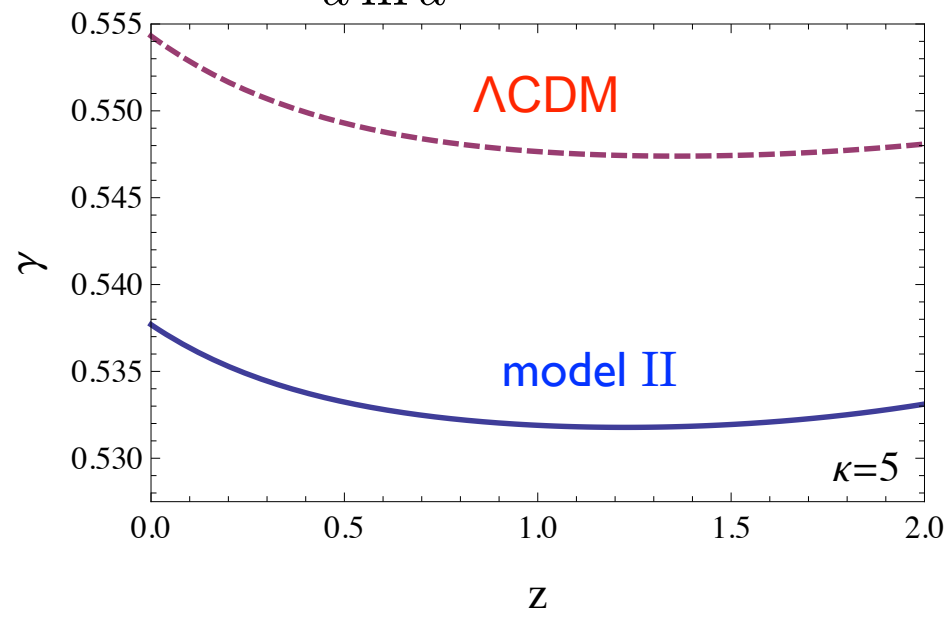
4% difference at z=0.5



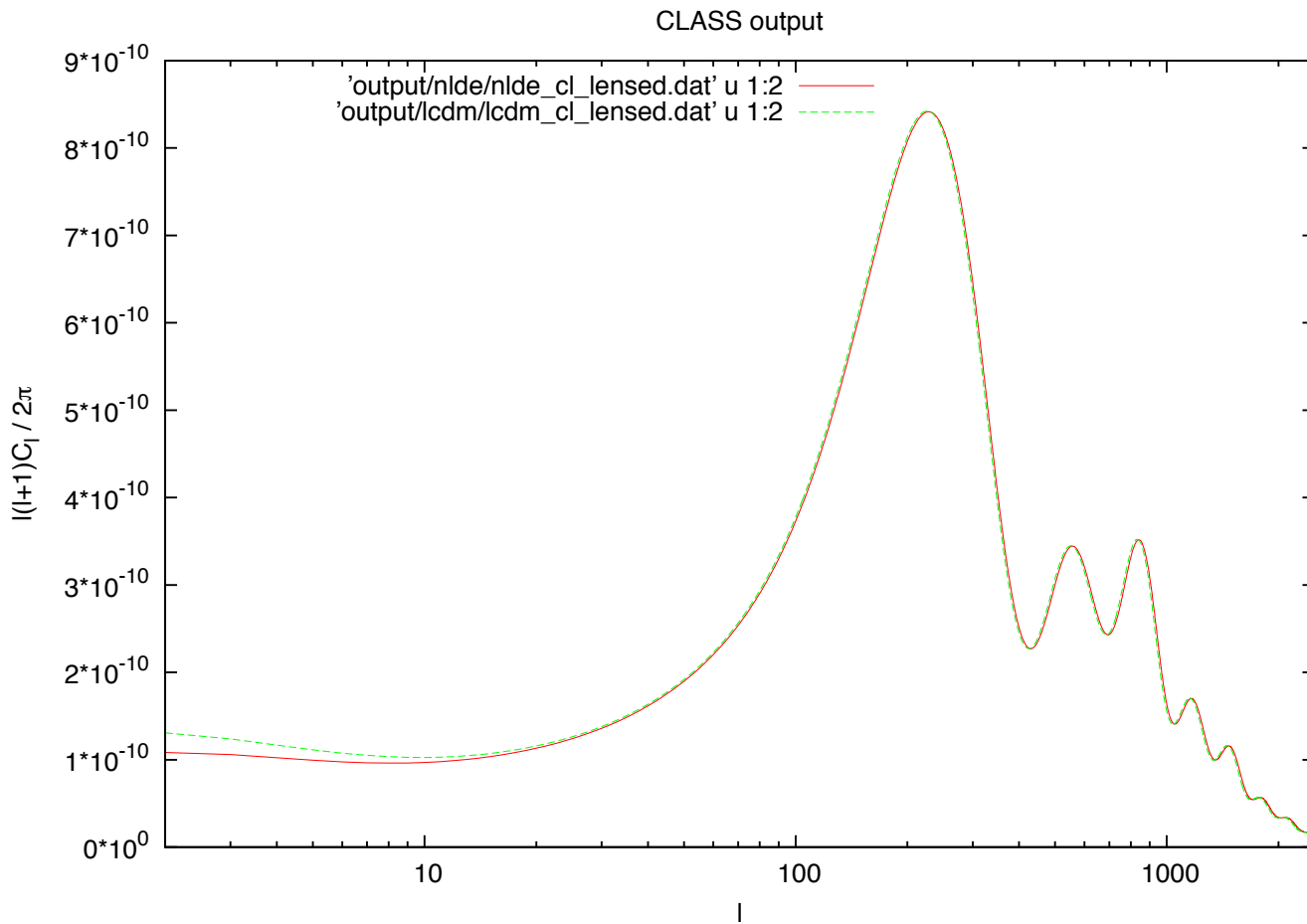
$$k^2 \Phi = 4\pi G_{\text{eff}} \delta \rho$$



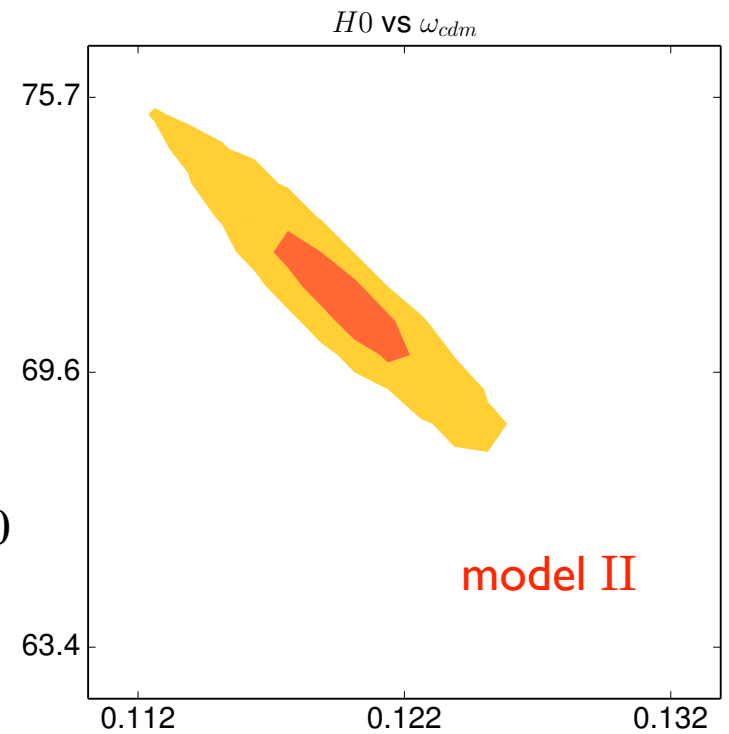
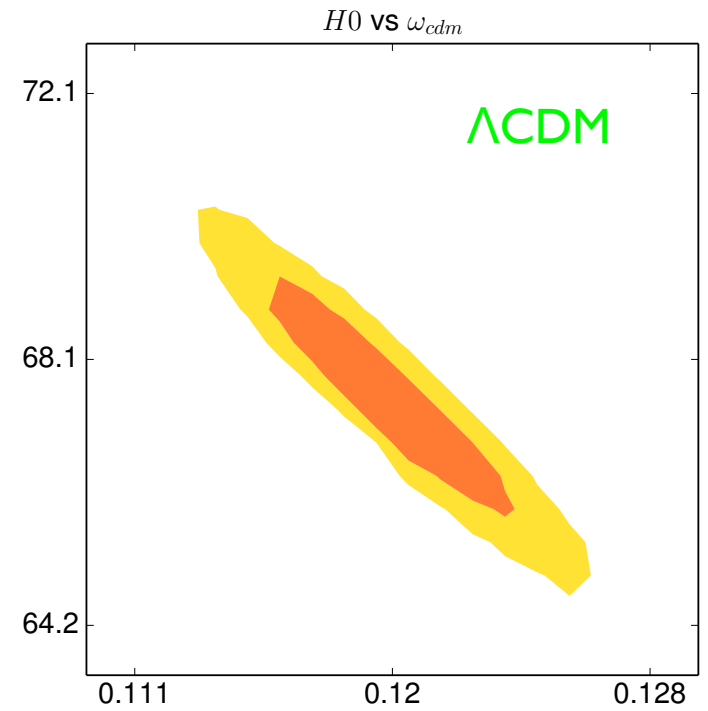
$$\frac{d \log \delta_M(a; k)}{d \ln a} = [\Omega_M]^{\gamma(z; k)}$$



CMB (preliminary)



nonlocal models prefer high values for H_0



Conclusions

Nonlocal massive gravity is phenomenological approach providing:

- attenuation of coincidence problem
- phantom e.o.s. $w < -1$
- good structure formation,
with % deviations form Λ CDM
- good agreement with CMB and high H_0

Can it be derived from a more fundamental theory?

...see for instance TSAMIS, WOODARD: 1405.4470
and CUSIN, FUMAGALLI, MAGGIORE: 1407.5580

A fake ghost in massless GR

$$S_{\text{EH}}^{(2)} = \frac{1}{2} \int d^{d+1}x h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma}$$

$$h_{\mu\nu} = h_{\mu\nu}^{\text{TT}} + \frac{1}{2}(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) + \frac{1}{d} \eta_{\mu\nu} s$$

$$S_{\text{EH}}^{(2)} = \frac{1}{2} \int d^{d+1}x \left[h_{\mu\nu}^{\text{TT}} \square (h^{\mu\nu})^{\text{TT}} - \frac{d-1}{d} s \square s \right]$$

$$S_{\text{int}} = \frac{\kappa}{2} \int d^{d+1}x h_{\mu\nu} T^{\mu\nu} = \frac{\kappa}{2} \int d^{d+1}x \left[h_{\mu\nu}^{\text{TT}} (T^{\mu\nu})^{\text{TT}} + \frac{1}{d} s T \right]$$

$$\square h_{\mu\nu}^{\text{TT}} = -\frac{\kappa}{2} T_{\mu\nu}^{\text{TT}}, \quad \square s = \frac{\kappa}{2(d-1)} T$$

It looks as if there are many more propagating d.o.f

Furthermore s seems a ghost !

S. F. Hassan, R. A. Rosen, and A. Schmidt-May 2012