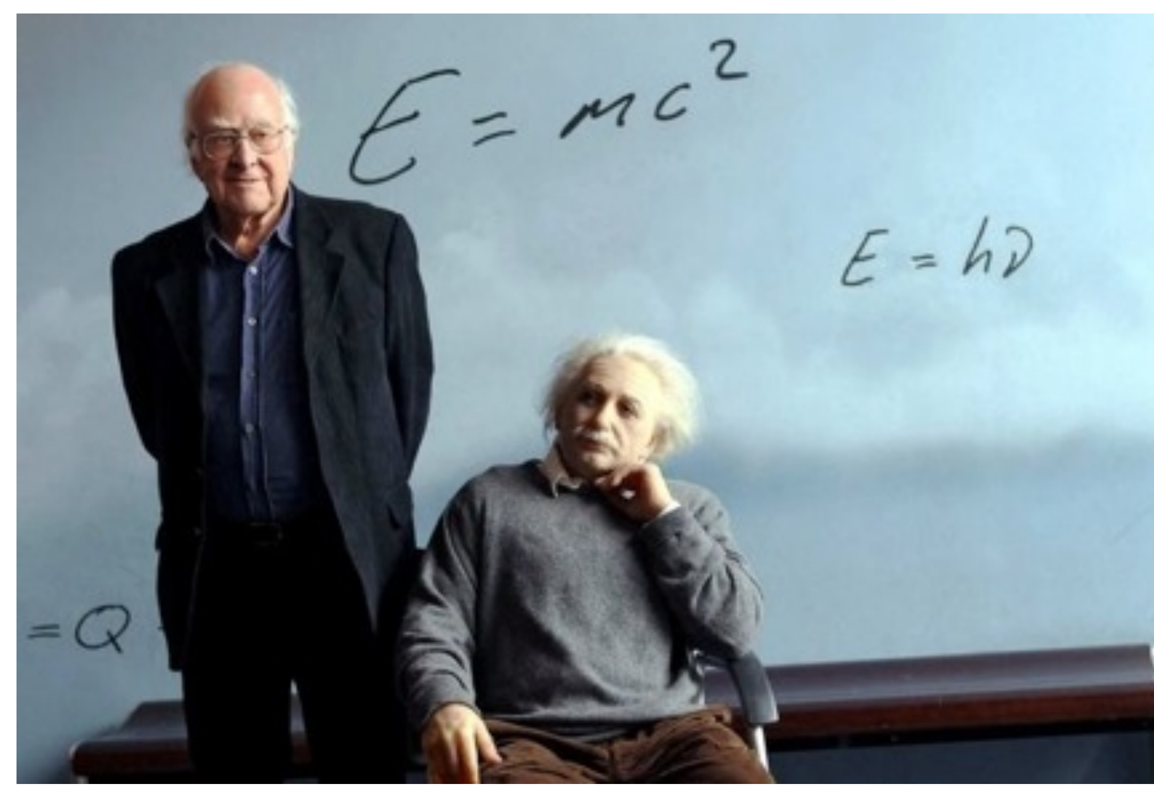
Subtleties in Higgs inflation

with Damien George, Sander Mooij & Marco Volponi
1207.6963, 1310.2157, 1407.6874

Marieke Postma
Nikhef, Amsterdam

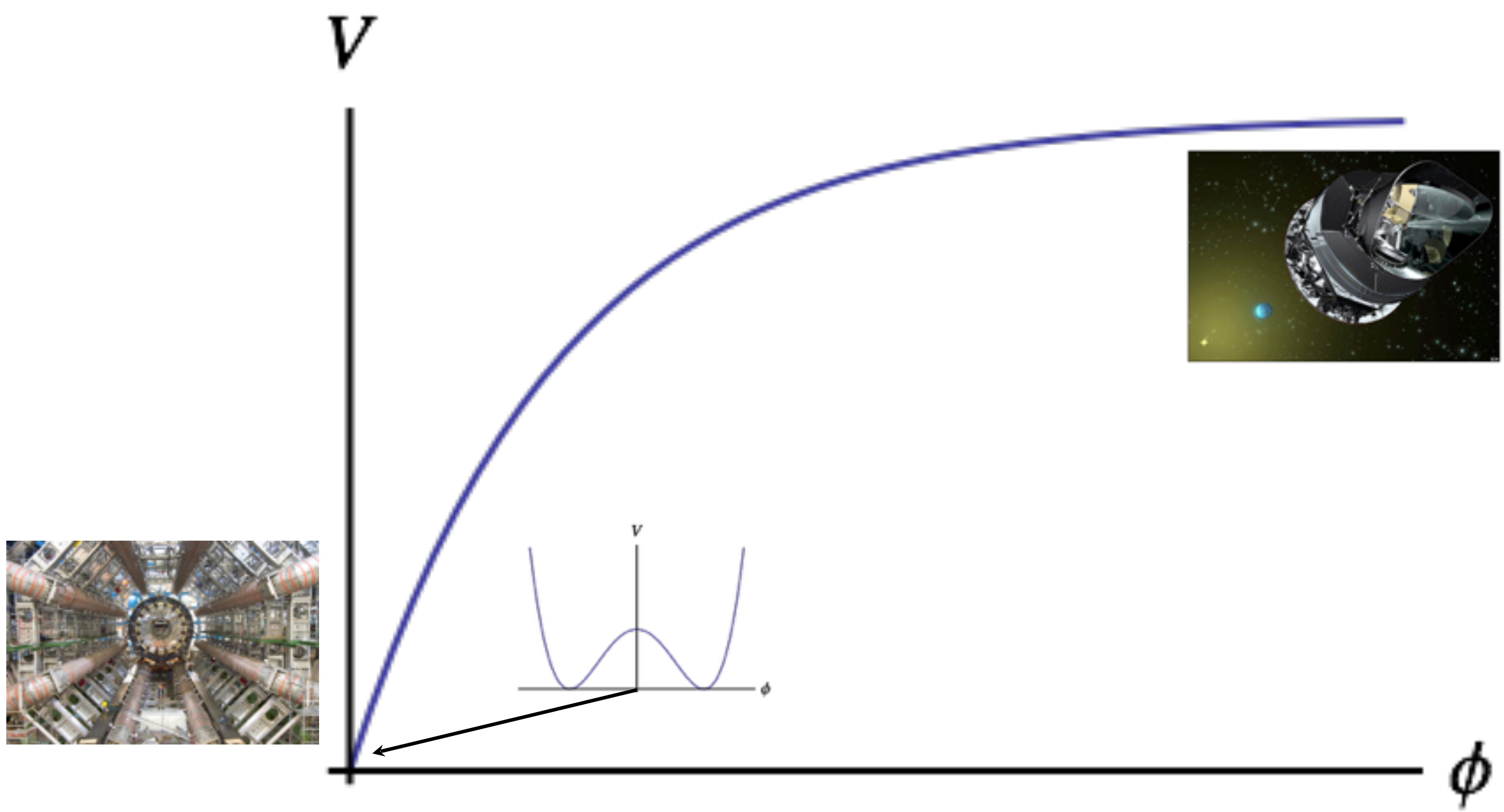


Benasque
August 2014

Higgs inflation

Fakir ’83, Salopek, Bond, Bardeen ’89, Bezrukov & Shaposhnikov ’08

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}M^2\left(1+\xi\frac{\phi^2}{M^2}\right)R + |\partial\phi|^2 - V(\phi)$$



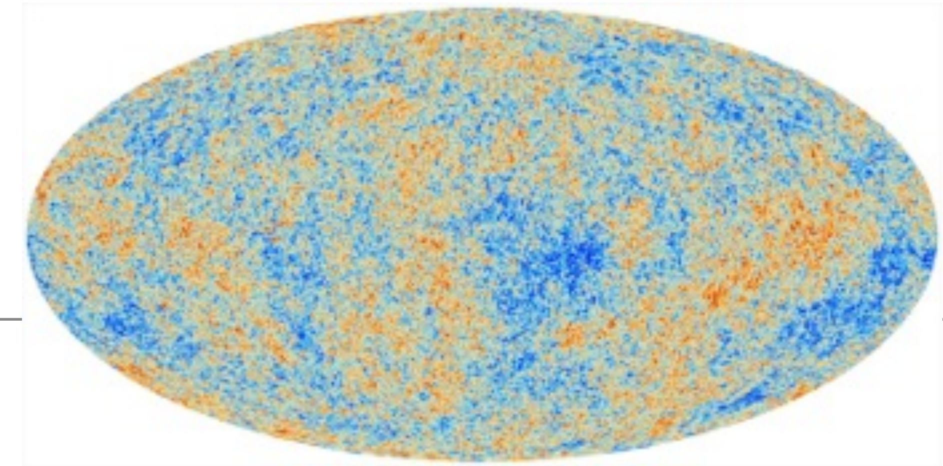
Plan

- Higgs inflation: review

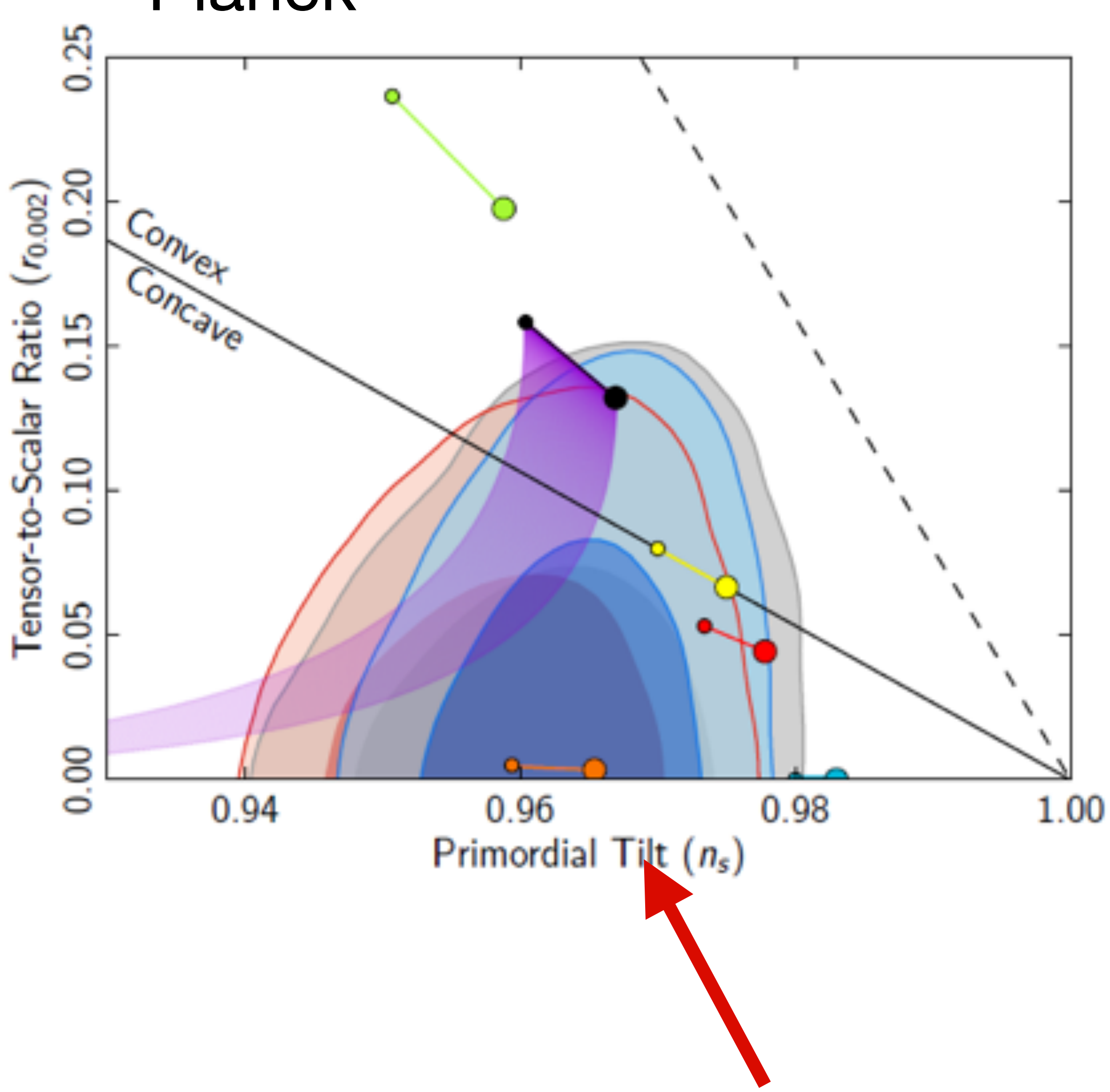
- Jordan vs. Einstein frame

- Renormalizability: goldstone bosons

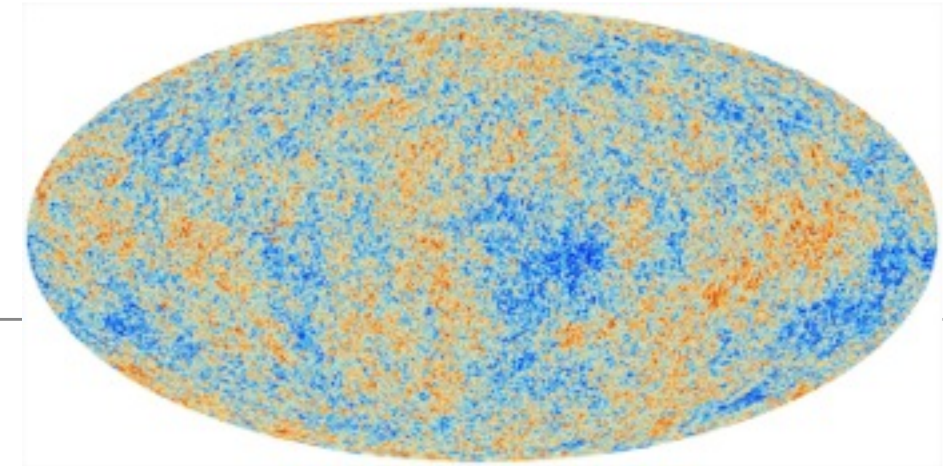
Status: last year



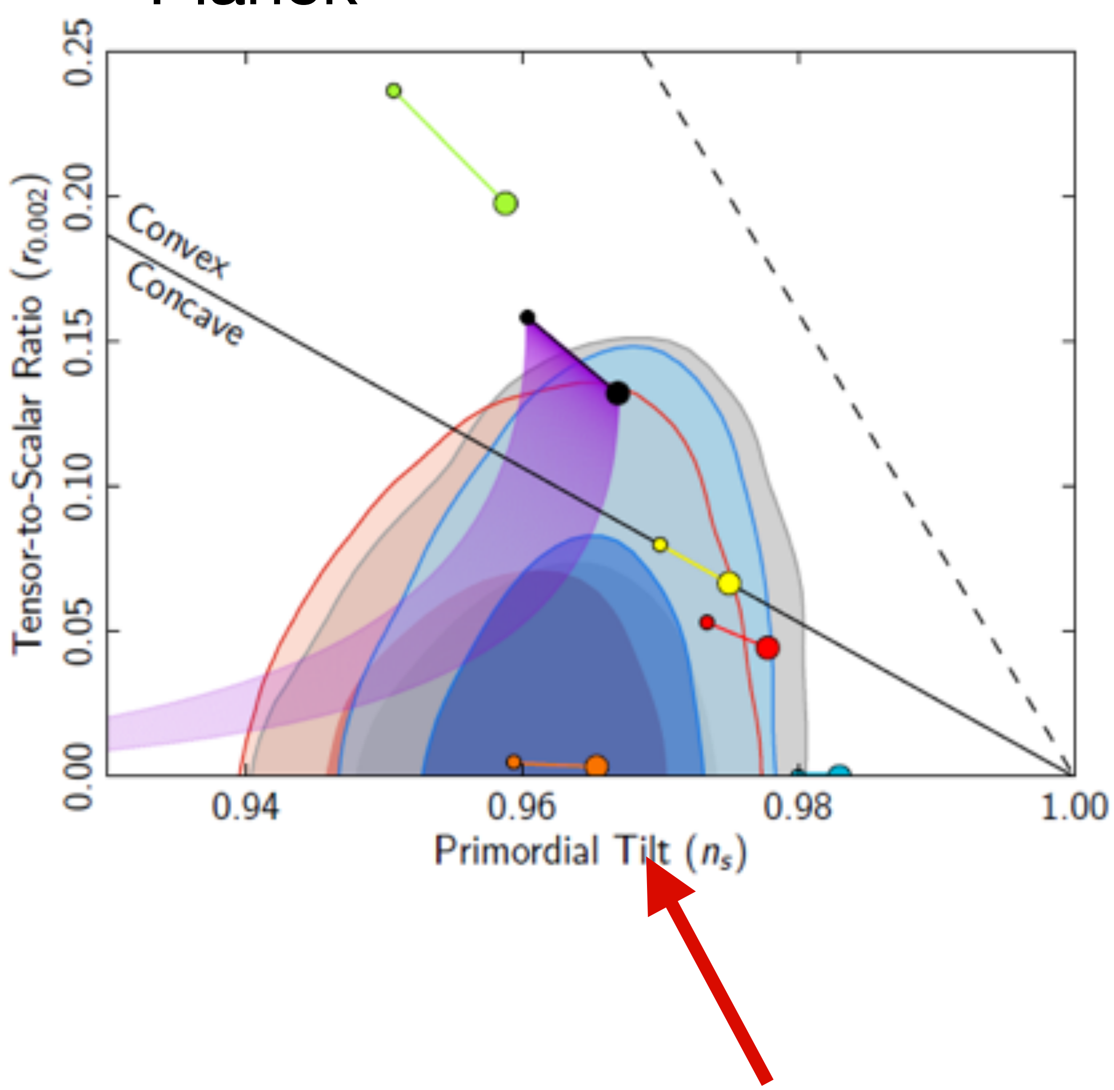
Planck



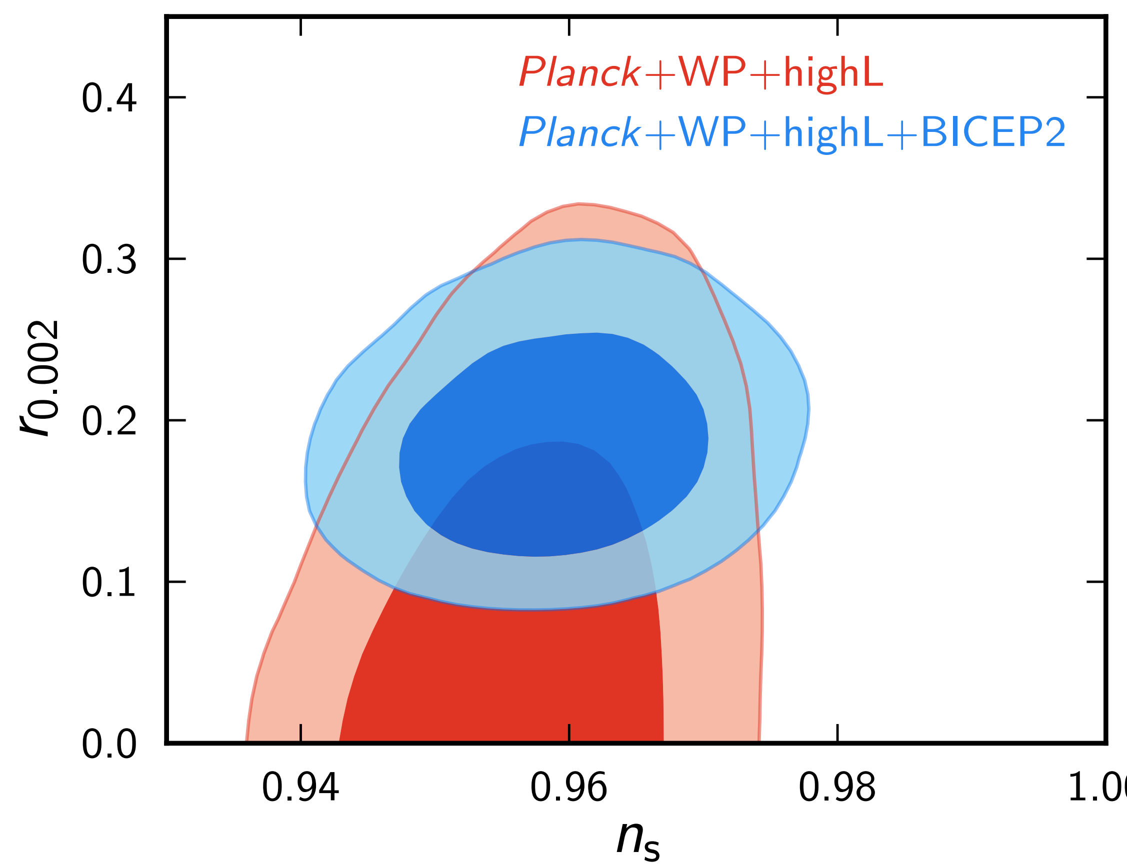
Status: this year



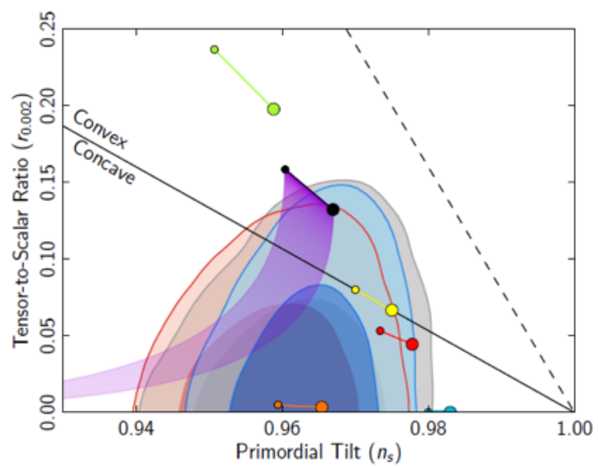
Planck

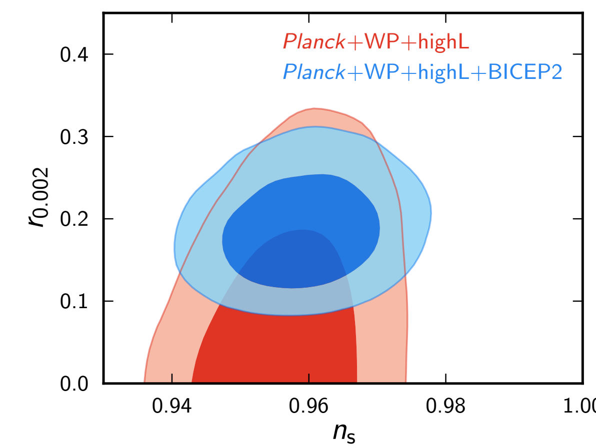


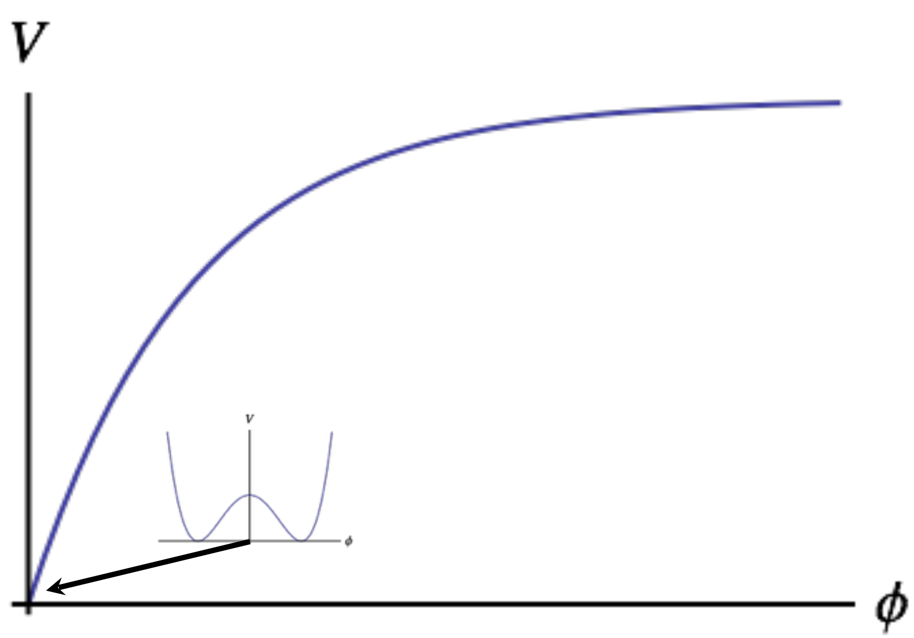
BICEP2

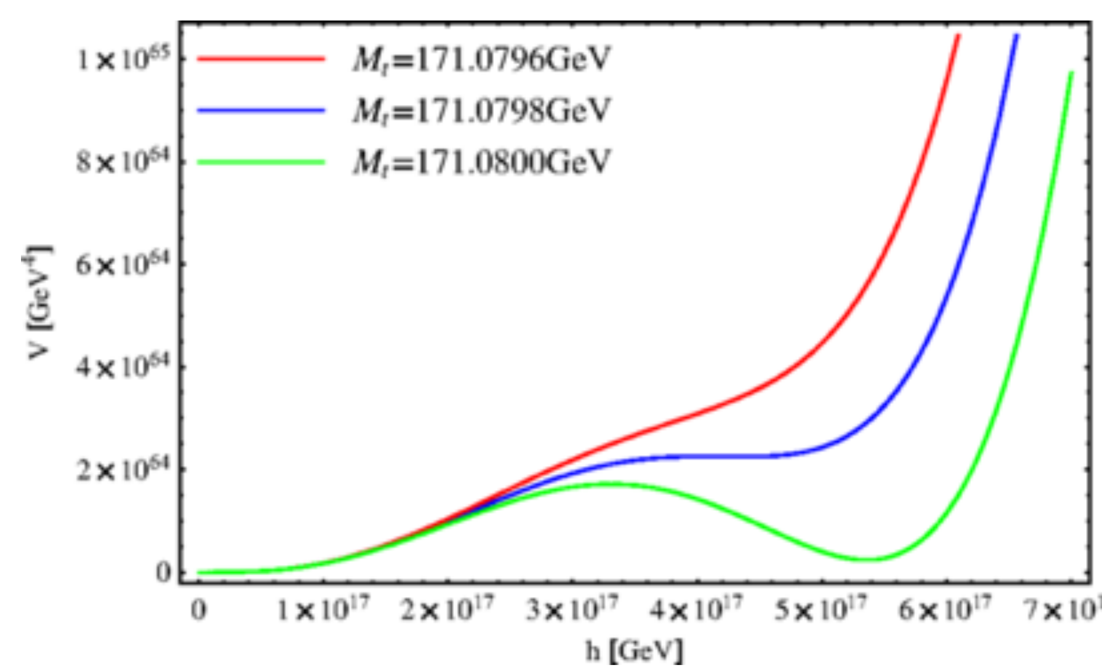


Status: this year





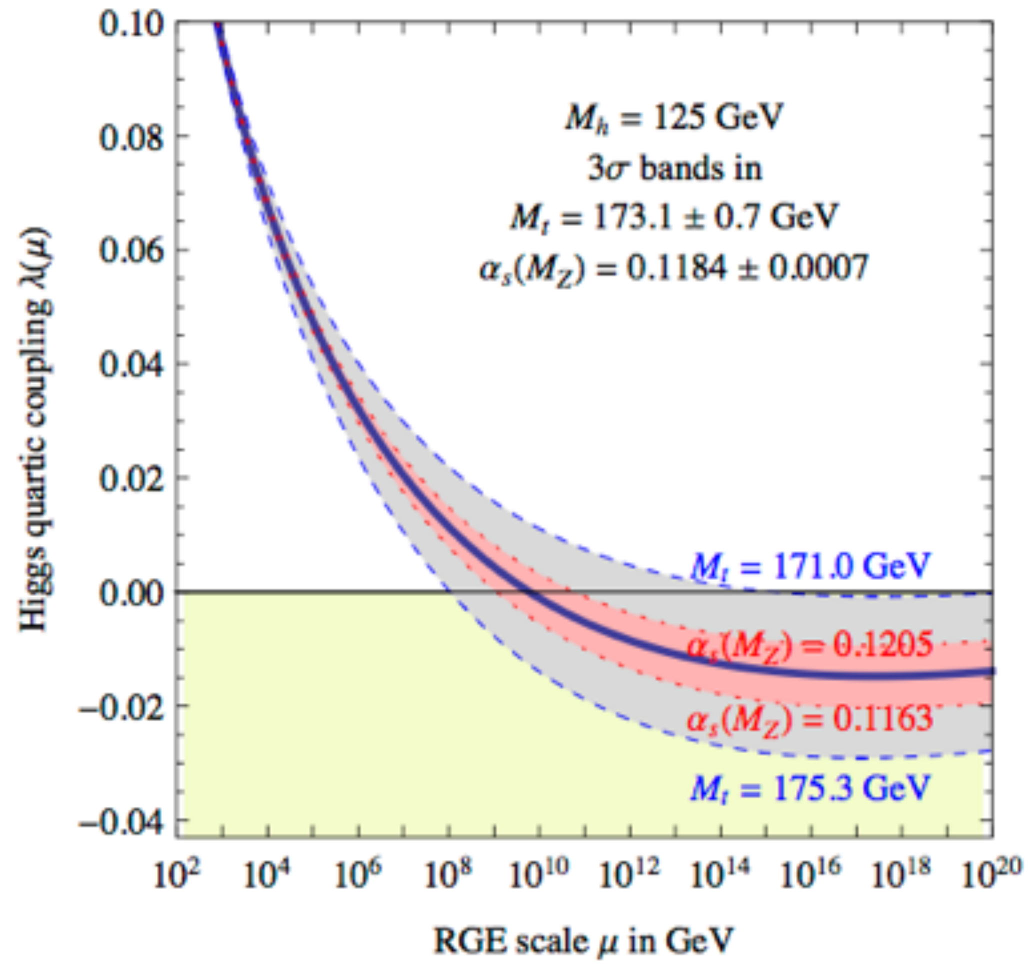




Potential problems with SM Higgs inflation

- stability bound

$$\lambda(\mu) < 0$$

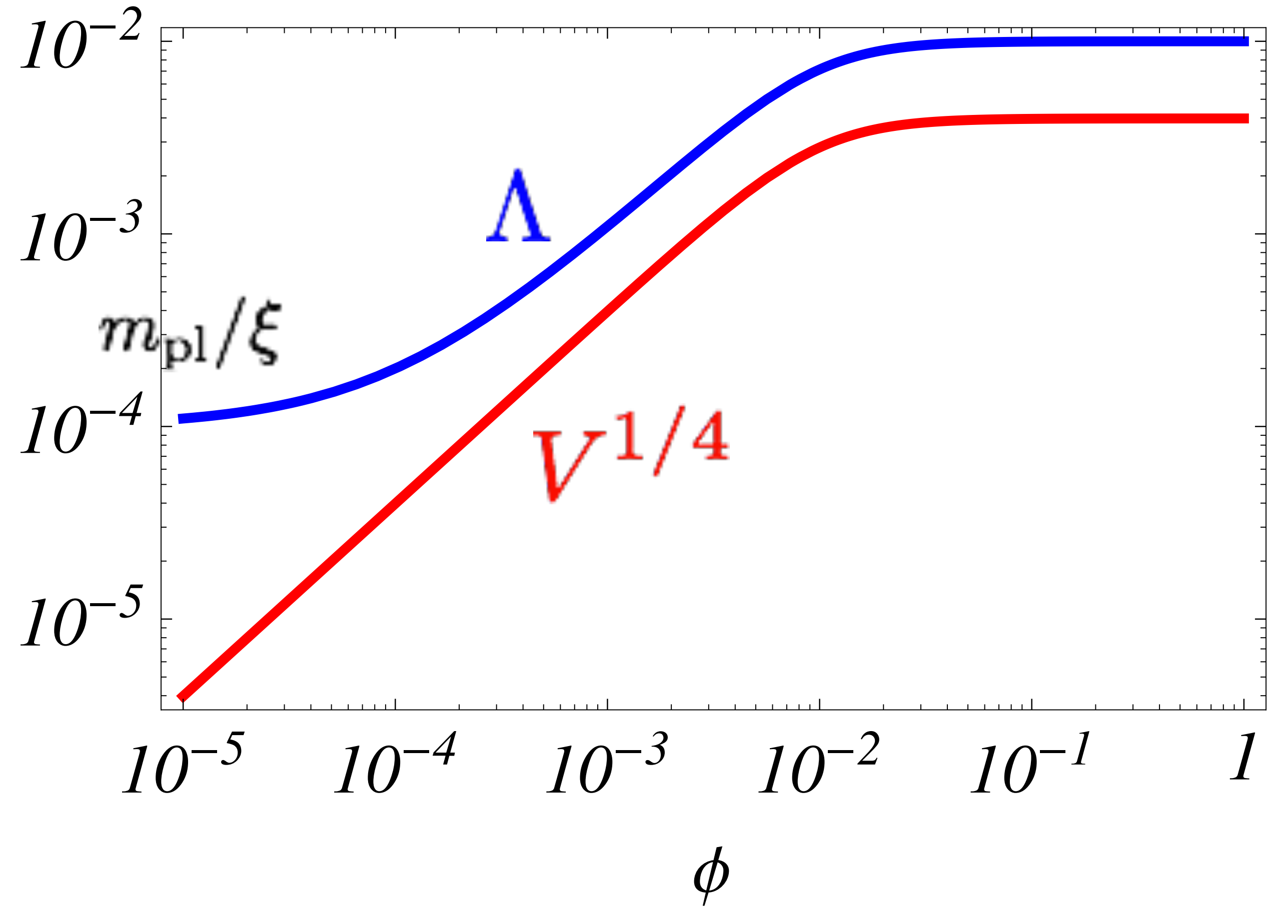


Degrassi et al. ’12, Bezrukov et al. ’12

Potential problems with SM Higgs inflation

- unitarity bound

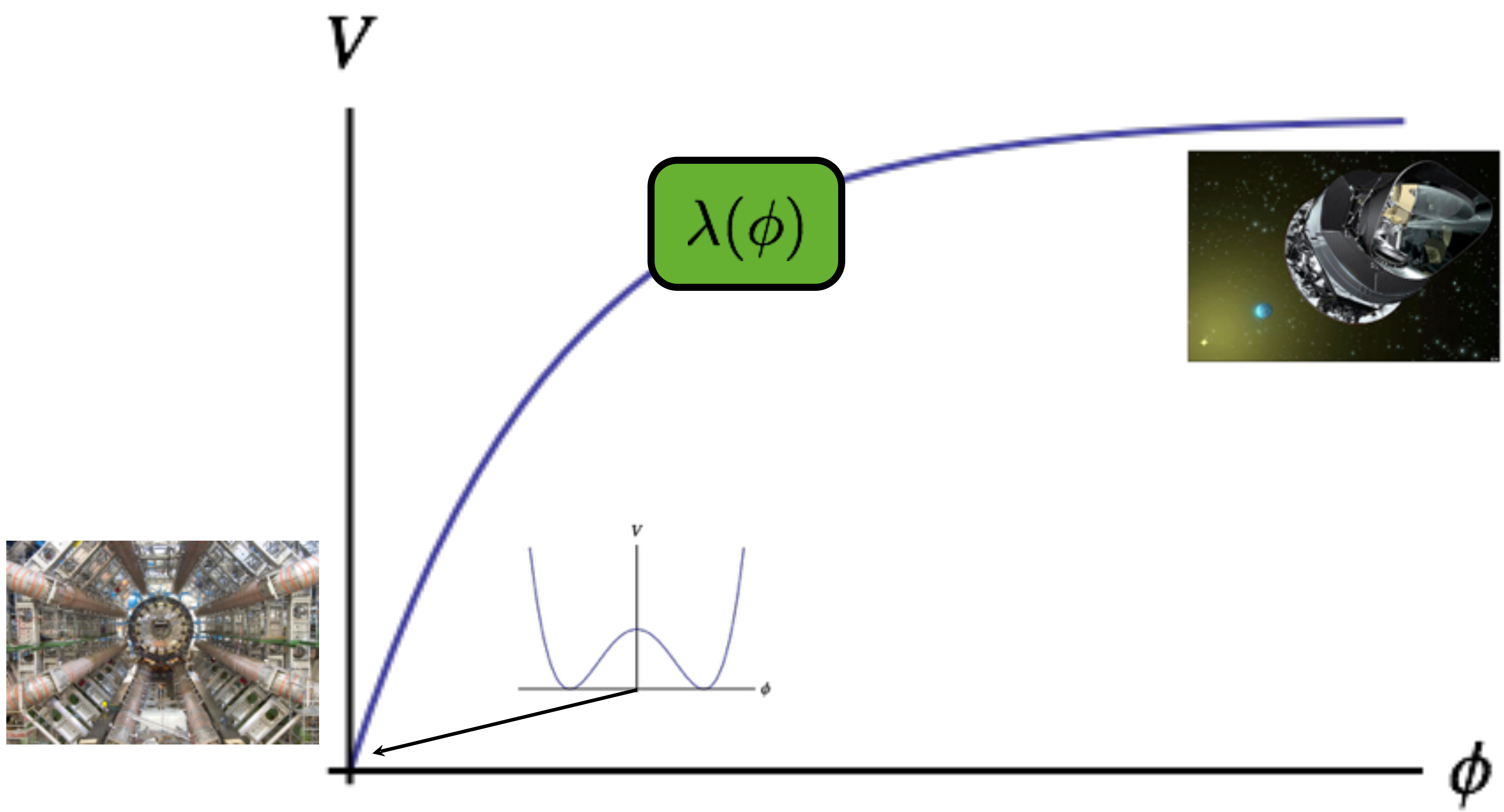
$$\mathcal{M}(\phi\phi \to \phi\phi) > 1$$



Ferrara et al. '11, Bezrukov et al. '11, Burgess '14

Running couplings

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}M^2\left(1 + \xi\frac{\phi^2}{M^2}\right)R + |\partial\phi|^2 - V(\phi)$$



Running couplings

RGEs: $$\mu \frac{\partial \lambda_i(\mu)}{\partial \mu} = \beta_i(\lambda)$$

Bezrukov, Grubinov, Shaposhnikov
Barvinsky, Kamenshchik, Kiefer Starobinsky, Steinwachs
Simone, Hertzberg, Wilzcek
etc.

Plan

- Higgs inflation: review
- Jordan vs. Einstein frame
- Renormalizability: goldstone bosons

Frames

Jordan frame

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = \frac{1}{2} M^2 \underbrace{\left(1 + 2\xi |\frac{\Phi|^2}{M^2}\right)}_{\Omega^2} R[g_J] - |\partial\Phi|^2 - V_J(\Phi_J)$$

$$g^E_{\mu\nu} = \Omega^2 g^J_{\mu\nu}$$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = \frac{1}{2} M^2 R[g_e] - \frac{1}{\Omega^2} |\partial\Phi|^2 + \frac{3\xi^2}{M^2\Omega^4} |\partial|\Phi|^2|^2 - \frac{V_J(\Phi_J)}{\Omega^4}$$

Einstein frame

Frames

Jordan frame

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = \frac{1}{2} M^2 \underbrace{\left(1 + 2\xi |\frac{\Phi|^2}{M^2}\right)}_{\Omega^2} R[g_J] - |\partial\Phi|^2 - V_J(\Phi_J)$$

what one measures:

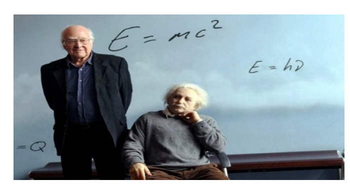
$$m^2_{\mathrm{pl},J} \mathrm{d}s^2_J = m^2_{\mathrm{pl},E} \mathrm{d}s^2_E$$

$$\frac{m^2_J}{m^2_{\mathrm{pl},J}} = \frac{m^2_E}{m^2_{\mathrm{pl},E}}$$

dimensionless quantities invariant !

$$g^E_{\mu\nu} = \Omega^2 g^J_{\mu\nu}$$

overall scaling:



$$\frac{\mathcal{L}}{\sqrt{-g_E}} = \frac{1}{2} M^2 R[g_e] - \frac{1}{\Omega^2} |\partial\Phi|^2 + \frac{3\xi^2}{M^2\Omega^4} |\partial|\Phi|^2|^2 - \frac{V_J(\Phi_J)}{\Omega^4}$$

Einstein frame

Damien George, Sander Mooij & MP 1310.2157

Error 1 — treat gravity as a classical background

Jordan frame

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = \frac{1}{2}M^2\left(1 + 2\xi|\frac{\Phi|^2}{M^2}\right)R[g_J] - |\partial\Phi|^2 - V_J(\Phi_J)$$

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Einstein frame

Error 1 — treat gravity as a classical background

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$$\frac{\mathcal{L}}{\sqrt{-g_J}} = \frac{1}{2}M^2\left(1 + 2\xi|\frac{\Phi|^2}{M^2}\right)R[g_J] - |\partial\Phi|^2 - V_J(\Phi_J)$$

error large: no inflation $m_h^2 > H^2$

error sub-leading $m_h^2 = -2H^2(2 - 3\eta - \epsilon + 6\epsilon)$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = \frac{1}{2}M^2 R[g_e] - \frac{1}{\Omega^2}|\partial\Phi|^2 + \frac{3\xi^2}{M^2\Omega^4}|\partial|\Phi|^2|^2 - \frac{V_J(\Phi_J)}{\Omega^4}$$

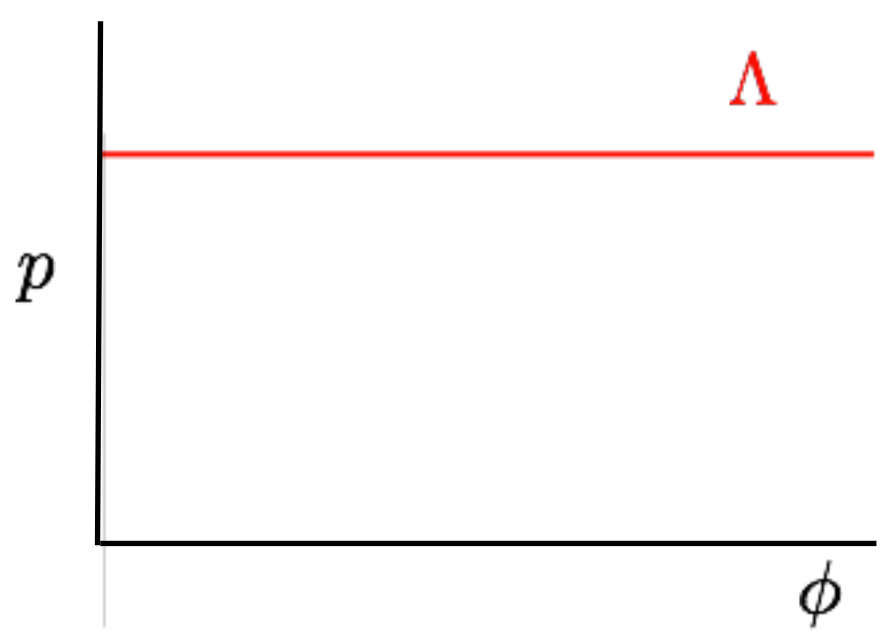
Einstein frame

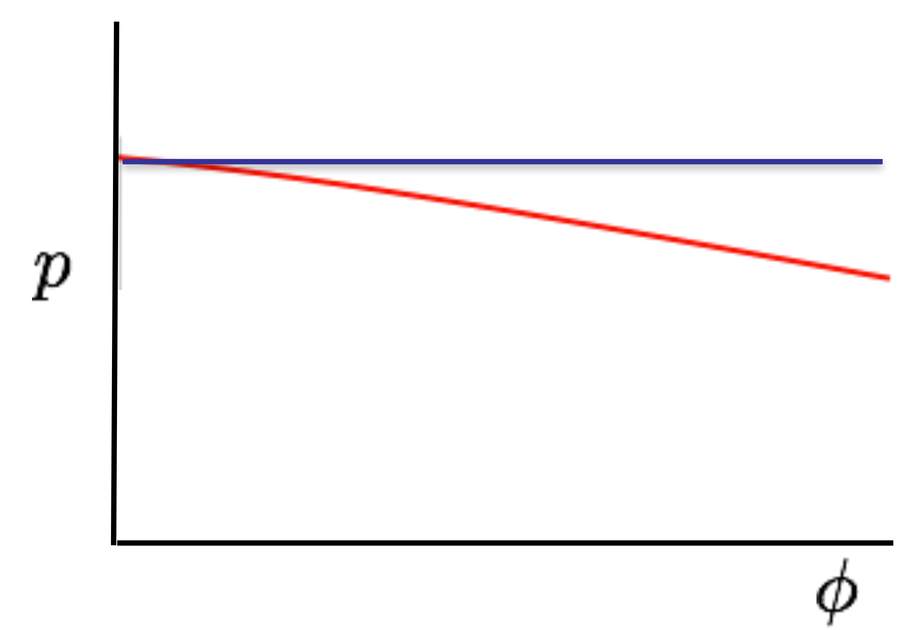
Error 2 — field dependent cutoff

UV dependence?

$$\delta V_E = c m_E^4 \ln\left(\frac{\Lambda^2}{m_E^2}\right)$$

$$\delta V_J = c m_J^4 \ln\left(\frac{\Lambda^2}{m_J^2}\right) \quad \underset{m_J = \Omega m_E}{\Longrightarrow} \quad \delta V_E = c m_E^4 \ln\left(\frac{\Lambda^2}{\Omega^2 m_E^2}\right)$$





Jordan frame

Einstein frame

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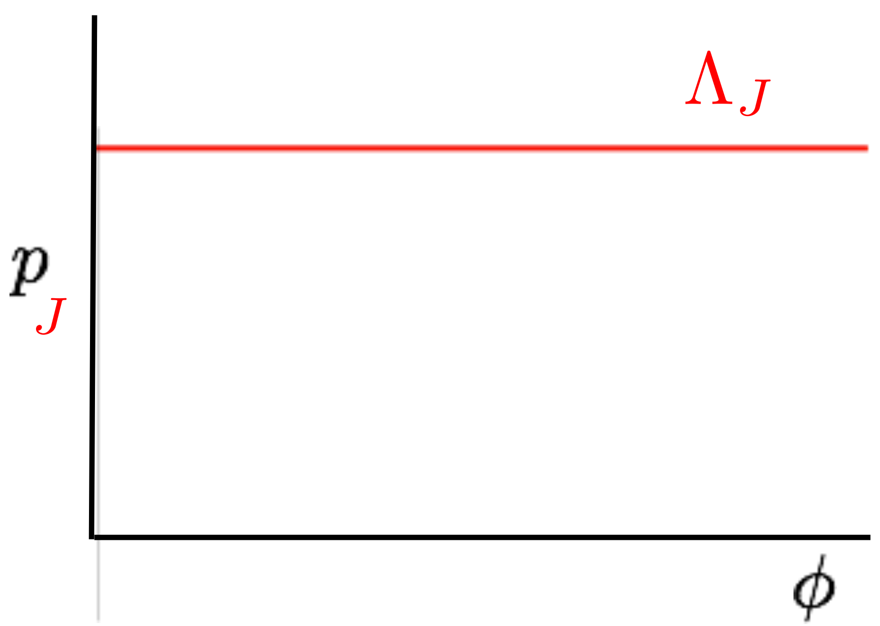
Error 2 — field dependent cutoff

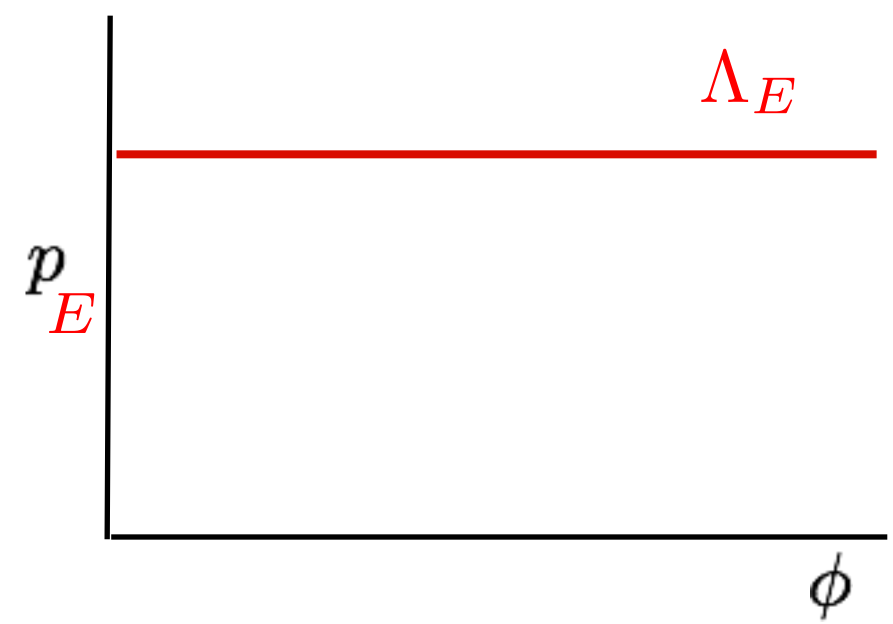
UV dependence?

$$\delta V_J = c m_J^4 \ln\left(\frac{\Lambda_J^2}{m_J^2}\right) \quad \Longrightarrow \quad \delta V_E = c m_E^4 \ln\left(\frac{\Lambda_E^2}{m_E^2}\right)$$

$$m_J = \Omega m_E$$

$$\Lambda_J = \Omega \Lambda_E$$





Jordan frame

Einstein frame

Dimensionless & frame-independent action

$$S = \int d^4x \sqrt{-\bar{g}} \left(\frac{1}{2}\bar{R} - \frac{1}{2}\bar{S}_{ab}\bar{g}_{\mu\nu}\nabla^{\mu}\bar{\phi}^a\nabla^{\nu}\bar{\phi}^b - \bar{V} \right)$$

Planck units

$$m_J^2 \mathrm{d}s_J^2 = m_J^2[-N_J^2\mathrm{d}t^2 + a_J^2\mathrm{d}x^2] = m_E^2[-N_E^2\mathrm{d}t^2 + a_E^2\mathrm{d}x^2] = m_E^2\mathrm{d}s_E^2$$

dimensionless quantities: $\bar{N} = m_i N_i, \qquad \bar{a} = m_i a_i, \qquad i = J, E$

$$\sqrt{-\bar{g}} = \sqrt{-g_i}\, m_i^4$$

$$\bar{H} = \frac{\bar{a}'}{\bar{a}} = \frac{1}{\bar{a}}\frac{\partial_t \bar{a}}{\bar{N}} = \frac{1}{a_i m_i}\frac{1}{m_i N_i}\partial_t(a_i m_i)$$

Plan

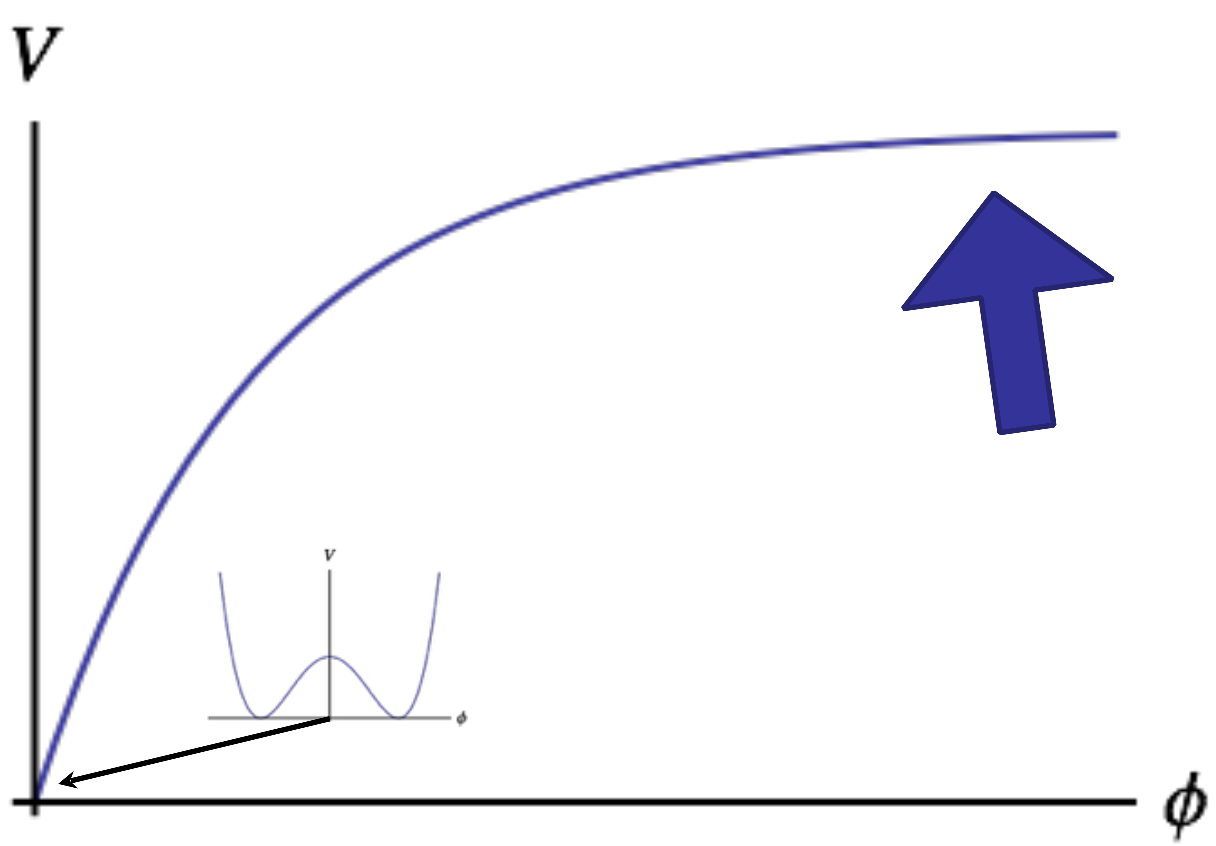
- Higgs inflation: review

- Jordan vs. Einstein frame

- Renormalizability: goldstone bosons

Renormalizability

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}M^2\left(1+\xi\frac{\phi^2}{M^2}\right)R + |\partial\phi|^2 - V(\phi)$$



renormalizable EFT:

expansion in $\delta = \frac{M^2}{\xi\phi_0^2} \ll 1$

demand: at every order finite number of counterterms

Lagrangian

Complex Higgs plus fermion

$$\Phi = (\phi + i\theta)/\sqrt{2}$$

$$\frac{\mathcal{L}_E}{\sqrt{-g}} = \frac{1}{2}M^2 R - \frac{1}{2}\gamma_{ab}\partial_\mu \phi^a \partial^\mu \phi^b + i\bar{\psi}\gamma \cdot \partial\psi - V(\phi^a) - \bar{\psi}F(\phi^a)\psi$$

$$V(\phi^a) = \frac{\lambda}{4}\frac{(\phi + i\theta)^2}{\Omega^4}$$

$$F(\phi^a) = \frac{y}{\sqrt{2}}\frac{\phi + i\gamma^5\theta}{\Omega}$$

expand

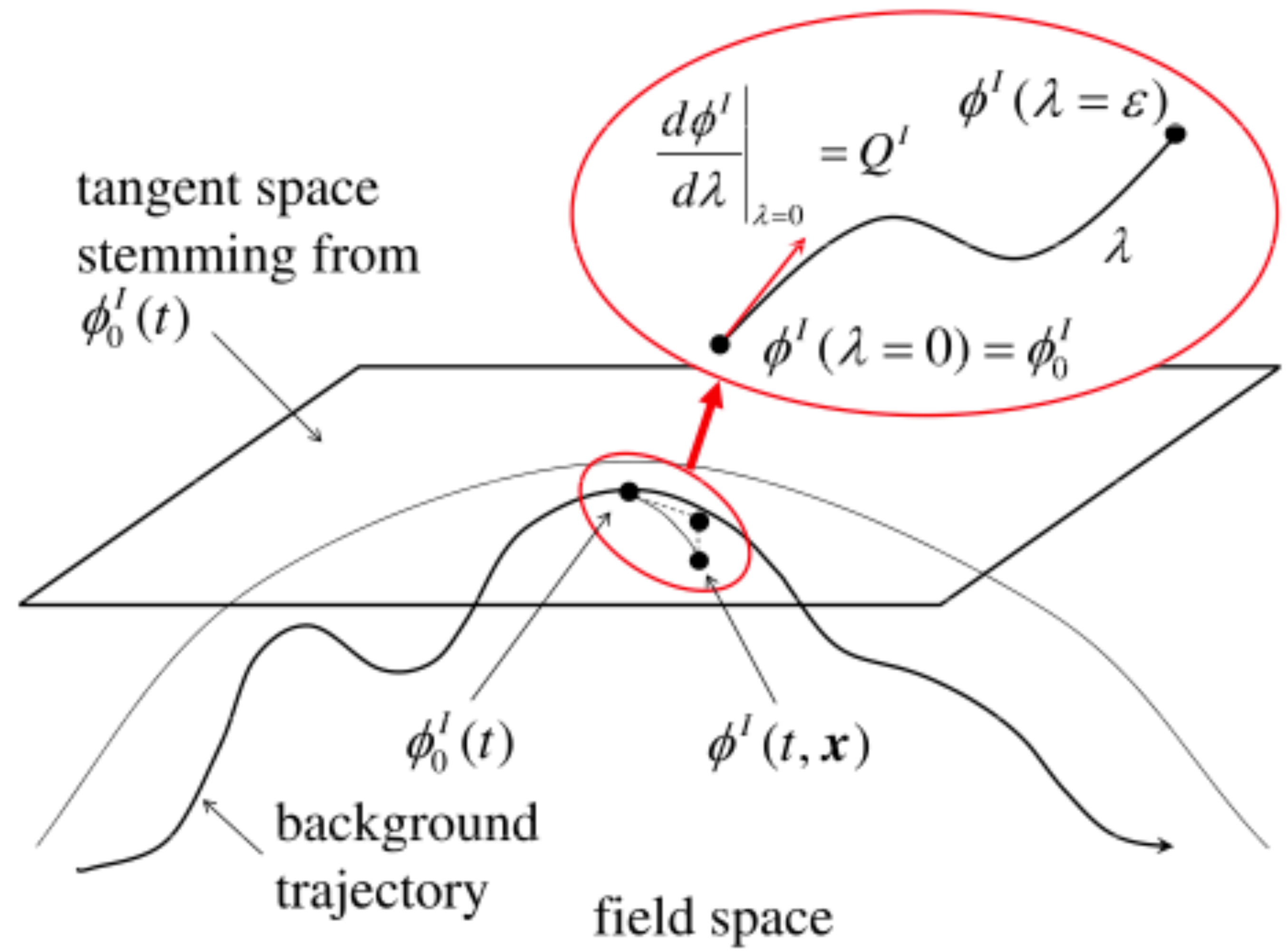
$$\phi^a = (\phi_0(t) + \delta\phi(x,t), \delta\theta(x,t))$$

not covariant!

Lagrangian: covariant formulation

$$\delta\phi^a = Q^a - \frac{1}{2!}\Gamma^a_{bc}Q^bQ^c + \frac{1}{3!}\left(\Gamma^a_{be}\Gamma^e_{cd} - \Gamma^a_{bc,d}\right)Q^bQ^cQ^d + \ldots$$

covariant!



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covariant!

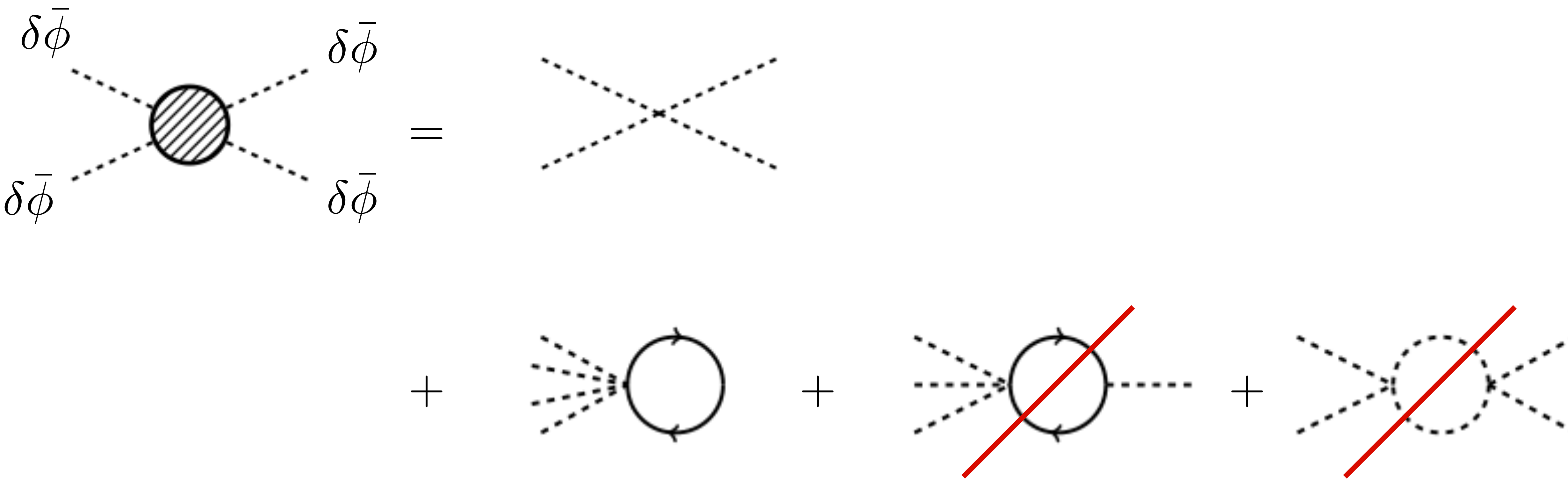
$$-\left(V + V_{;a}Q^a + \frac{1}{2}V_{;ab}Q^a Q^b + ..\right) - \bar{\psi}\left(F + F_{;a}Q^a + F_{;ab}Q^a Q^b + ..\right)\psi$$

notation

$$\frac{\mathcal{L}_E}{\sqrt{-g}} \supset -\lambda_{4\phi}(\delta\bar{\phi})^4 - y_\phi(\delta\bar{\phi})\bar{\psi}\psi - y_{2\theta}(\bar{\delta}\theta)^2\bar{\psi}\psi + \ldots$$

Damien George, Sander Mooij & MP in progress

Higgs scattering



$$= i8\delta^3 \left(\lambda + y^4 \frac{1}{8\pi^2} \frac{1}{\epsilon} \right)$$

$$\lambda_{4\phi} \sim \delta^3, \quad y_{4\phi} \sim \delta^3, \quad y_{3\phi} \sim \delta^{5/2}, \quad y_{1\phi} \sim \delta^{3/2}$$

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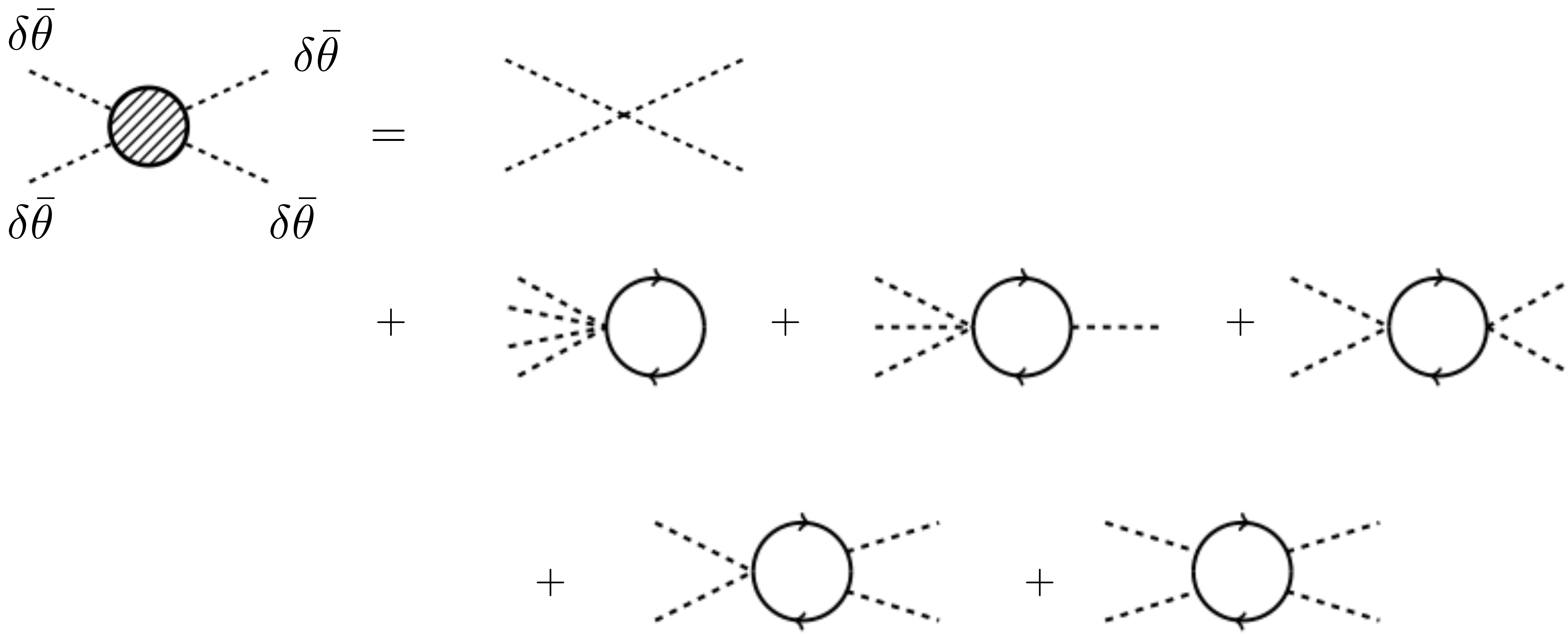
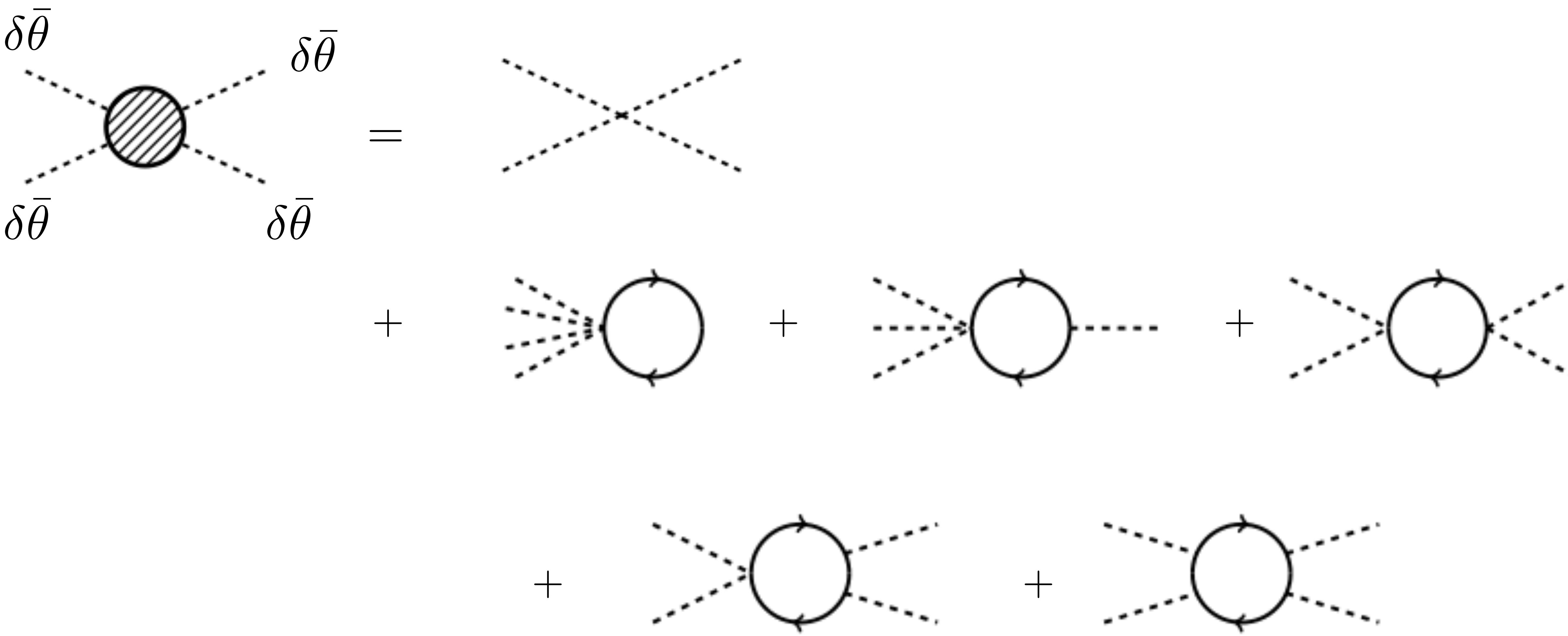
Goldstone boson scattering



$$\lambda_{4\theta} = \frac{\delta^5}{\xi^2}, \quad y_{n\theta} = \delta^{n/2}\xi^{(n-1)/2}$$

Goldstone boson scattering



not-renormalizable!!

$$= i\,\frac{2}{9\xi^2}\left[\lambda\delta^5 + \frac{y^4}{8\pi^2}\,\frac{1}{\epsilon}\left(\delta^2\xi^2 4!3^3 + \delta\xi^2 3!3^2 k^2\right)\right]$$

Damien George, Sander Mooij & MP in progress

Goldstone boson scattering

not-renormalizable!!

$$= i\frac{2}{9\xi^2}\left[\lambda\delta^5 + \frac{y^4}{8\pi^2}\frac{1}{\epsilon}\left(\delta^2\xi^2 4!3^3 + \delta\xi^2 3!3^2 k^2\right)\right]$$

Goldstone boson scattering

corrections suppressed

$$\frac{\mathcal{L}_E}{\sqrt{-g}} \supset -\frac{1}{2}\left(V_{;ab} - R_{cabd}\dot{\phi}^c\dot{\phi}^d - \frac{1}{a^3}D_t\left(\frac{a^3}{H}\dot{\phi}_a\dot{\phi}_b\right)\right)Q^aQ^b$$

used

time-dependence

$\phi_0(t)$

FRW & back reaction

Conclusions

- Higgs inflation: review

some open questions (waiting for data)

- Jordan vs. Einstein frame

both frames describe the same physics

- Renormalizability: Goldstone bosons

fermion coupling: theory non-renormalizable