Relativistic effects in large-scale structure

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Outline

- How do **relativistic effects** distort our observables?

  → Effect on: the galaxy number counts Δ

  the convergence κ (or magnification)

- How can we **measure** relativistic effects?

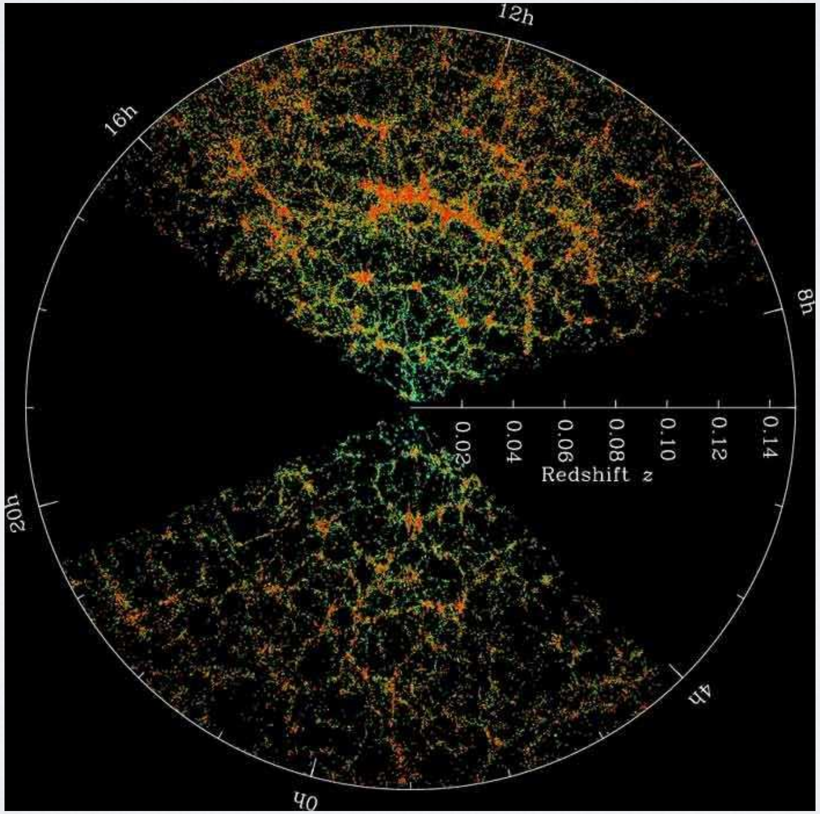
  → We can isolate these effects by looking at **anti-symmetries** in the correlation function.

- How can we use them to **test gravity**?

Galaxy survey

- We want to measure **fluctuations** in the distribution of galaxies.
- We pixelise the map.
- We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N - \bar{N}}{\bar{N}}$
- Question: what are the effects contributing to Δ ?

Credit: M. Blanton, SDSS



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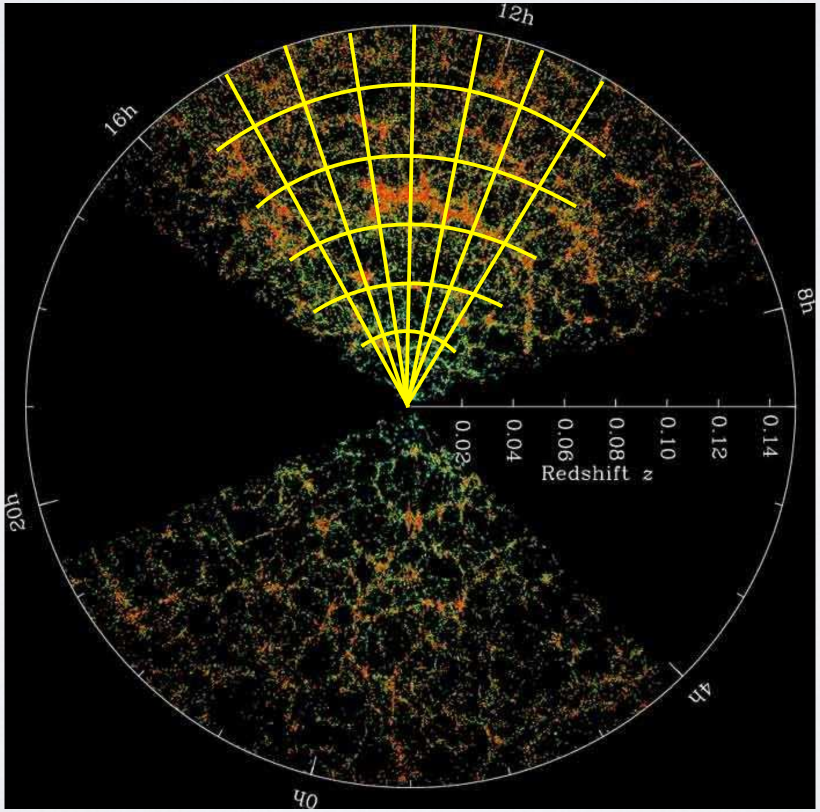

Galaxy number counts

We observe $\Delta(z, \mathbf{n}) = \dfrac{N(z,\mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}$

$N(z,\mathbf{n}) = \rho(z,\mathbf{n})V(z,\mathbf{n})$ and $\bar{N} = \bar{\rho}(z)\bar{V}(z)$

$$\Delta = \frac{\rho(z,\mathbf{n}) \cdot V(z,\mathbf{n}) - \bar{\rho}(z)\cdot\bar{V}(z)}{\bar{\rho}(z)\cdot\bar{V}(z)}$$

$$\Delta(z,\mathbf{n}) = b\cdot\delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}$$

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$$\rho = \bar{\rho} + \delta\rho \qquad V = \bar{V} + \delta V \qquad z = \bar{z} + \delta z$$

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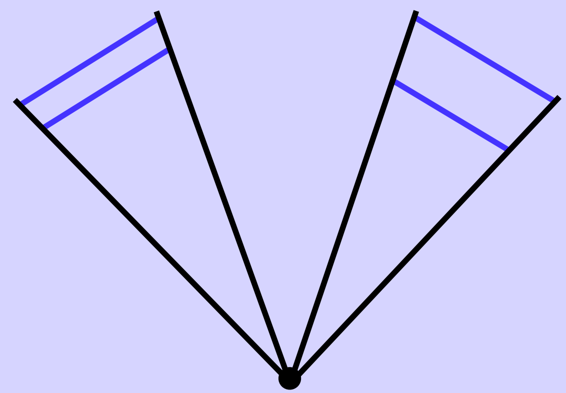
$$\Delta = \frac{\rho(z,\mathbf{n})\cdot V(z,\mathbf{n}) - \bar{\rho}(z}{\bar{\rho}(z)\cdot\bar{V}(z)}$$

same redshift bin
different physical volume



Observer $\quad dzd\Omega$

$$\Delta(z,\mathbf{n}) = b\cdot\delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}$$

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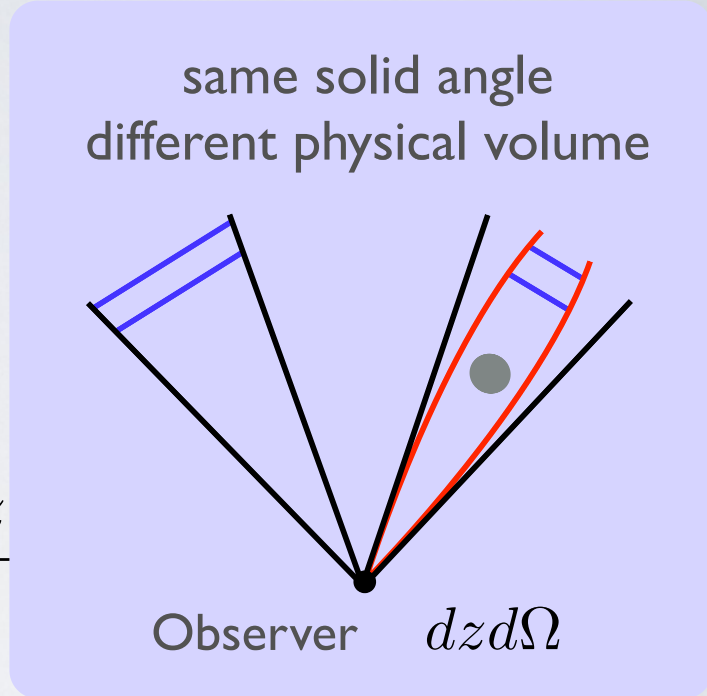

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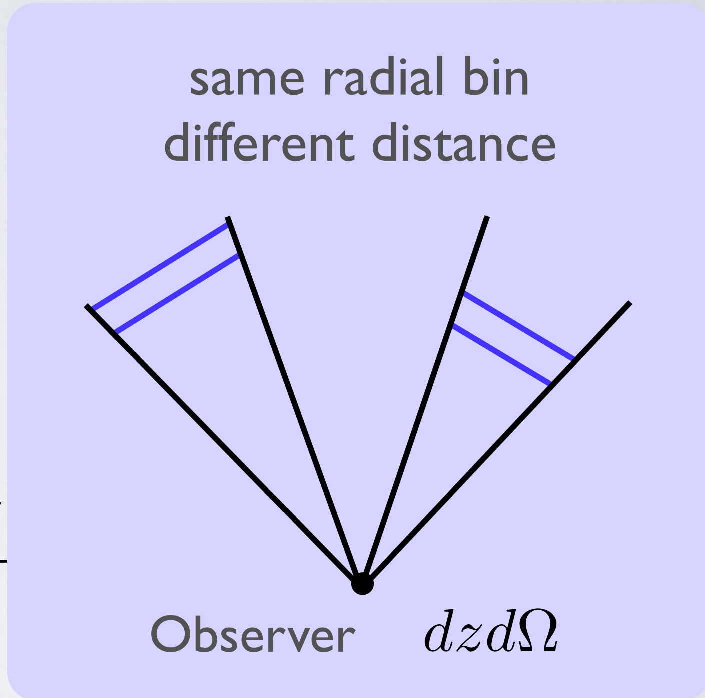


$\rho = \bar{\rho} + \delta\rho$ $\qquad$ $V = \bar{V} + \delta V$

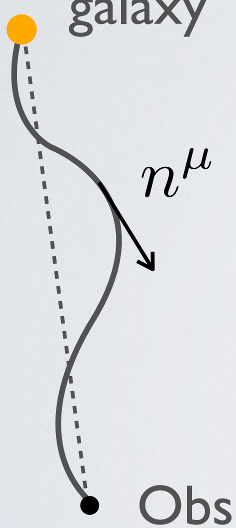
$$\Delta = \frac{\rho(z,\mathbf{n}) \cdot V(z,\mathbf{n}) - \bar{\rho}(z}{\bar{\rho}(z) \cdot \bar{V}(z)}$$

$$\Delta(z,\mathbf{n}) = b \cdot \delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}$$

Fluctuations

Perturbed Friedmann universe:

$$ds^2 = -a^2\Big[\big(1+2\Psi\big)d\eta^2 + \big(1-2\Phi\big)\delta_{ij}dx^i dx^j\Big]$$



We follow the propagation of photons from the galaxies to the observer and calculate:

- Changes in energy
- Changes in direction

Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

density — redshift space distortion — lensing — Doppler — gravitational redshift

$$\Delta(z,\mathbf{n}) = b\cdot D - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$
$$- \int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$
$$+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$
$$+ \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$
$$+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]$$

→ potential

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

Result

standard expression

lensing: important at high z

relativistic contributions: important at large scale

$$\Delta(z,\mathbf{n}) = b\cdot D - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

$$-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

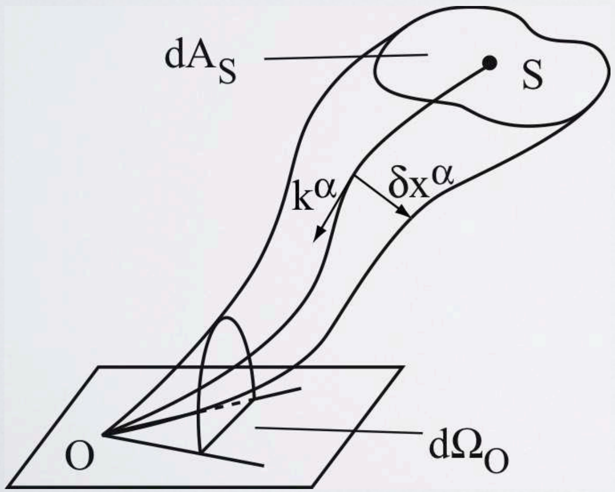
$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi \qquad \frac{\mathcal{H}}{k}D$$

$$+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$

$$+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right] \qquad \left(\frac{\mathcal{H}}{k}\right)^2 D$$

Convergence

- Galaxy surveys observe also the shape and the luminosity of galaxies → measure of the **convergence**.
- The convergence κ measures distortions in the **size**.
- The shear γ measures distortions in the **shape**.
- **Relativistic** distortions affect the convergence at **linear** order.



We solve Sachs equation

$$\frac{D^2 \delta x^\alpha(\lambda)}{D\lambda^2} = R^\alpha{}_{\beta\mu\nu} k^\beta k^\mu \delta x^\nu$$

Convergence

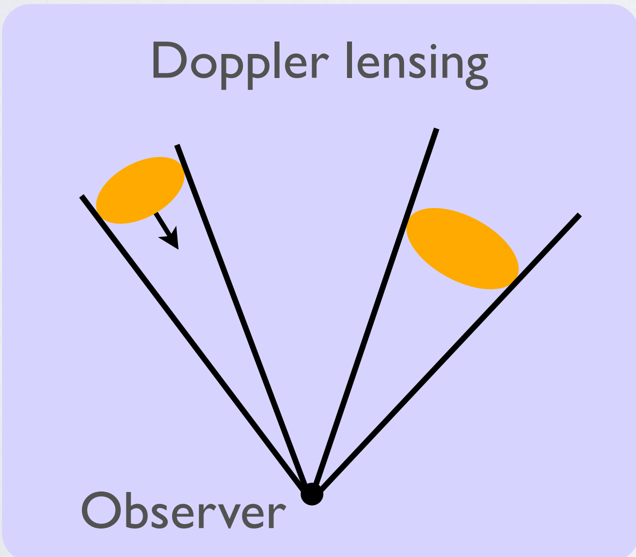
CB (2008)
Bolejko et al (2013)
Bacon et al (2014)

Gravitational lensing ↖   Doppler lensing ↑

$$\kappa = \frac{1}{2r}\int_0^r dr' \frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi) + \left(\frac{1}{r\mathcal{H}}-1\right)\mathbf{V}\cdot\mathbf{n}$$
$$-\frac{1}{r}\int_0^r dr'(\Phi+\Psi) + \left(1-\frac{1}{r\mathcal{H}}\right)\int_0^r dr'(\dot\Phi+\dot\Psi)$$
$$+\left(1-\frac{1}{r\mathcal{H}}\right)\Psi+\Phi$$

→ Integrated terms

→ Sachs Wolfe



The moving galaxy is further away → it looks smaller, i.e. demagnified

Observations

- Due to relativistic effects, Δ and κ contain additional **information**.

$$\delta, V, \Phi, \Psi \qquad (\Phi+\Psi), V, \Phi, \Psi$$

- This can help **testing gravity** by probing the **relation** between density, velocity and gravitational potentials.

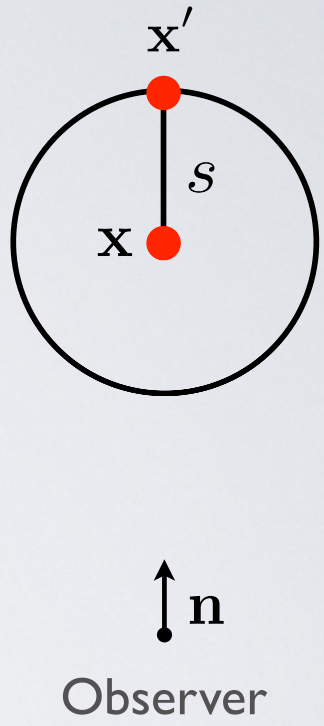
- Two **difficulties**:
  - The relativistic effects are small: we need to go to large scales.
  - We always measure the sum of all the effects.

- We need a way of **isolating** relativistic effects

  → look for **anti-symmetries** in the correlation function.

Density

The **density** contribution $\Delta = b \cdot \delta$, generates an **isotropic** correlation function.



$\xi(s) = \langle \Delta(\mathbf{x}) \Delta(\mathbf{x}') \rangle$ depends only on the **separation** $s = |\mathbf{x} - \mathbf{x}'|$

$$\xi(s) = \frac{Ab^2 D_1^2}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0} \right)^{n_s - 1} T_\delta^2(k)\, j_0(k \cdot s)$$

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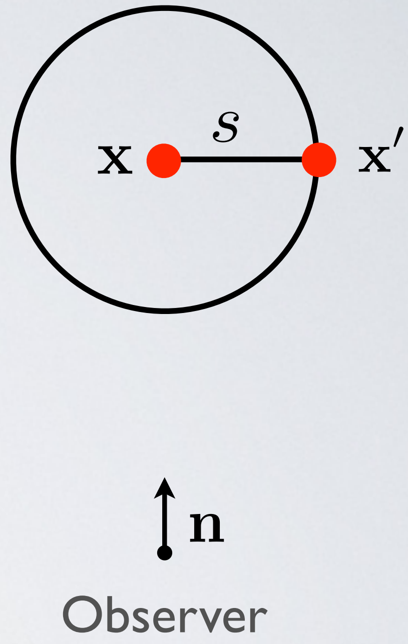

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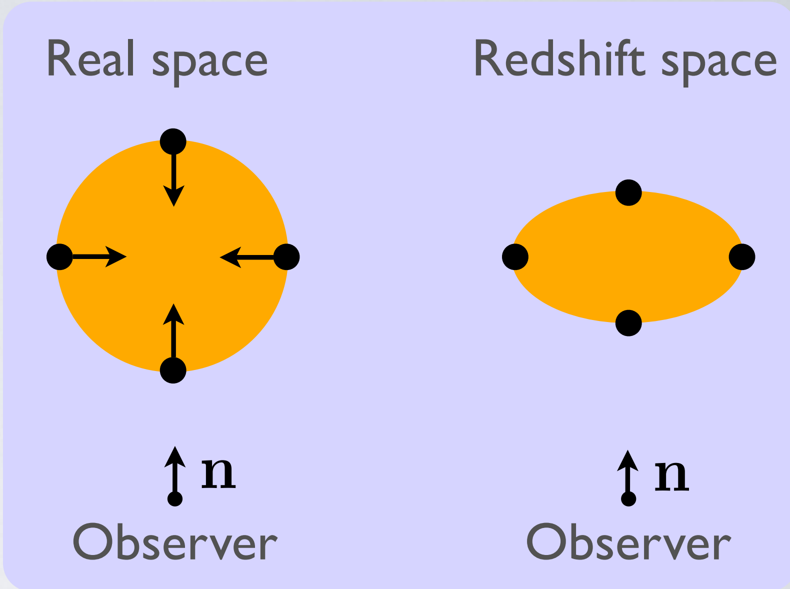
Redshift distortions

Redshift distortions **break** the **isotropy** of the correlation function.

$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$



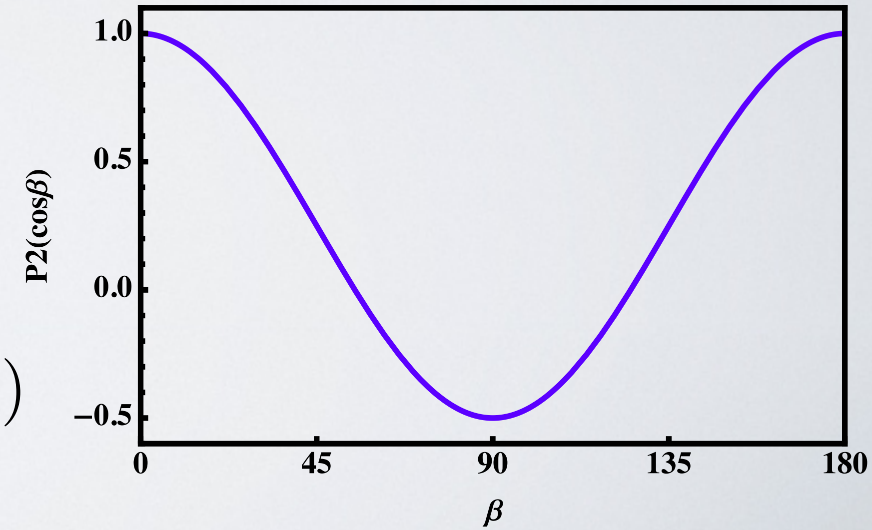
Quadrupole

Hamilton (1992)

$$\xi_2 = -D_1^2 \left(\frac{4fb}{3} + \frac{4f^2}{7} \right) \mu_2(s) P_2(\cos\beta)$$

$$\mu_2(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0} \right)^{n_s - 1} T_\delta^2(k)\, j_2(k \cdot s)$$

$$P_2(\cos\beta) = \frac{3}{2}\cos^2\beta - \frac{1}{2}$$



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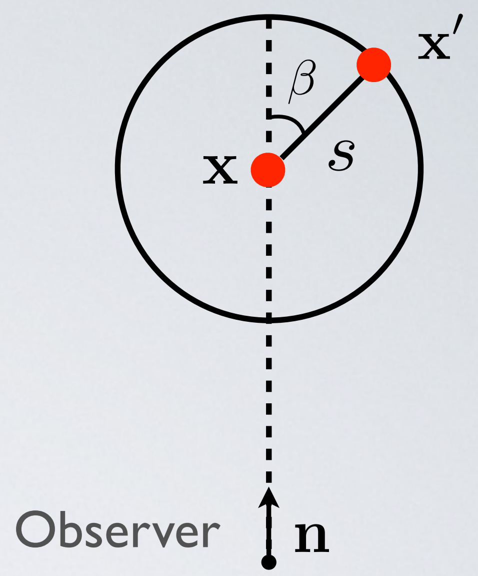

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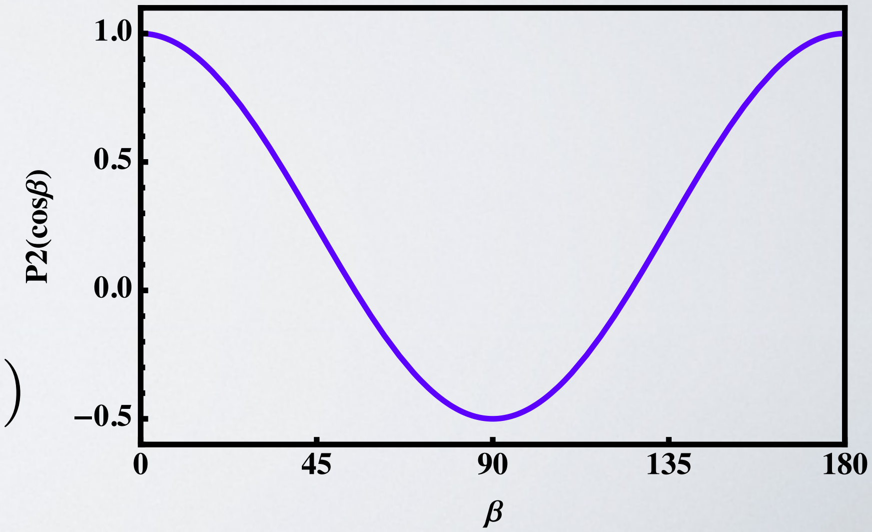

$$\mu_2(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1}T_\delta^2(k)\,j_2(k\cdot s)$$

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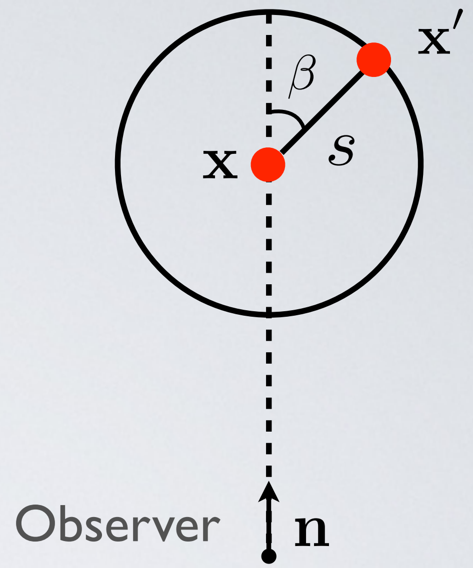
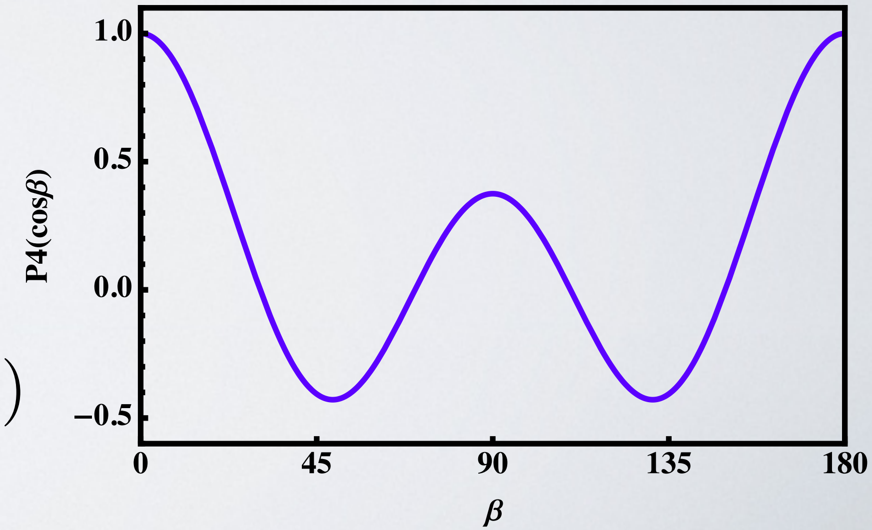


Hexadecapole Hamilton (1992)

$$P_4(\cos\beta) = \frac{1}{8}\left[35\cos^4\beta - 30\cos^2\beta + 3\right]$$



$$\xi_4 = D_1^2 \frac{8f^2}{35} \mu_4(s) P_4(\cos\beta)$$

$$\mu_4(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0}\right)^{n_s - 1} T_\delta^2(k)\, j_4(k \cdot s)$$

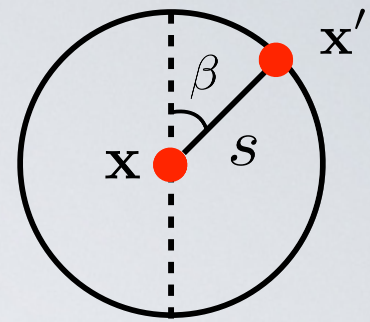
Redshift distortions

$$\xi_0(s) = \frac{1}{2}\int_{-1}^{1} d\mu\ \xi(s,\mu)$$

$$\xi_2(s) = \frac{5}{2}\int_{-1}^{1} d\mu\ \xi(s,\mu) P_2(\mu)$$

$$\xi_4(s) = \frac{9}{2}\int_{-1}^{1} d\mu\ \xi(s,\mu) P_4(\mu)$$

$$\mu = \cos\beta$$

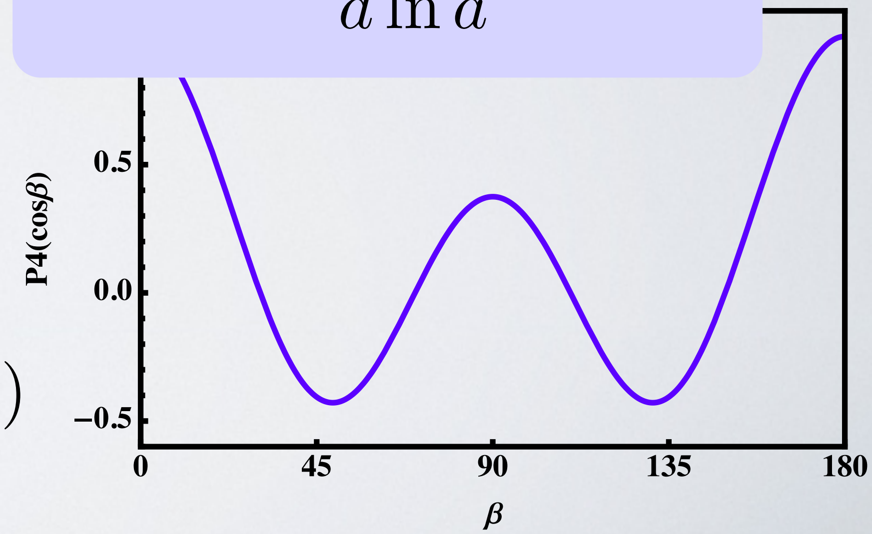


→

Measure separately:

$b \cdot \sigma_8$ and $f \cdot \sigma_8$

$$f = \frac{d \ln D_1}{d \ln a}$$

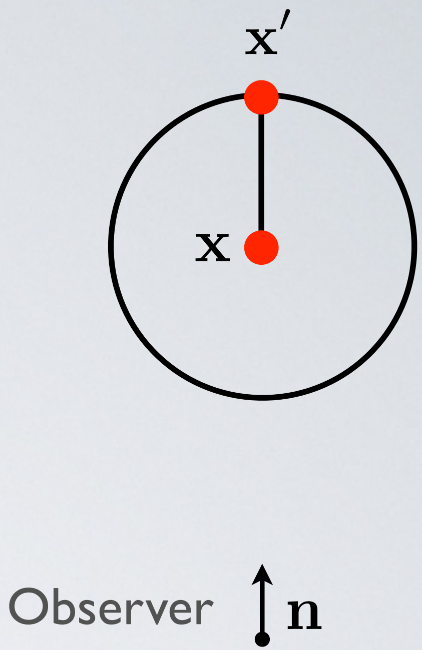


$$\mu_4(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_4(k\cdot s)$$

Relativistic effects

The relativistic effects break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.



This differs from the breaking of **isotropy**, due to redshift distortions, which is symmetric.

To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

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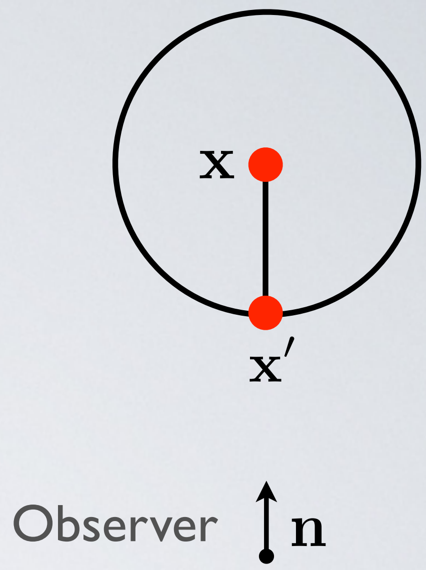

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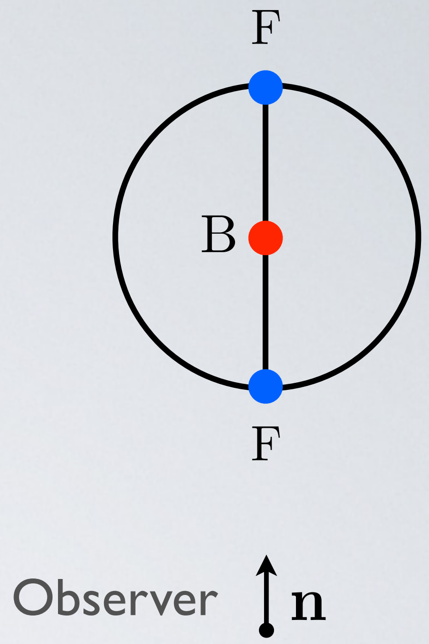

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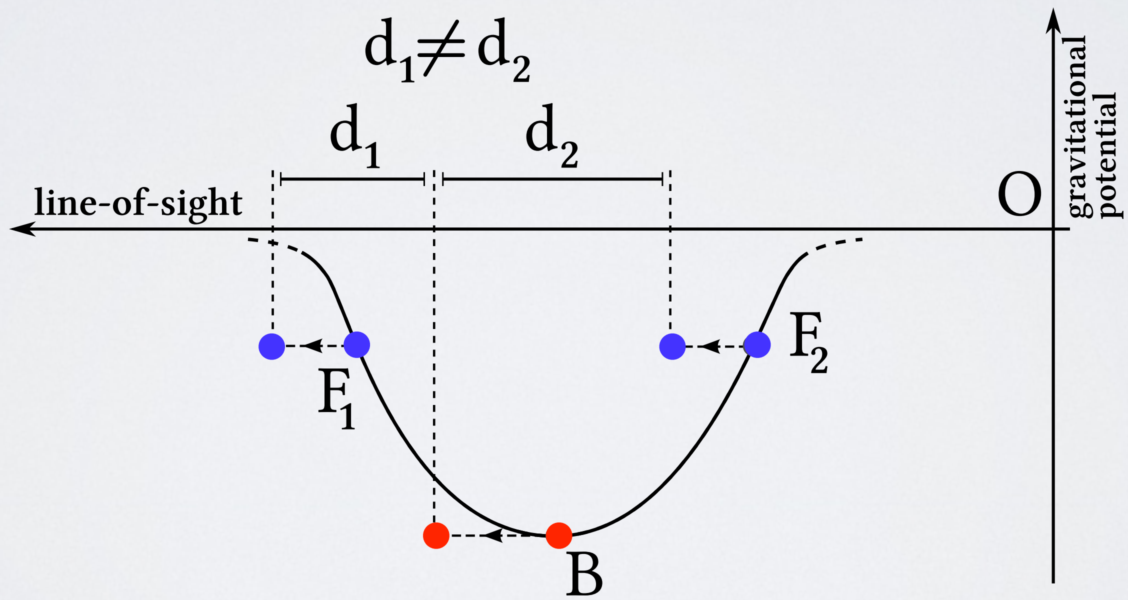
Cross-correlation

The following terms **break** the **symmetry**:

$$\Delta_{\rm rel} = \frac{1}{\mathcal{H}}\partial_r\Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$

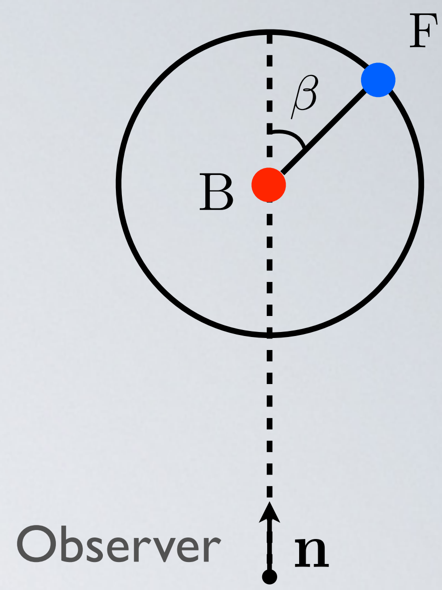


Dipole in the correlation function

CB, Hui and Gaztanaga (2013)

$$\xi(s,\beta) = D_1^2\, f \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\rm B} - b_{\rm F})\nu_1(s) \cdot \cos(\beta)$$

$$\nu_1(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k) T_\Psi(k)\, j_1(k\cdot s)$$



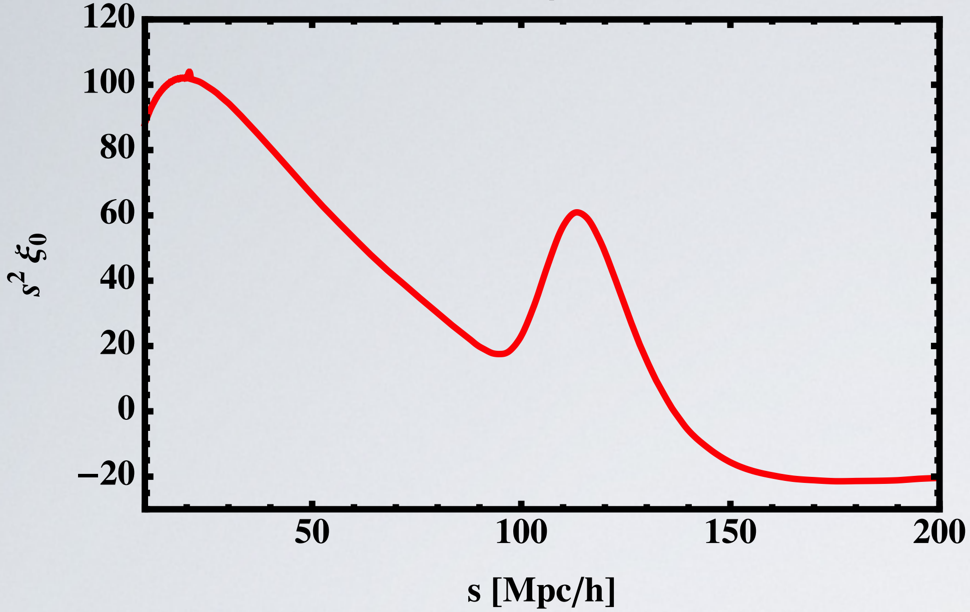
By fitting for a **dipole** in the correlation function, we can measure **relativistic effects**, and separate them from the density and redshift space distortions.

$$\xi_1(s) = \frac{3}{2}\int_{-1}^{1} d\mu\, \xi(s,\mu)\cdot\mu \qquad \mu = \cos\beta$$

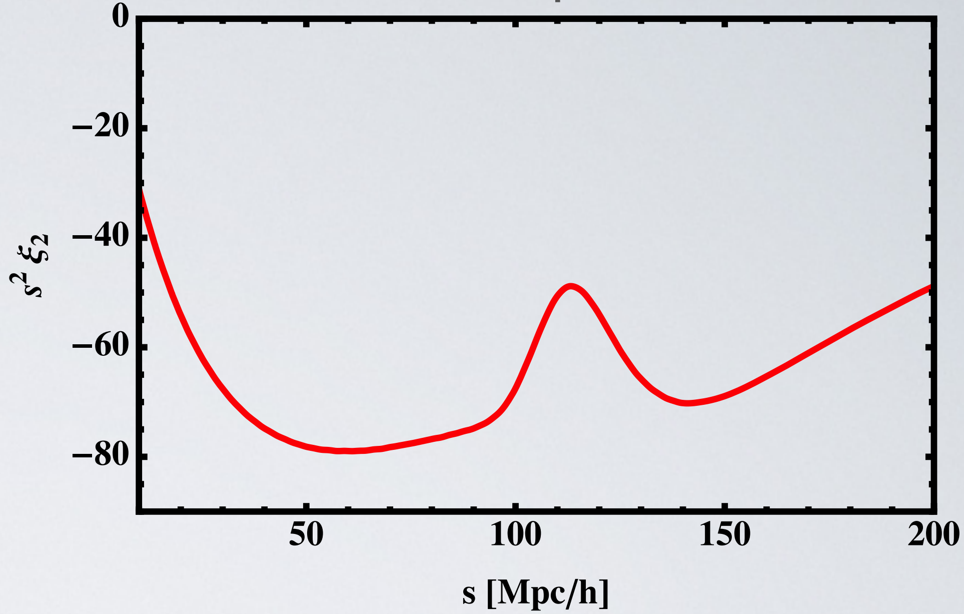
Multipoles

$z = 0.25$

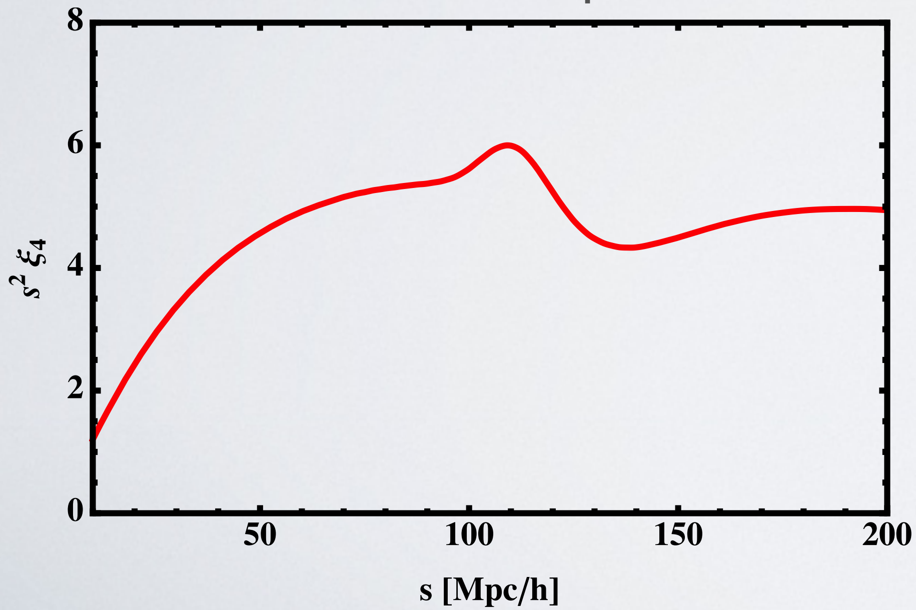
Monopole



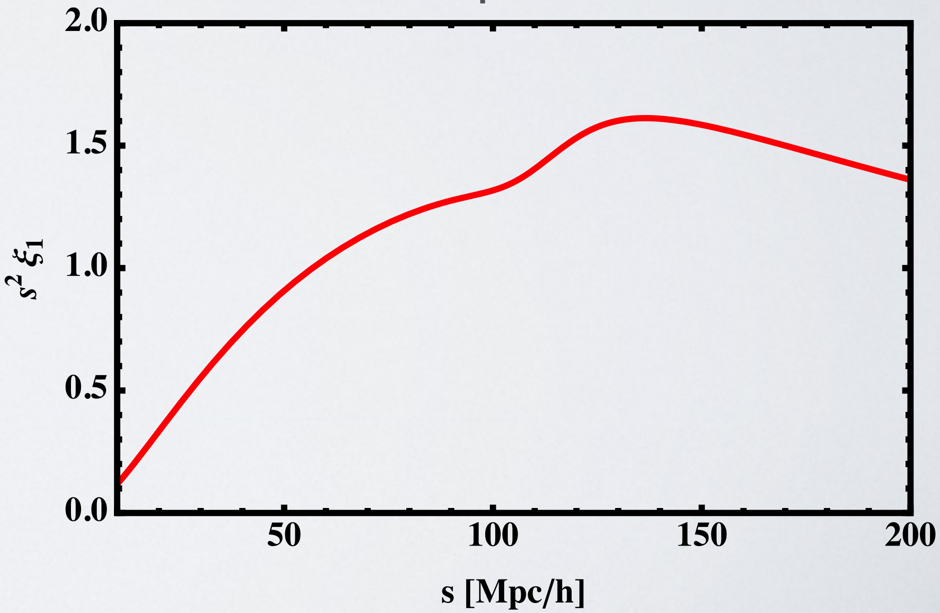
Quadrupole



Hexadecapole



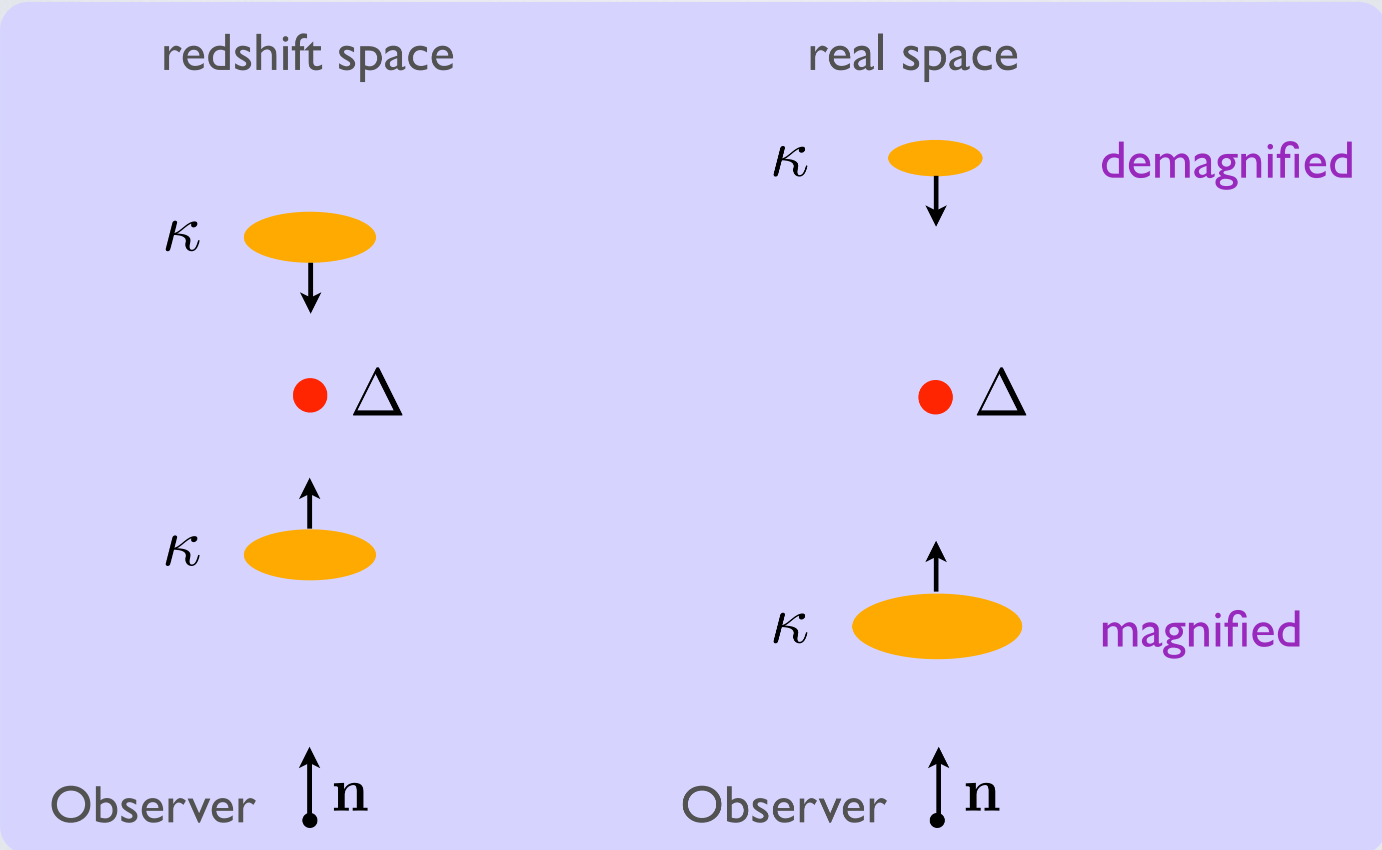
Dipole



Convergence

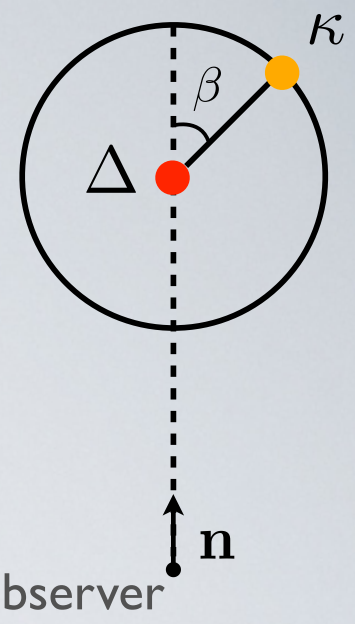
$$\kappa_g = \frac{1}{2r}\int_0^r dr' \frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi) \qquad \kappa_v = \left(\frac{1}{r\mathcal{H}}-1\right)\mathbf{V}\cdot\mathbf{n}$$

We can isolate the Doppler lensing by looking for **anti-symmetries** in $\langle \Delta\kappa \rangle$

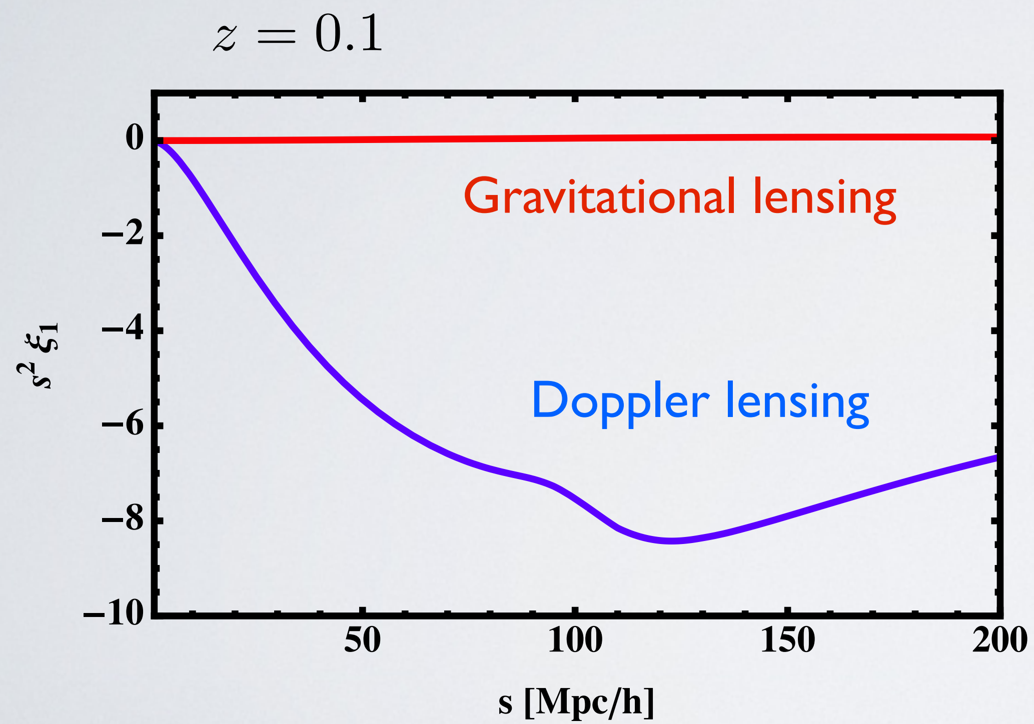


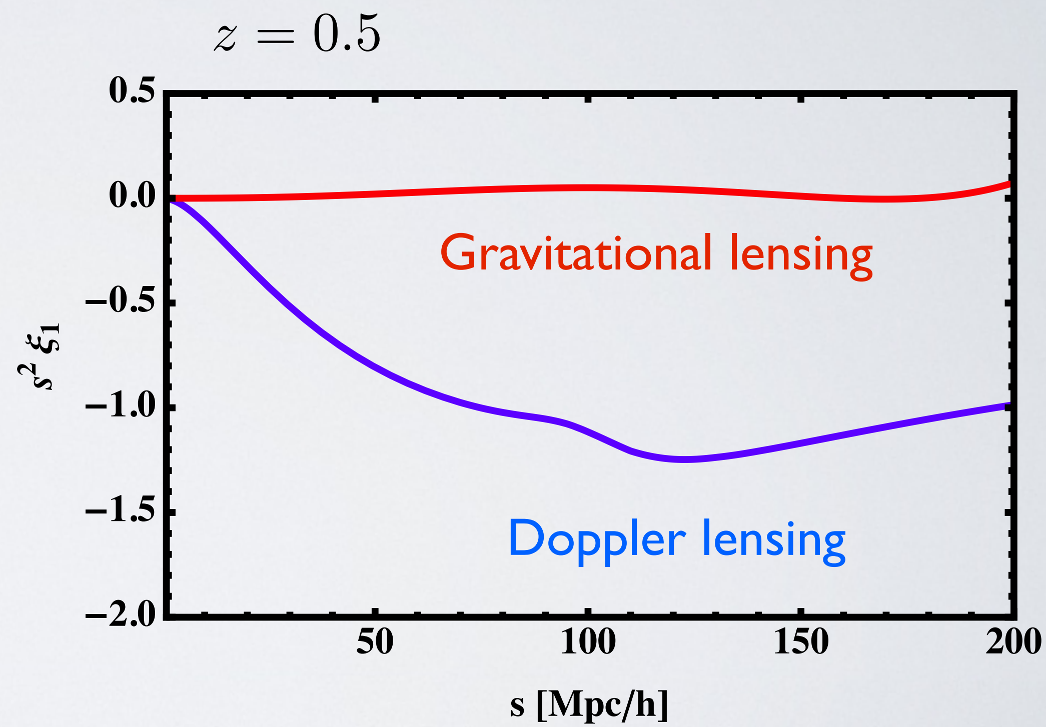
Dipole

CB, Bacon, Clarkson, Andrianomena and Maartens (in preparation)



$$\xi(s,\beta) = \frac{2A}{9\pi^2\Omega_m^2} D_1^2 \frac{\mathcal{H}}{\mathcal{H}_0} f \left(1 - \frac{1}{\mathcal{H}r}\right)\left(b + \frac{3f}{5}\right)\nu_1(s)\cos(\beta)$$





The dipole due to gravitational lensing is completely subdominant.

Testing Euler equation

- The monopole and quadrupole in Δ allow to measure V

- The **dipole** allows to measure:

$$\Delta_{\rm rel} = \frac{1}{\mathcal{H}}\partial_r\Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$

- If **Euler** equation is valid: $\dot{\mathbf{V}}\cdot\mathbf{n} + \mathcal{H}\mathbf{V}\cdot\mathbf{n} + \partial_r\Psi = 0$

$$\Delta_{\rm rel} = -\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}$$

→ With the dipole, we can **test** Euler equation.

Measuring the anisotropic stress

- The dipole in the convergence is sensitive to:

$$\kappa_{\mathrm{v}} = \left(\frac{1}{r\mathcal{H}} - 1\right) \mathbf{V} \cdot \mathbf{n}$$

- The standard part $\kappa_{\mathrm{g}} = \frac{1}{2r}\int_0^r dr' \frac{r - r'}{r'} \Delta_\Omega(\Phi + \Psi)$

  can be measure through $\langle\kappa\kappa\rangle$ and $\langle\gamma\gamma\rangle$

- Assuming Euler equation, we can test the relation between the two metric potentials Φ and Ψ.

Conclusion

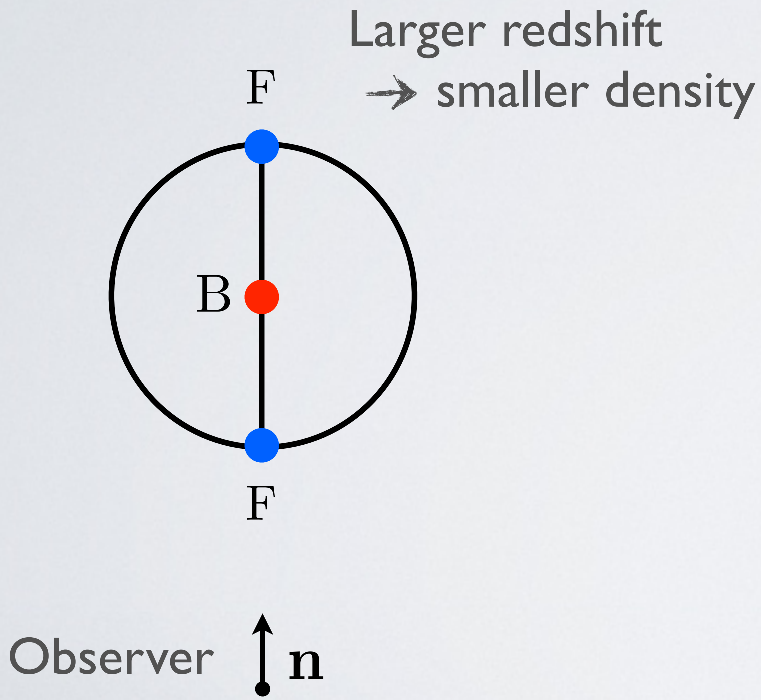
- Our **observables** are affected by relativistic effects.

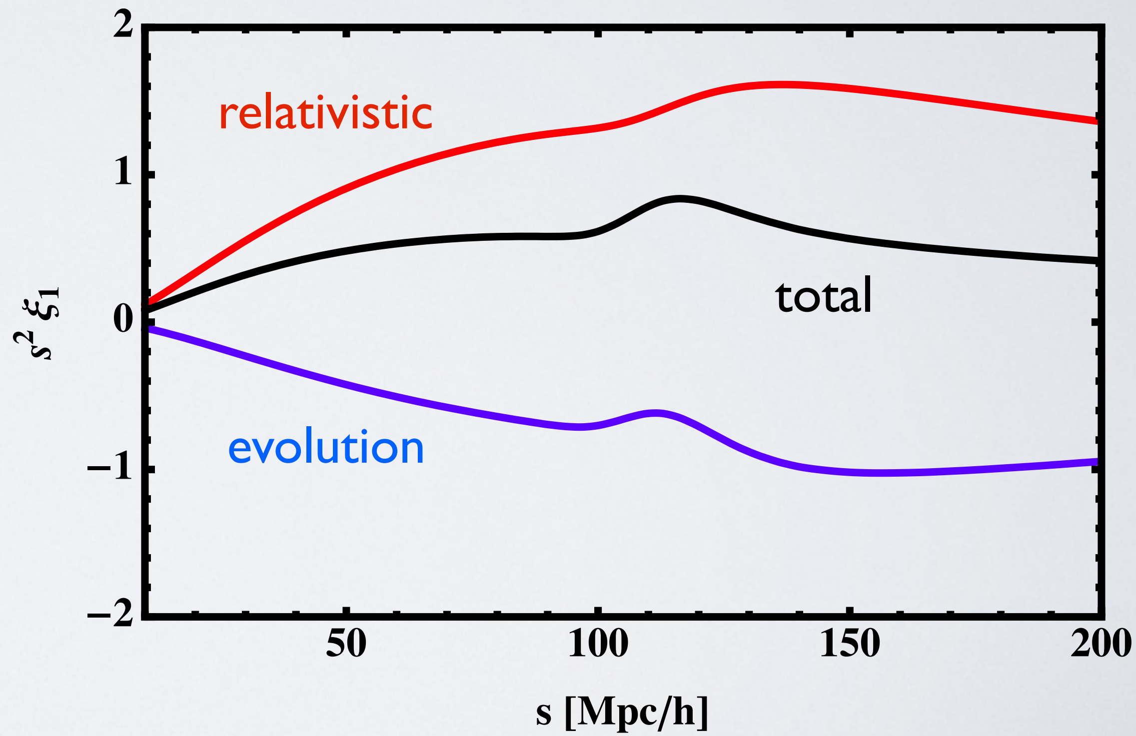
- These effects have a different **signature** in the **correlation** function: they induce anti-symmetries.

- By measuring these anti-symmetries we can isolate the relativistic effects and use them to **test** the **relations** between the density, velocity and gravitational potential.

Contamination

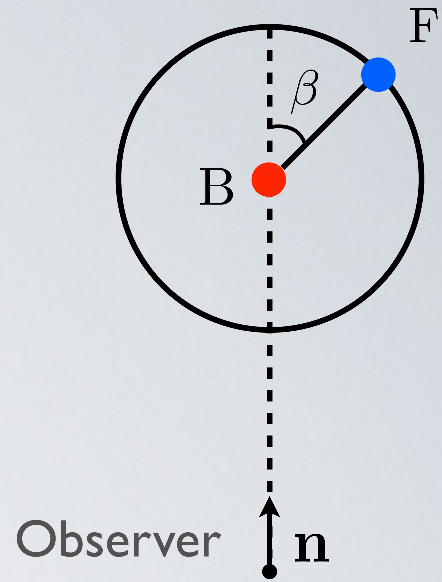
The density and velocity **evolve** with time: the density of the faint galaxies in front of the bright is larger than the density behind. This also induces a **dipole** in the correlation function.



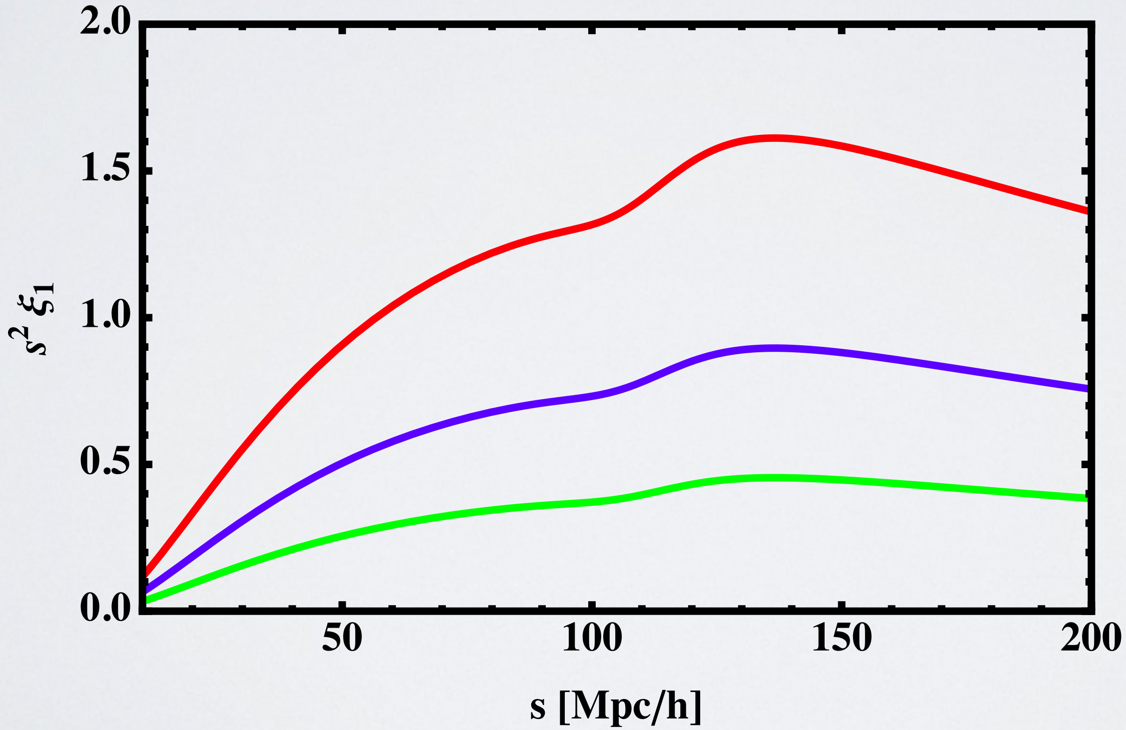


Dipole in the correlation function

$$\xi(s,\beta) = D_1^2\, f\, \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\rm B} - b_{\rm F})\nu_1(s)\cdot \cos(\beta)$$



$$\nu_1(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k) T_\Psi(k)\, j_1(k\cdot s)$$



$z = 0.25$

$z = 0.5$

$z = 1$

$b_{\rm B} - b_{\rm F} \simeq 0.5$

CB, Hui and Gaztanaga (2013)