Center for Astroparticle Physics
GENEVA

UNIVERSITÉ
DE GENÈVE
FACULTÉ DES SCIENCES

HeII-ionizing background fluctuations from clustered sources

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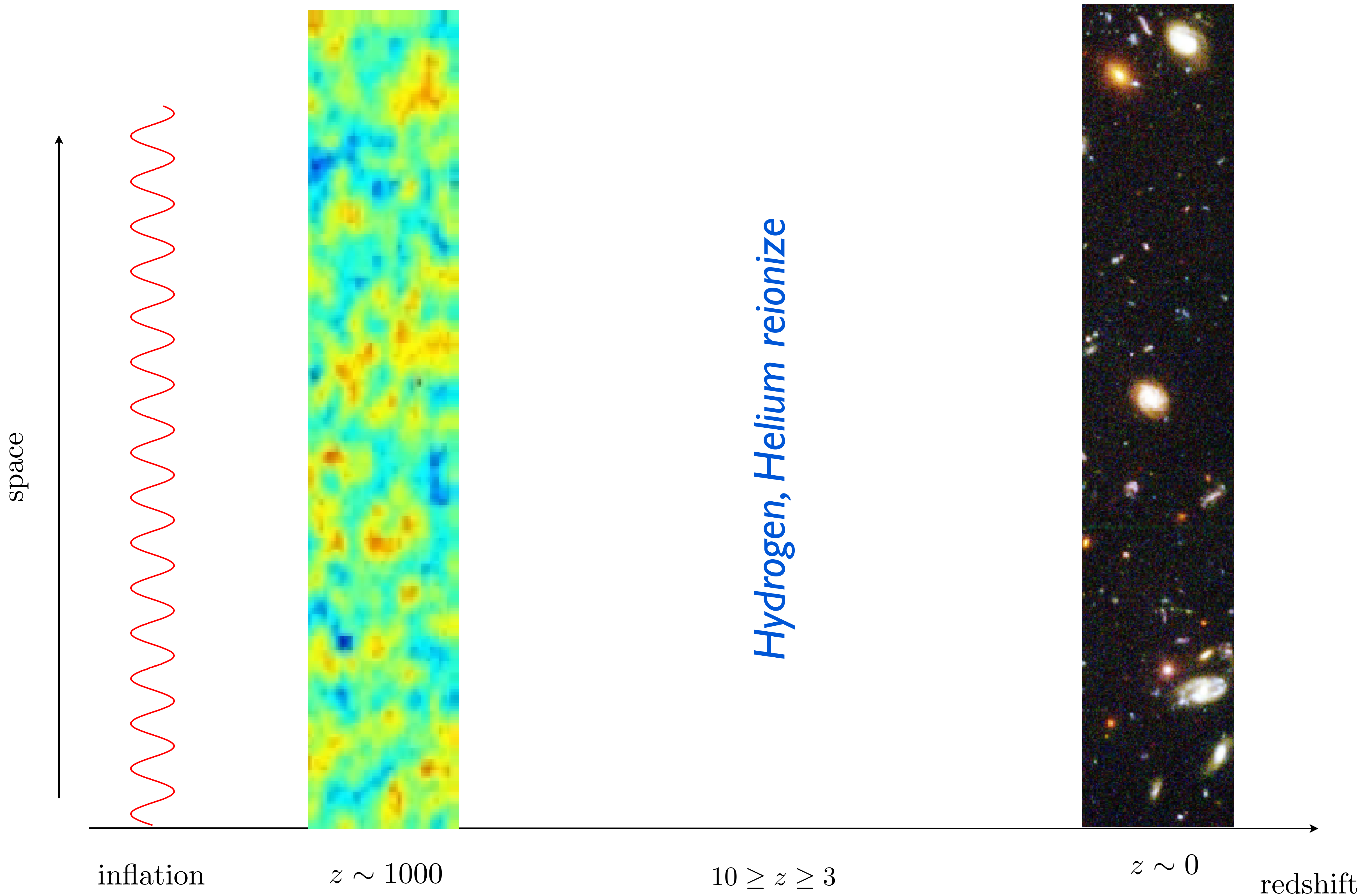
Benasque cosmology 2014, August 5, 2014

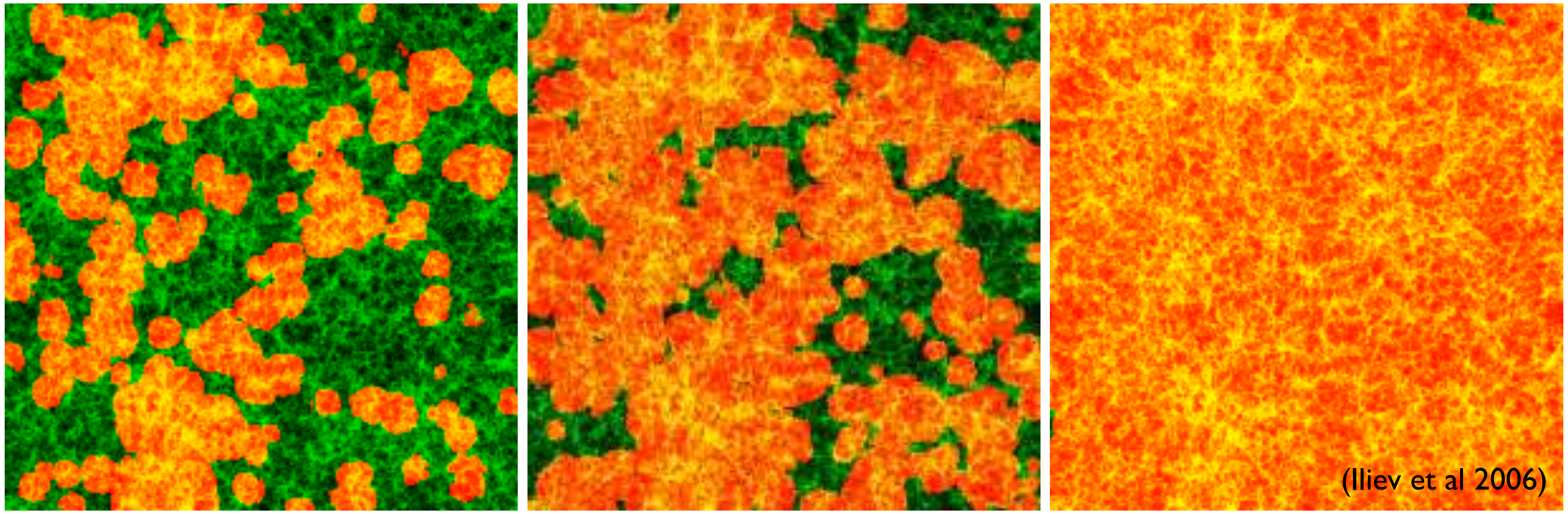
1406.6379, with A. Moradinezhad Dizgah, M. Biagetti


space
Hydrogen, Helium reionize
inflation
$z \sim 1000$
$10 \geq z \geq 3$
$z \sim 0$
redshift

Z	Element	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
												Spectrum		
1	H	13.598												
2	He	24.587	54.416											
3	Li	5.392	75.638	122.451										
4	Be	9.322	18.211	153.893	217.713									
5	B	8.298	25.154	37.930	259.368	340.217								
6	C	11.260	24.383	47.887	64.492	392.077	489.981							
7	N	14.534	29.601	47.448	77.472	97.888	552.057	667.029						
8	O	13.618	35.116	54.934	77.412	113.896	138.116	739.315	871.387					
9	F	17.422	34.970	62.707	87.138	114.240	157.161	185.182	953.886	1103.089				
10	Ne	21.564	40.962	63.45	97.11	126.21	157.93	207.27	239.09	1195.797	1362.164			
11	Na	5.139	47.286	71.64	98.91	138.39	172.15	208.47	264.18	299.87	1465.091	1648.659		
12	Mg	7.646	15.035	80.143	109.24	141.26	186.50	224.94	265.90	327.95	367.53	1761.802	1962.613	
13	Al	5.986	18.828	28.447	119.99	153.71	190.47	241.43	284.59	330.21	398.57	442.07	2085.983	2304.080
14	Si	8.151	16.345	33.492	45.141	166.77	205.05	246.52	303.17	351.10	401.43	476.06	523.50	2437.67
15	P	10.486	19.725	30.18	51.37	65.023	230.43	263.22	309.41	371.73	424.50	479.57	560.41	611.85
16	S	10.360	23.33	34.83	47.30	72.68	88.049	280.93	328.23	379.10	447.09	504.78	564.65	651.63
17	Cl	12.967	23.81	39.61	53.46	67.8	98.03	114.193	348.28	400.05	455.62	529.26	591.97	656.69
18	Ar	15.759	27.629	40.74	59.81	75.02	91.007	124.319	143.456	422.44	478.68	538.95	618.24	686.09
19	K	4.341	31.625	45.72	60.91	82.66	100.0	117.56	154.86	175.814	503.44	564.13	629.09	714.02
20	Ca	6.113	11.871	50.908	67.10	84.41	108.78	127.7	147.24	188.54	211.270	591.25	656.39	726.03
21	Sc	6.54	12.80	24.76	73.47	91.66	111.1	138.0	158.7	180.02	225.32	249.832	685.89	755.47
22	Ti	6.82	13.58	27.491	43.266	99.22	119.36	140.8	168.5	193.2	215.91	265.23	291.497	787.33
23	V	6.74	14.65	29.310	46.707	65.23	128.12	150.17	173.7	205.8	230.5	255.04	308.25	336.267
24	Cr	6.766	16.50	30.96	49.1	69.3	90.56	161.1	184.7	209.3	244.4	270.8	298.0	355
25	Mn	7.435	15.640	33.667	51.2	72.4	95	119.27	196.46	221.8	248.3	286.0	314.4	343.6
26	Fe	7.870	16.18	30.651	54.8	75.0	99	125	151.06	235.04	262.1	290.4	330.8	361.0

Challenging:

- *Need small scales: IGM physics*
- *Need large scales: quasars (QSOs) are rare*



Analytic/semi-numeric methods can provide physical insights into certain aspects of the problem

- *How can we incorporate source clustering in (semi-)analytic models of the UV ionizing background (UVB) evolution ?*

- *Is quasar clustering important for the UVB at the end of HeII reionization ?*

Scales

- *(Comoving) attenuation length or photon mean free path*

$$r_0 \sim 30 - 50 \text{ Mpc}$$

(Bolton & Haehnelt 2006; Furlanetto & Oh 2008)

- *Quasar (QSO) clustering length*

$$r_\xi \sim 15 - 30 \text{ Mpc}$$

(Shen et al 2007; Francke et al 2008)

- *QSO number density*

$$l = \bar{n}^{-1/3}$$

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Quasar clustering important if:

i) $r_\xi / r_0 \gtrsim 1$

ii) $r_0 / l \gg 1$

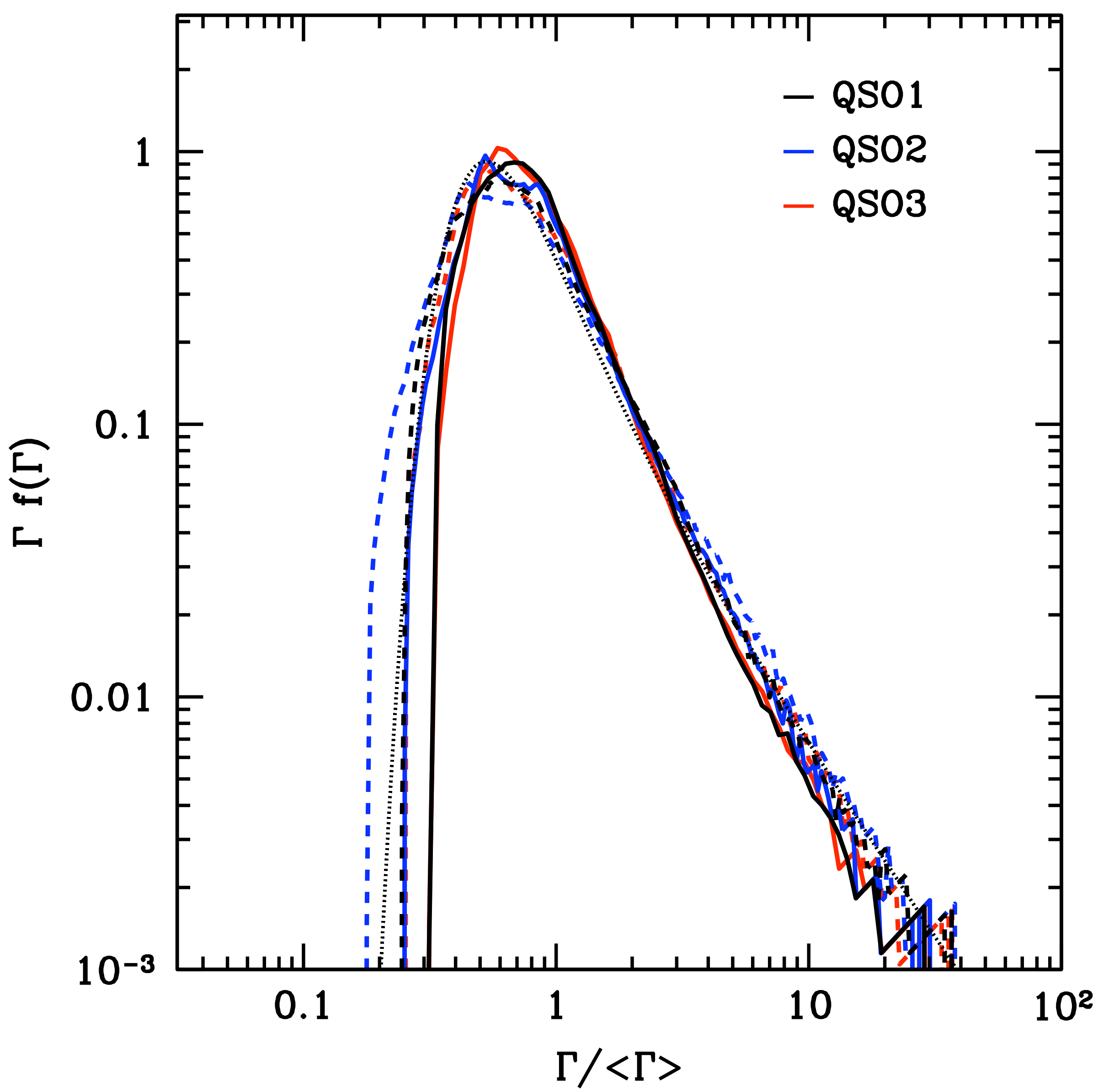
The only attempt so far:

$$\text{Photoionization rate}: \quad \Gamma = \int_{\nu_{\rm HeII}}^{\infty} d\nu \, \frac{J_\nu}{h\nu} \, \sigma_{\rm HeII}(\nu)$$

$\mathrm{QSO1} = \text{sub-sample of halos with } M \geq 5 \times 10^{11} M_\odot$

$\mathrm{QSO2} = \text{most massive halos}$

$\mathrm{QSO3} = \text{randomly distributed}$

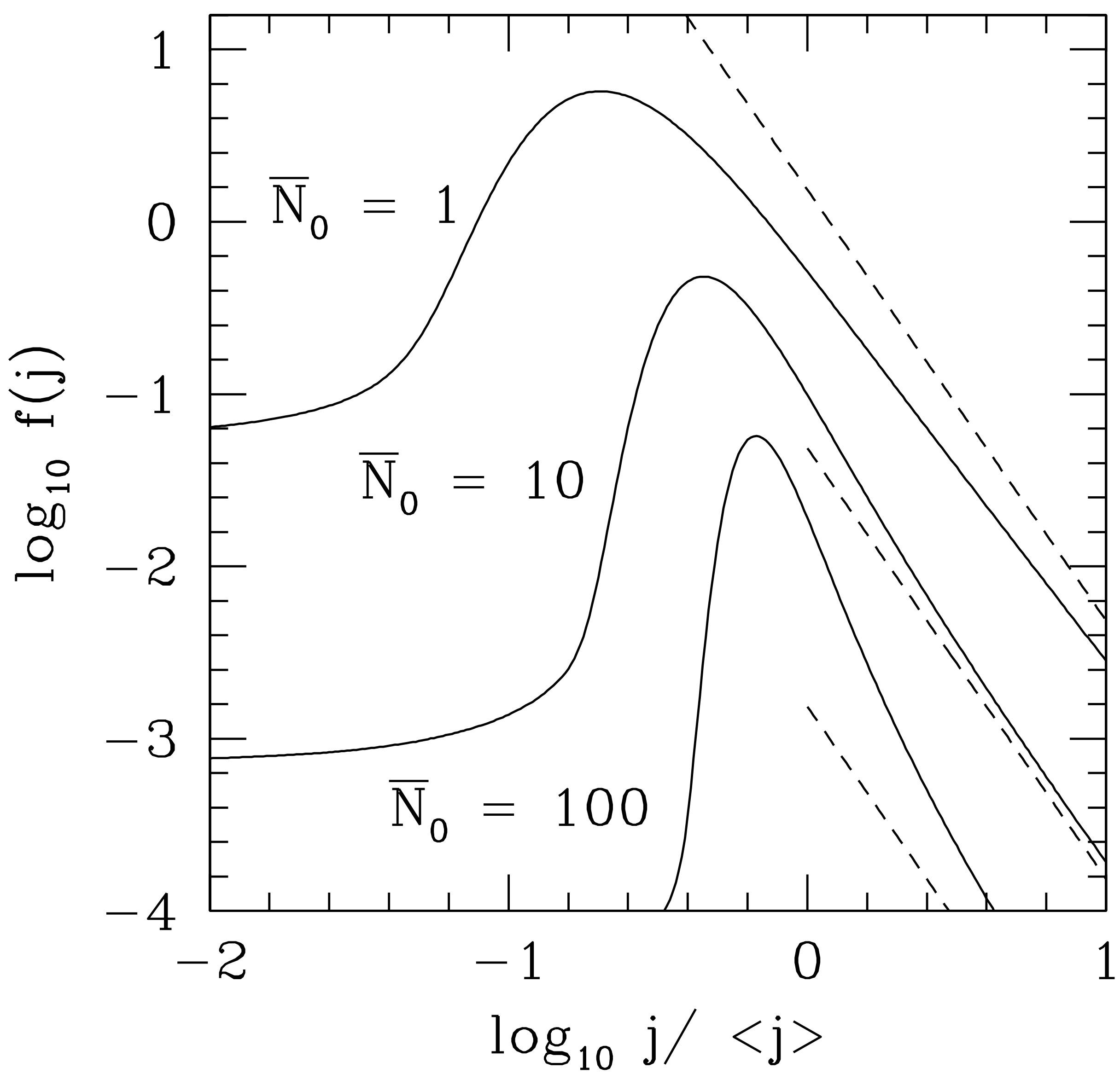


(Dixon, Furlanetto & Mesinger 2013)

Plan of attack

Extend the work of Zuo 1992; Fardall & Shull 1993, Meiksin & White 1993; who worked out P(J) analytically for randomly distributed sources

$$\bar{N}_0 = \frac{4\pi}{3}\bar{n}r_0^3$$



(Meiksin & White 2003)

Count-in-cells

Consider randomly-located cells of volume V.

- *Probability to have an empty cell:*

$$P_0 = P\big(\Phi_0(V)\big) = \exp\big(\mathcal{W}_0(V)\big)$$

- *Conditional void correlation:*

$$\mathcal{W}_0(V) = \sum_{k=1}^{\infty} \frac{(-\bar{n})^k}{k!} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k \, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k)$$
$$= \sum_{k=1}^{\infty} \frac{(-\bar{N})^k}{k!} \bar{\xi}_k(V)$$

$$\bar{N} = \bar{n} V$$

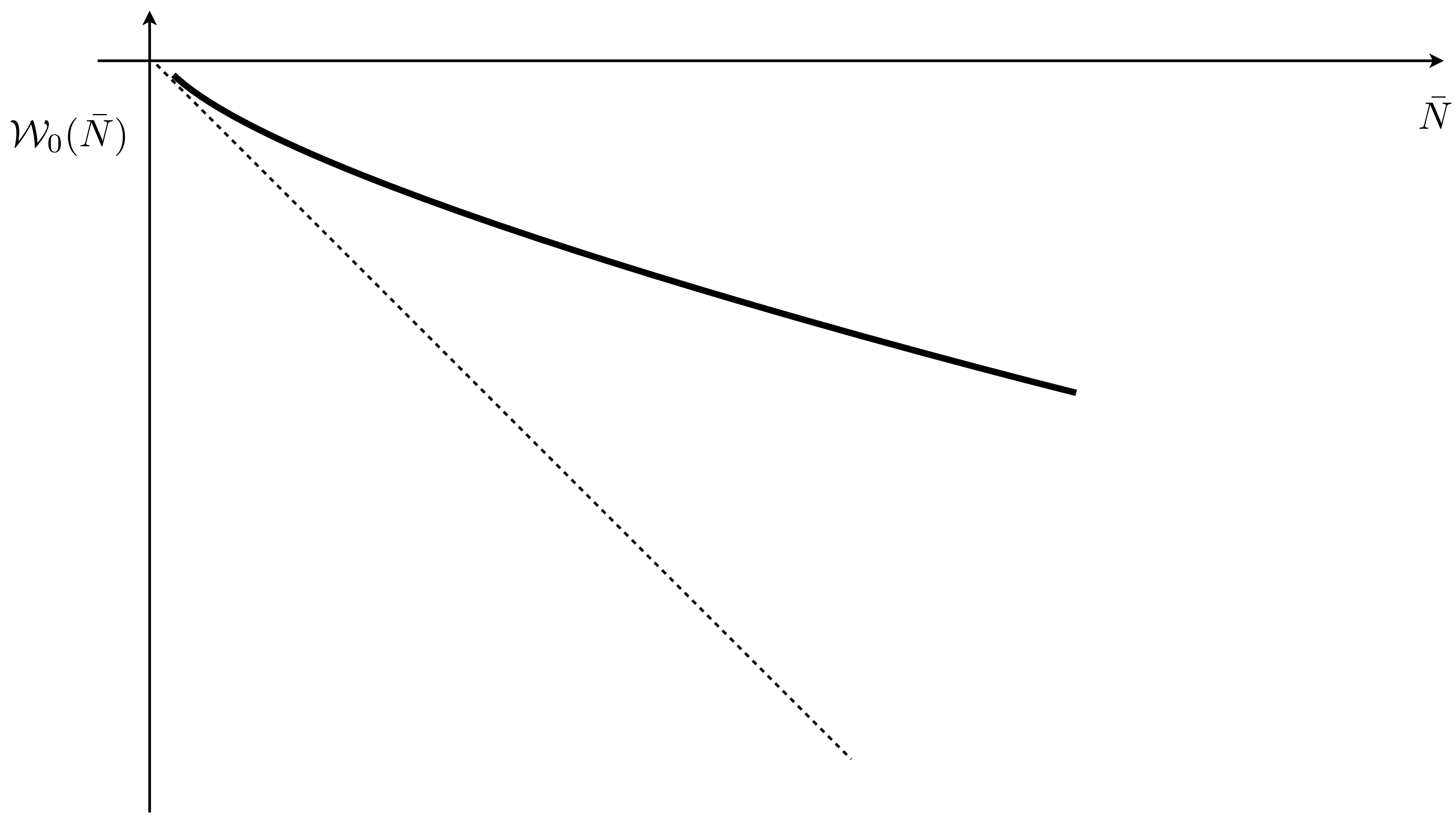
- *Volume-averaged irreducible correlations:*

$$\bar{\xi}_k(V) \equiv \frac{1}{V^k} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k \, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k)$$

Fall et al. 1976; White 1979; Peebles 1980; Fry 1985; Balian & Schaefer 1989; Szapudi & Colombi 1996

Void conditional probability

- *Poisson distribution:* $\bar{\xi}_1(V) \equiv 1, \qquad \mathcal{W}_0(V) = -\bar{N}$



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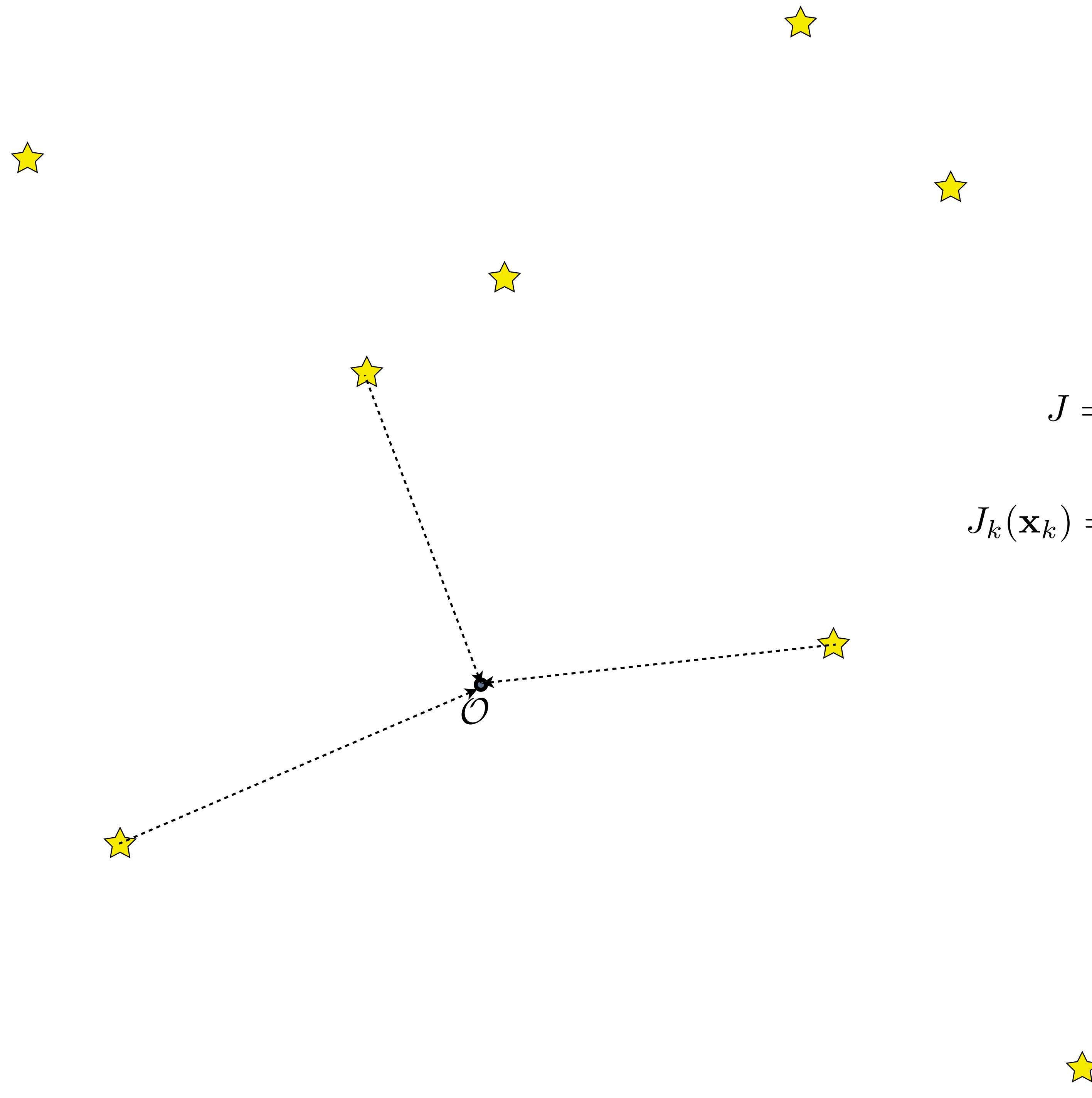
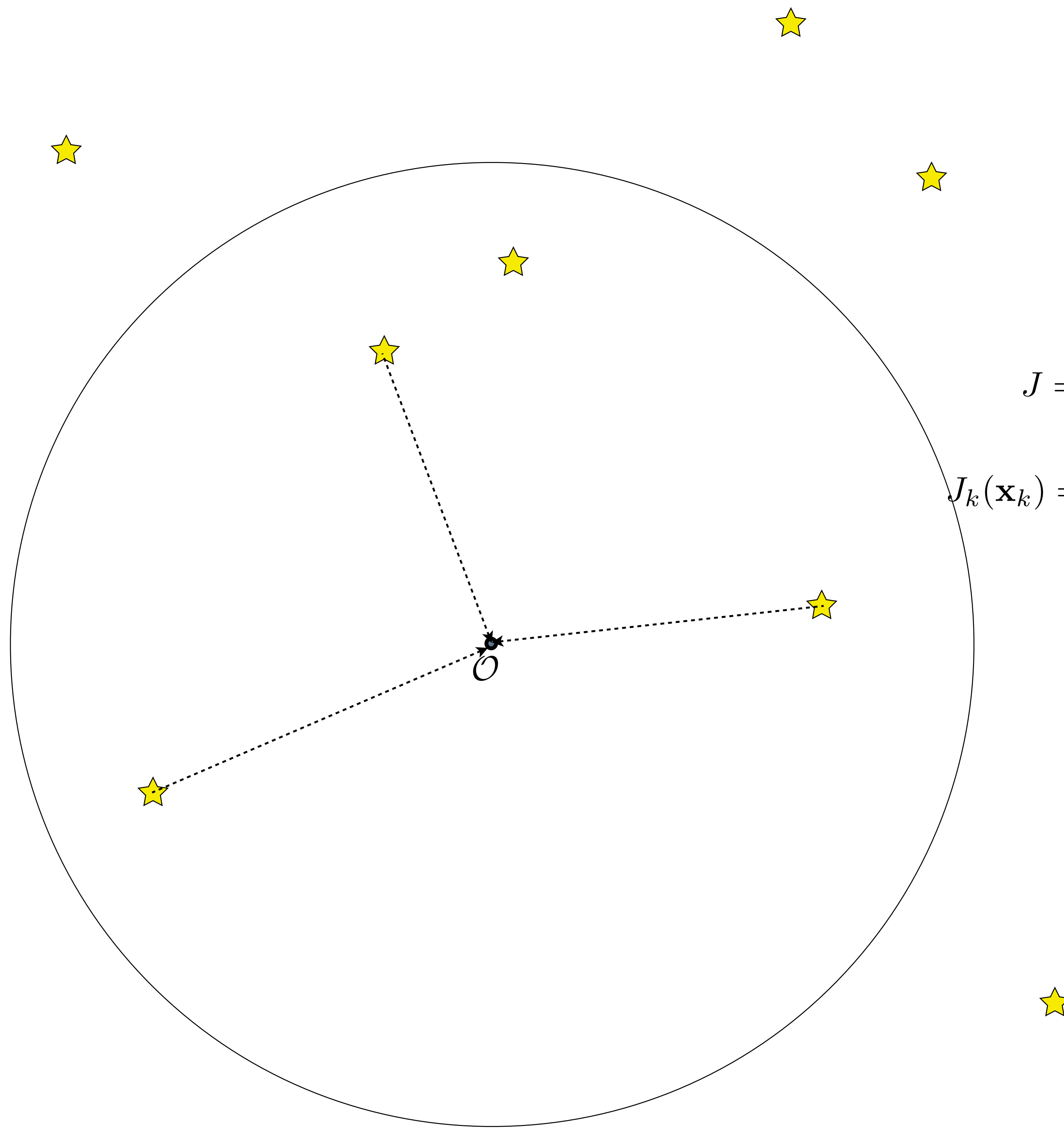
Intensity distribution



$$J = \sum_k J_k(\mathbf{x}_k)$$

$$J_k(\mathbf{x}_k) = (1+z)^2 \frac{L_k}{(4\pi r_k)^2} e^{-r_k/r_0}$$

Intensity distribution



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Count-in-cells + weight

- *Assign a weight to each point:*

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$$1 = \sum_{N=0}^{\infty} \frac{1}{N!} \int \dots \int P\{X_1 \dots X_N \big| \Phi_0(V)\}\, e^{\mathcal{W}_0(V)}$$

↓

$$P_\omega(V) = \sum_{N=0}^{\infty} \frac{1}{N!} \int \dots \int P\{X_1 \dots X_N \big| \Phi_0(V)\}\, \omega(\mathbf{x}_1) \dots \omega(\mathbf{x}_N)\, e^{\mathcal{W}_0(V)}$$

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$$\downarrow$$

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- *"Weighted" probability distribution:*

$$P_\omega(V) = e^{\mathcal{W}_\omega(V)} - e^{\mathcal{W}_0(V)}$$

$$\mathcal{W}_\omega(V) = \sum_{k=1}^{\infty} \frac{(-\bar{n})^k}{k!} \int_V d^3\mathbf{x}_1 \dots \int_V d^3\mathbf{x}_k\, \xi_k(\mathbf{x}_1, \dots, \mathbf{x}_k)\, \big(1 - \omega(\mathbf{x}_1)\big) \dots \big(1 - \omega(\mathbf{x}_k)\big)$$

Application to UV background

- *Weight is provided by the Quasar contribution to the specific intensity at x=0*

- *Each configuration of N quasars in cells of volume V contributes*

$$\int d\alpha_1 \dots d\alpha_N \, \phi(\alpha_1) \dots \phi(\alpha_N)$$
$$\times P\{X_1 \dots X_N \Phi_0(V)\}$$
$$\times \delta_D(J_1 + \dots + J_N - J)$$

$$L = \alpha \, L_\star$$

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$$\int d\alpha_1 \dots d\alpha_N \, \phi(\alpha_1) \dots \phi(\alpha_N) \times P\{X_1 \dots X_N \Phi_0(V)\} \times \delta_D(J_1 + \dots + J_N - J)$$

- *Substitute Laplace/Fourier representation:*

$$\delta_D(J_1 + \dots + J_N - J) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} ds \, e^{s(J - J_1 - \dots - J_N)}$$

$$\omega(\mathbf{x}_k) = \Theta_H(R - |\mathbf{x}_k|) \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha_k \, \phi(\alpha_k) \, e^{-sJ_k(\mathbf{x}_k)}$$

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- *Intensity distribution is*

$$P(J) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} ds \, e^{sJ + \mathcal{W}_\omega(V)}$$

Hierarchical ansatz

- *Volume-averaged correlation functions are of the form*

$$\bar{\xi}_k = S_k \, \bar{\xi}_2^{k-1}$$

- *Under this assumption, we can recast the conditional void correlation into the form*

$$\mathcal{W}_0(V) = -\bar{N} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k!} S_k \left(\bar{N}\bar{\xi}_2\right)^{k-1}$$
$$\equiv -\bar{N}\chi(\bar{N}\bar{\xi}_2)$$

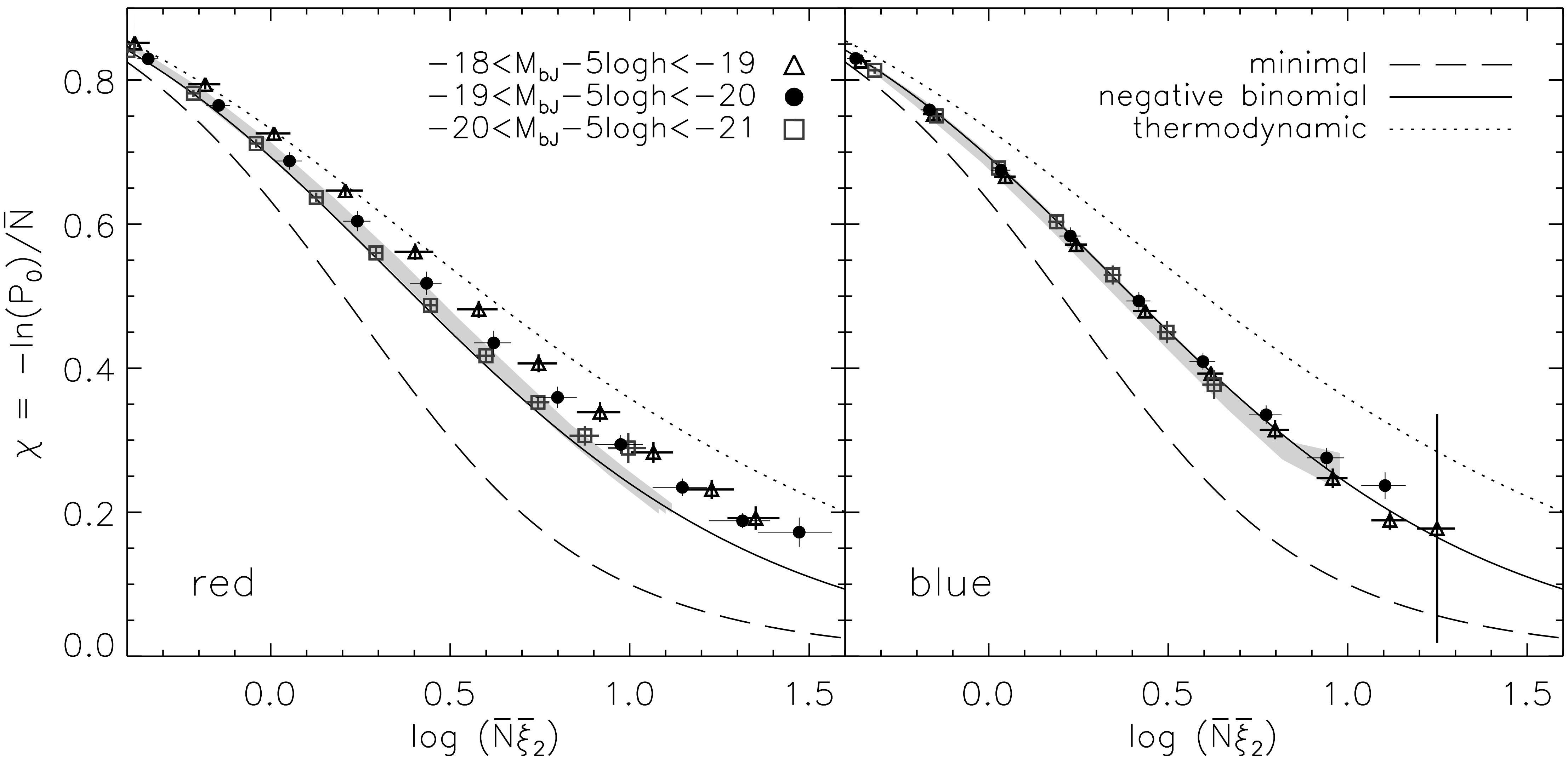
$$\text{void scaling function :} \quad \chi = -\frac{\ln\left(P_0\right)}{\bar{N}} = -\frac{\mathcal{W}_0(V)}{\bar{N}}$$

e.g.

$$\chi(\bar{N}\bar{\xi}_2) = \frac{\ln\left(1 + \bar{N}\bar{\xi}_2\right)}{\bar{N}\bar{\xi}_2} \qquad \text{(Negative Binomial)}$$

$$\chi(\bar{N}\bar{\xi}_2) = \frac{1}{1 + \frac{1}{2}\bar{N}\bar{\xi}_2} \qquad \text{(Geometric Hierarchical)}$$

Galaxies

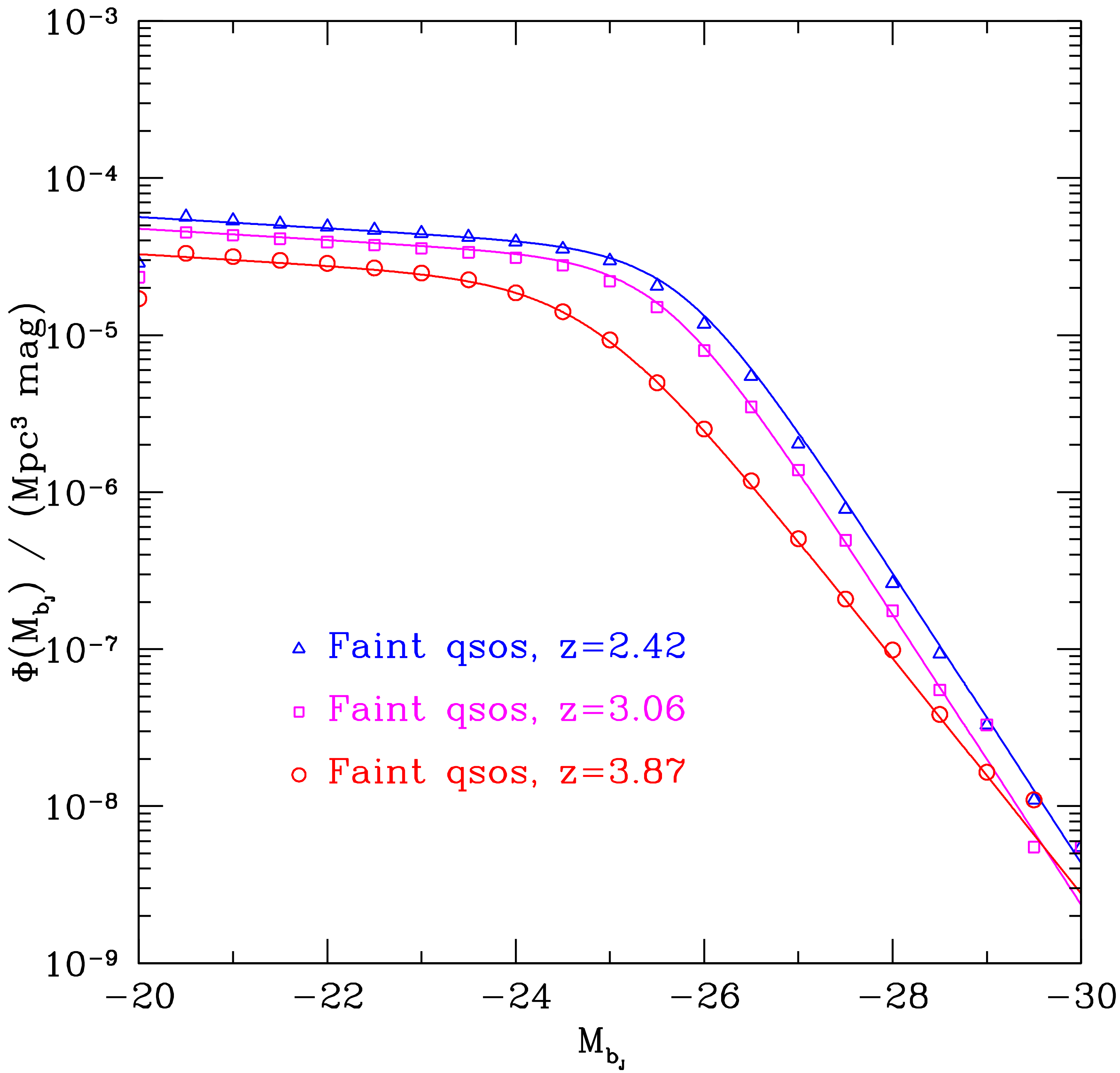


(Croton, Norberg et al 2006)

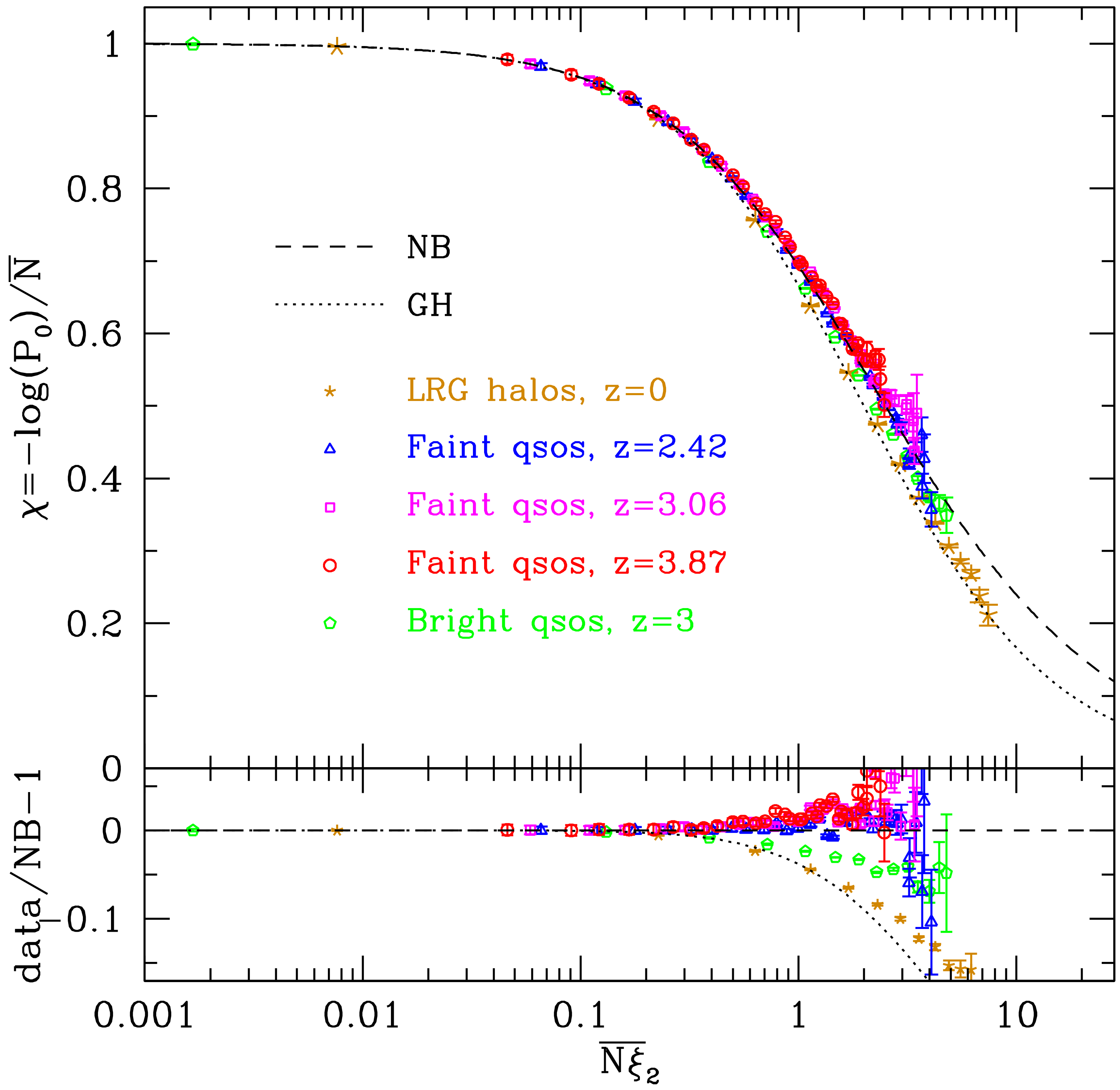
Test with mock quasars

Synthetic QSO catalogues constructed from the Millennium simulation

(Croton 2009)



Quasar void scaling function

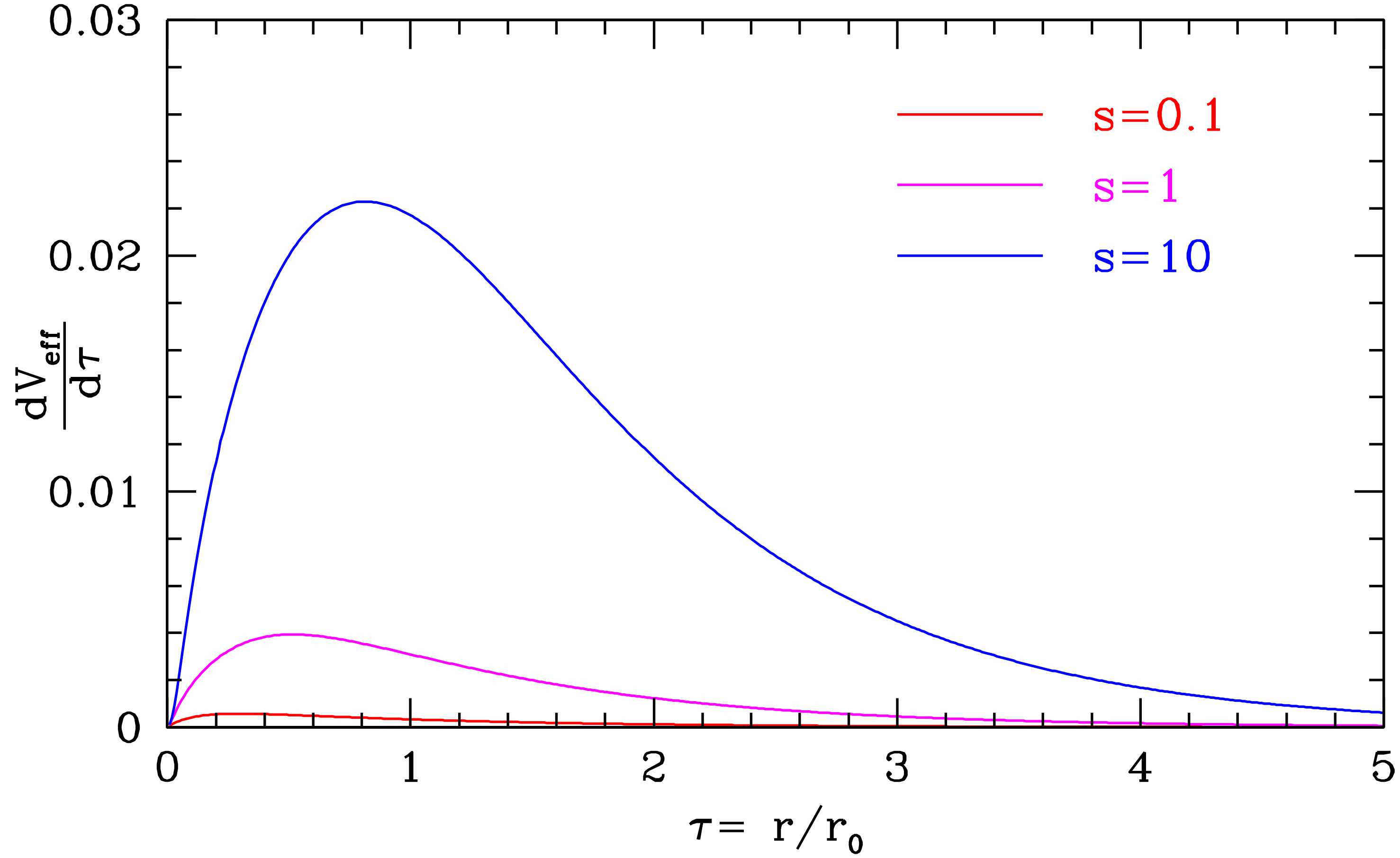


Window

Hierarchical ansatz holds regardless the shape of the window function

$$V_{\rm e}(s,V) \equiv \int d^3\mathbf{x}\,\big(1-\omega(\mathbf{x})\big)\Theta_H(|\mathbf{x}|-R)$$
$$= \int_0^{R/r_0} d\tau\, \frac{dV_e}{d\tau}(s,\tau)$$

$$\tau = r/r_0 = \text{optical depth}$$



Intensity distribution in hierarchical models

PDF is:

$$P(J) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} ds\, e^{sJ+\mathcal{W}_\omega(V)}$$

where:

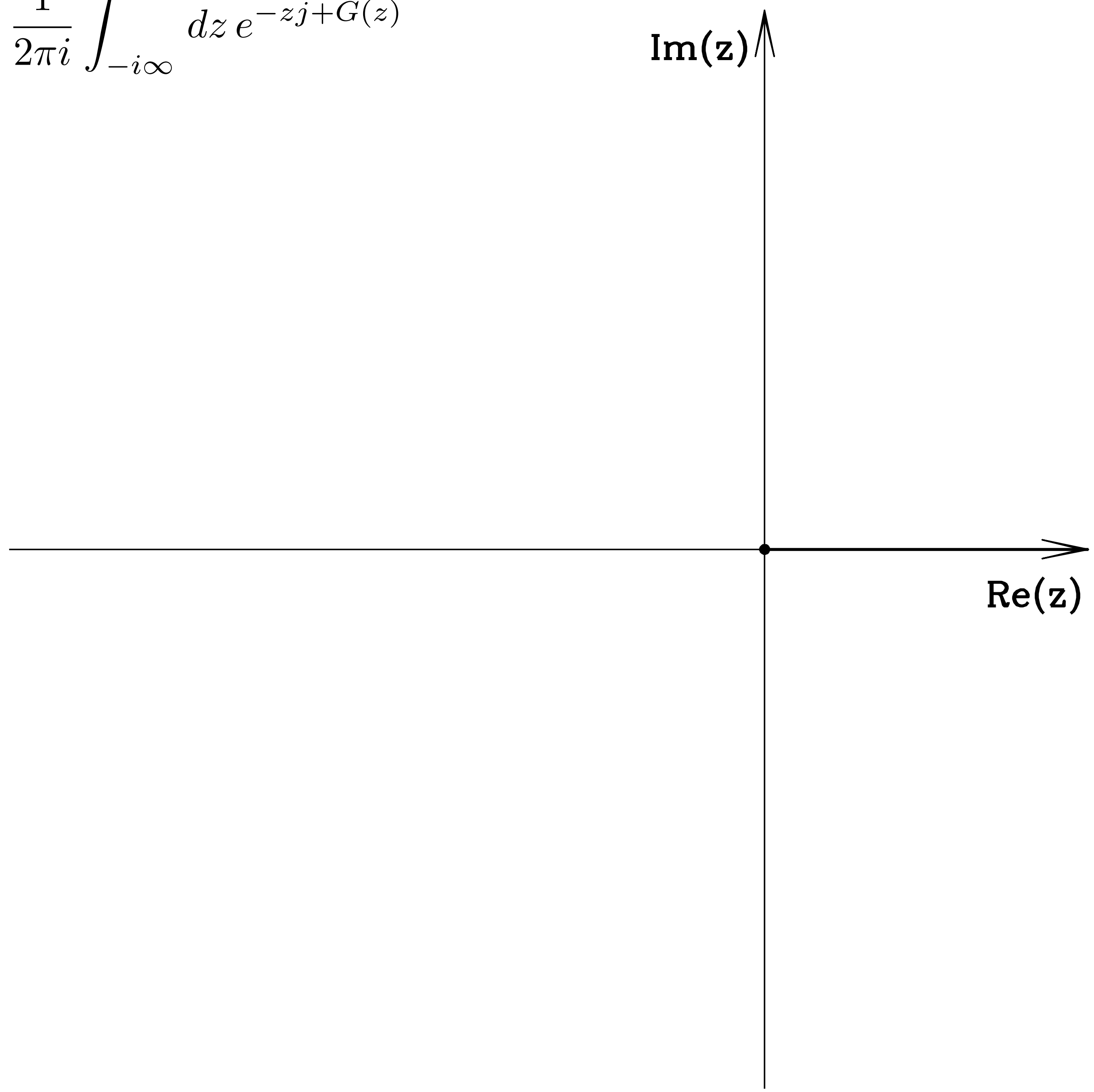
$$\mathcal{W}_\omega(V) \equiv -\bar{N}_{\rm e}\,\chi\big[\bar{N}_{\rm e}\bar{\xi}_2(V_{\rm e})\big]$$

$$\bar{N}_e\bar{\xi}_2 \equiv \left(\frac{\bar{n}}{V_e}\right)\int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2\, \xi_2(\mathbf{x}_1,\mathbf{x}_2)\big(1-\omega(\mathbf{x}_1)\big) \times \big(1-\omega(\mathbf{x}_2)\big)\Theta_H(|\mathbf{x}_1|-R)\Theta_H(|\mathbf{x}_2|-R)$$

Numerical evaluation

- *Normalized intensity:* $j \equiv J/J_\star, \quad J_\star = L_\star/(4\pi r_0^2)$

$$P(j) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dz\, e^{-zj+G(z)}$$



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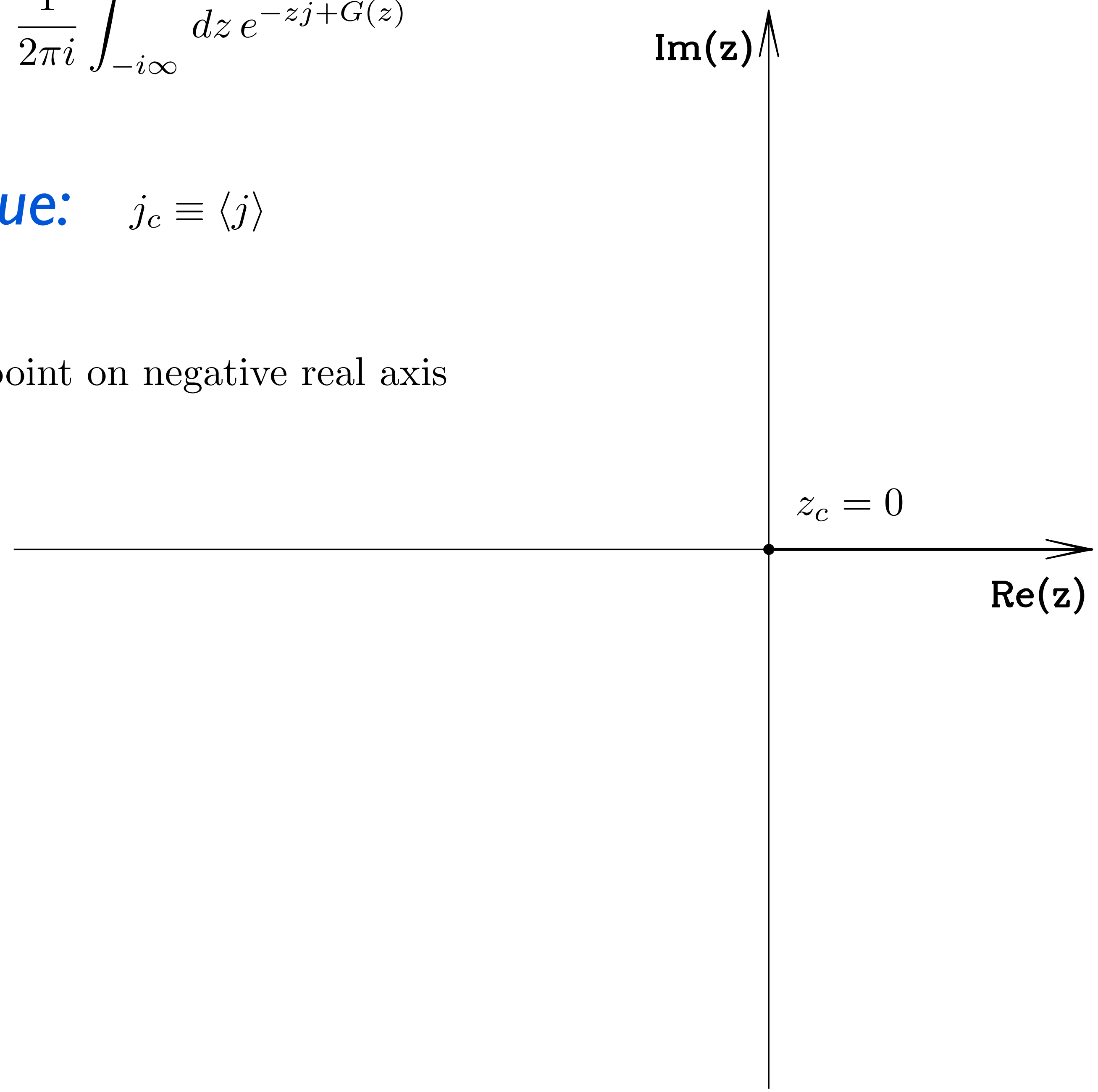
- *There is a critical intensity value:* $j_c \equiv \langle j \rangle$

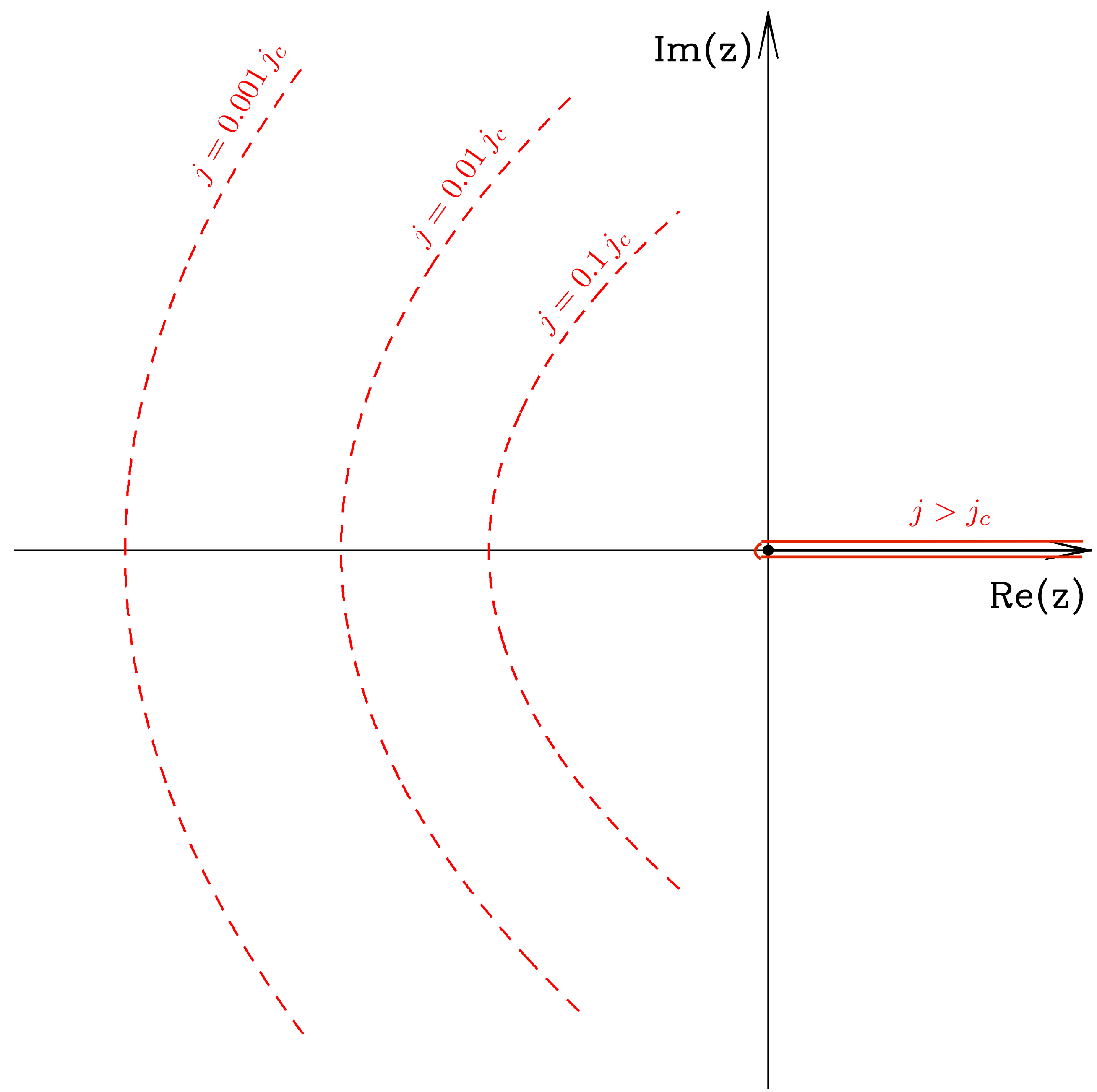
i) $j \leq j_c \;\Rightarrow\; -zj + G(z)$ admits a saddle point on negative real axis

ii) $j > j_c \;\Rightarrow\;$ critical point j_c dominates



Saddle point approximation

Construct paths in the complex plane such that $\delta(-zj + G(z)) \in \mathbb{R}$



Asymptotics

- *Low-intensities:*

$$j \le j_c: \quad P(j) \sim \sqrt{F''(j)}\, e^{-F(j)}$$

$$\sim e^{-(\ln j)^m}$$

- *High-intensities:*

$$j > j_c: \quad -zj + G(z) \approx -(j - j_c)z + c_{3/2} z^{3/2}$$

$$P(j) \sim j^{-5/2}$$

Model inputs

- *Standard double power-law form for the bolometric QLF:*

$$\Phi(L,z) = \frac{\Phi_\star(z)/L_\star(z)}{(L/\ L_\star(z))^{\beta_1(z)} + (L/L_\star(z))^{\beta_2(z)}}$$

(Boyle, Shanks & Peterson 1988)

- *Power-law form for the QSO correlation function:*

$$\xi_2(r) = \left(\frac{r}{r_\xi}\right)^{-\gamma}$$

$$\gamma \approx 2$$

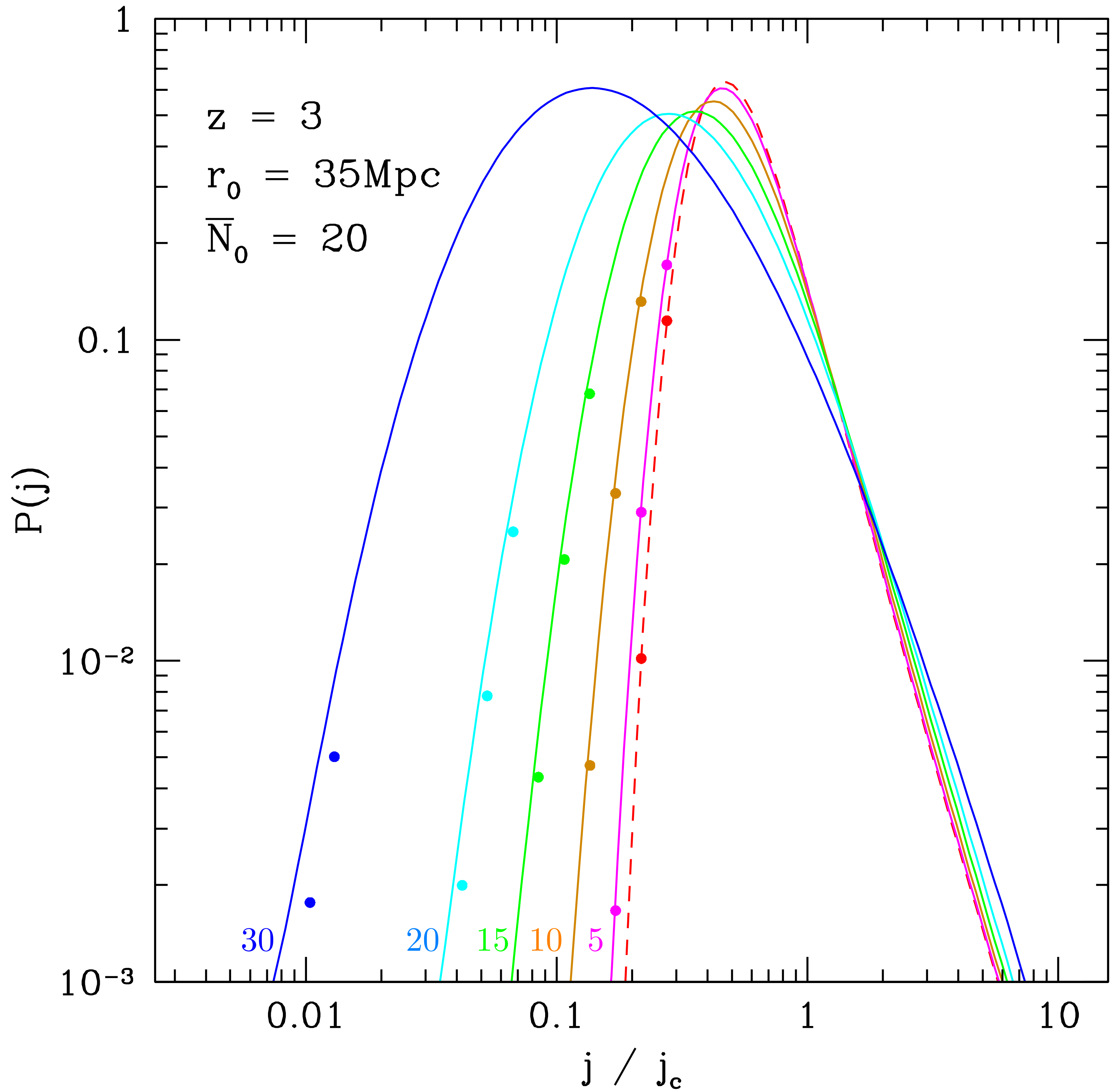
$$r_\xi \sim 24\mathrm{Mpc} \quad (2.9 \le z \le 3.5)$$

$$r_\xi \sim 35\mathrm{Mpc} \quad (z \ge 3.5)$$

(Shen, Strauss et al 2007)

(Shen et al 2007)

Intensity distribution



Varying assumptions

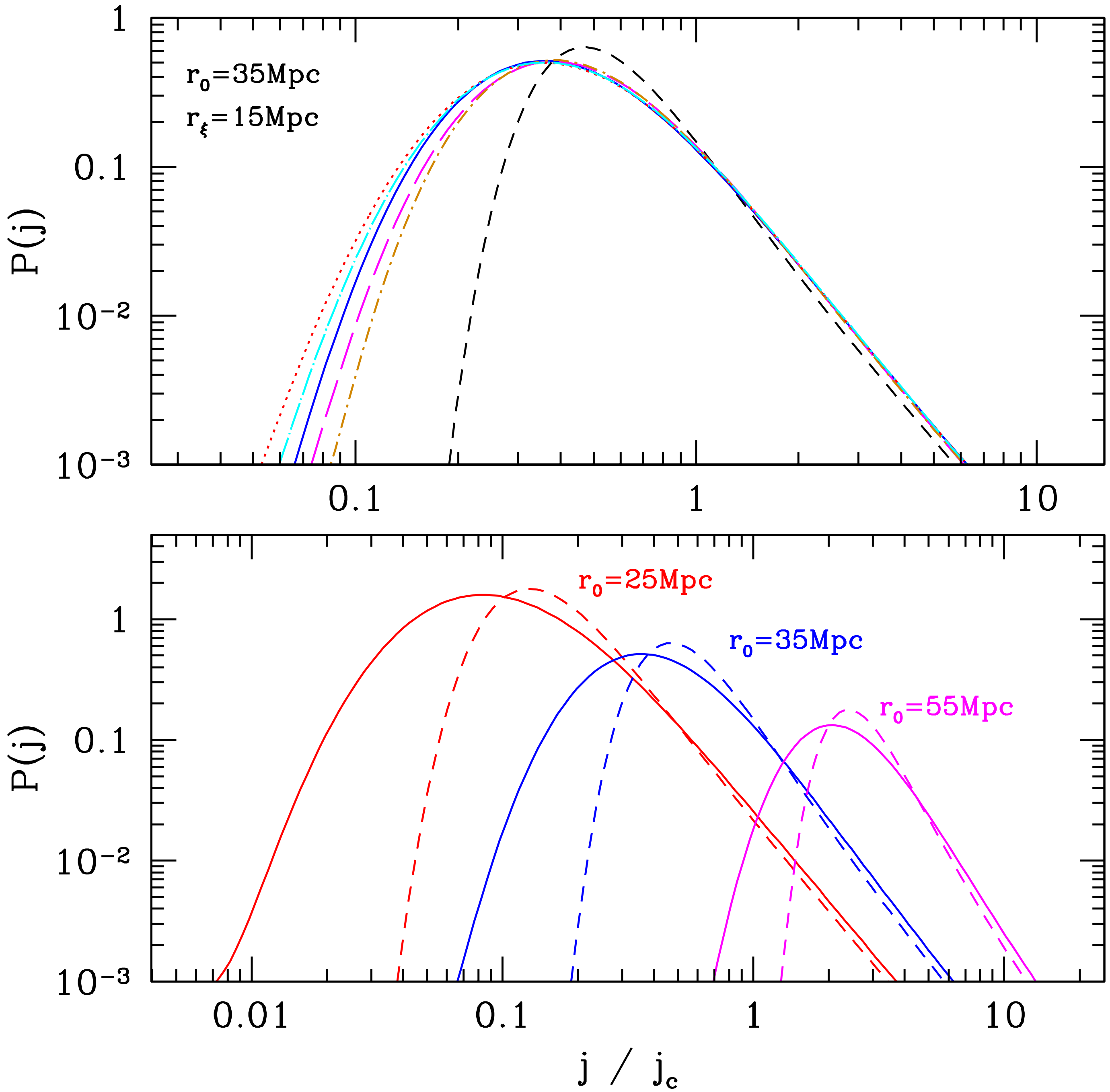


Figure 5. *Top panel* : Effect of changing the behaviour of the quasar correlation function on the distribution $P(j)$. The solid (blue) curve is our fiducial model, the dotted (red) curve was obtained using the GH rather than the NB void scaling function, the long-dashed (magenta) curve has $\xi_2 = 0$ for $r < 1$ Mpc while the dotted-short dashed (orange) assumes $\xi_2 = 0$ outside the range $1 < r < 150$ Mpc. Finally, the dotted-long dashed (cyan) curve assumes a powerlaw slope $\gamma = 1.9$ rather than 2.1. The correlation and attenuation lengths are $r_\xi = 15$ Mpc and $r_0 = 35$ Mpc, respectively. *Bottom panel* : $P(j)$ for 3 different attenuation lengths. Results are shown for randomly distributed (dashed curves) and clustered sources with $r_\xi = 15$ Mpc (solid curves).

Variance of intensity fluctuations

$$\frac{\langle \Delta j^2 \rangle\big|_{\rm clus}}{\langle \Delta j^2 \rangle\big|_{\rm ran}} = \frac{\langle j^2 \rangle - \langle j \rangle^2\big|_{\rm clus}}{\langle j^2 \rangle - \langle j \rangle^2\big|_{\rm ran}}$$

	$r_\xi = 5$	$r_\xi = 10$	$r_\xi = 15$	$r_\xi = 20$	$r_\xi = 30$
$r_0 = 25$	1.02	1.08	1.19	1.35	1.56
$r_0 = 35$	1.03	1.10	1.23	1.41	1.95
$r_0 = 55$	1.05	1.15	1.32	1.56	2.25

Environmental dependence

- *Environmental dependence of conditional void probability:*

$$\mathcal{W}_0(V|\delta) = \sum_{k=0}^{\infty} \frac{(-\bar{n})^k}{k!} \int_V d^3\mathbf{x}_1 \ldots \int_V d^3\mathbf{x}_k \, \xi_k(\mathbf{x}_1, \ldots, \mathbf{x}_k|\delta)$$

$$\equiv \sum_{k=1}^{\infty} \frac{(-\bar{N})^k}{k!} \bar{\xi}_k(V|\delta)$$

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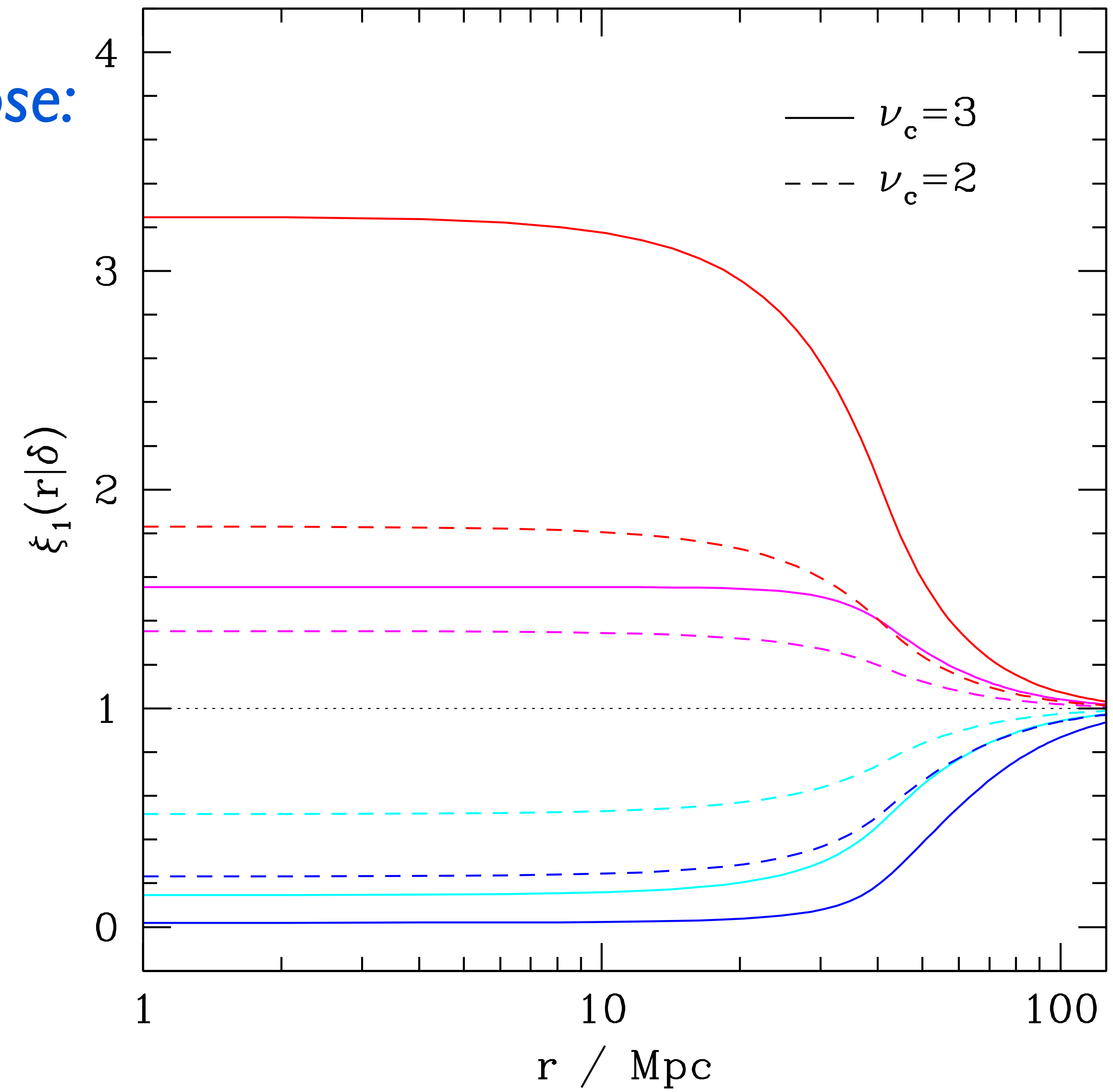
$$\equiv \sum_{k=1}^{\infty} \frac{(-\bar{N})^k}{k!} \bar{\xi}_k(V|\delta)$$

- *Poisson distribution + spherical collapse:*

$$\mathcal{W}_0(V|\delta) = -\bar{N}\bar{\xi}_1(V|\delta)$$



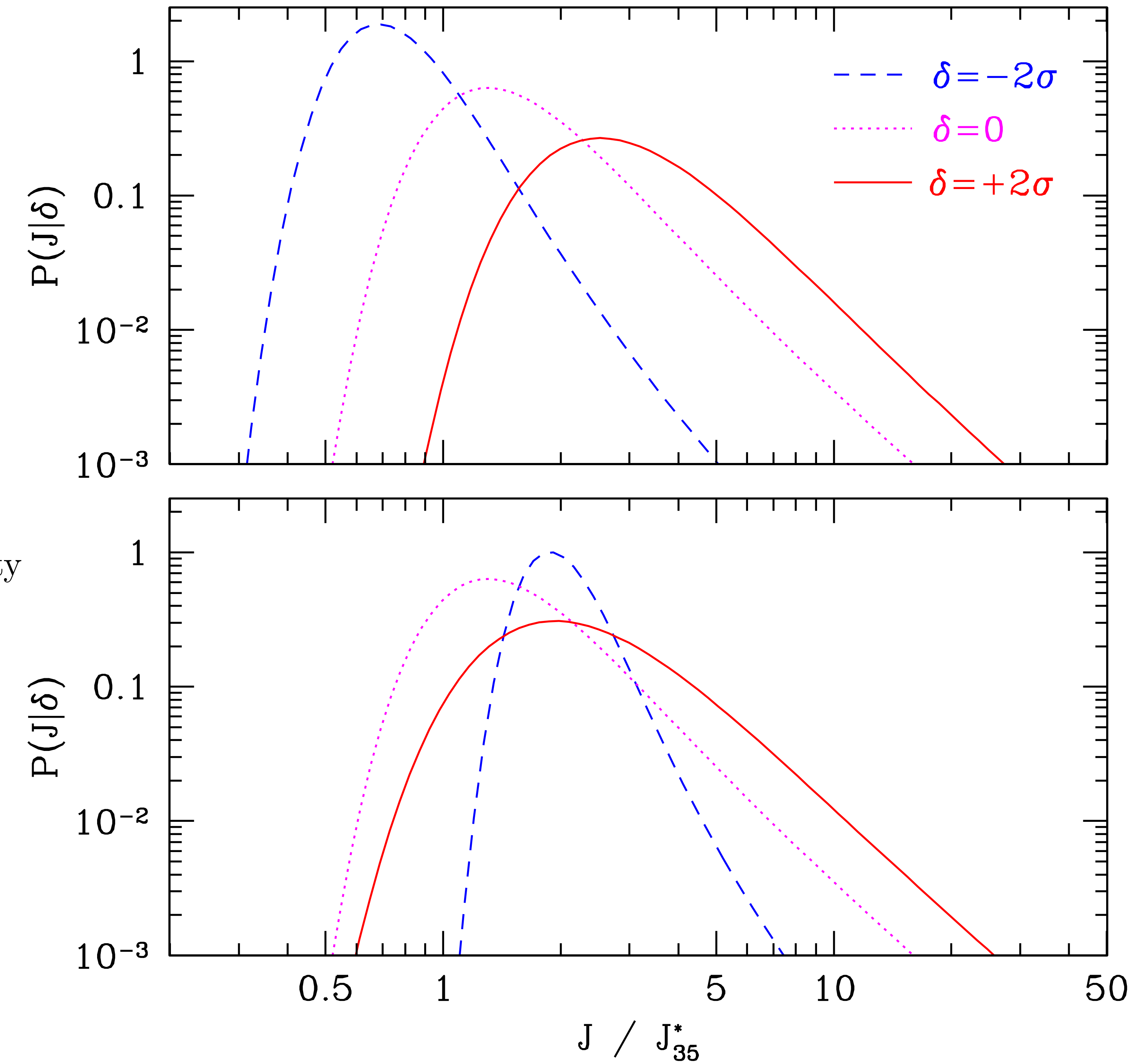
Environmental dependence

constant attenuation length

$r_0 = 35\text{Mpc}$

r_0 depends on environmental density

$29 < r_0 < 57\text{Mpc}$



Take home message

- *Count-in-cells + Hierarchical ansatz can be very powerful to describe strongly clustered distributions*