Mildly non-linear effects in the large scale structure: resummation vs effective approaches

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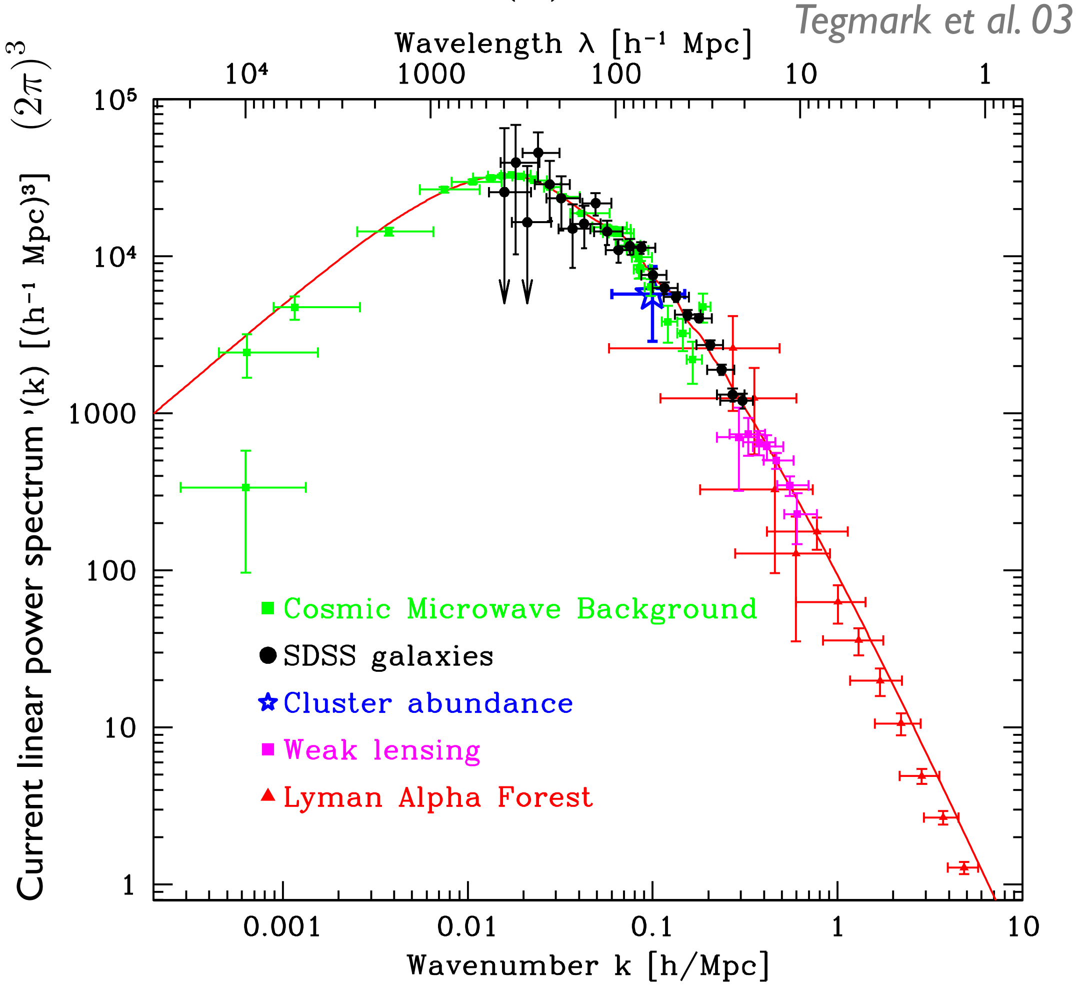
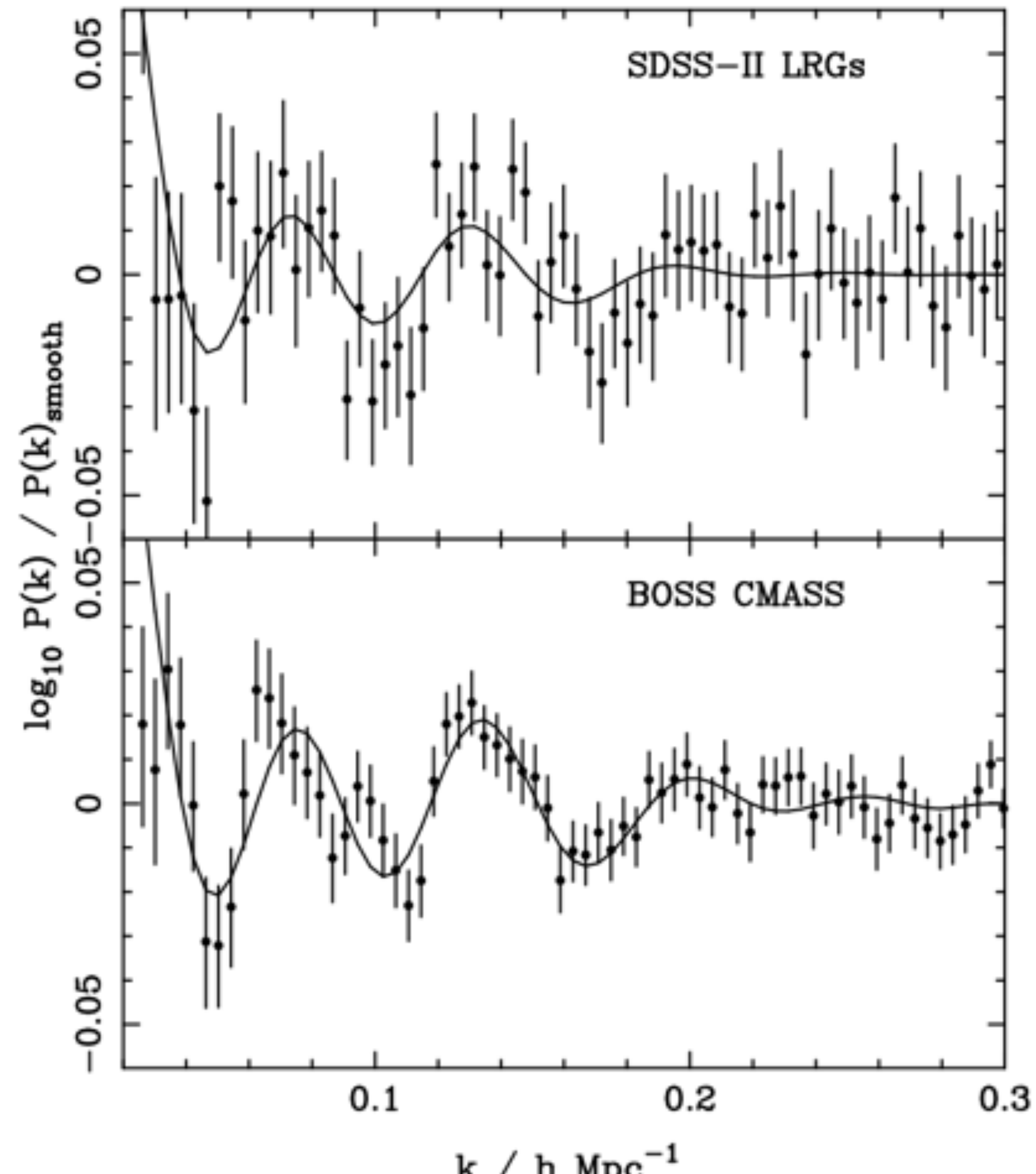
Precision cosmology: power spectrum

$$\delta_n \equiv \frac{\rho_n(x,t)}{\bar{\rho}_n(t)} - 1 \qquad \langle \delta(k,t)\delta(k',t)\rangle = P(k,t)\delta^{(3)}(k+k')$$

Tegmark et al. 03



Anderson et al. 12



DES, Euclid, LSST,...: 1% level at different redshifts!
(also higher n-point correlation functions)

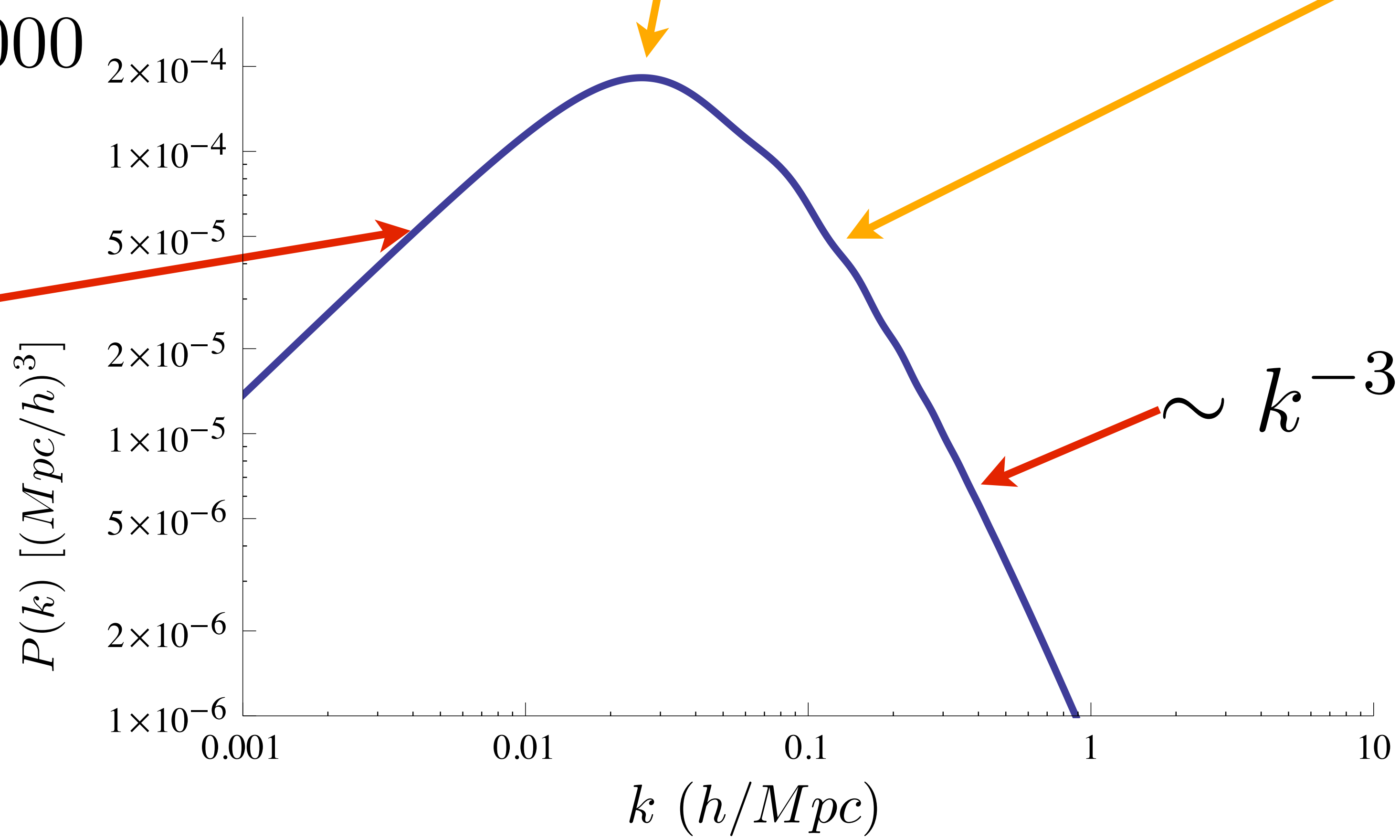
Matter power spectrum at decoupling

gaussian initial scale invariant PS +
radiation-matter transition + BAO imprint

CLASS CMB Code
Blas, Lesgourgues, Tram 11

$z = 1000$

$\sim k$



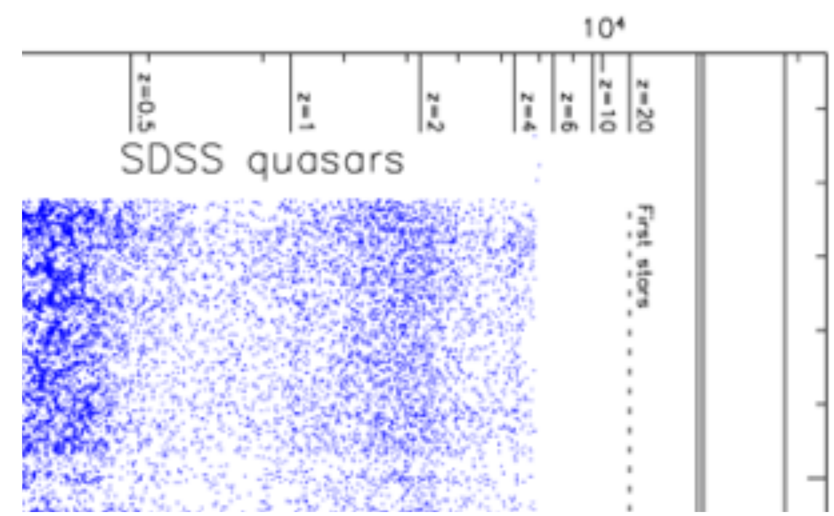
$\sim k^{-3}$

$$P_k \sim \frac{k}{(1 + k^2/k_0^2)^2}$$

Small quantity for PT: $\delta_k(z = 1000)$

Gravity makes matter clump: $\delta_k \sim a(t)$

perturbations grow! PT will break down



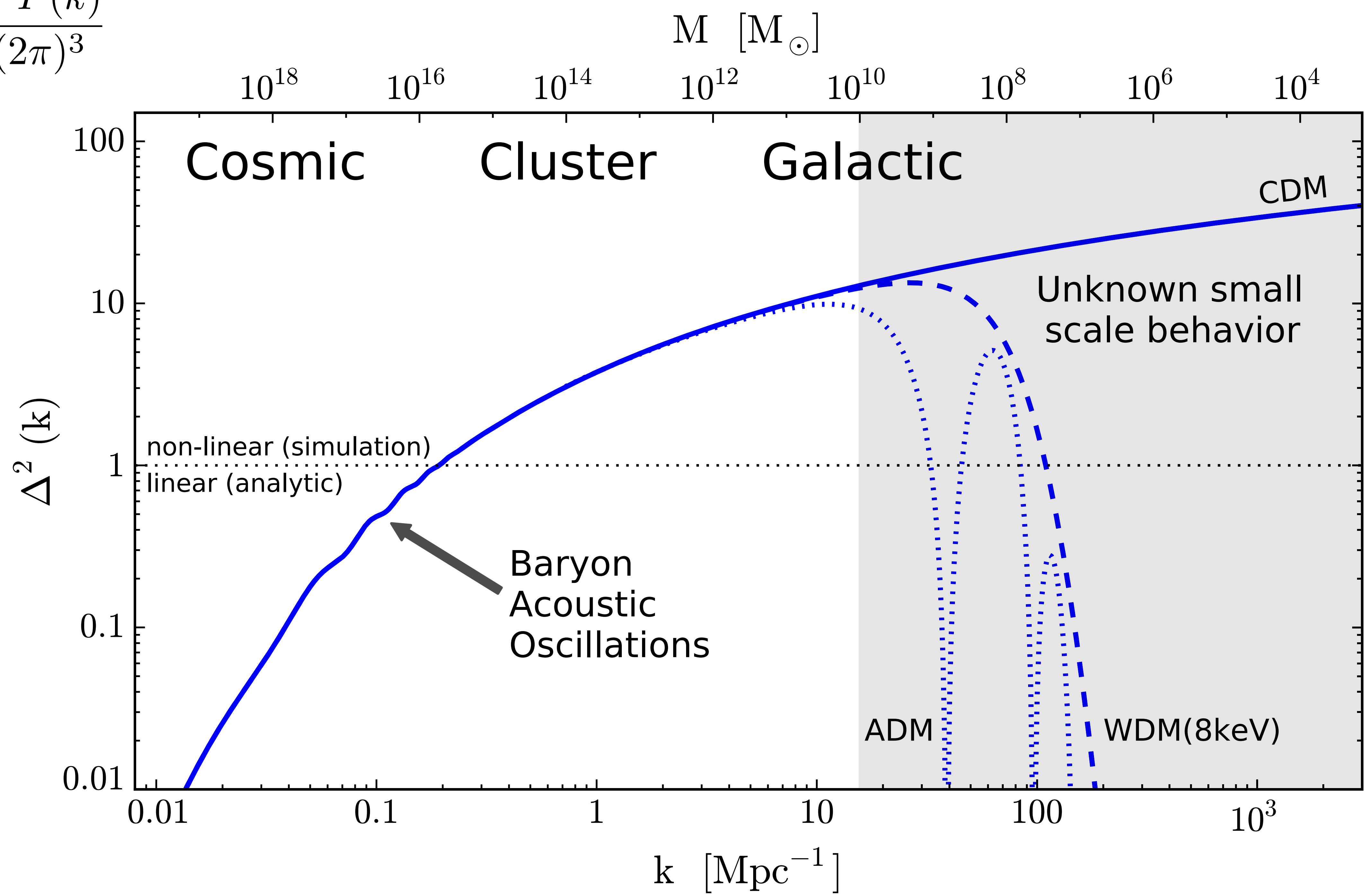
Gott et al. 05

what can we learn from PT?

Non-linearity in the Universe

M. Kuhlen et al. / Dark Universe 1 (2012) 50–93

$$\Delta^2 \equiv \frac{k^3 P(k)}{(2\pi)^3}$$



Theoretical framework

$$8\pi G T^{m}_{\mu\nu} = G_{\mu\nu}$$

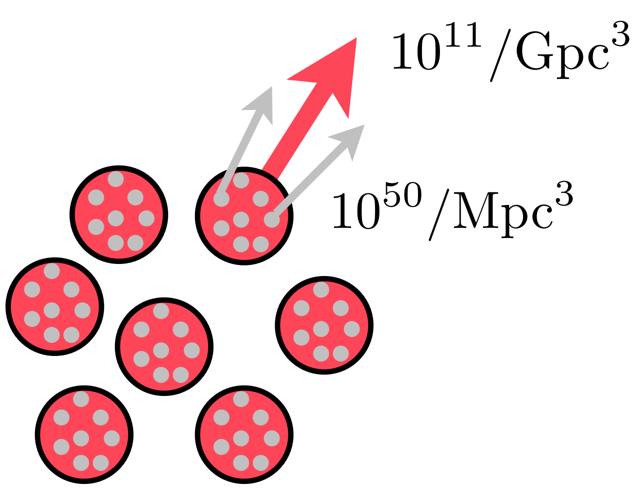
Matter

Non-relativistic and small ϕ

Micro: 'Particles' (sampling δ_{DM}) interacting through gravity

$$p^i_A \equiv a m_A v^i_A\,, \qquad \frac{\mathrm{d}p^i_A}{\mathrm{d}t} = -a m_A \partial_i \phi$$

$$\frac{\mathrm{d}f(x,p,t)}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{p^i_A}{a m_A}\partial_i f - a m_A \partial_i \phi \frac{\partial f}{\partial p^i_A} = 0$$



Linear: free streaming particles

Fully non-linear: *N*-body

Mildly non-linear PT?

Fluid description

Taking moments:

Particles per volume $$\int \mathrm{d}^3p f(x,p,t) \equiv \rho(x,t) \;, \quad \delta_n \equiv \frac{\rho_n(x,t)}{\bar{\rho}_n(t)} - 1$$

Velocity field $$\int \mathrm{d}^3p \frac{p^i}{am} f(x,p,t) \equiv \rho(x,t)\, v^i(x,t)$$

$$\frac{\mathrm{d}f(x,p,t)}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{p_A^i}{am_A}\partial_i f - am_A \partial_i \phi \frac{\partial f}{\partial p_A^i} = 0 \;, \qquad \Delta\phi = \frac{3}{2}\mathcal{H}^2 \sum \Omega_n \delta_n$$

$$\dot{\delta} + \partial_i([1+\delta]v^i) = 0$$

$$\dot{v}^i + \mathcal{H}v^i + v^j\partial_j v^i = -\partial_i\phi - \frac{1}{\rho}\partial_j\left[\int \mathrm{d}^3p \frac{p^i p^j}{(am)^2} f - \rho\, v^i v^j\right]$$

Pressureless perfect fluid
interacting through gravity

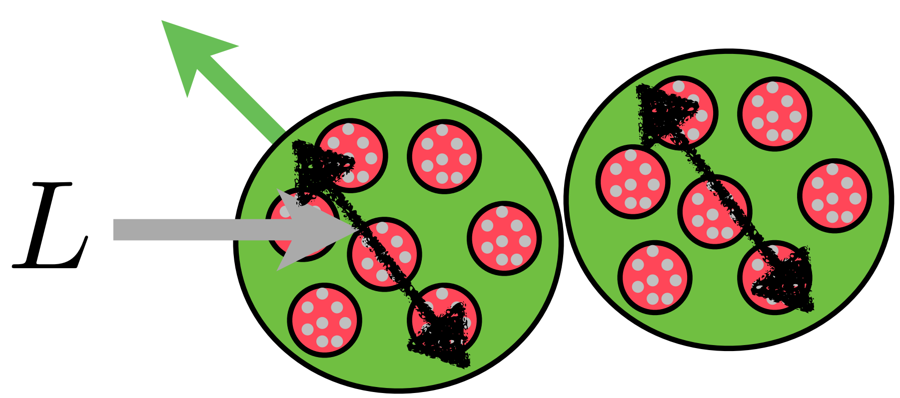
Deviation from single flow
Suppressed by
$k v_p \mathcal{H}^{-1}$

Dealing with short scales

$$\dot{v}^i + \mathcal{H}v^i + v^j\partial_j v^i = -\partial_i\phi - \frac{1}{\rho}\partial_j\left[\int d^3p\,\frac{p^ip^j}{(am)^2}f - \rho\, v^iv^j\right]$$

- Short scales: non-linear and no-fluid before.
- Hard to really 'integrate-out' modes

To restrict to long 'quasi-linear' scales, can the short scales be cut off (e.g. mean field) and get the right predictions?



$$\dot{v}^i_L + \mathcal{H}v^i_L + v^j_L\partial_j v^i_L = - \partial_i\phi_L + \mathcal{O}_L\mathcal{O}_S + \mathcal{O}_S + \mathcal{O}_{mf}$$

$v^j\partial_j v^i$

EFT approach: encapsulate these effects in operators of ***L***

$$\dot{v}^i_L + \mathcal{H}v^i_L + v^j_L\partial_j v^i_L = -\partial_i\phi_L + c_{L;s}(t)^2\partial_i\delta_L + c_{L;bv}(t)^2\partial_i\partial_j v^j_L + \ldots$$

Pietroni et al 11, Carrasco et al 12

‘Effective’ coefficients

$$\dot{v}^i_L + \mathcal{H} v^i_L + v^j_L \partial_j v^i_L = -\partial_i \phi_L + c_{L;s}(t)^2 \partial_i \delta_L + c_{L;bv}(t)^2 \partial_i \partial_j v^j_L + \ldots$$

- Related to non-linear scales: not clear power-counting
- Functions of time
- Some effects are known (decoupling of virialized structures, k^2 tail, measured non-linearities,...)

measurements

Pueblas, Scoccimarro 09

Shell-crossing

$$\dot{\theta}(k,t) + \mathcal{H}\theta(k,t) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(k,t) = q_\theta(k,t)$$

$$\theta \equiv \partial_i v^i$$

Carrasco et al 13

EFT matching

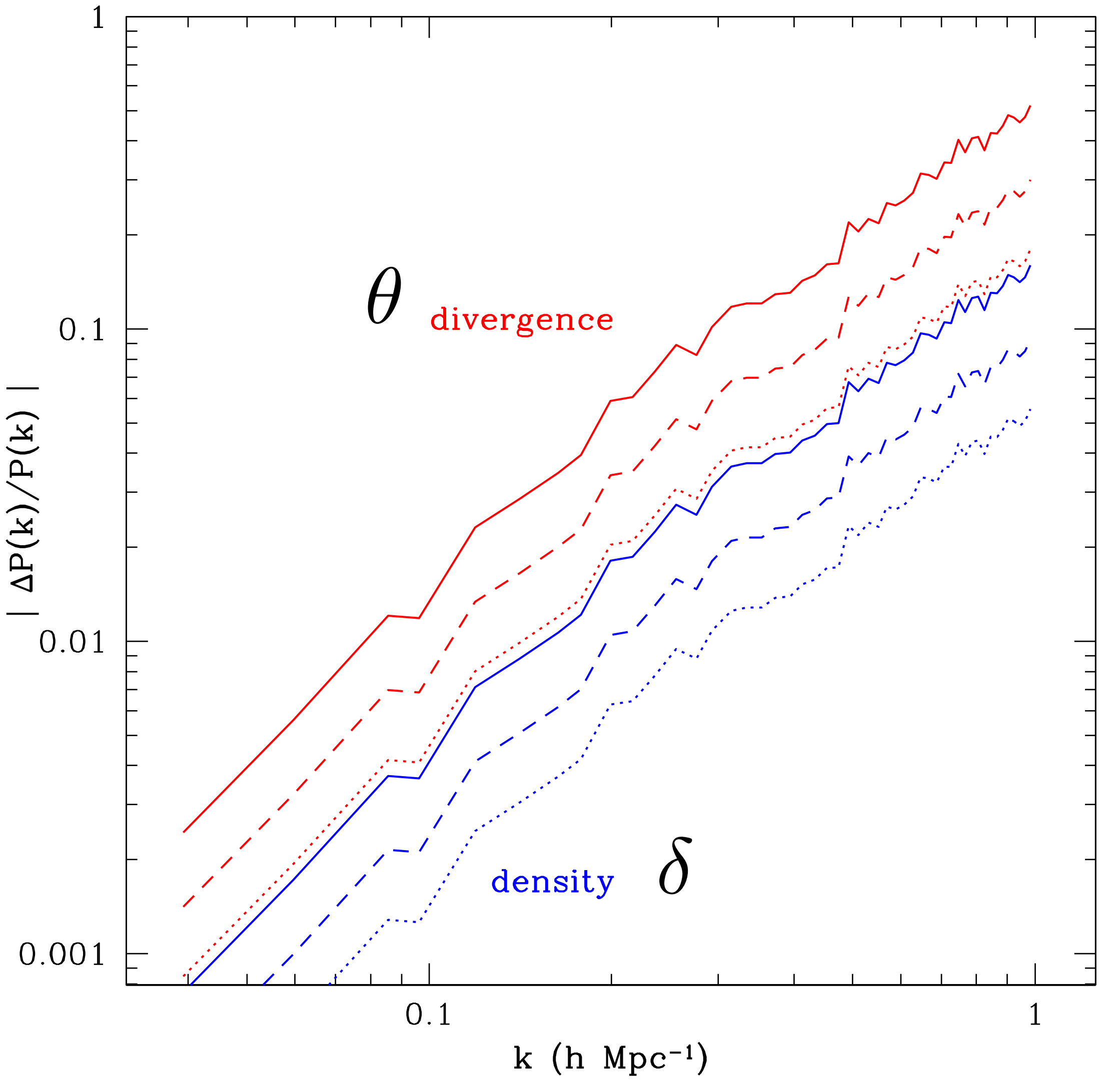
$O(k^2)$ small effect: can only be measured at mildly non-linear scales

not exactly the same

Effects of effective coefficients

Pueblas, Scoccimarro 09

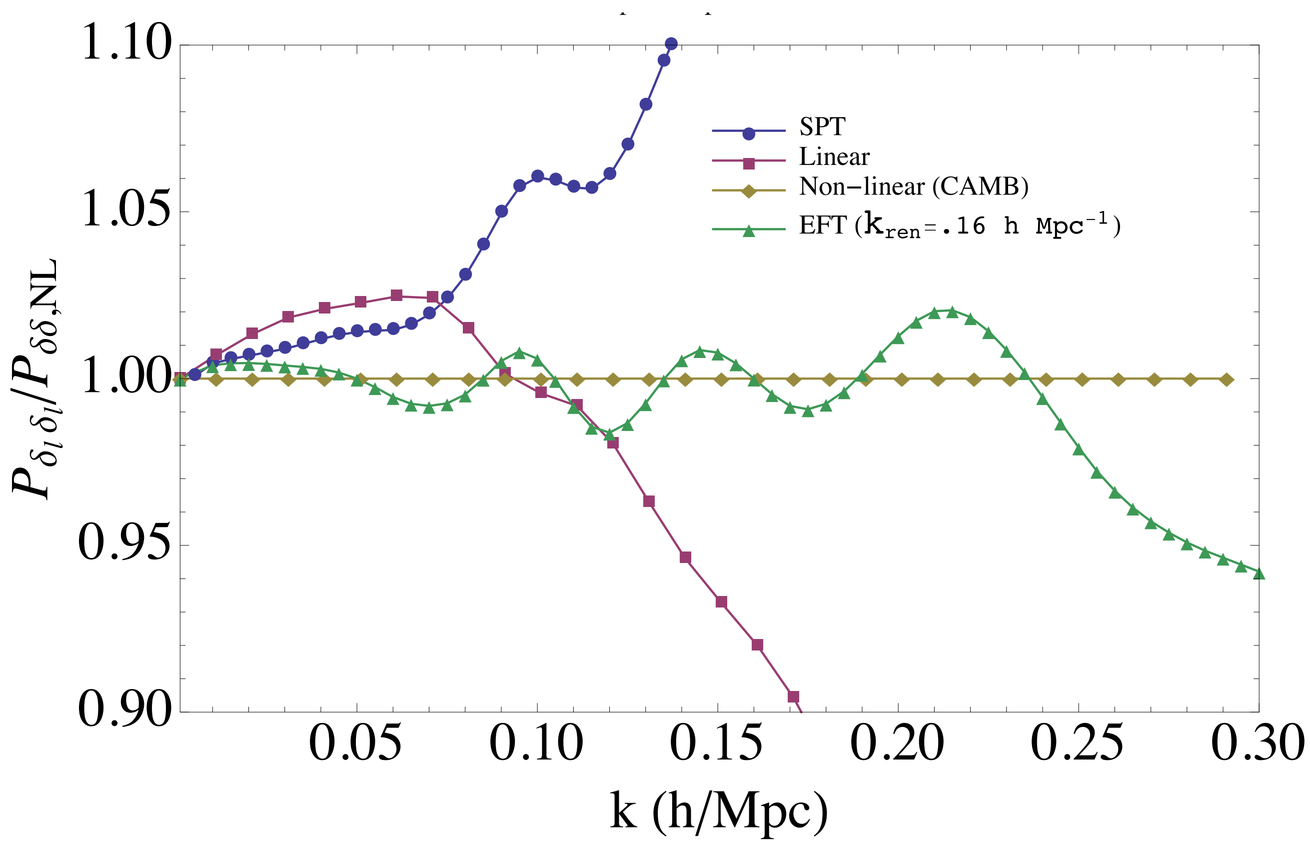
$$\dot{\theta}(k,t) + \mathcal{H}\theta(k,t) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta(k,t) = q_\theta(k,t)$$



Negligible!

Carrasco et al 13

After many approximations and matching,



Crucial!

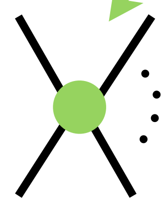
NN…LO formalism

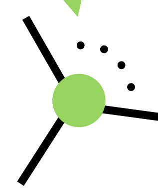
$\Omega_m = 1$

$$\delta(k,t) = \sum_n a(t)^n F_n(k_1,...,k_n)\delta(k_1,t_0)\ldots\delta(k_n,t_0)\delta^{(3)}(k - \sum k_i)$$

Power spectrum $\langle \delta(k,t)\delta(k',t)\rangle = P(k,t)\delta^{(3)}(k+k')$

$F_i\, \delta^{(3)}(\sum k_i - k)$





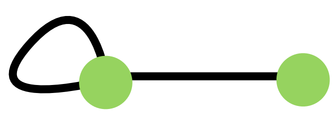
$F_j\, \delta^{(3)}(\sum q_i + k)$

$F_1(k_1) = 1$

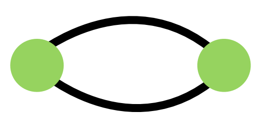
Gaussian IC

$\langle \delta(k,t_0)\delta(k',(t_0))\rangle = P_0(k)\delta^{(3)}(k+k')$

NLO (1L) : $\sim \delta(k,t_0)^4$

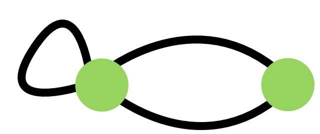


$$P_{13} = 3P_0(k)\int d^3q F_3^s(k,q,-q)P_0(q)$$



$$P_{22} = 2\int d^3q \left[F_2^s(q,k-q)\right]^2 P_0(q)P_0(|k-q|)$$

NNLO (2L): $\sim \delta(k,t_0)^6$



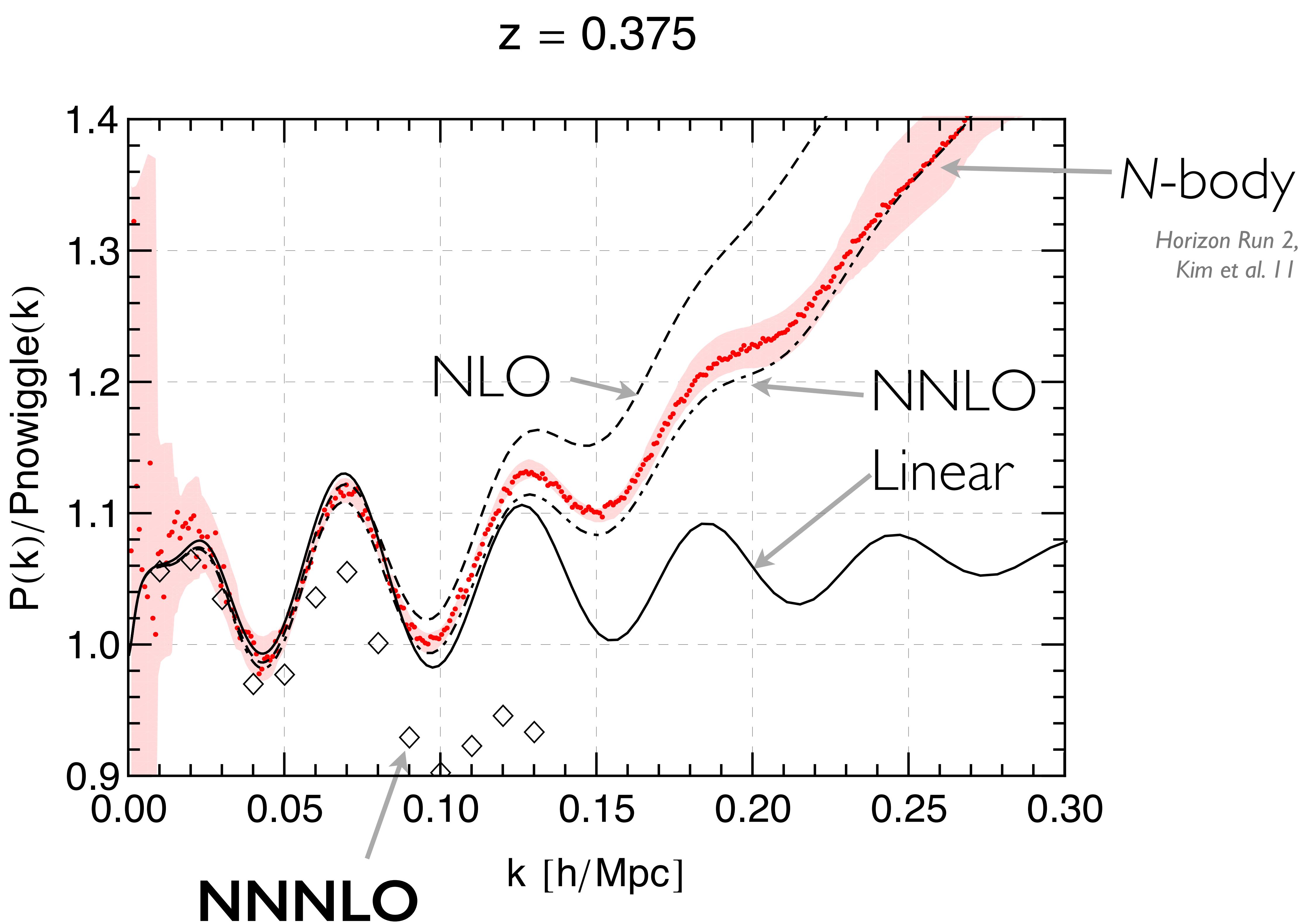
$+\ldots$

$$P(k,t) = a^2 P_0 + a^4(2P_{13} + P_{22}) + a^6(2P_{15} + 2P_{24} + P_{33}) + \ldots$$

NLO NNLO

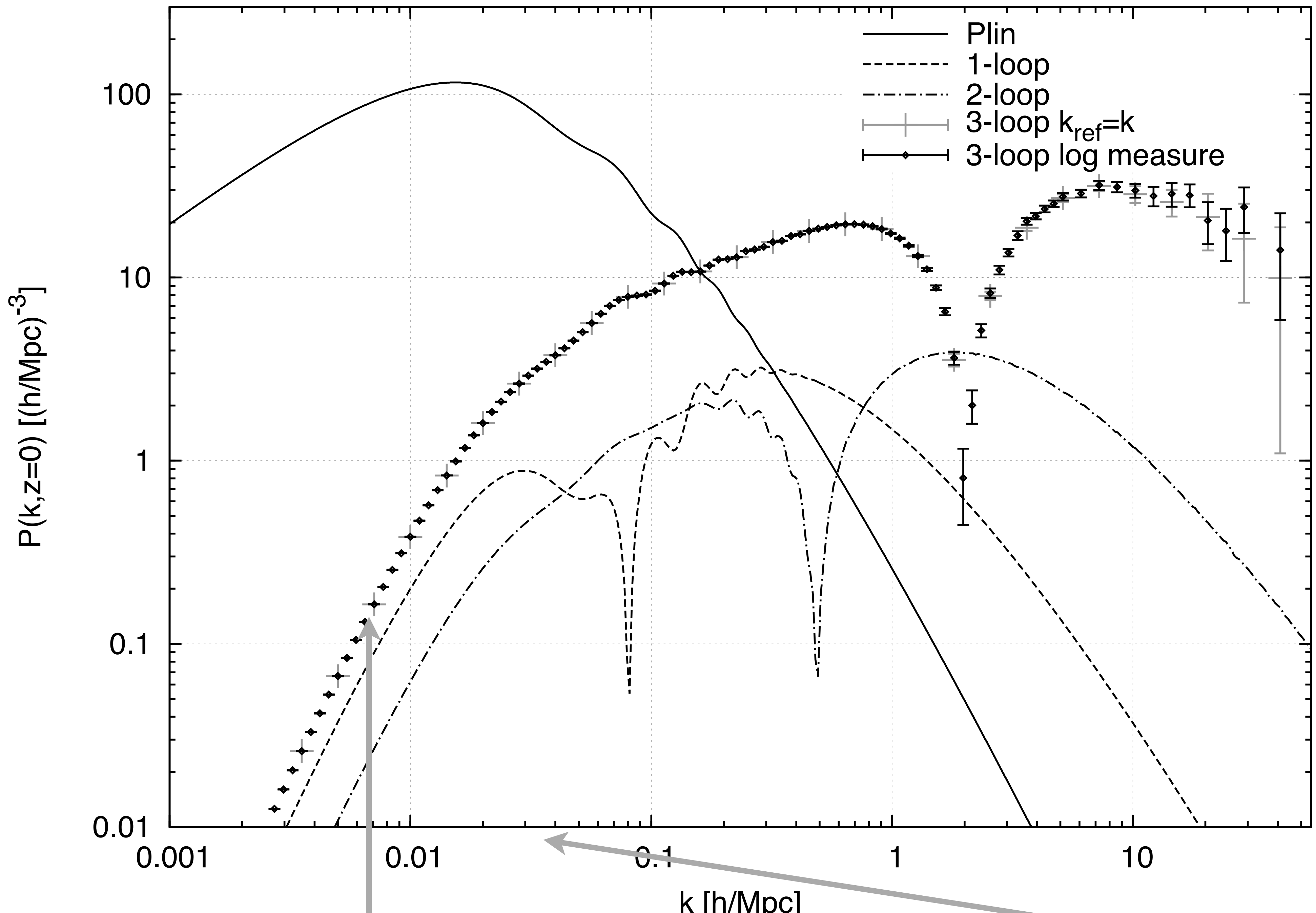
3 Loop ‘disaster’

DB, Garny and Konstandin 13B



NNNLO (3 Loop)

DB, Garny and Konstandin 13B



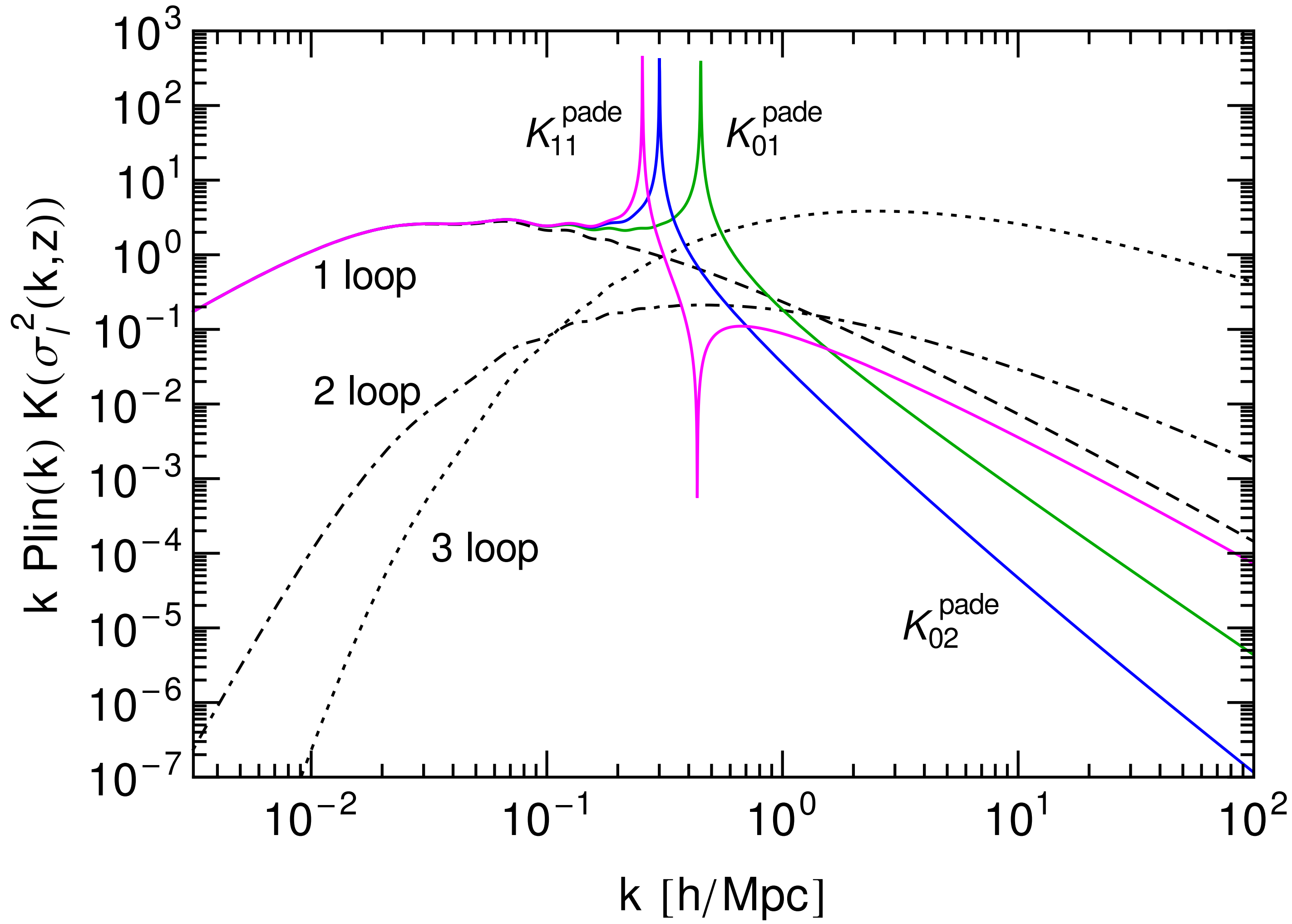
3L surpasses 1L at low k!

k_{NL}
shifted to lower k

A resummation: Padé integrands

$$P_{low-k} = -\frac{244\pi}{315} k^2 P_{lin}(k,z) \int dq P_{lin}(k,z) \sum_L C_L \left[4\pi \int_0^q dp\, p^2 P_{lin}(p,z)\right]^{2(L-1)}$$

$$K(x) = \sum C_L x^{L-1} \quad \Rightarrow \quad K(x)^{Padé}_{nm} = \frac{1+\sum_{i=1}^n a_i x^i}{1+\sum_{j=1}^m b_j x^j}$$



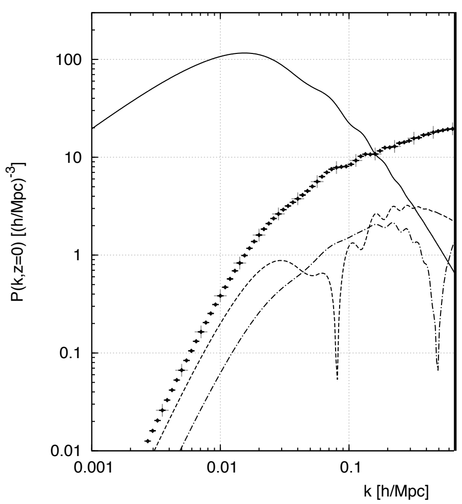
$z = 0$

The resummation **damps** the UV dependence!

May made the series **convergent**! (1% target attainable)

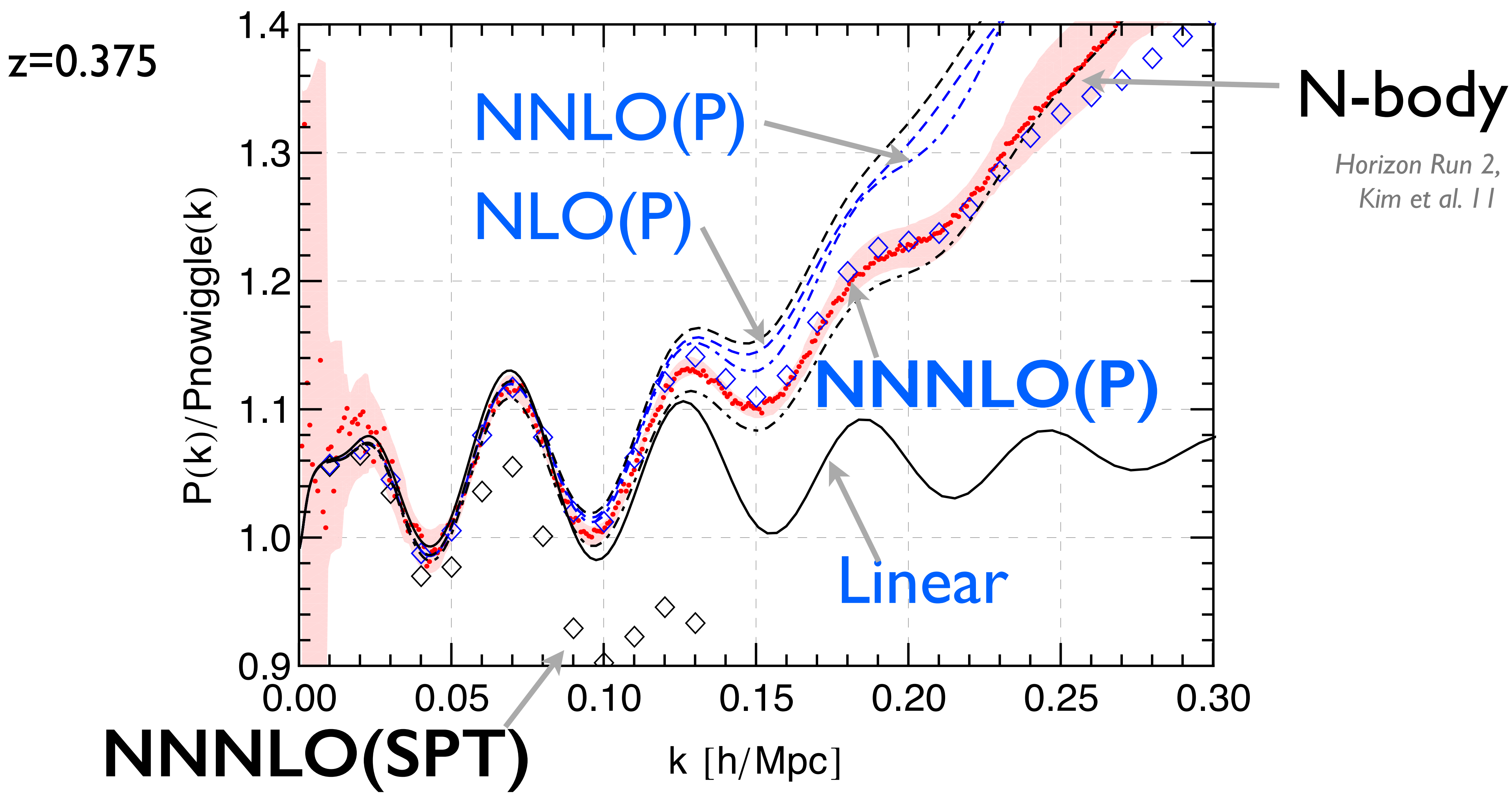
Padé results: perturbation theory

DB, Garny and Konstandin 13B



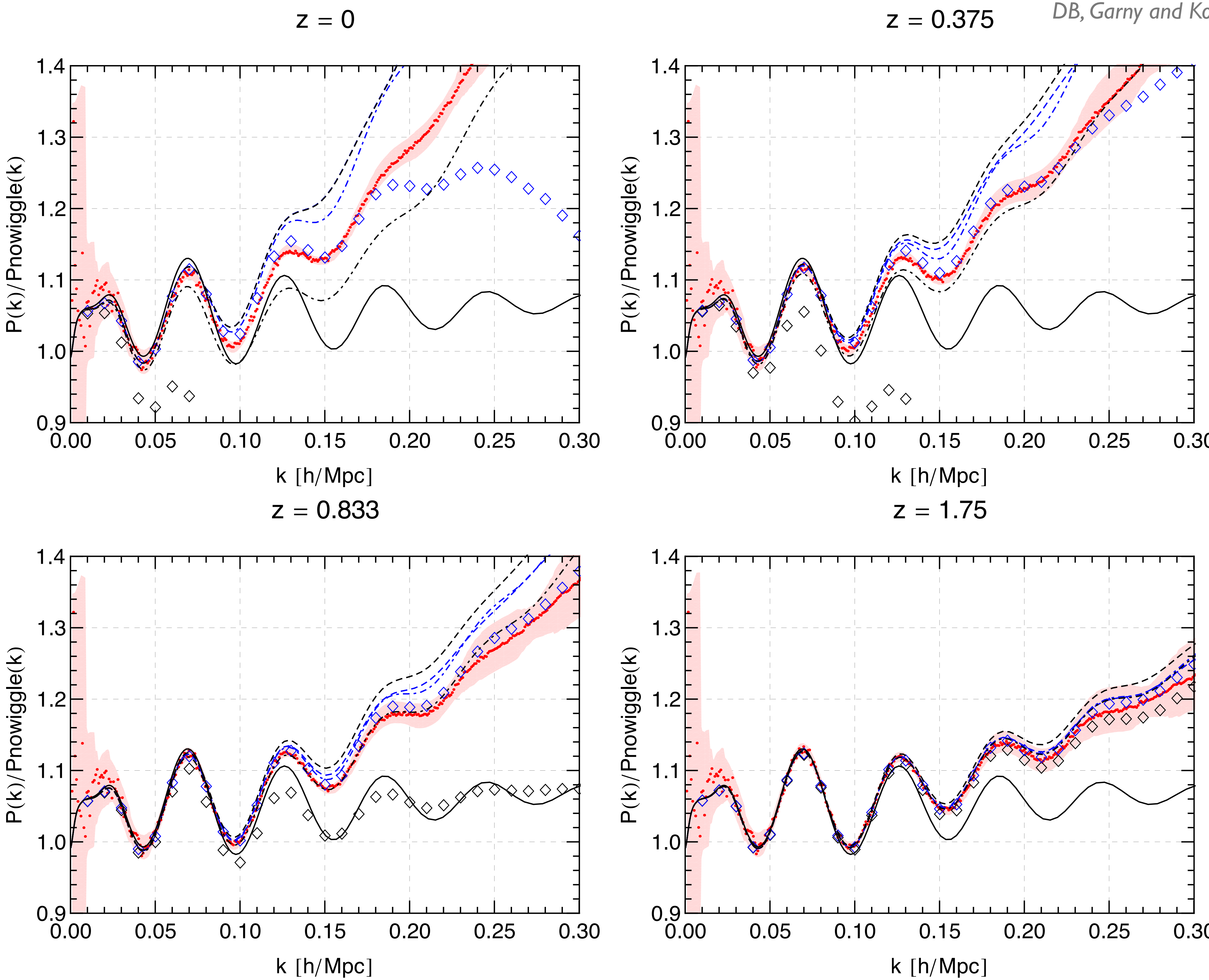
$$P(k,t) = P_l + P_{1-loop} + ... = P_l + P^{Padé}_{low-k} + \Delta P_{1-loop} \cdots$$

$$\Delta P_{L-loop} \equiv P_{L-loop} - P^{small-k}_{L-loop}\,, \quad P^{Padé}_{low-k} \equiv \sum P^{small-k}_{L-loop}$$



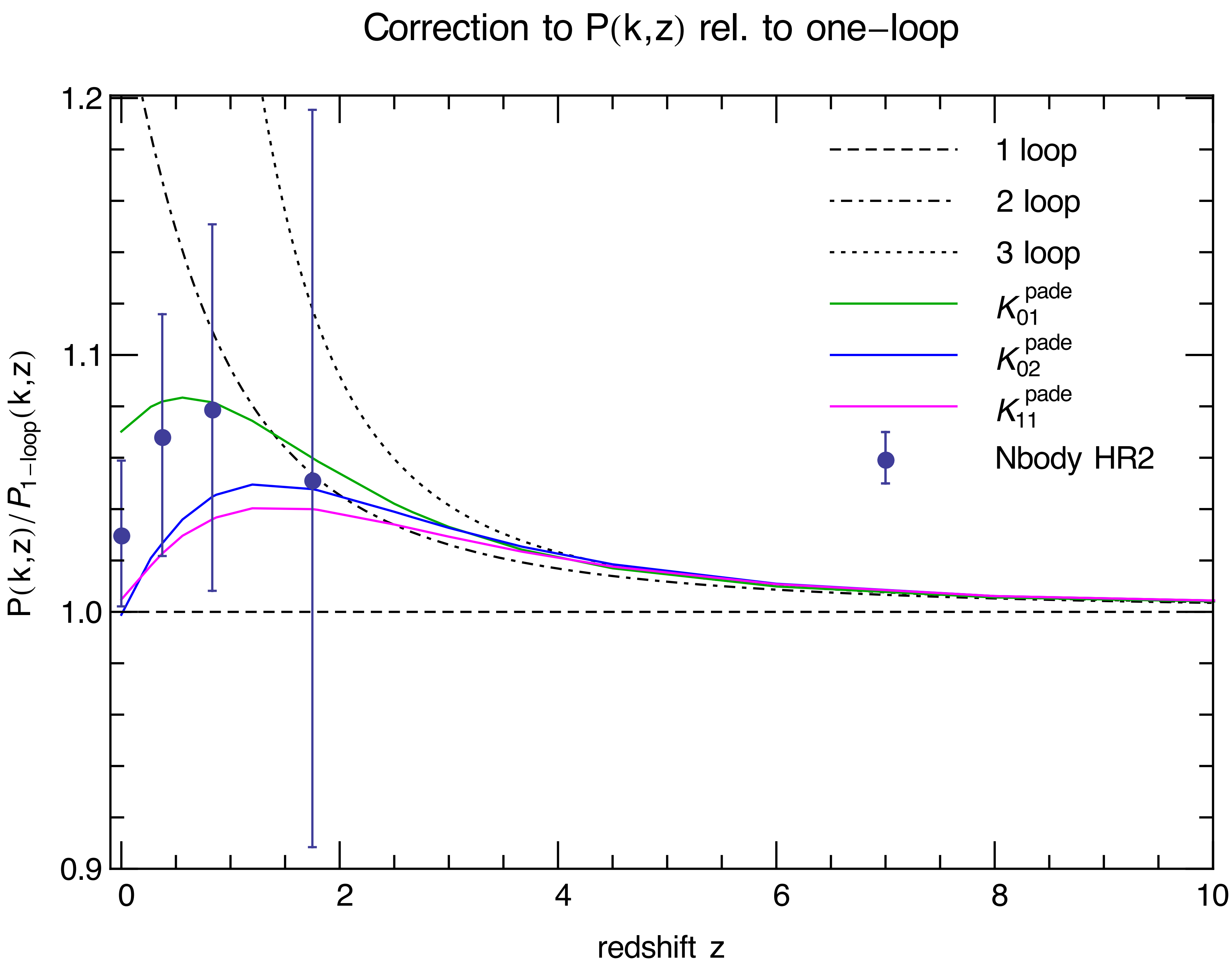
Padé results: redshift dependence

DB, Garny and Konstandin 13B



Further checks

DB, Chan, Garny, Konstandin Unpublished



Conclusions

- Future surveys will test cosmological expansion and structure formation to **percent level**.
- At this precision, the Universe at large scales behaves **almost** as pressureless perfect fluid.
- On-going discussion on the importance of short modes. **Necessary/irrelevant** for convergence at semi-linear k
- PT series is **not convergent**! Reminds asymptotic series (result at 3 loop).
- **Padé ansatz**: parameter free resummation. Much better convergence properties and agreement with *N*-body. (percent accuracy at BAO scales and $z = 0$ reachable)

For the future

- More analytical understanding. Borel-Padé. UV sens. Other basis…
- Other observables ($P_{\theta\theta}$, bispectrum,...), other IC (NG).
- Predictions for observations: results in redshift space, parametrization of BAOs, bias...
- Putting all together? **EFTofLSS + resummations**
- Including neutrinos…
- Lagrangian space. Work in phase space.