NON-SUSY BSM: LECTURE 1/2

GENERALITIES

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Introduction

- There are a number of theoretical and experimental issues motivating BSM: the hierarchy problem (HP), the flavour problem, the strong CP-problem, the existence of Dark Matter (and possibly Dark energy), the origin of baryons, ...
- The main motivation for BSM has always been the hierarchy problem. The others have been treated within the proposed solutions to HP
- The hierarchy problem has two generic solutions:
  - Either there is an extra symmetry such that the quadratic sensitivity of the Higgs mass to the scale cancels: the prototype is supersymmetry
  - Or the Higgs is a composite field such that at high scales it dissolves into its constituents: the original prototype has been technicolor or its modern formulation in extra dimensions motivated by the AdS/CFT duality.
- As supersymmetry has already been covered at this School (M. Kramer's talks) I will try to motivate the issue of extra dimensions as a solution to the hierarchy (and others) problems

Bibliography

- Some review papers

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Outline

The outline of this lecture is

- The 5D metric background
  - The superpotential method
  - Particular cases: flat background, AdS_5 background, general warped backgrounds
- Scalars in the bulk
  - EOM & BC's
  - KK decomposition
- Gauge bosons in the bulk
  - EOM & BC's
  - KK decomposition
- Fermions in the bulk
  - EOM & BC's
  - KK decomposition

The metric background

- The most general five-dimensional metric consistent with four-dimensional Poincaré symmetry is

$$ds^2 = g_{MN} dx^M dx^N = e^{-2A(y)}\, \eta_{\mu\nu}\, dx^\mu dx^\nu - dy^2$$

  with $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1, -1\}$
- We include a scalar field $\phi(x, y)$ in the action (units $M_5 \equiv 1$):

$$S = \int_M d^5x \sqrt{g} \left[-\frac{1}{4} R + \frac{1}{2} (\partial_M \phi)^2 - V(\phi) \right] - \int_{\partial M} d^4x \sqrt{\bar{g}} \lambda_\alpha(\phi)$$

  where M is the full five-dimensional spacetime, ∂M is the codimension one hypersurface where each brane is located and $\bar{g}$ is the induced metric. It will always be assumed that the branes are at definite values of $y = y_\alpha,\ \alpha = 0, 1$

- The Ricci tensor:

$$R_{\mu\nu} = e^{-2A}\left(4A'^2 - A''\right)\eta_{\mu\nu} \quad R_{55} = -4A'^2 + 4A''$$

- The EOM are

$$\delta S/\delta\phi : \quad \phi'' - 4A'\phi' = \frac{\partial V(\phi)}{\partial\phi} + \sum_\alpha \frac{\partial\lambda_\alpha(\phi)}{\partial\phi}\delta(y - y_\alpha)$$

$$\delta S/\delta g_{\mu\nu} : \quad A'' = \frac{2}{3}\phi'^2 + \frac{2}{3}\sum_\alpha \lambda_\alpha(\phi)\delta(y - y_\alpha), \qquad \{' \equiv d/dy\}$$

$$\delta S/\delta g_{55} : \quad A'^2 = -\frac{1}{3}V(\phi) + \frac{1}{6}\phi'^2$$

- By integrating the first two equations on $(y_\alpha - \epsilon, y_\alpha + \epsilon)$ the jump conditions are

$$A'\Big|_{r_\alpha-\epsilon}^{r_\alpha+\epsilon} = \frac{2}{3}\lambda_\alpha(\phi(r_\alpha)), \qquad \phi'\Big|_{r_\alpha-\epsilon}^{r_\alpha+\epsilon} = \frac{\partial\lambda_\alpha}{\partial\phi}(\phi(r_\alpha))$$

THE SUPERPOTENTIAL METHOD

- Solving these EOM is quite hard. But there is a class of potentials for which EOM are analytically tractable
- Suppose $V(\phi)$ has the special form, for some $W(\phi)$

$$V(\phi) = \frac{1}{8}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2 - \frac{1}{3}W(\phi)^2$$

- Then it is straightforward to verify that a solution to

$$\phi' = \frac{1}{2}\frac{\partial W(\phi)}{\partial \phi}\,, \qquad A' = \frac{1}{3}W(\phi)$$

is also a solution to EOM, provided we have

$$\frac{1}{2}W(\phi)\Big|_{y_\alpha-\epsilon}^{y_\alpha+\epsilon} = \lambda_\alpha(\phi(y_\alpha))\,, \qquad \frac{1}{2}\frac{\partial W(\phi)}{\partial \phi}\Big|_{y_\alpha-\epsilon}^{y_\alpha+\epsilon} = \frac{\partial \lambda_\alpha}{\partial \phi}(\phi(y_\alpha))$$

- The jump conditions are written in the language of the $S^1/\mathbb{Z}_2$ orbifold
- As the metric A is even under the action of $\mathbb{Z}_2$ the superpotential W is odd
- The jump conditions become

$$A'(y_0+\epsilon) = \frac{1}{3}\lambda_0(\phi_0)\,, \quad \phi'(y_0+\epsilon) = \frac{1}{2}\frac{\partial\lambda_0(\phi_0)}{\partial\phi}$$

$$A'(y_1-\epsilon) = -\frac{1}{3}\lambda_1(\phi_1)\,, \quad \phi'(y_1-\epsilon) = -\frac{1}{2}\frac{\partial\lambda_1(\phi_1)}{\partial\phi}$$

where $\phi_\alpha \equiv \phi(y_\alpha)$ $(\alpha = 0,1)$ are constants to be interpreted as boundary field values

- From where (here $' = \partial/\partial\phi$)

$$\lambda_0(\phi) = W(\phi_0) + W'(\phi_0)(\phi-\phi_0) + \frac{1}{2}W''(\phi_0)(\phi-\phi_0)^2 + \cdots$$

$$\lambda_1(\phi) = -W(\phi_1) - W'(\phi_1)(\phi-\phi_1) - \frac{1}{2}W''(\phi_0)(\phi-\phi_0)^2 + \cdots$$

Particular cases

Flat case

- The simplest case corresponds to a flat extra dimension, i.e.

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

- This case corresponds to $W = 0 \;\Rightarrow \{A = 0,\; \phi \equiv 0;\; \lambda_0 = \lambda_1 = 0\}$

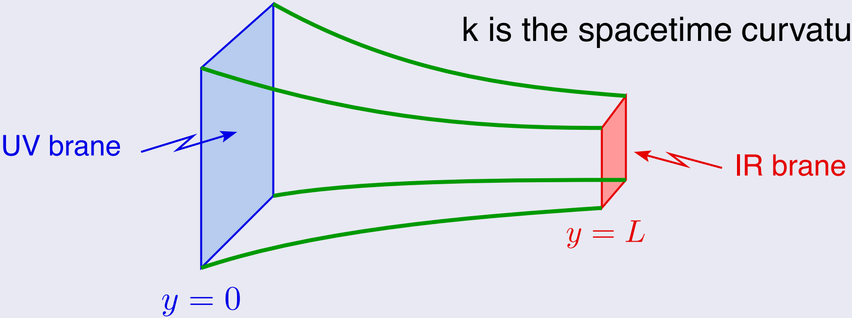
AdS_5

- This case corresponds to $W = 3k \;\Rightarrow \{A = ky,\; \phi \equiv 0;\; \lambda_0 = -\lambda_1 = 3k\}$, i.e. a negative brane tension in the IR.
- The vacuum energy

$$V = -\frac{1}{3} W^2 = -3k^2$$

corresponds to Anti-de-Sitter in 5D (AdS_5)

RS1 setup



k is the spacetime curvature

$$L \equiv y_1$$

General warped: soft-walls

- One can introduce a more general superpotential as

$$W = 6k\left(1 + e^{\nu\phi/\sqrt{6}}\right)$$

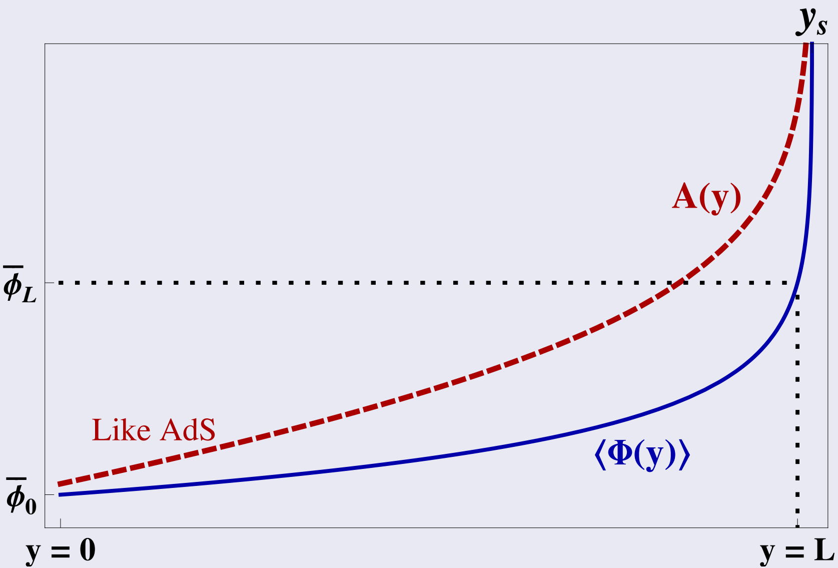
- The bulk solutions from the superpotential method are

$$\phi(y) = -\frac{\sqrt{6}}{\nu}\log\left\{\nu^2 k(y_s - y)\right\}$$

$$A(y) = ky - \frac{1}{\nu^2}\log\left(1 - \frac{y}{y_s}\right)$$

  controlled by two parameters: ν and y_s
- The bulk solutions exhibit a (naked) singularity at $y = y_s > y_1$ (beyond y_1 but in its vicinity)

Singular background



Scalars in the bulk

- In this section we will consider a scalar field (typically the SM Higgs) propagating in the bulk of a given gravitational background $A(y)$
- In other words we will neglect the back-reaction of the scalar field on the metric
- We will consider the 5D action

$$S=\int d^5x\sqrt{g}\left\{\frac{1}{2}g_{MN}\partial^M\Phi\partial^N\Phi-V(\Phi)\right\}$$

- We will use the notation for the metric

$$g_{\mu\nu}=e^{-2A(y)}\eta_{\mu\nu},\quad g_{55}=-1$$

- and for the induced metric at the brane y_α

$$\bar{g}_{\mu\nu}=e^{-2A(y_\alpha)}\eta_{\mu\nu},\quad A(0)=0$$

EOM & BC

- Under a variation $\delta\Phi$ one finds: $\delta S = \delta S_M + \delta S_{\partial M}$
- where

$$\delta S_M = -\int d^5x\sqrt{g}\left\{\frac{1}{\sqrt{g}}\partial_M\left[\sqrt{g}\,g^{MN}\partial_N\Phi\right] + \frac{\partial V}{\partial\Phi}\right\}\delta\Phi$$

$$\delta S_{\partial M} = \int d^4x\sqrt{g}\,g^{5N}\partial_N\Phi\cdot\delta\Phi\Big|_{y=0}^{y=L}, \quad \text{from integration by parts}$$

- We have to impose $\delta S_M = \delta S_{\partial M} = 0$ for arbitrary $\delta\Phi$, i.e.

EOM

$$e^{2A}\partial_\mu\partial^\mu\Phi - e^{4A}\partial_y\left[e^{-4A}\partial_y\Phi\right] + V' = 0$$

BC's

$$-\partial_y\Phi\,\delta\Phi\Big|_{y=y_1} + \partial_y\Phi\,\delta\Phi\Big|_{y=0} = 0$$

- The boundary conditions can be satisfied in essentially two ways

$$\begin{aligned} \text{Neumann (N or +):} \qquad \partial_y \Phi\Big|_{y=y_\alpha} &= 0 \,, \\ \text{Dirichlet (D or -):} \qquad \Phi\Big|_{y=y_\alpha} &= 0 \,, \end{aligned}$$

- Various combinations are then: $(+,+),(+,-),(-,+),(--)$
- The most general BC's are (Robin BC's)

$$\partial_y \Phi\Big|_{y=y_\alpha} - m\Phi\Big|_{y=y_\alpha} = 0$$

- This indicates that there is a *source* on the boundary. For instance, adding a boundary term to the action

$$\Delta S_{\rm boundary} = -\int d^4x\, \sqrt{\bar{g}}\, \frac{1}{2}\, m\Phi^2\Big|_{y=y_\alpha}$$

KK DECOMPOSITION

- A central concept in extra-dimensional physics is that the theory of a bulk field propagating in a *compact* extra dimension can be *rewritten* as a 4D theory involving an infinite number of 4D fields.
- For a free scalar

$$S = \sum_n \int d^4x \, \frac{1}{2} \left\{ \partial_\mu \phi_n \partial^\mu \phi_n - m_n^2 \phi_n^2 \right\}$$

- The $\phi_n(x^\mu)$ are known as Kaluza-Klein (KK) modes.
- The information about the background is fully contained in the KK spectrum, m_n
- We write the Fourier-mode decomposition as

$$\Phi(x^\mu, y) = \frac{e^{A(y)}}{\sqrt{y_1}} \sum_n \phi_n(x^\mu) f_n(y)$$

- KK decomposition for *free action*, interactions treated perturbatively

- In the KK expansion we pulled out $e^{A(y)}$, which gives the f_n's a direct physical interpretation in terms of *localization properties of the KK-modes along the extra dimension ("physical wavefunctions")*.
- The interpretation of ϕ_n as a (free) 4D scalar field of mass m_n means that

$$\partial_\mu \partial^\mu \phi_n + m_n^2 \phi_n = 0$$

- Replacing the KK decomposition into the bulk EOM yields

$$f_n'' - 2A' f_n' + \left[A'' - 3(A')^2 - M^2 + e^{2A} m_n^2 \right] f_n = 0$$

while the functions f_n form a complete set of orthonormal functions satisfying the relation

$$\frac{1}{y_1} \int_0^{y_1} dy\, f_m f_n = \delta_{mn} \ ,$$

- The KK masses, m_n^2, are found by solving the EOM for the f_n and requiring the desired BC's

Gauge bosons in the bulk

- We will consider here gauge fields propagating in 5D
- The action

$$S_A = \int d^5x\sqrt{g}\left\{-\frac{1}{4}g^{MN}g^{KL}F_{MK}F_{NL}\right\} = \int d^5x \left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right.$$

$$\left. + \frac{1}{2}e^{-2A}\partial_\mu A_5\partial^\mu A_5 - \partial_5\left[e^{-2A}A_5\right]\partial_\mu A^\mu + \frac{1}{2}e^{-2A}\partial_5 A_\mu\partial_5 A^\mu\right\} + S_{\partial M}$$

$$S_{\partial M} = \int d^4x\; e^{-2A}A_5\partial_\mu A^\mu\Big|_{y=0}^{y=y_1}$$

- There is a bulk term mixing A_μ and A_5. We cancel it by the gauge fixing action

$$S_{\rm GF} = -\int d^5x\frac{1}{2\xi}\left\{\partial_\mu A^\mu - \xi\,\partial_5\left[e^{-2A}A_5\right]\right\}^2$$

- The total gauge action then becomes

$$S_A = \int d^5x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2 + \frac{1}{2}\, e^{-2A} \partial_5 A_\mu \partial_5 A^\mu \right.$$
$$\left. + \frac{1}{2}\, e^{-2A} \partial_\mu A_5 \partial^\mu A_5 - \frac{1}{2}\, \xi \left(\partial_5 \left[e^{-2A} A_5 \right] \right)^2 \right\} + S_{\partial M}$$

- A couple of comments
  - We will be interested in the quadratic part of the action so that there should be no distinction between abelian and non-abelian cases
  - The Faddeev-Popov procedure, when applied to our gauge-fixing action S_{GF} leads to ghost fields

EOM & BC

- The variational procedure leads

Under δA_μ

$$\delta S_A^M = \int d^5x\, \delta A_\mu \left\{ \left[\eta^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu \right] - \eta^{\mu\nu} \partial_5\, e^{-2A} \partial_5 \right\} A_\nu$$

$$\delta S_A^{\partial M} = \int d^4x\, e^{-2A} \delta A_\mu \left\{ \partial_5 A^\mu - \partial^\mu A_5 \right\} \Big|_{y=0}^{y=y_1}$$

Under δA_5

$$\delta S_A^M = \int d^5x\, e^{-2A} \delta A_5 \left\{ -\Box + \xi\, \partial_5^2 e^{-2A} \right\} A_5$$

$$\delta S_A^{\partial M} = -\int d^4x\, e^{-2A} \delta A_5 \left\{ \xi\, \partial_5 \left[e^{-2A} A_5 \right] + \partial_\mu A^\mu \right\} \Big|_{y=0}^{y=y_1}$$

- Therefore the EOM are

$$\left[\eta^{\mu\nu}\Box - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]A_\nu - \partial_5\left[e^{-2A}\partial_5 A^\mu\right] = 0$$

$$\Box A_5 - \xi\,\partial_5^2\left[e^{-2A}A_5\right] = 0$$

- And the BC's are

$$\partial_5 A^\mu - \partial^\mu A_5 \;\Big|_{y=0}^{y=y_1} = 0$$

$$\xi\,\partial_5\left[e^{-2A}A_5\right] + \partial_\mu A^\mu\;\Big|_{y=0}^{y=y_1} = 0$$

- Two interesting BC's are

$$(+) \;\equiv\; A_5\Big|_{y_\alpha} = 0 \;\Rightarrow\; \partial_5 A_\mu\Big|_{y_\alpha} \;=\; 0\,,$$

<u>or</u>

$$(-) \;\equiv\; A_\mu\Big|_{y_\alpha} = 0 \;\Rightarrow\; \partial_5\left[e^{-2A}A_5\right]\Big|_{y_\alpha} \;=\; 0\,.$$

KK decomposition

- We write the KK decomposition

$$A_{\mu,5}(x^\mu, y) = \frac{1}{\sqrt{y_1}} \sum_n A^n_{\mu,5}(x^\mu) f^n_{A,5}(y)$$

- Replacing the KK decomposition into the bulk EOM's yields to

$$\partial_y \left[e^{-2A} \partial_y f^n_A \right] + m_n^2 f^n_A = 0$$

$$\partial_y^2 \left(e^{-2A} f^n_5 \right) + m_n^2 f^n_5 = 0$$

- The KK wavefunctions are normalized as expected

$$\frac{1}{y_1} \int_0^{y_1} dy \, f^m_{A,5} f^n_{A,5} = \delta_{mn}$$

- A solution f_A^n with $m_n^2 \neq 0$ gives a solution for $f_5^n = \partial_5 f_A^n / m_n$
- The reason is that A_5^n with such f_5^n provides the longitudinal polarization of the massive A_μ^n: compactification breaks the gauge invariance spontaneously and there is a Higgs mechanism at work in the unitary gauge at each KK level
- The BC's are important to identify physical theories

NN

For $(+,+)$ BC's, i.e. $\partial_y f_A^n| = f_5^n| = 0$ on both branes, one finds a spin-1 zero-mode with $f_A^0(y) = 1$ and $f_5^0(y) = 0$. Presumably, such a state could be identified with one of the observed SM gauge bosons

DD

For $(-,-)$ BC's, i.e. $\partial_y (e^{-2A} f_5^n)\big| = f_A^n| = 0$ on both branes, there is no spin-1 zero-mode, but there is a spin-0 zero-mode with $f_5^0(y) \propto e^{2A(y)}$. In certain constructions, such scalars can be identified with the SM Higgs field

Fermions in the bulk

- First we need *five* anticommuting Dirac Γ-matrices, for which we can take

$$\Gamma^A = (\gamma^\alpha, -i\gamma_5)$$

- In the Weyl representation,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \quad \begin{array}{rcl} \sigma^\mu & = & (1, \vec\sigma) \\ \bar\sigma^\mu & = & (1, -\vec\sigma) \end{array}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

which obey $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$

- We also need the fünfbein, ${e_M}^A$ defined by

$$g_{MN} = {e_M}^A {e_N}^B \eta_{AB}, \quad {e^M}_A {e_M}^B = \delta_A^B$$
$${e^\mu}_\alpha = e^{+A(y)} \delta^\mu_\alpha, \quad {e^y}_5 = 1$$

with all other components vanishing

- We also need a spin connection ω_{MAB} to define the covariant derivative with respect to general coordinate and local Lorentz transformations in 5D

$$\omega_\mu{}^{a5} = -\omega_\mu{}^{5a} = \partial_5(e^{-A})\delta_\mu^a$$

- The covariant derivative is

$$D_M = \partial_M + \frac{1}{8}\omega_{MAB}[\Gamma^A, \Gamma^B] \quad \left\{ \begin{array}{l} D_\mu = \partial_\mu - \frac{i}{2}\, e^{-A} A' \gamma_\mu \gamma_5 \\ D_5 \; = \; \partial_5 \end{array} \right.$$

- The fermion action is

$$S_\Psi = \int d^5x \sqrt{g} \left\{ \frac{i}{2} \overline{\Psi} e_A^M \Gamma^A D_M \Psi - \frac{i}{2} (D_M \Psi)^\dagger \Gamma^0 e_A^M \Gamma^A \Psi - M \overline{\Psi}\Psi \right\} =$$

$$\int d^5x\, e^{-3A} \left\{ i\, \overline{\Psi} \gamma^\mu \partial_\mu \Psi + \frac{1}{2} e^{-A} \left[\overline{\Psi} \gamma_5 \partial_5 \Psi - (\partial_5 \overline{\Psi}) \gamma_5 \Psi \right] - e^{-A} M \overline{\Psi}\Psi \right\}$$

EOM & BC

- Taking the variation with respect to $\delta\bar{\Psi}$ we obtain the EOM and BC's

$$\delta\bar{\Psi}\left\{i\, e^{A}\gamma^{\mu}\partial_{\mu}+\left(\partial_5-2A'\right)\gamma_5-M\right\}\Psi=0$$

$$-\,\delta\overline{\Psi}\gamma_5\Psi\Big|_{y=y_1}+\delta\overline{\Psi}\gamma_5\Psi\Big|_{y=0}=0$$

- It is useful to express them in terms of chiral components

$$\Psi_{L,R}=P_{L,R}\Psi\equiv\frac{1}{2}(1\mp\gamma_5)\Psi$$

$$i\, e^{A}\gamma^{\mu}\partial_{\mu}\Psi_L+\left[\left(\partial_5-2A'\right)-M\right]\Psi_R=0$$

$$i\, e^{A}\gamma^{\mu}\partial_{\mu}\Psi_R+\left[-\left(\partial_5-2A'\right)-M\right]\Psi_L=0$$

$$\delta\overline{\Psi}_L\Psi_R-\delta\overline{\Psi}_R\Psi_L\Big|_{y=0}^{y=y_1}=0$$

We can use as BC

$$\Psi_L\Big| = 0 \qquad \underline{\text{or}} \qquad \Psi_R\Big| \; = \; 0$$

- We do not have the right to impose Dirichlet b.c.'s conditions on *both* chiralities at a given boundary!
- For instance if $\Psi_L\big|_{y_\alpha} = 0$, then the first equation implies

$$\partial_5 \Psi_R\Big|_{y_\alpha} = \left(2A' + M\right) \Psi_R\Big|_{y_\alpha}$$

- If we were to also impose $\Psi_R\big|_{y_\alpha} = 0$, we would automatically have $\partial_5 \Psi_R\big|_{y_\alpha} = 0$, and then –from the second equation– we would find $\partial_5 \Psi_L\big|_{y_\alpha} = 0$
- But then the only allowed solution of the system of two first-order differential equations (in y) is $\Psi \equiv 0$

- Only two possibilities then

$$
(-) \equiv \Psi_L\Big|_{y_\alpha} = 0\ , \quad \text{hence} \quad \partial_5 \Psi_R\Big|_{y_\alpha} = \left(\tfrac{1}{2}A' + M\right)\Psi_R\Big|_{y_\alpha}\ ,
$$

<u>or</u>

$$
(+) \equiv \Psi_R\Big|_{y_\alpha} = 0\ , \quad \text{hence} \quad \partial_5 \Psi_L\Big|_{y_\alpha} = \left(\tfrac{1}{2}A' - M\right)\Psi_L\Big|_{y_\alpha}\ .
$$

- *Compactification on an interval necessarily leads to boundary conditions that distinguish L from R*, which will allow us to easily embed the SM structure
- Similar to the scalar case, specifying the boundary conditions on *both* boundaries leads to four possibilities that we label as

$$
(+,+) \qquad (+,-) \qquad (-,+) \qquad (-,-)
$$

- Also, as in the scalar case, one can generalize these boundary conditions by including localized terms in the action which contribute directly to $\delta S_\Psi^{\partial M}$.

KK DECOMPOSITION

- We write the KK decomposition as

$$\Psi_{L,R}(x^\mu, y) = \frac{e^{\frac{3}{2}A(y)}}{\sqrt{y_1}} \sum_n \psi^n_{L,R}(x^\mu) f^n_{L,R}(y)$$

- The wave-functions f^n satisfy the EOM and orthogonality relations

$$(i\gamma^\mu \partial_\mu - m_n)\,\psi^n = 0$$

$$\left(\partial_y + M - \frac{1}{2}A'\right) f^n_L = m_n e^A f^n_R$$

$$\left(\partial_y - M - \frac{1}{2}A'\right) f^n_R = -m_n e^A f^n_L$$

$$\frac{1}{y_1}\int_0^{y_1} dy\, f^m_{L,R} f^n_{L,R} = \delta_{mn}$$

- And the two types of BC's

$$(-) \quad : \quad \partial_y f_R^n - \left(M + \frac{1}{2} A' \right) f_R^n \Big|_{y_\alpha} \; = \; 0 \quad \& \quad f_L^n \big|_{y_\alpha} = 0 \, ,$$

or

$$(+) \quad : \quad \partial_y f_L^n + \left(M - \frac{1}{2} A' \right) f_L^n \Big|_{y_\alpha} \; = \; 0 \quad \& \quad f_R^n \big|_{y_\alpha} = 0 \, ,$$

- A special case for the zero-mode $m_0 = 0$ happens when $M = \pm c\, A'(y)$ for [(+,+),(-,-)] BC's

$$f_{L,R}^0(y) \equiv \; N_0 \, e^{-(c-\frac{1}{2})A(y)}, \quad f_{R,L}^0(y) \equiv 0$$

- The localization properties can then be conveniently described by c
- For instance, in the case of AdS$_5$, with $A' = k$, c is just the Dirac mass M in units of the curvature scale k.

Singular background