The Anatomy of the Pion Loop Hadronic Light by Light Scattering Contribution to the Muon Magnetic Anomaly

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Magnetic Anomaly

$$\boldsymbol{\mu} = \frac{e}{2m}\mathbf{L}$$

$$\boldsymbol{\mu} = g_s \frac{q}{2m}\mathbf{S}$$

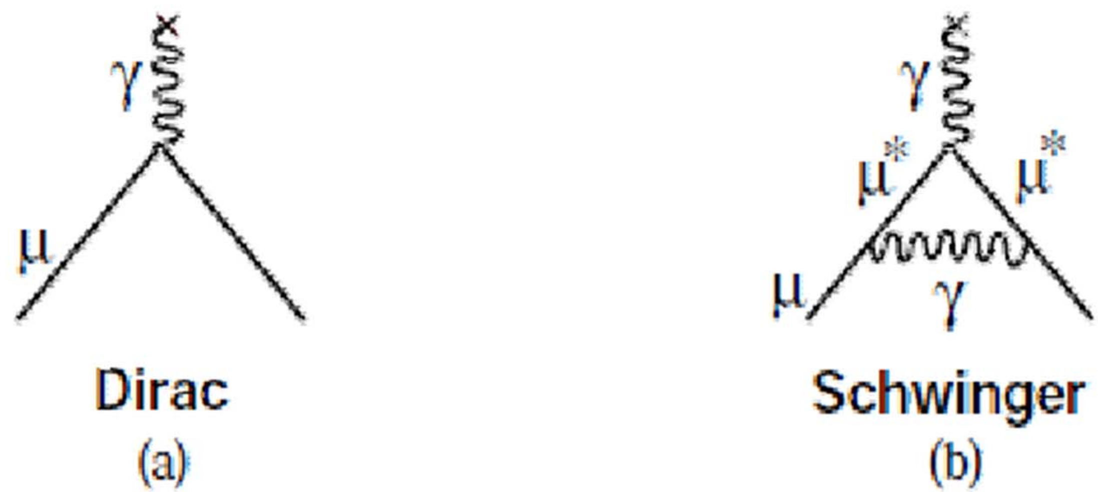
$$\mu = (1 + a)\frac{q\hbar}{2m}$$

The first piece, called the Dirac moment

The second piece is called the anomalous (Pauli) moment

In 1947, Schwinger,

the deviation of g_s from 2 can be ascribed to radiative corrections



More generally

Standard-Model corrections $a(SM)$

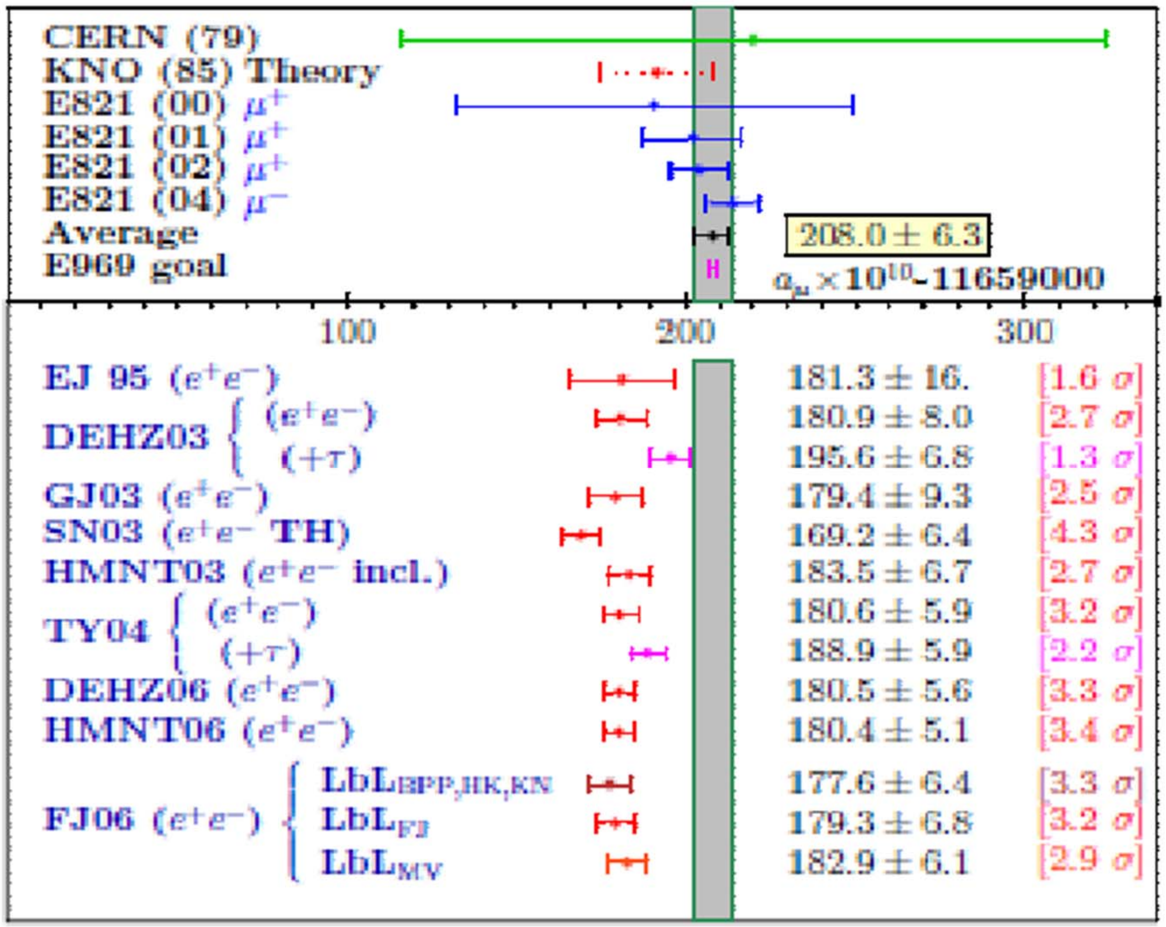
virtual leptons, hadrons and gauge bosons

$$a_l = a_l^{QED} + a_l^{Had} + a_l^{Weak}$$

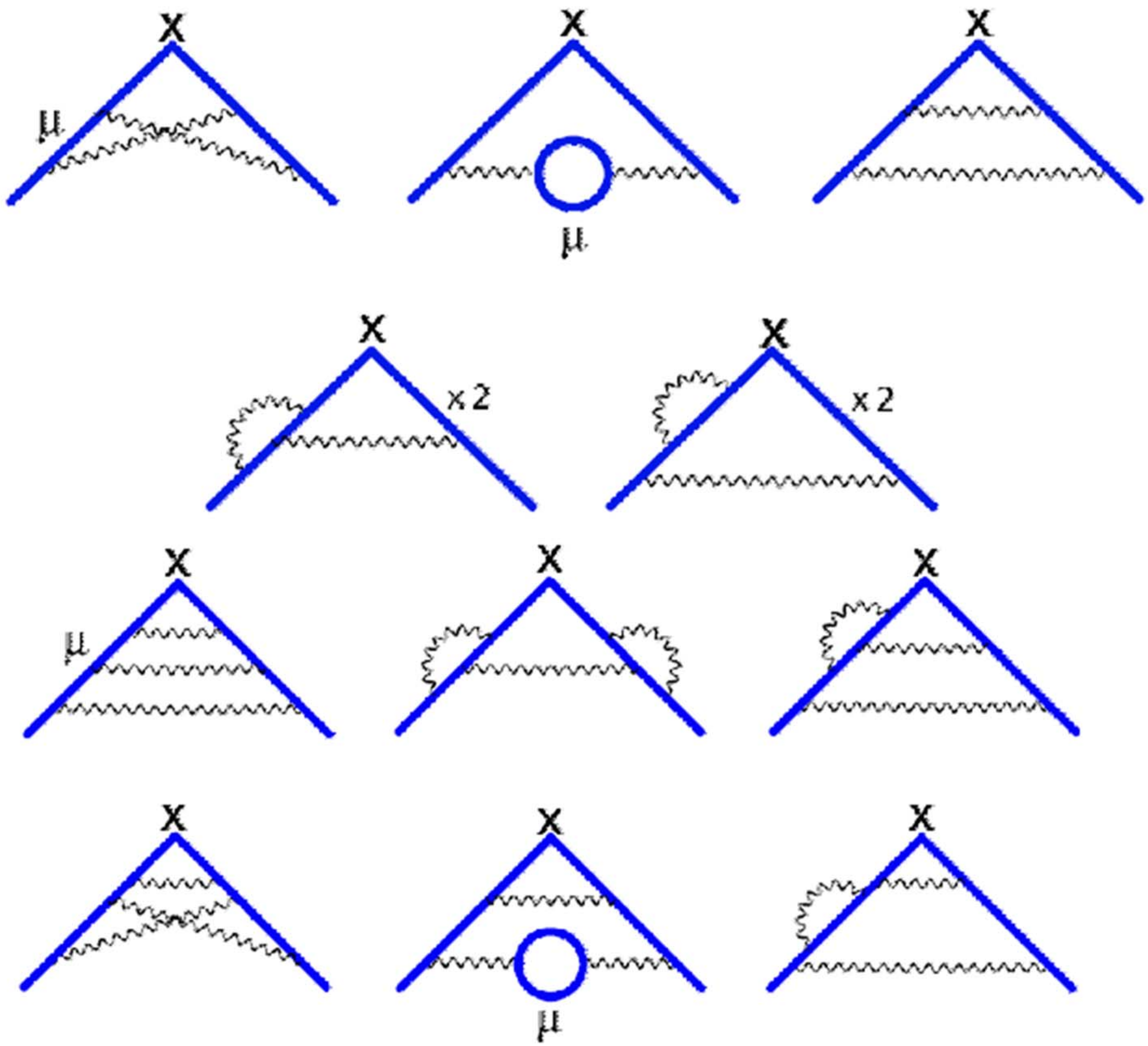
Before the E821 experiment at Brookhaven national laboratory

$$a_\mu^{exp} = 1165924.0(8.5) \times 10^{-9} \qquad a_\mu^{th} = 1165921(8.3) \times 10^{-9}$$

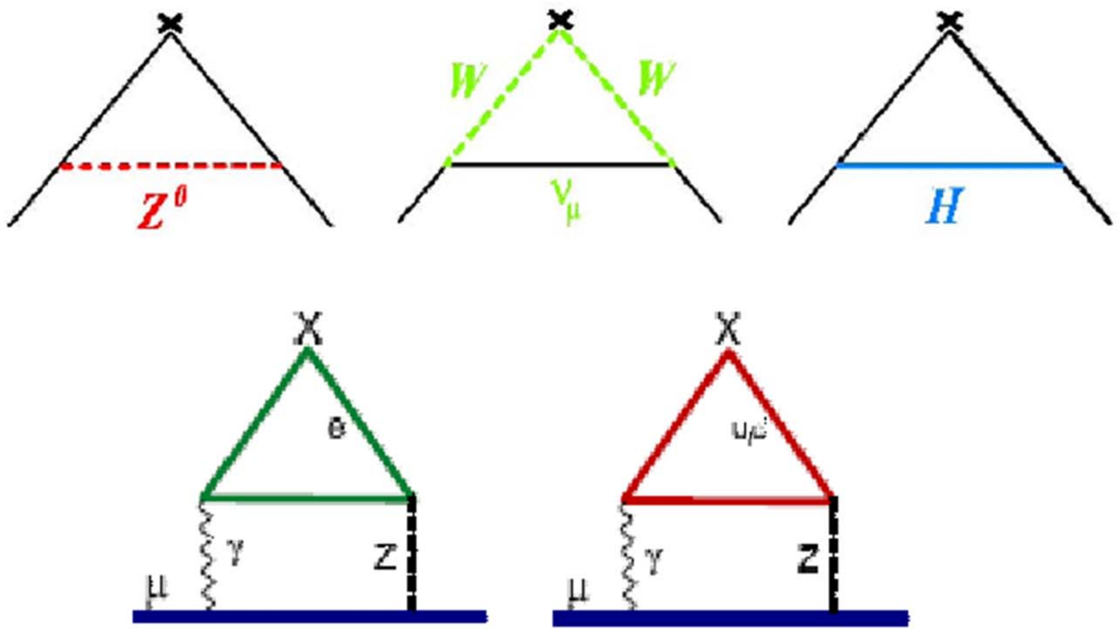
$$a_\mu = 11659208.0(3.3)[6.3] \times 10^{-10}$$



the dominant QED terms, which contain only leptons and photons

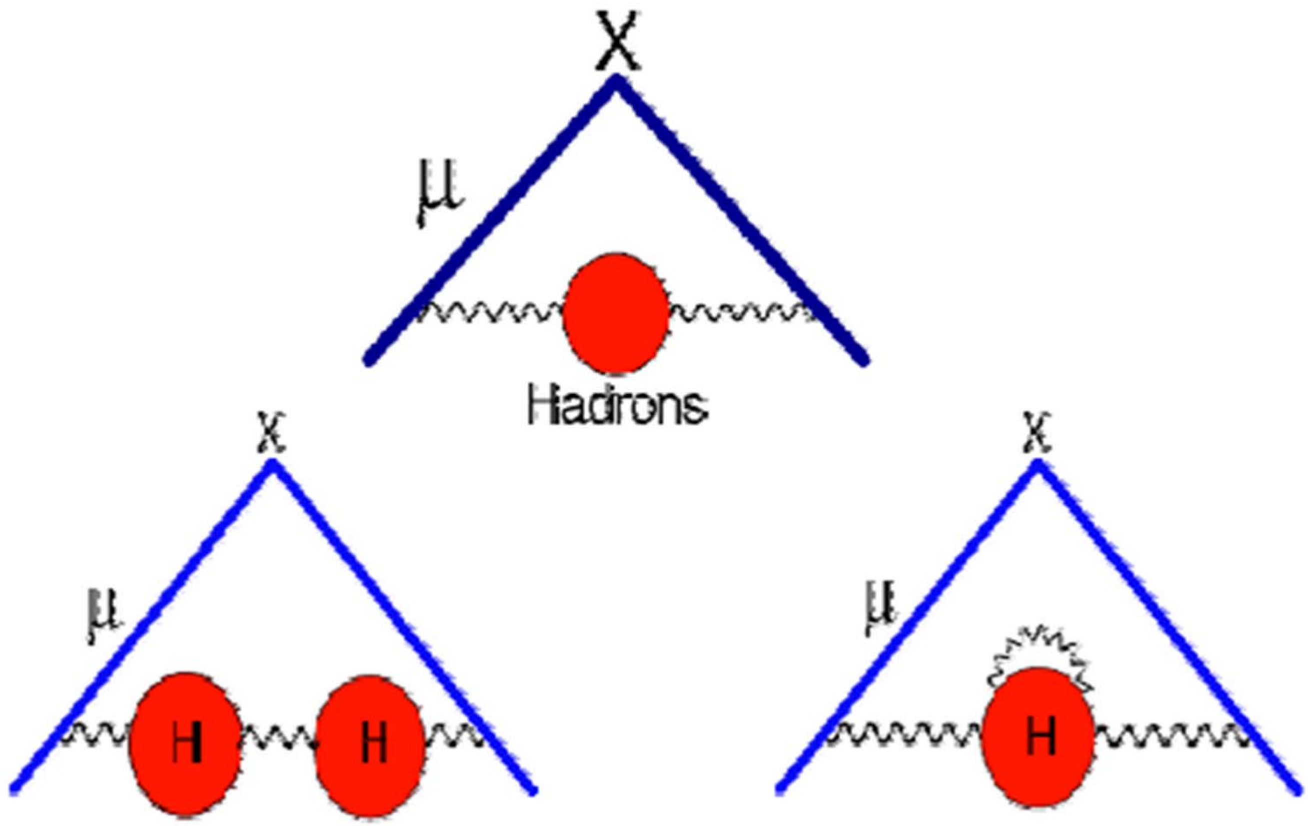


Electroweak one loop and two loop contributions to a_μ

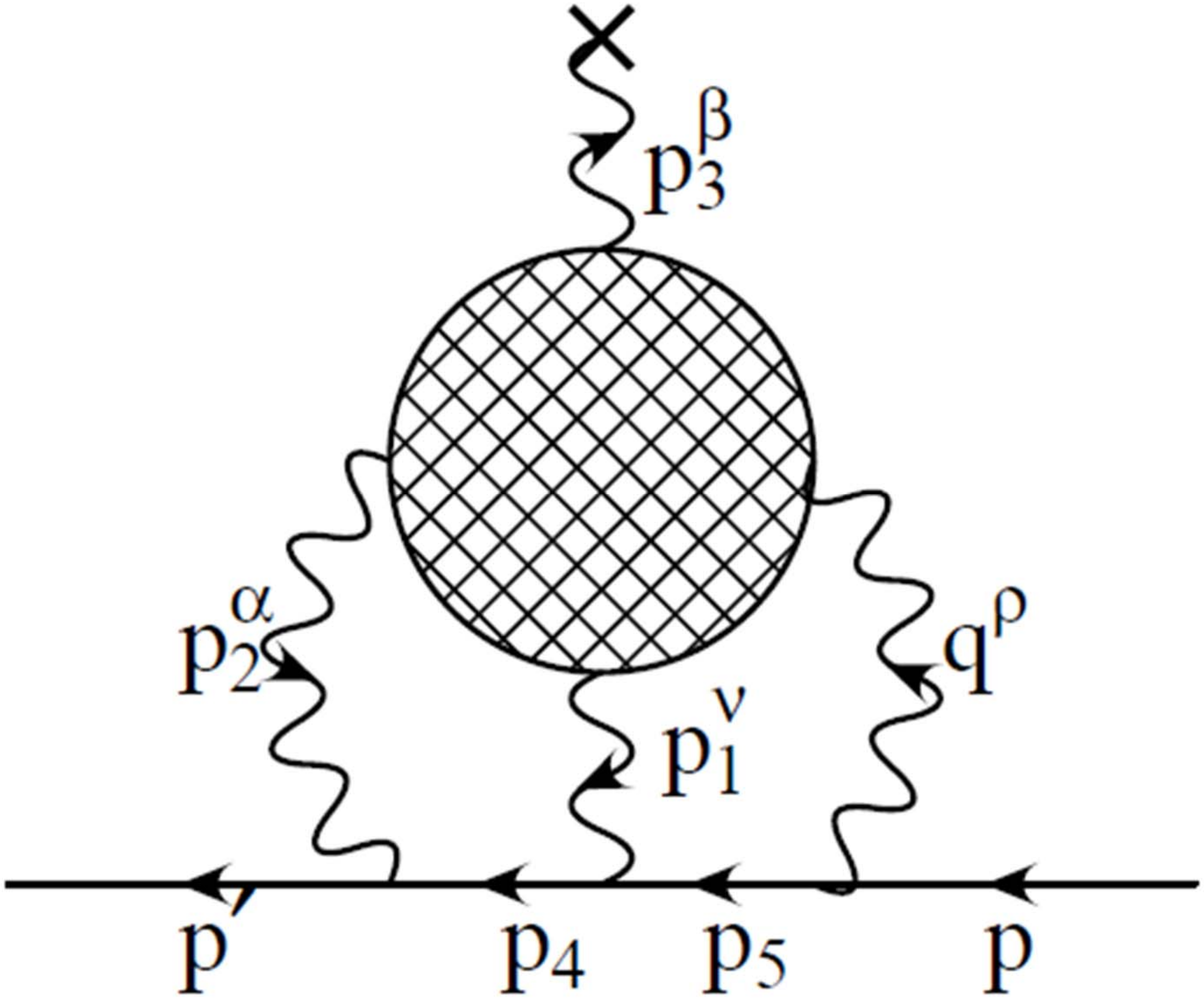


$$a_\mu^{had} = a_\mu^{(hvp)} + a_\mu^{(HLL)}$$

The hadronic vacuum polarization contribution



Hadronic Light by Light (HLL)



Both the QED and electroweak contributions can be calculated to high precision

$$a_\mu^{Had,LO} = 690.9(4.4) \times 10^{-10}$$

$$a_\mu^{Had,HO} = -9.8(0.1) \times 10^{-10}$$

$$a_\mu^{(h.L\times L)} = (10.5 \pm 2.6) \times 10^{-10}$$

which is suffering from a large error

Calculating the HLL part is the trickiest the hadronic contribution to a_μ cannot be accurately evaluated

the relevant QCD contributions to a_μ

non perturbative regime.

dominant theoretical uncertainty

ChPT

HLL contribution consists of three parts

quark loop, the pion exchange and the charged pion loop

charged pion loop correction in this work

Charged pion and Kaon Loop Contributions	$a_\mu \times 10^{10}$
Bijnens, Pallante and Prades(Full VMD)	-1.9 ± 0.5
Hayakawa and Kinoshita (HGS)	-0.45 ± 0.85
Kinoshita, Nizic and Okamoto(Naive VMD)	-1.56 ± 0.23
Kinoshita, Nizic and Okamoto(Scalar QED)	-5.47 ± 4.6

full VMD result three times larger than the one from the HLS

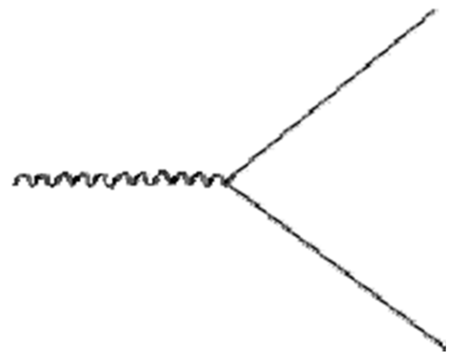
Why?

full VMD

HLS

$$\rho_\mu = \frac{v_\mu^a}{g} T_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\rho_\mu^0 + \omega_\mu\right) & \rho_\mu^+ & K_\mu^{\star,+} \\ \rho_\mu^- & -\frac{1}{\sqrt{2}}\left(\rho_\mu^0 + \omega_\mu\right) & K_\mu^{\star,0} \\ K_\mu^{\star,-} & \bar{K}_\mu^{\star,0} & \phi_\mu \end{pmatrix}$$

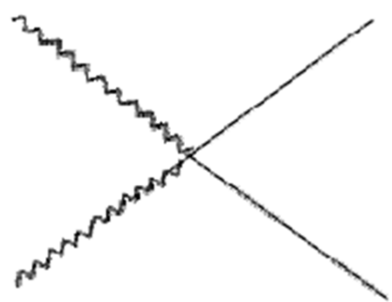
$$\gamma(q,\varepsilon) \rightarrow \pi^+(p) + \pi^-(p')$$



$\mathcal{M} = ie\varepsilon \cdot (p + p')$

$ie(p_\mu + p'_\mu)$

$$\gamma(q,\varepsilon) + \gamma(q',\varepsilon') \rightarrow \pi^+(p) + \pi^-(p'),$$

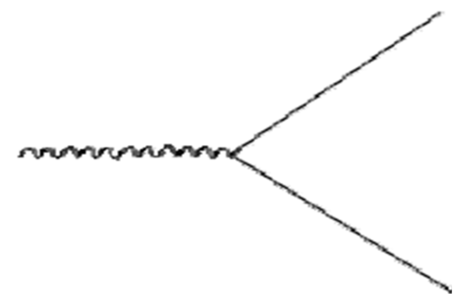


$\mathcal{M} = 2ie^2 \varepsilon'^{\star} \cdot \varepsilon$

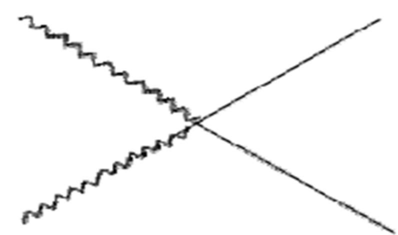
$2ie^2 g_{\mu\nu}$

full VMD

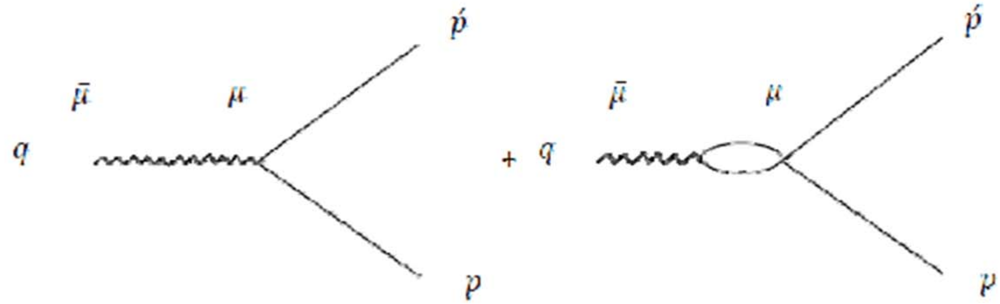
$$\frac{g_{\mu\bar{\mu}} m_\rho^2 - q_\mu q_{\bar{\mu}}}{q^2 - m_\rho^2}$$



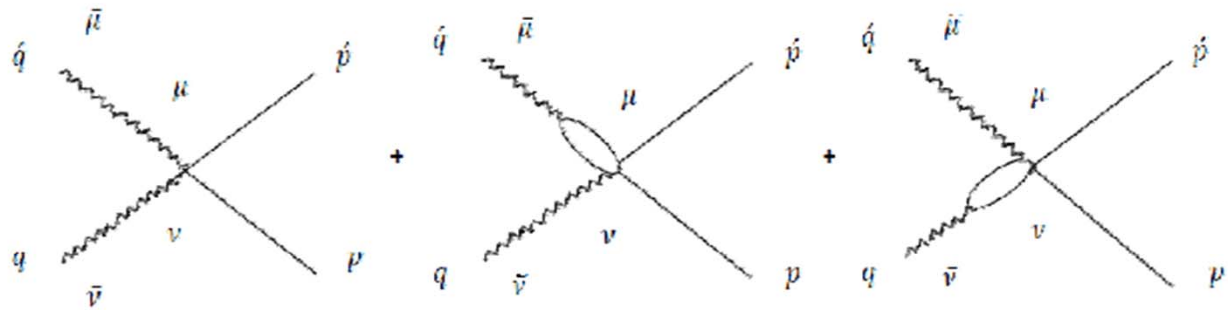
$$\frac{m_\rho^2 g_{\nu\bar{\nu}} - p_{\bar{\nu}} p_\nu}{m_\rho^2 - p^2} \frac{m_\rho^2 g_{\mu\bar{\mu}} - q_{\bar{\mu}} q_\mu}{m_\rho^2 - q^2}$$



$\gamma\pi^+\pi^-$ and $\gamma\gamma\pi^+\pi^-$ vertices HLS



$$(1-\frac{a}{2})g_{\mu\bar{\mu}} + \frac{a}{2}\frac{m_\rho^2 g_{\mu\bar{\mu}} - q_\mu q_{\bar{\mu}}}{q^2 - m_\rho^2} = g_{\mu\bar{\mu}} + \frac{q^2 g_{\mu\bar{\mu}} - q_\mu q_{\bar{\mu}}}{q^2 - m_\rho^2}$$



$$2(1-a)g_{\mu\bar{\mu}}g_{\nu\bar{\nu}} + a g_{\nu\bar{\nu}}\Big(\frac{m_\rho^2 g_{\mu\bar{\mu}} - q_\mu q_{\bar{\mu}}}{q^2 - m_\rho^2}\Big) + a g_{\mu\bar{\mu}}\Big(\frac{m_\rho^2 g_{\nu\bar{\nu}} - q_\nu q_{\bar{\nu}}}{q^2 - m_\rho^2}\Big)$$

$$= 2\Big[g_{\mu\bar{\mu}}g_{\nu\bar{\nu}} + g_{\mu\bar{\mu}}\frac{a}{2}\frac{p^2 g_{\nu\bar{\nu}} - p_{\bar{\nu}}p_\nu}{m_\rho^2 - p^2} + g_{\nu\bar{\nu}}\frac{a}{2}\frac{q^2 g_{\mu\bar{\mu}} - q_{\bar{\mu}}q_\mu}{m_\rho^2 - q^2}\Big]$$

Muon magnetic anomaly from light by light amplitude

$$\mathcal{M} \equiv - \mid e \mid A_\rho \bar{u}(p')\Gamma^\rho(\acute{p},p)u(p)$$

$$\Gamma^\rho(\acute{p},p) = F_1(p_3^2)\gamma^\rho + \frac{i}{2m_l}F_2(p_3^2)\sigma^{\rho\nu}p_{3\nu} - F_3(p_3^2)\gamma_5\sigma^{\rho\nu}p_{3\nu} + F_4(p_3^2)[p_3^2\gamma^\rho - 2m_l p_3^\rho]\gamma_5$$

$$a \equiv (g-2)/2 = F_2(0)$$

$$\Gamma^{\lambda\beta}(p_3) = |e|^6 \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m^2)\,(p_5^2 - m^2)}$$
$$\times \left[\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right] \gamma_\alpha (\not{p}_4 + m)\gamma_\nu (\not{p}_5 + m)\gamma_\rho \ .$$

$$a_\mu^{\mathrm{light-by-light}} = \frac{1}{48m} \mathrm{tr}[(\not{p} + m)\Gamma^{\lambda\beta}(0)\,(\not{p} + m)[\gamma_\lambda, \gamma_\beta]]$$

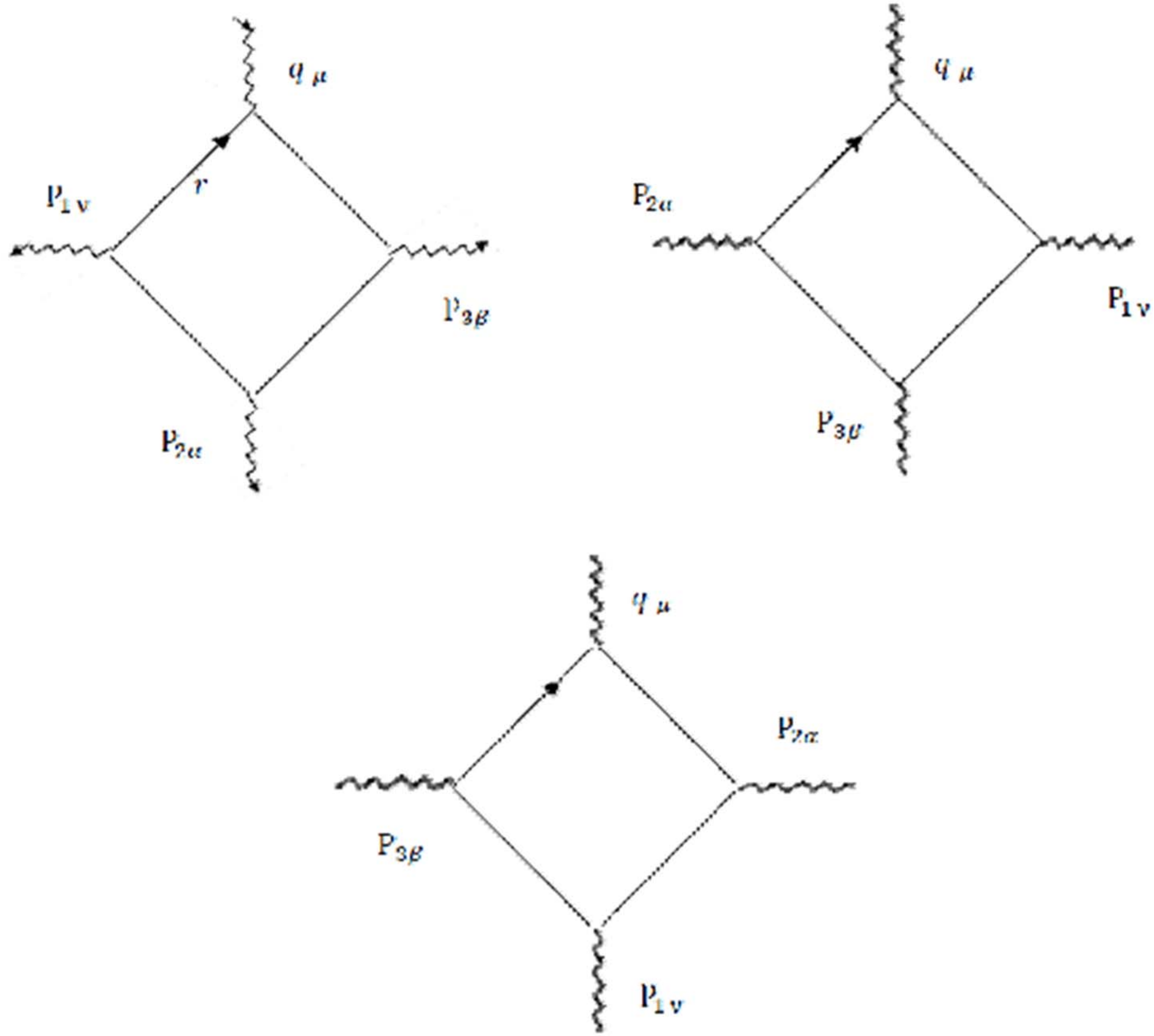
$$\Pi^{\rho\nu\alpha\lambda}(p_1, p_2, p_3) = i^3 \int d^4x_1 \int d^4x_2 \int d^4x_3 \exp i\,(p_1 \cdot x_1 + p_2 \cdot x_2 + p_3 \cdot x_3)$$
$$\times \langle 0 \mid T j_\rho(0) j_\nu(x_1) j_\alpha(x_2) j_\lambda(x_3) \mid 0 \rangle$$

find all functions, add them up, derive them with respect to p_3, then set $p_3 = 0$

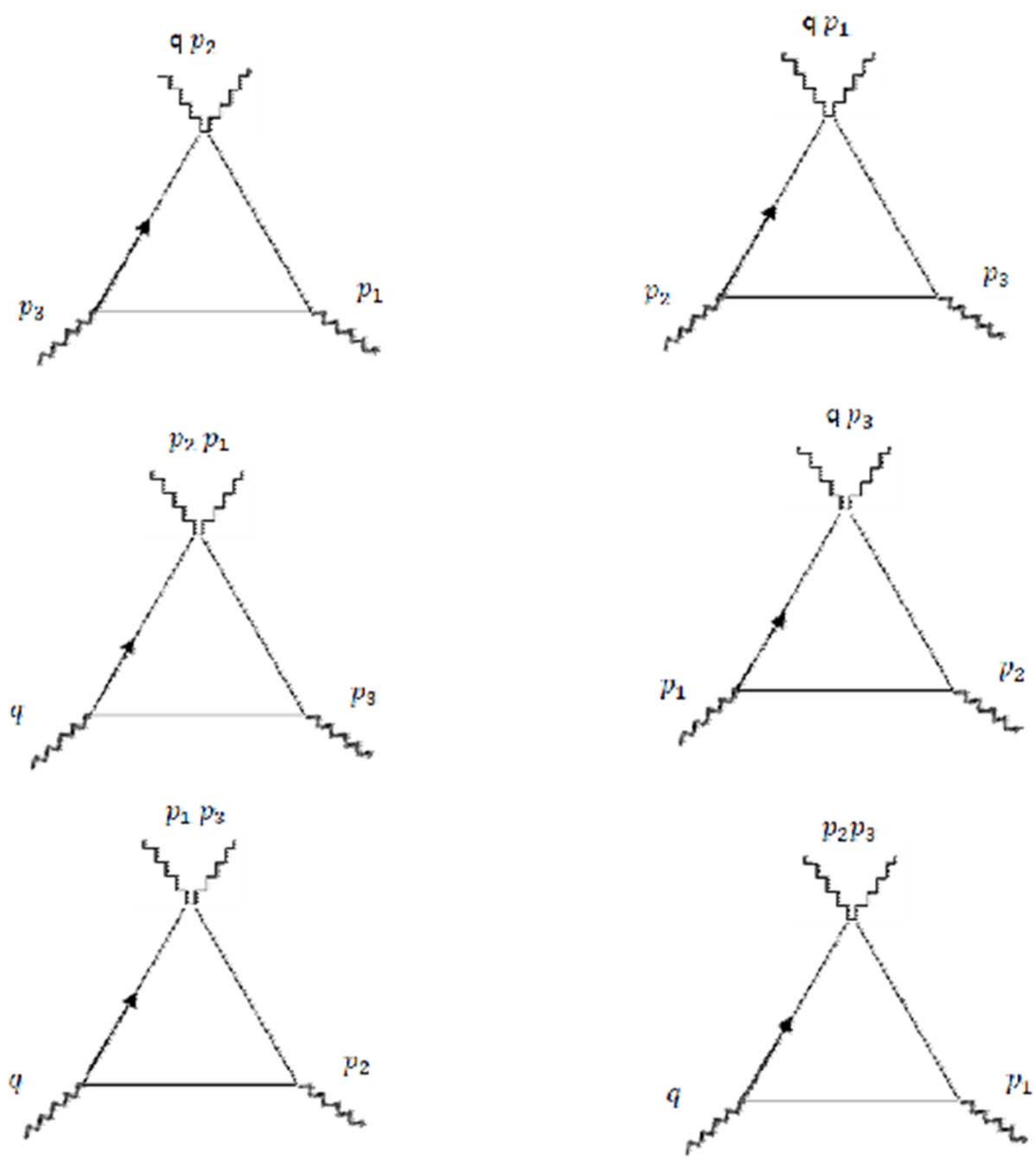
into the integral

The finale integral

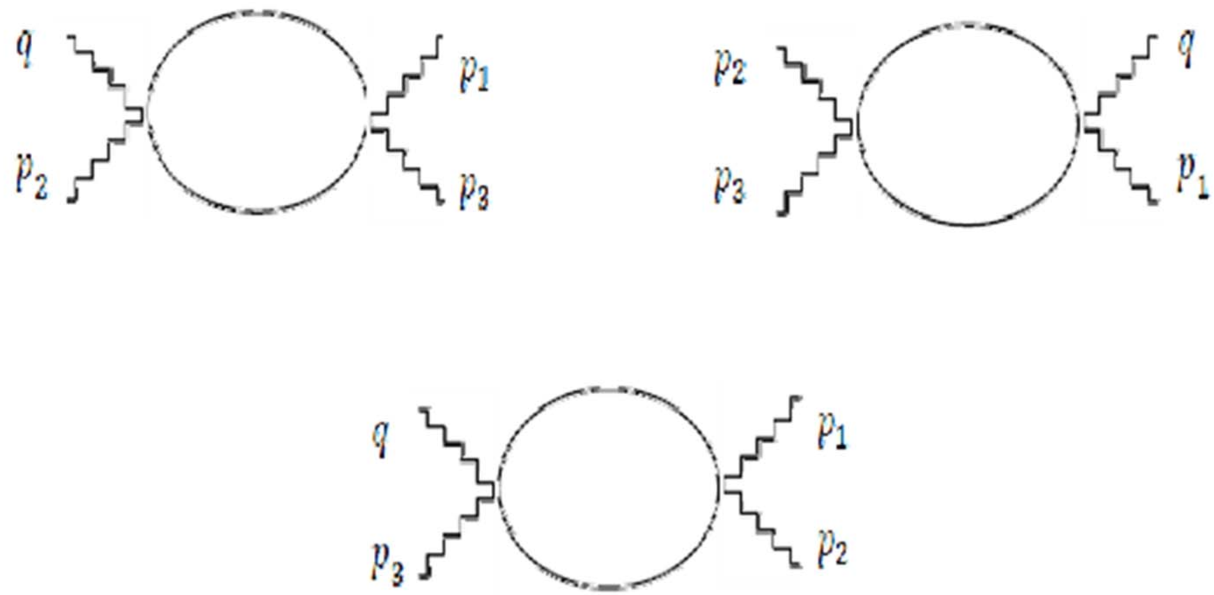
to be done is a five or four dimensional integral, which we have dealt with using the Monte Carlo routine VEGAS.



q μ
P1 ν
r
P3β
P2α
q μ
P2α
P1 ν
P3β
q μ
P2α
P3β
P1 ν



q p_2
p_3
p_1
q p_1
p_2
p_3
p_2 p_1
q
p_3
q p_3
p_1
p_2
p_1 p_3
q
p_2
$p_2 p_3$
q
p_1


q
p_1
p_2
p_3
p_2
q
p_3
p_1
q
p_1
p_3
p_2

HLS

$$
\begin{aligned}
&\Pi_{\mu\nu\alpha\beta} = \\
&\tfrac{1}{i}\int \frac{d^4r}{(2\pi)^4}\frac{i^4\times i^4}{(r^2-m^2)((r+p_1)^2-m^2)((r+p_1+p_2)^2-m^2)((r+p_1+p_2+p_3)^2-m^2)} \\
\times&(2r+p_1+p_2+p_3)_\mu\left(g_{\mu\bar{\mu}}+\frac{q^2g_{\mu\bar{\mu}}-q_\mu q_{\bar{\mu}}}{q^2-m_\rho^2}\right) \\
&(2r+p_1)_\nu\left(g_{\nu\bar{\nu}}+\frac{p_1^2g_{\nu\bar{\nu}}-p_{1\nu}p_{1\bar{\nu}}}{p_1^2-m_\rho^2}\right) \\
&(2r+2p_1+p_2)_\alpha\left(g_{\alpha\bar{\alpha}}+\frac{p_2^2g_{\alpha\bar{\alpha}}-p_{2\alpha}p_{2\bar{\alpha}}}{p_2^2-m_\rho^2}\right) \\
&(2r+2p_1+2p_2+p_3)_\beta\left(g_{\beta\bar{\beta}}+\frac{p_3^2g_{\beta\bar{\beta}}-p_{3\beta}p_{3\bar{\beta}}}{p_3^2-m_\rho^2}\right) \quad .
\end{aligned}
$$

Full VMD

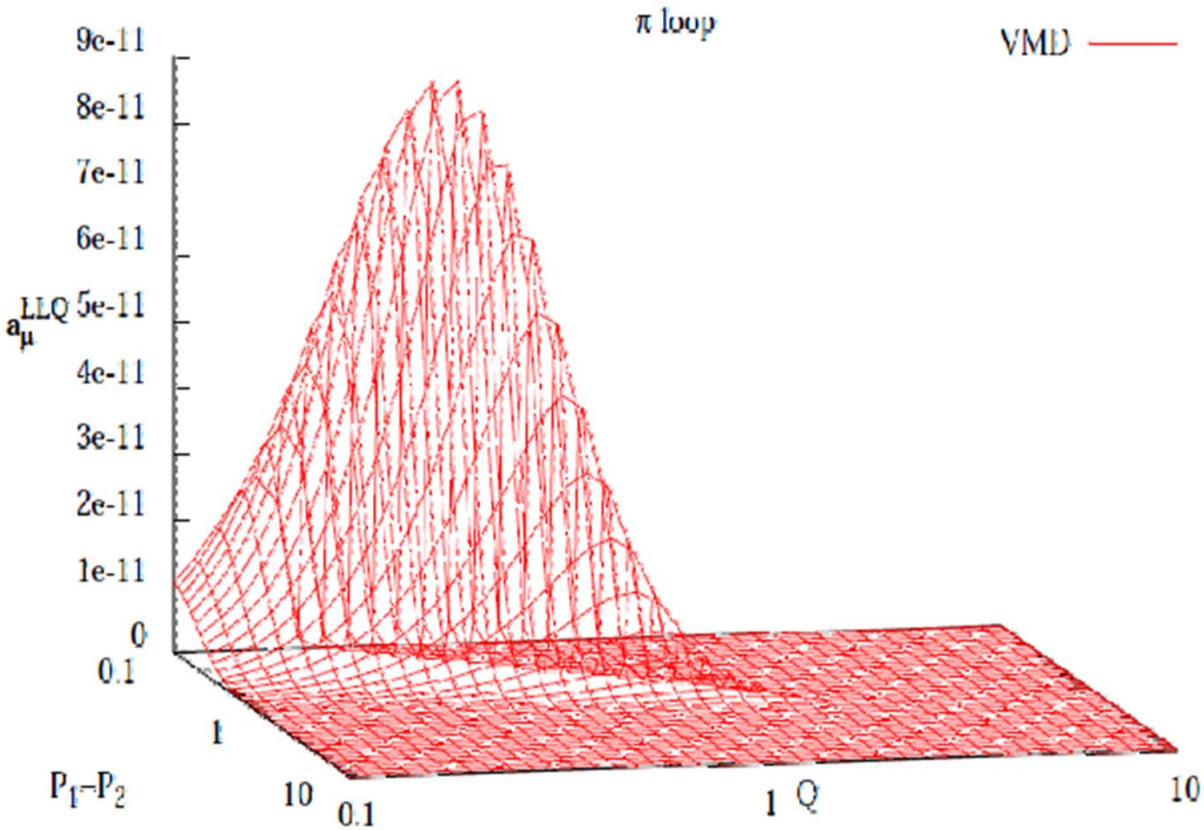
$$
\begin{aligned}
&\Pi_{\mu\nu\alpha\beta} = \\
&\frac{1}{i}\int \frac{d^4r}{(2\pi)^4}\frac{i^4 \times i^4}{(r^2-m^2)((r+p_1)^2-m^2)((r+p_1+p_2)^2-m^2)((r+p_1+p_2+p_3)^2-m^2)} \\
&\times(2r+p_1+p_2+p_3)_\mu\left(\frac{g_{\mu\bar{\mu}}m_\rho^2 - q_\mu q_{\bar{\mu}}}{q^2-m_\rho^2}\right) \\
&(2r+p_1)_\nu\left(\frac{g_{\bar{\nu}\nu}m_\rho^2 - p_{1\nu}p_{1\bar{\nu}}}{p_1^2-m_\rho^2}\right) \\
&(2r+2p_1+p_2)_\alpha\left(\frac{g_{\alpha\bar{\alpha}}m_\rho^2 - P_{2\alpha}p_{2\bar{\alpha}}}{p_2^2-m_\rho^2}\right) \\
&(2r+2p_1+2p_2+p_3)_\beta\left(\frac{g_{\beta\bar{\beta}}m_\rho^2 - p_{3\beta}p_{3\bar{\beta}}}{p_3^2-m_\rho^2}\right) \quad .
\end{aligned}
$$

Cut-off	$10^{10} a_\mu$		
GeV	bare	VMD	HLS
0.5	$-1.71(7)$	$-1.16(3)$	$-1.05(0.01)$
0.6	$-2.03(8)$	$-1.41(4)$	$-1.15(0.01)$
0.7	$-2.41(9)$	$-1.46(4)$	$-1.17(0.01)$
0.8	$-2.64(9)$	$-1.57(6)$	$-1.16(0.01)$
1.0	$-2.97(12)$	$-1.59(15)$	$-1.07(0.01)$
2.0	$-3.82(18)$	$-1.70(7)$	$-0.68(0.01)$
4.0	$-4.12(18)$	$-1.66(6)$	$-0.50(0.01)$

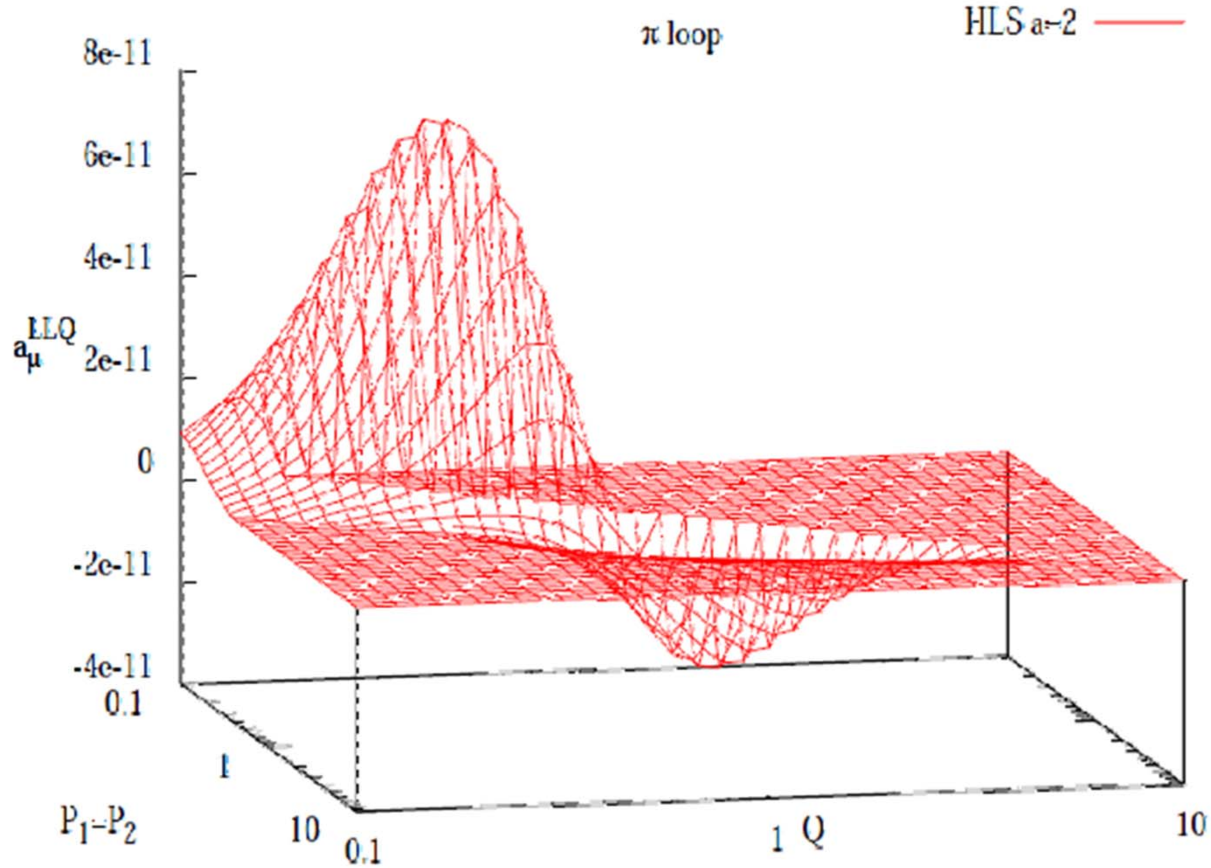
Relevant Momentum Regions for the pion Loop Contribution.

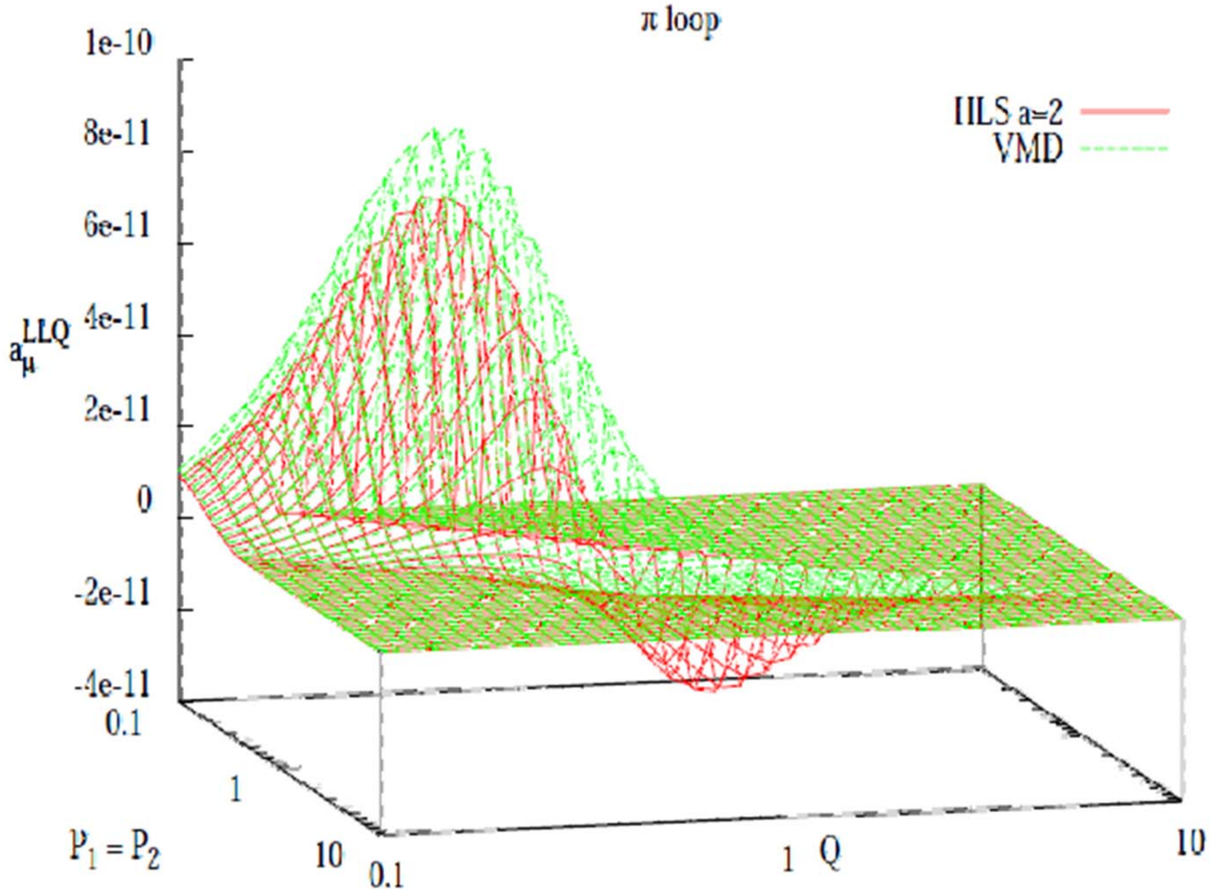
$$\begin{aligned} a_\mu &= \int dl_1 dl_2 a_\mu^{LL}(l_1, l_2) \\ &= \int dl_1 dl_2 dl_q a_\mu^{LLQ}(l_1, l_2, l_q) \end{aligned}$$

$l_1 = log(P_1/GeV)$, $l_2 = log(P_2/GeV)$ and $l_q = log(Q/GeV)$



π loop
VMD
9e-11
8e-11
7e-11
6e-11
5e-11
4e-11
3e-11
2e-11
1e-11
0
a_μ^{LLQ}
0.1
1
10
P_1–P_2
0.1
1
Q
10


π loop
HLS a=2
8e-11
6e-11
4e-11
a_μ^{LLQ}
2e-11
0
-2e-11
-4e-11
0.1
1
$P_1=P_2$
10
0.1
1 Q
10



π loop
HLS a=2
VMD
1e-10
8e-11
6e-11
4e-11
2e-11
0
-2e-11
-4e-11
a_μ^{LLQ}
0.1
1
10
$P_1 = P_2$
0.1
1
Q
10

Thank you!