Taller de Altas Energías, Benasque
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Supersymmetry: a very basic, biased and completely incomplete introduction

Michael Krämer (RWTH Aachen University)

Outline

- The supersymmetric harmonic oscillator
- Motivation for SUSY: Symmetry & the hierarchy problem
- The MSSM
- SUSY searches

References

- *Supersymmetry and the MSSM: An elementary introduction*
  I.J.R. Aitchison, hep-ph/0505105
- *A supersymmetry primer*
  S. Martin, hep-ph/9709356
- *Theory and phenomenology of sparticles*
  M. Drees, R. Godbole, P. Roy, World Scientific
- *An introduction to supersymmetry*
  M. Drees, hep-ph/9611409
- *Hide and seek with supersymmetry*
  H. Dreiner, hep-ph/9902347
- *Supersymmetry phenomenology*
  H. Murayama, hep-ph/0002332

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The supersymmetric harmonic oscillator

Recall raising and lowering operators in quantum mechanics

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\begin{aligned}
b^+|n_B\rangle &= \sqrt{n_B+1}\,|n_B+1\rangle \\
b^-|n_B\rangle &= \sqrt{n_B}\,|n_B-1\rangle
\end{aligned}
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where $b^-|0\rangle = 0$ and $[b^-, b^+] = 1; [b^-, b^-] = [b^+, b^+] = 0$

$\rightarrow b^+/b^-$ creates/annihilates bosons

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Analogously for fermions

$$\begin{aligned} f^+|n_F\rangle &= \sqrt{n_F+1}|n_F+1\rangle \\ f^-|n_F\rangle &= \sqrt{n_F}|n_F-1\rangle \end{aligned}$$

But fermions obey Pauli exclusion principle

$\rightarrow$ only two states $|0\rangle$ and $f^+|0\rangle = |1\rangle$

So for fermions

$$f^+|0\rangle = |1\rangle, f^-|1\rangle = |0\rangle \quad \text{and} \quad f^-|0\rangle = f^+|1\rangle = 0$$

For fermions

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Matrix representation:

with
$$|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

one has
$$f^+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad f^- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

and
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and $$\{f^-, f^+\} = 1; \{f^-, f^-\} = \{f^+, f^+\} = 0\,.$$

Thus, bosonic and fermionic Hamilton operators take the form

$$\begin{aligned} H_B &= \omega_B \left(b^+ b^- + \frac{1}{2} \right) \\ H_F &= \omega_F \left(f^+ f^- - \frac{1}{2} \right) \end{aligned}$$

SUSY transformations

SUSY operators act on product space

$$|n_B\rangle|n_F\rangle \equiv |n_B n_F\rangle \quad \text{where} \quad n_B = 0, 1, \ldots, \infty; \quad n_F = 0, 1$$

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Need to construct operators with

$$\begin{aligned} Q_+|n_B n_F\rangle &\propto |n_B - 1, n_F + 1\rangle \\ Q_-|n_B n_F\rangle &\propto |n_B + 1, n_F - 1\rangle \end{aligned}$$

so that

$$\begin{aligned} Q_+|\text{boson}\rangle \propto |\text{fermion}\rangle \qquad & Q_+|\text{fermion}\rangle = 0 \\ Q_-|\text{fermion}\rangle \propto |\text{boson}\rangle \qquad & Q_-|\text{boson}\rangle = 0\,. \end{aligned}$$

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A simple choice is

$$\begin{aligned} Q_+ &= b^- f^+ \\ Q_- &= b^+ f^- \end{aligned}$$

where $(f^+)^2 = (f^-)^2 = 0 \quad \Rightarrow \quad Q_+^2 = Q_-^2 = 0\,.$

We now want to construct a SUSY invariant Hamilton operator so that

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[Check e.g. $[H_{\mathrm{SUSY}}, Q_+] = Q_+ Q_- Q_+ + Q_- Q_+ Q_+ - Q_+ Q_+ Q_- - Q_+ Q_- Q_+ = 0\,.$]

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Now recall

$$\begin{aligned} Q_+ &= \sqrt{\omega}\; b^- f^+ \\ Q_- &= \sqrt{\omega}\; b^+ f^- \end{aligned}$$

so that

$$\begin{aligned} H_{\rm SUSY} &= \omega\{b^-f^+, b^+f^-\} \\ &= \omega(b^-f^+b^+f^- + b^+f^-b^-f^+) \\ &= \omega((1+b^+b^-)f^+f^- + b^+b^-(1-f^+f^-)) \\ &= \omega(f^+f^- + b^+b^-) \\ &= H_B + H_F \end{aligned}$$

provided we set $\omega_B = \omega_F = \omega\,.$

The energy spectrum of the SUSY oscillator has remarkable features

$$H_{\mathrm{SUSY}}|n_B n_F\rangle = \omega(N_B + N_F)|n_B n_F\rangle$$

$\rightarrow E = \omega(n_B + n_F)$

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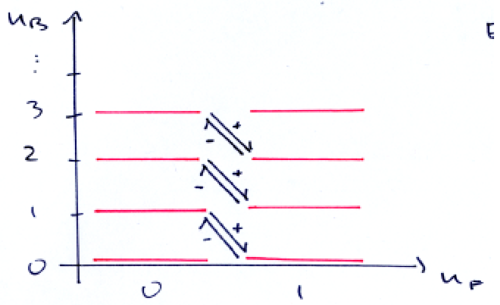
The spectrum of the SUSY oscillator:



Energies

$E_{20} = E_{11} = 2\omega$

$E_{10} = E_{01} = \omega$

$E_{00} = 0$

Summary of the SUSY oscillator

- If we start with a bosonic system we need to introduce fermions (and vice versa)
- We need identical couplings: $\omega_F = \omega_B$
- The spectrum consists of pairs of states (bosonic/fermionic) with the same energy
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Summary of the SUSY oscillator

- If we start with a bosonic system we need to introduce fermions (and vice versa)

  $\rightarrow$ for a SUSY extension of the SM we will have to introduce SUSY partners for all SM particles

- We need identical couplings: $\omega_F = \omega_B$

  $\rightarrow$ SUSY extensions of the SM do not introduce new couplings

- The spectrum consists of pairs of states (bosonic/fermionic) with the same energy

  $\rightarrow$ SM particles and SUSY partners have the same mass (and internal quantum numbers)

- The energy of the ground state is zero

  $\rightarrow$ SUSY QFTs have less divergences

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Why supersymmetric quantum field theory?

SUSY is a symmetry which relates fermions and bosons:

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Example: electron $(\psi_e)_L(s = 1/2) \;\leftrightarrow\; \phi_{\tilde{e}_L}(s = 0)$ (scalar electron $\tilde{e}_L$)
$(\psi_e)_R(s = 1/2) \;\leftrightarrow\; \phi_{\tilde{e}_R}(s = 0)$ (scalar electron $\tilde{e}_R$)

Note: $\tilde{e}_{L/R}$ are called "left/right-handed" selectron to indicate SUSY partner (scalar particle has no helicity).

How do we characterize a particle?

Consider Lorentz group (rotations & boosts) with invariants

$$P_\mu P^\mu = m^2 \quad \text{and} \quad W_\mu W^\mu = -m^2 s(s+1)\,.$$

P_μ: energy momentum operator
$W_\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu M_{\rho\sigma}$: Pauli-Lubanski spin vector

where $M_{\mu\nu}$ = angular momentum tensor $= x^\mu P^\nu - x^\nu P^\mu + \frac{1}{2}\Sigma^{\mu\nu}$

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$$\text{The} \left\{ \begin{array}{c} \text{Lorentz} \\ \text{Gauge} \end{array} \right\} \text{symmetry is an} \left\{ \begin{array}{c} \text{external} \\ \text{internal} \end{array} \right\} \text{symmetry.}$$

→ invariants of gauge symmetries ("charges") do not change in space and time

→ the generators of the gauge group T^a commute with the generators of the Lorentz group $[T^a, P^\mu] = 0$ and $[T^a, M^{\mu\nu}] = 0$

The Coleman-Mandula theorem

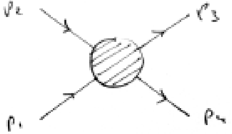
Coleman & Mandula, "*All Possible Symmetries of the S Matrix*",
PRD 159 (1967):

> *The only possible conserved quantities that transform as tensors under the Lorentz group are the generators of the Lorentz group ($P_\mu, M_{\mu\nu}$) and Lorentz scalars (internal symmetries).*

According to Coleman & Mandula, if we add to the Lorentz symmetry any further external symmetry, whose generators are tensors, then the scattering process must be trivial, i.e. there is no scattering at all.

Let us work this out in an example. . .

We consider



$2 \to 2$ spinless scattering

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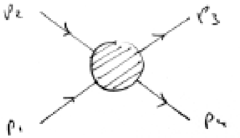

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Momentum conservation implies $p_1 + p_2 = p_3 + p_4$.

Now let us postulate an additional external symmetry,
e.g. a conserved tensor $R_{\mu\nu} = p_\mu p_\nu - \frac{1}{4} g_{\mu\nu} m^2$.

If $R_{\mu\nu}$ is conserved, then

$$\begin{aligned} R^1_{\mu\nu} + R^2_{\mu\nu} &= R^3_{\mu\nu} + R^4_{\mu\nu} \\ \text{and thus} \quad p^1_\mu p^1_\nu + p^2_\mu p^2_\nu &= p^3_\mu p^3_\nu + p^4_\mu p^4_\nu \,. \end{aligned}$$

Specifically, in the center-of-mass frame we have

$$\begin{aligned} p_1 &= (E, 0, 0, p) \\ p_2 &= (E, 0, 0, -p) \\ p_3 &= (E, 0, p\sin\theta, p\cos\theta) \\ p_4 &= (E, 0, -p\sin\theta, -p\cos\theta) \end{aligned}$$

Let us look at e.g. $\mu = \nu = 4$. We find

$$2p^2 = 2p^2 \cos\theta\,.$$

$\Rightarrow \theta = 0$, i.e. no scattering

The Haag-Lopuszanski-Sohnius theorem

Tensors $a_{\mu_1\cdots\mu_N}$ are combinations of Lorentz vector indices, which each transform like a vector:

$$a'_{\mu_1\cdots\mu_N} = \Lambda_{\mu_1}^{\ \nu_1}\cdots\Lambda_{\mu_N}^{\ \nu_N} a_{\mu_1\cdots\mu_N}$$

$\rightarrow$ tensors are bosons

This points to the loop-hole in the Coleman-Mandula "no-go" theorem:

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How could nature have ignored this last possible external symmetry?

What is the algebra of the SUSY generators Q_α?

One can work out that

$$\begin{aligned} [P^\mu, Q_\alpha] &= 0 \\ [M^{\mu\nu}, Q_\alpha] &= -i(\sigma^{\mu\nu})_\alpha^\beta Q_\beta \\ \{Q_\alpha, Q_\beta\} &= 0 \\ \{Q_\alpha, Q_\beta^\dagger\} &= 2(\sigma^\mu)_{\alpha\beta} P_\mu \end{aligned}$$

where $\sigma^\mu = (1, \sigma^i)$, $\bar{\sigma}^\mu = (1, \sigma^i)$, $\sigma^{\mu\nu} = (\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)/4$.

Supersymmetry

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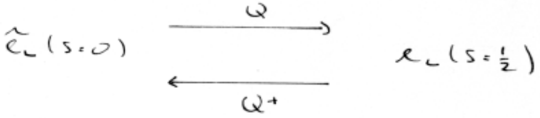
Q raises by spin $1/2$, $Q^\dagger$ lowers by spin $1/2$



What are the immediate consequences of SUSY invariance?

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But we have not seen a 511 keV$= m_{\tilde{e}}$ charged ($[Q, T^a] = 0$) scalar

$\rightarrow$ SUSY must be broken

At what scale?

What is the mass of the supersymmetric particles?

The hierarchy problem and the scale of SUSY breaking

Let us first look at electrodynamics:

The Coulomb field of the electron is $E_{\mathrm{self}} = \frac{3}{5}\frac{e^2}{r_e}$.

This can be interpreted as a contribution to the electron mass:

$$m_e c^2 = m_{e,0} c^2 + E_{\mathrm{self}} \,.$$

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However, with $r_e \lesssim 10^{-17}$ cm (exp. bound on point-like nature) one has

$$m_e c^2 = 0.511\,\text{MeV} = (-9999.489 + 10000.000)\,\mathrm{MeV}$$

$\rightarrow$ fine-tuning!

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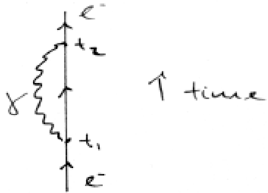
Coulomb self-energy in time-ordered perturbation theory:



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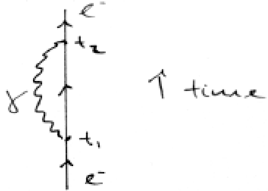

But also have positron e^+ with $Q(e^+) = -Q(e^-)$ and $m(e^+) = m(e^-)$

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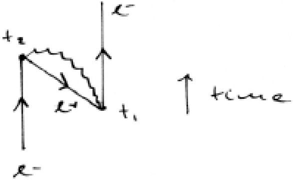

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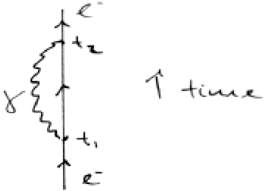

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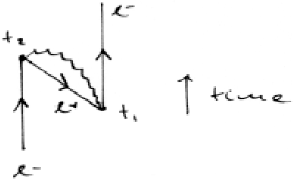

$$\rightarrow m_e c^2 = m_{e,0} c^2 \left(1 + \frac{3\alpha}{4\pi} \ln\left(\frac{\hbar}{m_e c r_e}\right)\right)$$

We found that $m_e c^2 = m_{e,0} c^2 \left(1 + \frac{3\alpha}{4\pi} \ln\left(\frac{\hbar}{m_e c r_e}\right)\right)$.

So even if $r_e = 1/M_{\rm Planck} = 1.6 \times 10^{-33}\,\text{cm}$, the corrections to the electron mass are small

$$m_e c^2 \approx m_{e,0} c^2 \left(1 + 0.1\right) .$$

Also, if $m_{e,0} = 0$ then $m_e = 0$ to all orders:

the mass is protected by a (chiral) symmetry

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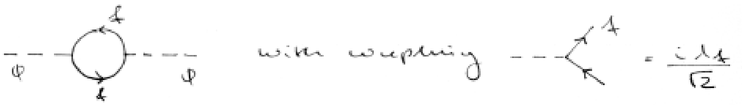
Recall 't Hooft's naturalness argument

> A dimensionless number x is allowed to be very small iff
>
> *The value $x = 0$ would imply an exact symmetry*

Now let us look at the scalar (=Higgs) self-energy:



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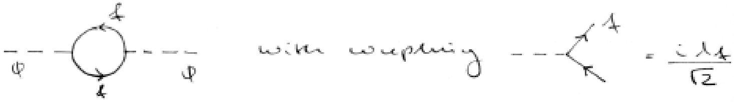

$$\Rightarrow \Delta m_\phi^2 = 2N(f)\,\lambda_f^2 \int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2}\right)$$

The integral is divergent, so we introduce a momentum cut-off.

[Recall that $d^4k \sim k^3 dk \to \int^\Lambda dk k^3/(k^2 - m_f^2) \sim \Lambda^2$ and $\int^\Lambda dk k^3/(k^2 - m_f^2)^2 \sim \ln\Lambda$.]

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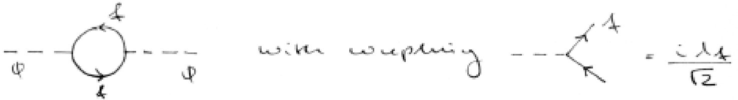

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Straightforward calculation gives

$$\Delta m_\phi^2 = \frac{N(f)\,\lambda_f^2}{8\pi^2}\left(\Lambda^2 + 3m_f^2\ln\left(\frac{\Lambda^2+m_f^2}{m_f^2}\right) + 2m_f^2\frac{\Lambda^2}{\Lambda^2+m_f^2}\right)\,.$$

Because of the quadratic divergence we find

$$\Delta m_\phi^2(\Lambda = M_{\rm Planck}) \approx 10^{35}\mathrm{GeV}^2 = (3 \times 10^{17}\,\mathrm{GeV})^2$$

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and so

$$m_\phi^2 \lesssim 1\,\mathrm{TeV}^2 = m_{\phi,0}^2 + \Delta m_\phi^2$$

implies a huge fine-tuning:

$$\begin{array}{lr} & 1,735,405,204,836,950,645,958,932,812,557,642,954 \\ - & 1,735,405,204,836,950,645,958,932,812,557,642,829 \\ = & 125 \end{array}$$

Comment: it is essential that $\Lambda < \infty$, i.e. we assume that new physics sets in at $E \sim \Lambda$. Is this a tautology? No: we assume new physics at some very high scale Λ and find that the standard model needs new physics well below Λ.

The natural mass scale of a scalar field is the highest scale in nature.

The SUSY solution to the hierarchy problem

The SUSY solution to the hierarchy problem

Let us increase the particle content (as for the e^- self-energy)

Before we had 

Now we include in addition two scalars $\tilde{f}_L, \tilde{f}_R$ with couplings

$$\mathcal{L}_{\phi\tilde{f}} = -\frac{\tilde{\lambda}_f^2}{2}\phi^2\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right) - v\tilde{\lambda}_f^2\phi\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right) + \left(\frac{\lambda_f}{\sqrt{2}}A_f\phi\tilde{f}_L\tilde{f}_R^* + \text{h.c.}\right)$$

which lead to additional contributions to the self-energy:

The additional contributions to the Higgs mass are:

$$\begin{aligned}
\Delta m_\phi^2 &= \tilde{\lambda}_f^2\, N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) \\
&+ (\tilde{\lambda}_f^2 v)^2\, N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{(k^2 - m_{\tilde{f}_L}^2)^2} + \frac{1}{(k^2 - m_{\tilde{f}_R}^2)^2} \right) \\
&+ (\lambda_f A_f)^2\, N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{\tilde{f}_L}^2)(k^2 - m_{\tilde{f}_R}^2)}
\end{aligned}$$

.

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$$
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&+ (\lambda_f A_f)^2\, N(\tilde{f}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{\tilde{f}_L}^2)(k^2 - m_{\tilde{f}_R}^2)}
\end{aligned}
$$

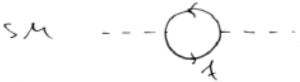
.

The first term cancels the SM Λ^2-contribution if

$$\tilde{\lambda}_f = \lambda_f \quad \text{and} \quad N(\tilde{f}) = N(f)$$

as required in SUSY.

The cancellation happens because of spin-statistics:



fermion loop $\to$ (-1)

boson-loop $\to$ $(+1)$

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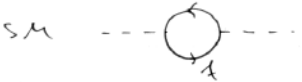

fermion loop $\rightarrow$ (-1)

boson-loop $\rightarrow$ (+1)

Note:

- the cancellation of quadratic divergences is independent of $m_{\tilde{f}_L}$, $m_{\tilde{f}_R}$, A_f.
- the term $\propto A_f \phi \tilde{f}_L \tilde{f}_R^*$ breaks SUSY but does not lead to Λ^2 divergences

  $\rightarrow$ "soft" SUSY breaking

Let us look at the finite SM + SUSY contributions:

$$\Delta m_\phi^2 = \frac{\lambda_f^2 N(f)}{16\pi^2}\left(-2m_f^2\left(1-\ln\frac{m_f^2}{\mu^2}\right)+4m_f^2\ln\frac{m_f^2}{\mu^2}\right.$$
$$\left.+\,2m_{\tilde{f}}^2\left(1-\ln\frac{m_{\tilde{f}}^2}{\mu^2}\right)-4m_{\tilde{f}}^2\ln\frac{m_{\tilde{f}}^2}{\mu^2}-|A_f|^2\ln\frac{m_{\tilde{f}}^2}{\mu^2}\right)\,,$$

where we have assumed $m_{\tilde{f}_L}=m_{\tilde{f}_R}=m_{\tilde{f}}$.

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where we have assumed $m_{\tilde f_L} = m_{\tilde f_R} = m_{\tilde f}$.

One has

$$\Delta m_\phi^2 = 0 \quad \text{for} \quad A_f = 0 \;\text{ and }\; m_{\tilde f} = m_f \quad \text{(SUSY)}$$

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where we have assumed $m_{\tilde f_L} = m_{\tilde f_R} = m_{\tilde f}$.

One has

$$\Delta m_\phi^2 = 0 \quad \text{for} \quad A_f = 0 \ \text{and} \ m_{\tilde f} = m_f \quad \text{(SUSY)}$$

But SUSY is broken, i.e. $m_{\tilde f}^2 = m_f^2 + \delta^2$. Thus

$$\Delta m_\phi^2 = \frac{\lambda_f^2 N(f)}{8\pi^2}\,\delta^2\left(2+\ln\frac{m_f^2}{\mu^2}\right)+\mathcal{O}(\delta^4)$$

To have Δm_ϕ^2 small, we thus need $m_{\tilde f}^2 = m_f^2 + \delta^2 = \mathcal{O}(1\,\mathrm{TeV}^2)$

Motivation for supersymmetry

A Priori:

- SUSY is the unique maximal external symmetry in Nature.
- Weak-scale SUSY provides a solution to the hierarchy problem.

A Posteriori:

- SUSY allows for unification of Standard Model gauge interactions.
- SUSY provides dark matter candidates.
- SUSY explains EWSB dynamically.
- SUSY QFT's allow for precision calculations.
- SUSY provides a rich phenomenology and is testable at the LHC.

Outline

- The supersymmetric harmonic oscillator
- Motivation for SUSY: Symmetry & the hierarchy problem
- The MSSM
- SUSY searches

The Minimal Supersymmetric extension of the SM

- external symmetries: Poincare symmetry & supersymmetry
- internal symmetries: SU(3)⊗SU(2)⊗U(1) gauge symmetries
- minimal particle content

Gauge Bosons $S=1$ gluon, $W^{\pm}, Z, \gamma$	Gauginos $S=1/2$ gluino, $\widetilde{W}, \tilde{Z}, \tilde{\gamma}$
Fermions $S=1/2$ $\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}$ u_R, d_R, e_R	Sfermions $S=0$ $\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \begin{pmatrix} \tilde{\nu}_L^e \\ \tilde{e}_L \end{pmatrix}$ $\tilde{u}_R, \tilde{d}_R, \tilde{e}_R$
Higgs $\begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix}$	Higgsinos $\begin{pmatrix} \widetilde{H}_2^0 \\ \widetilde{H}_2^- \end{pmatrix} \begin{pmatrix} \widetilde{H}_1^+ \\ \widetilde{H}_1^0 \end{pmatrix}$

Gauge coupling unification

In QFT the gauge couplings "run":

$$\frac{d\alpha_i(\mu)}{d\ln\mu^2} = \beta_i(\alpha_i(\mu))$$

The beta-functions β_i depend on the gauge group and on the matter multiplets to which the gauge bosons couple. Only particles with mass $< \mu$ contribute to the β_i and to the evolution of the coupling at any given mass scale μ.

The Standard Model couplings evolve with μ according to

$$\begin{array}{lcccl} \mathrm{SU}(3) & : & \beta_{3,0} & = & (33 - 4n_g)/(12\pi) \\ \mathrm{SU}(2) & : & \beta_{2,0} & = & (22 - 4n_g - n_h/2)/(12\pi) \\ \mathrm{U}(1) & : & \beta_{1,0} & = & (-4n_g - 3n_h/10)/(12\pi) \end{array}$$

where $n_g = 3$ is the number of quark and lepton generations and $n_h = 1$ is the number of Higgs doublet fields in the Standard Model.

Loop contributions of superpartners change the beta-functions. In the MSSM one finds:

$$
\begin{aligned}
\mathrm{SU}(3) &: \quad \beta_{3,0}^{\mathrm{SUSY}} = (27 - 6n_g)/(12\pi) \\
\mathrm{SU}(2) &: \quad \beta_{2,0}^{\mathrm{SUSY}} = (18 - 6n_g - 3n_h/2)/(12\pi) \\
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\end{aligned}
$$

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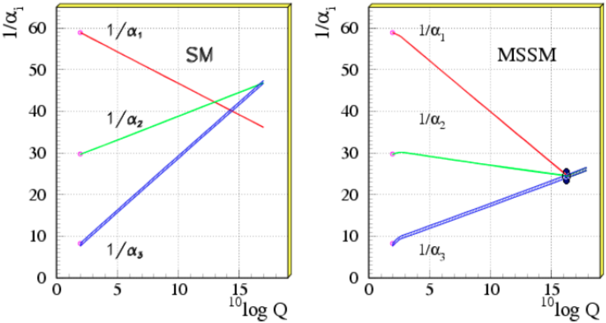
$$



R-parity

- In the SM baryon and lepton number are accidental symmetries
- The most general superpotential of the SUSY-SM contains baryon and lepton number violating terms:

$$W \in \underbrace{\lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \kappa_i L_i H_2}_{\text{lepton number violating}} \quad + \quad \underbrace{\lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k}_{\text{baryon number violating}}$$

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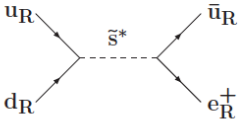
LQD and UDD couplings lead to rapid proton decay



$\rightarrow$ impose discrete symmetry: R-parity $R = (-1)^{3B+L+2S}$

$\rightarrow R_{\mathrm{SM}} = +$ and $R_{\mathrm{SUSY}} = -$

R-parity

R-parity conservation has dramatic phenomenological consequences:

- lightest SUSY particle (LSP) is absolutely stable
  $\rightarrow$ dark matter candidate if also electrically neutral
- in collider experiments SUSY particles can only be produced in pairs
- in many models SUSY collider events contain missing E_T

SUSY breaking

Supersymmetry: mass(e^-) = mass$(\tilde{e}^-_{L,R})$

$\rightarrow$ SUSY must be broken

No agreed model of supersymmetry breaking

$\rightarrow$ phenomenological ansatz

Must preserve solution to hierarchy problem

$\rightarrow$ "soft" SUSY breaking

SUSY breaking

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Must preserve solution to hierarchy problem

$\rightarrow$ "soft" SUSY breaking

Introduce

- gaugino masses $M_{1/2}\chi\chi$: $M_1\tilde{B}\tilde{B}$, $M_2\tilde{W}\tilde{W}$, $M_3\tilde{g}\tilde{g}$
- squark and slepton masses $M_0^2\phi^\dagger\phi$:
  $m^2_{\tilde{e}_L}\tilde{e}^\dagger_L\tilde{e}_L$, $m^2_{\tilde{e}_R}\tilde{e}^\dagger_R\tilde{e}_R$, $m^2_{\tilde{u}_L}\tilde{u}^\dagger_L\tilde{u}_L$, $m^2_{\tilde{u}_R}\tilde{u}^\dagger_R\tilde{u}_R$ etc.
- trilinear couplings $A_{ijk}\phi_i\phi_j\phi_k$: $A^e_{ij}\begin{pmatrix}\tilde{\nu}_i\\ \tilde{e}_j\end{pmatrix}_L h_1\tilde{e}_{jR}$ etc.
- Higgs mass terms $B_{ij}\phi_i\phi_j$: Bh_1h_2 etc.

SUSY breaking

MSSM w/o breaking: two additional parameters from Higgs sector

Soft SUSY breaking

- $A^e_{ij}, A^d_{ij}, A^u_{ij}$ $\rightarrow$ 27 real + 27 phases
- $M^2_{\tilde{Q}}$, $M^2_{\tilde{U}}$, $M^2_{\tilde{D}}$, $M^2_{\tilde{L}}$, $M^2_{\tilde{E}}$ $\rightarrow$ 30 real + 15 phases
- M_1, M_2, M_3 $\rightarrow$ 3 real + 1 phase

$\rightarrow$ 124 parameters in the MSSM!

(but strong constraints from FCNS's, flavour mixing and CP violation)

SUSY breaking

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- M_1, M_2, M_3 $\rightarrow$ 3 real + 1 phase

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(but strong constraints from FCNS's, flavour mixing and CP violation)

Simple framework constrained MSSM:

breaking is universal at GUT scale

- universal scalar masses: $M^2_{\tilde{Q}}$, $M^2_{\tilde{U}}$, $M^2_{\tilde{D}}$, $M^2_{\tilde{L}}$, $M^2_{\tilde{E}}$ $\rightarrow M^2_0$ at M_{GUT}
- universal gaugino masses: M_1, M_2, $M_3 \rightarrow M_{1/2}$ at M_{GUT}
- universal trilinear couplings $A^e_{ij}, A^d_{ij}, A^u_{ij} \rightarrow A \cdot h^e_{ij}, A \cdot h^d_{ij}, A \cdot h^u_{ij}$ at M_{GUT}

$\rightarrow$ 6 additional parameters: M_0, $M_{1/2}$, A, B, μ, $\tan(\beta)$

SUSY mass spectrum

In QFT the (s)particle masses "run": $\dfrac{dM_i(\mu)}{d\ln\mu^2} = \gamma_i M_i$

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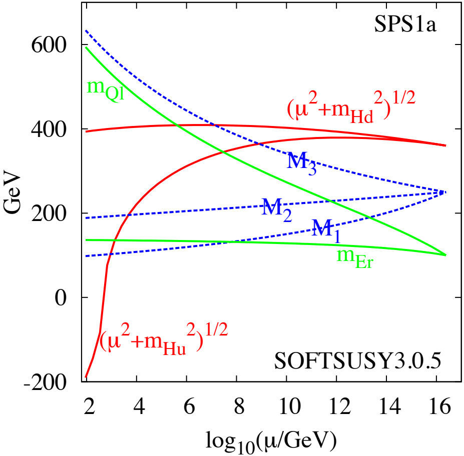

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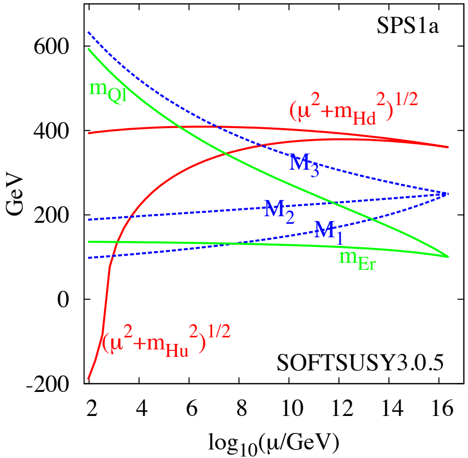

typical mass pattern e.g. from

$$\frac{M_1(\mu)}{\alpha_1(\mu)} = \frac{M_2(\mu)}{\alpha_2(\mu)} = \frac{M_3(\mu)}{\alpha_3(\mu)}$$

$\rightarrow M_3(M_Z) : M_2(M_Z) : M_1(M_Z)$
$\simeq 7 : 2 : 1$

Radiative EWK symmetry breaking

- RGE drives $(\mu^2 + m_{H_u^2})$ negative $\rightarrow$ EWK symmetry breaking
- Masses of W and Z bosons fix B and $|\mu|$
- cMSSM has 4 1/2 parameters:

  M_0, $M_{1/2}$, A, $\tan(\beta)$ and $\mathrm{sign}(\mu)$

Mixing

After $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ breaking, mixing will occur between any two or more fields which have the same color, charge and spin

- $(\tilde{W}^{\pm}, \tilde{H}^{\pm}) \rightarrow \tilde{\chi}^{\pm}_{i=1,2}$: charginos
- $(\tilde{B}, \tilde{W}^3, \tilde{H}^0_{1,2}) \rightarrow \tilde{\chi}^0_{i=1,2,3,4}$: neutralinos
- $(\tilde{t}_L, \tilde{t}_R) \rightarrow \tilde{t}_{1,2}$ etc.: sfermion mass eigenstates

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Note:

- mixing involves various SUSY parameters

  $\to$ cross sections and branching ratios become model dependent
- sfermion mixing $\propto m_f$

  $\to$ large only for 3rd generation $(\tilde{t}_{1,2}, \tilde{\tau}_{1,2})$

Outline

- The supersymmetric harmonic oscillator
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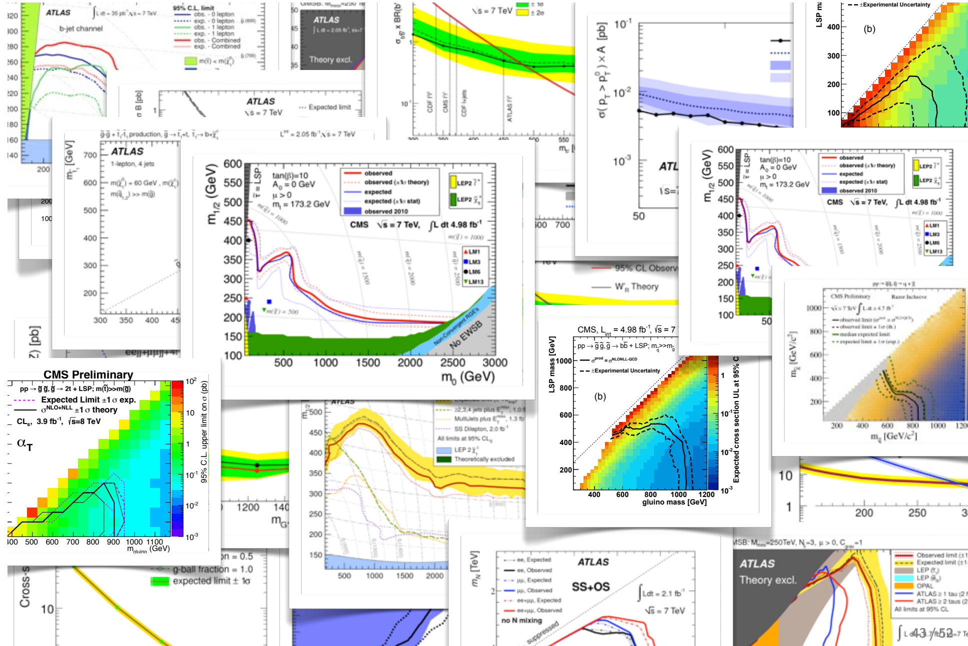
- The MSSM
- SUSY searches

Summary of SUSY searches: limits, limits and more limits...



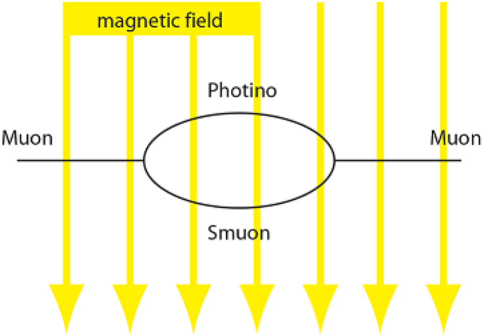
SUSY searches

The SUSY parameter space is strongly constrained by

- loop-induced effects:
  $\mathrm{BR}(b \to s\gamma)$, $\mathrm{BR}(B_s \to \mu\mu)$, $\mathrm{BR}(b \to \tau\nu)$, Δm_{B_s}, $(g-2)_\mu$, m_W, $\sin^2\theta_{\mathrm{eff}}$
- astrophysical observations:
  Ω_{DM}, direct and indirect DM detection limits
- direct sparticle and Higgs boson search limits from colliders:
  $m_{\tilde{\chi}^\pm}$, LEP limits on MSSM Higgs bosons
- LHC SUSY exclusions from jets+$E_{T\mathrm{miss}}$ searches
- the LHC Higgs signal

Indirect SUSY searches

- the anomalous magnetic moment of the muon $(g-2)_\mu$:

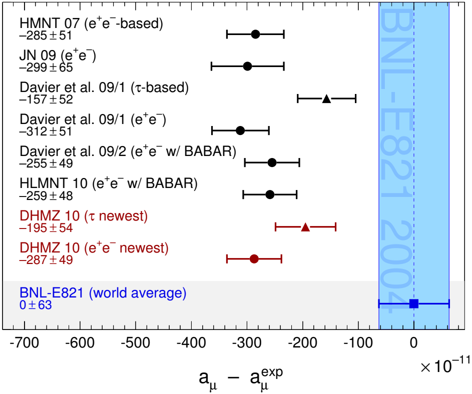


→ SUSY loops: $a_\mu^{\mathrm{SUSY}} \sim \mathrm{sgn}(\mu)\, \tan\beta\, M_{\mathrm{SUSY}}^{-2}$

Indirect SUSY searches

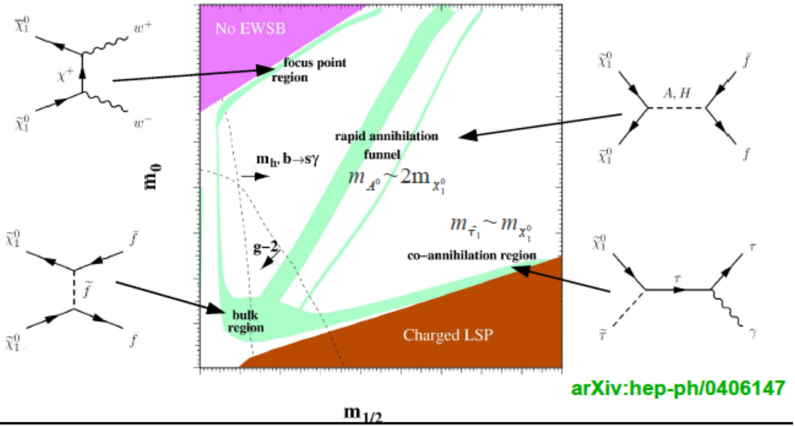
- the anomalous magnetic moment of the muon $(g-2)_\mu$:

Davier, Hoecker, Malaescu, Zhang, arXiv:1010.4180v1



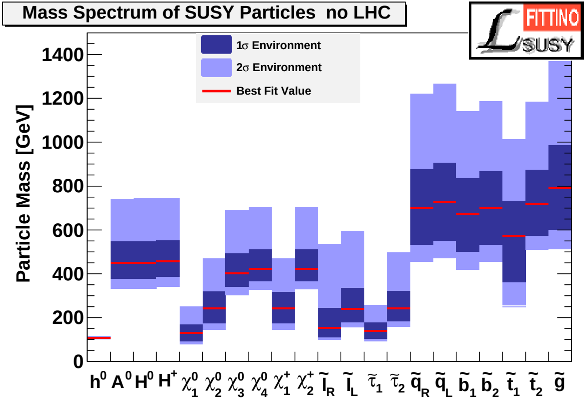
→ SUSY loops: $a_\mu^{\mathrm{SUSY}} \sim \mathrm{sgn}(\mu) \tan\beta \, M_{\mathrm{SUSY}}^{-2}$

- $\Omega_{\rm DM}$ is too large for large parts of the cMSSM parameter space, special annihilation mechanisms are needed:



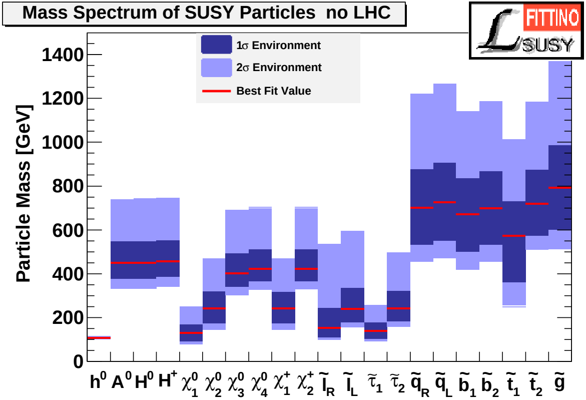
arXiv:hep-ph/0406147

→ CMSSM fit to B, K and EWK observables, $(g-2)_\mu$ and $\Omega_{\rm DM}$



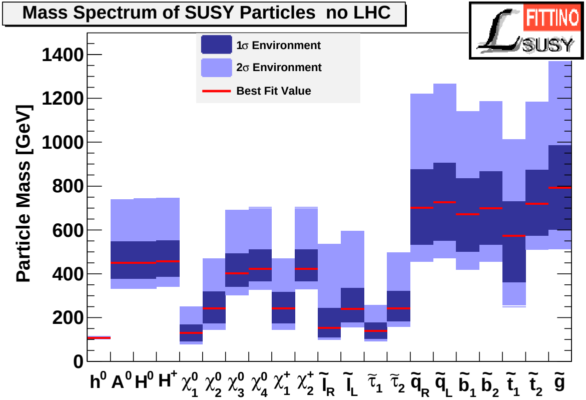
→ pre-LHC global fits point to light sparticle spectrum with $\tilde{m} < 1$ TeV

→ CMSSM fit to B, K and EWK observables, $(g-2)_\mu$ and $\Omega_{\rm DM}$



→ many of us thought SUSY is just around the corner...

→ CMSSM fit to B, K and EWK observables, $(g-2)_\mu$ and Ω_{DM}



→ Monica will tell us what we actually found at the LHC...

The landscape of new physics

. . . is it a natural supersymmetric Garden Eden?

or do we have to live with an anthropic big desert up to the Planck scale?