Top quark physics

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Outlook

1. Quantum numbers, interactions, mass
2. Top decay
3. Top decay beyond the SM
4. Extended quark sector and top mixing
5. Single top production
6. Single top beyond the SM
7. Top pair production
8. Top pair production beyond the SM

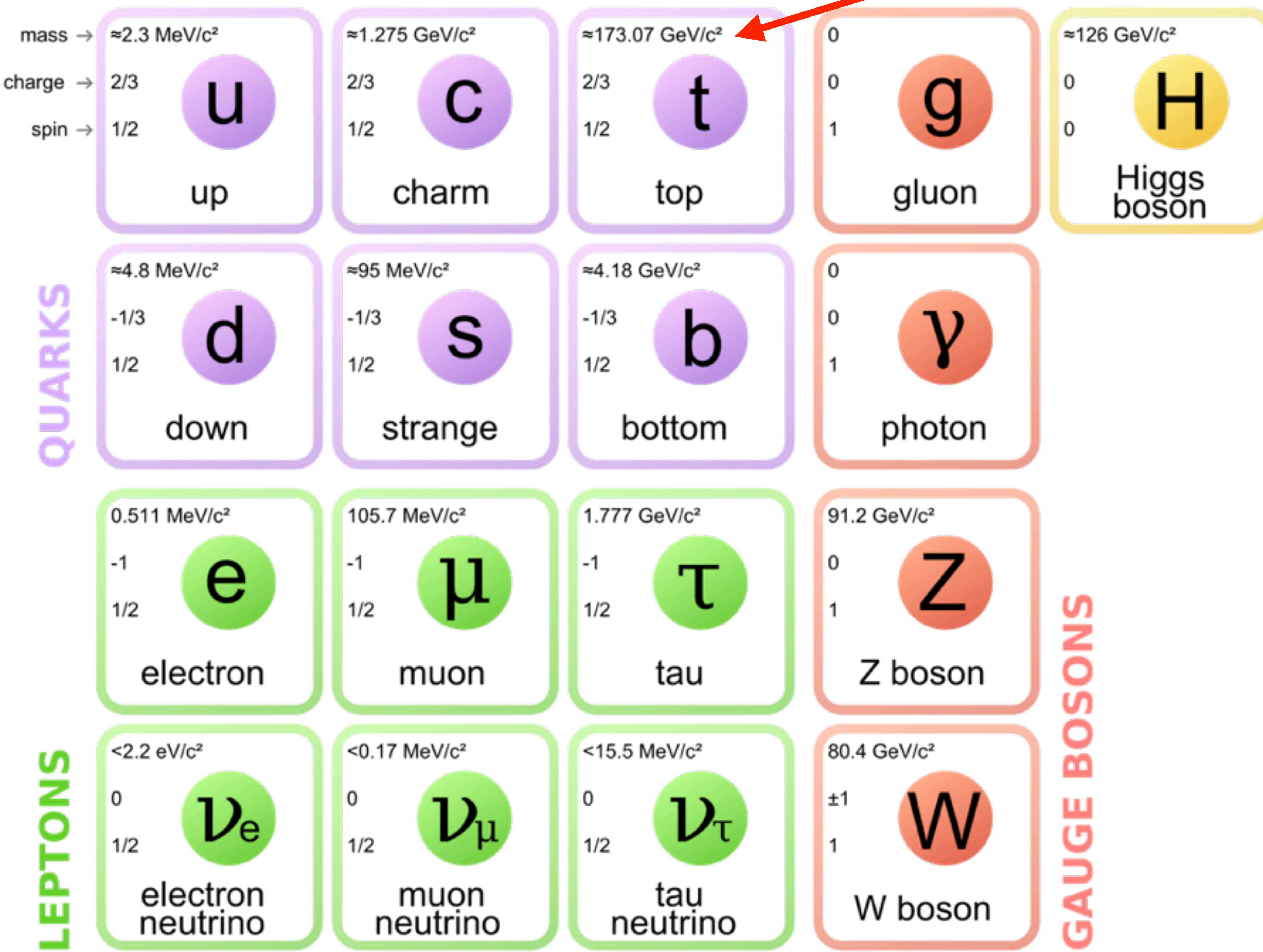
Top quantum numbers

The top quark is a massive spin-1/2 fermion that is a colour triplet and has electric charge 2/3.

- Spin 1/2? No undeniable evidence of this, but overwhelming indications that it has spin 1/2.
- Colour triplet? As for the rest of quarks, measurements tell us that top quarks come in three different colours.
- Charge 2/3? Yes, this has been directly verified in several experiments.

There are three known particles with these quantum numbers: the up (u), charm (c) and top (t) quarks. The top quark is the heaviest of them.

$m_t = 173.2 \pm 0.9\ \mathrm{GeV}$



Top interactions

The SM predicts that the top quark has interactions with the photon

$$-eQ_t\bar{t}\gamma^\mu tA_\mu$$

$Q_t = \frac{2}{3}$

the gluon

$$-g_s\bar{t}\frac{\lambda^a}{2}\gamma^\mu tG^a_\mu$$

γ^μ Dirac matrices
λ^a Gell-Mann matrices

the Z boson

$t_L = P_L\, t$, etc.

s_W sine of weak mixing angle

$$-\frac{g}{2c_W}\left[(1-2Q_ts^2_W)\bar{t}_L\gamma^\mu t_L - 2Q_ts^2_W\bar{t}_R\gamma^\mu t_R\right]Z_\mu$$

the W boson

V_{td}, V_{ts}, V_{tb} CKM matrix elements

$$-\frac{g}{\sqrt{2}}\left[V_{td}\,\bar{t}_L\gamma^\mu d_L + V_{ts}\,\bar{t}_L\gamma^\mu s_L + V_{tb}\,\bar{t}_L\gamma^\mu b_L\right]W^+_\mu + \text{h.c.}$$

and the Higgs boson

$$-\frac{1}{\sqrt{2}}y_t\,\bar{t}\,tH$$

y_t Yukawa coupling

Interactions: γ

The interactions with the photon are flavour-diagonal

$$-eQ_t\bar{t}\gamma^\mu t A_\mu$$

Renormalisable (γ^μ) *t-u* or *t-c* terms, for example

$$a\,\bar{t}\gamma^\mu c A_\mu$$

photon momentum

would conserve charge but violate Ward identity $\mathcal{M}^\mu q_\mu = 0$ in amplitudes:

$$\begin{aligned} 0 &= a\,\bar{u}(p_c)\gamma^\mu u(p_t) q_\mu \\ &= a\,\bar{u}(p_c)\gamma^\mu u(p_t)(p_{t\mu} - p_{c\mu}) \\ &= a\,(m_t - m_c)\bar{u}(p_c)u(p_t) \end{aligned} \quad \Longrightarrow \quad a = 0$$

Dirac equation

Analogous thing (but more complicated) happens with the gluon.

Interactions: Z

Gauge symmetry does not forbid flavour-changing interactions with the Z.
Still, they are flavour-diagonal:

$$-\frac{g}{2c_W}\left[(1-2Q_t s_W^2)\bar{t}_L\gamma^\mu t_L - 2Q_t s_W^2 \bar{t}_R\gamma^\mu t_R\right] Z_\mu$$

The reason is that <u>in the SM</u> the mass eigenstates are linear combinations of weak eigenstates with the same weak isospin.

Example: up sector. In the weak basis $u_i^0 = (u^0, c^0, t^0)$,

$2T_{3L}$ $\quad$ $2Q_t$

$$\begin{aligned}\mathcal{L}_Z = &-\frac{g}{2c_W}\begin{pmatrix}\overline{u_L^0} & \overline{c_L^0} & \overline{t_L^0}\end{pmatrix}\begin{pmatrix}1-\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & 1-\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & 1-\frac{4}{3}s_W^2\end{pmatrix}\gamma^\mu\begin{pmatrix}u_L^0 \\ c_L^0 \\ t_L^0\end{pmatrix} Z_\mu \\ &-\frac{g}{2c_W}\begin{pmatrix}\overline{u_R^0} & \overline{c_R^0} & \overline{t_R^0}\end{pmatrix}\begin{pmatrix}-\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & -\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & -\frac{4}{3}s_W^2\end{pmatrix}\gamma^\mu\begin{pmatrix}u_R^0 \\ c_R^0 \\ t_R^0\end{pmatrix} Z_\mu\end{aligned}$$

$T_{3R} = 0$ $\quad$ $2Q_t$

Mass eigenstates are related to weak eigenstates by unitary transformations $\mathcal{U}^{uL}$, $\mathcal{U}^{uR}$

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = \overbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}}^{\mathcal{U}^{uL}} \begin{pmatrix} u_L^0 \\ c_L^0 \\ t_L^0 \end{pmatrix} \qquad \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} = \overbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}}^{\mathcal{U}^{uR}} \begin{pmatrix} u_R^0 \\ c_R^0 \\ t_R^0 \end{pmatrix}$$

Obviously, the Z interactions remain diagonal in the mass eigenstate basis.

This is known as the GIM mechanism.

no flavour-changing neutral couplings

$$\mathcal{U}^{uL} \begin{pmatrix} 1-\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & 1-\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & 1-\frac{4}{3}s_W^2 \end{pmatrix} \mathcal{U}^{uL\dagger} = \begin{pmatrix} 1-\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & 1-\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & 1-\frac{4}{3}s_W^2 \end{pmatrix}$$

$$\mathcal{U}^{uR} \begin{pmatrix} -\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & -\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & -\frac{4}{3}s_W^2 \end{pmatrix} \mathcal{U}^{uR\dagger} = \begin{pmatrix} -\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & -\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & -\frac{4}{3}s_W^2 \end{pmatrix}$$

no flavour-changing neutral couplings

GIM breaking:
4th chapter

Interactions: *W*

Charged current interactions are left-handed and couple the top quark to the three charge -1/3 quarks *d*, *s*, *b*.

$$-\frac{g}{\sqrt{2}}\left[V_{td}\,\bar{t}_L\gamma^\mu d_L + V_{ts}\,\bar{t}_L\gamma^\mu s_L + V_{tb}\,\bar{t}_L\gamma^\mu b_L\right] W_\mu^+ + \text{h.c.}$$

These interactions are very important because they are responsible of the top quark decay $t \to W^+d$, $t \to W^+s$, $t \to W^+b$ with widths

$$\Gamma(t \to W^+d) : \Gamma(t \to W^+s) : \Gamma(t \to W^+b) = |V_{td}|^2 : |V_{ts}|^2 : |V_{tb}|^2$$

The SM predicts $|V_{td}|,|V_{ts}| \ll |V_{tb}| \simeq 1$, so the top quark almost always decays

$$\boxed{t \to W^+ b} \qquad W^+ \to \ell^+\nu, q\bar{q}$$

Experimentally, $|V_{td}|,|V_{ts}| \ll |V_{tb}|$ has been confirmed.

Interactions: *H*

The top interaction with the Higgs is

$$-\frac{1}{\sqrt{2}} y_t \, \bar{t} \, t H$$

Flavour-changing terms are possible but not present in the SM because:

- Only one scalar doublet introduced
- GIM mechanism

⇨ the unitary transformations that connect weak and mass eigenstates diagonalise the Higgs interactions too

Top mass

Everything so far mentioned is not very different from the other quarks.
What singles out the top quark?

the mass!

Indeed, the top quark is much heavier than the rest of fermions:

- 130x heavier than the next heaviest charge 2/3 quark (*c*)
- 36x heavier than its $SU(2)_L$ partner (*b*)
- 100x heavier than the heaviest lepton (τ)

Moreover, if its mass results from the Higgs mechanism with a single Higgs doublet [as it is predicted in the SM] its Yukawa coupling is remarkably close to one:

$$y_t \frac{v}{\sqrt{2}} = m_t \quad \Longrightarrow \quad y_t = 0.995$$

What does a heavy top mean to theorists?

- Maybe it is intrinsically different from the other quarks!
  - Top compositeness: the top quark is not elementary
  - Top partial compositeness: partly that…
  - …
- Maybe its detailed properties (interactions) are more sensitive to corrections from new heavy physics!
- Maybe it couples more strongly to new particles, so these new particles decay into top quarks!

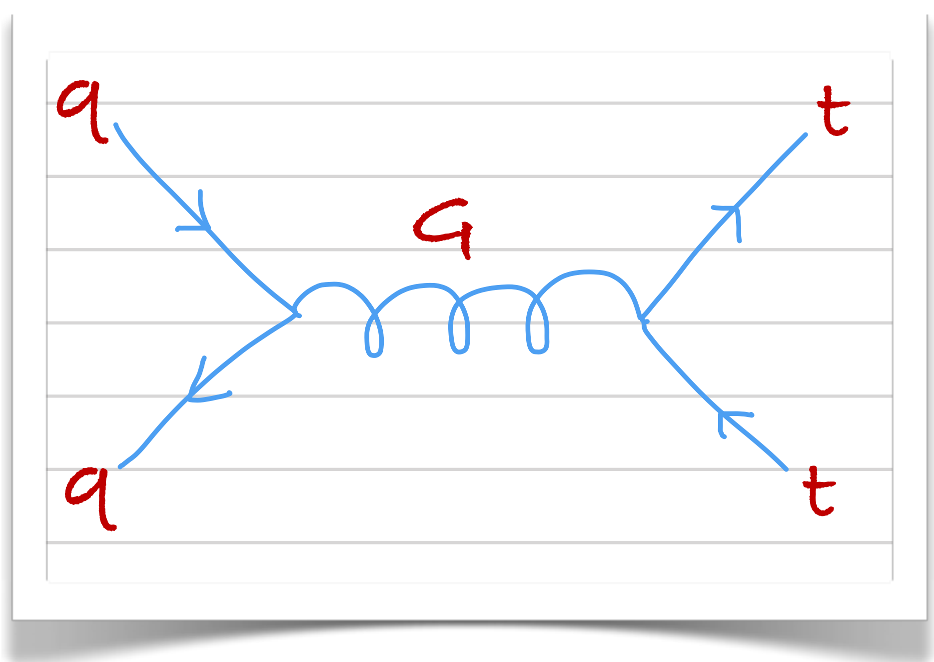
What does a heavy top mean for experimentalists?

- The top does not form hadrons $[t\bar{u},\ t\bar{t},\ \ldots\ ]$ because it decays $t \to W^{+}b$ before that can happen.

- Then, the information about how it was produced is preserved and can be investigated [analogue: the tau lepton].

- Then, there are many measurable quantities in top physics, that allow for detailed studies of its properties.

- On the other hand, top quarks are easy to tag and allow to probe the existence of new heavy particles ($G, Z', W', \ldots$)



Top as a window to new physics

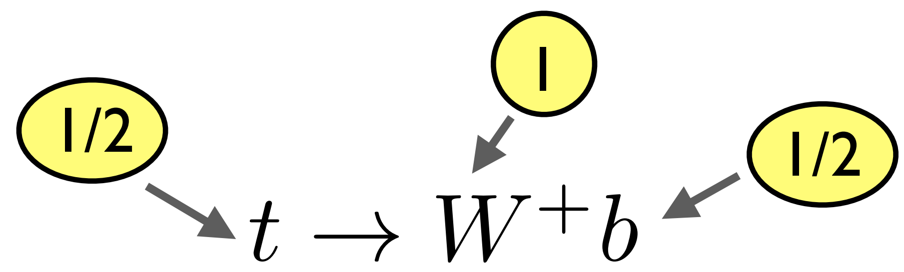
If new physics manifests in the top sector, it may appear in

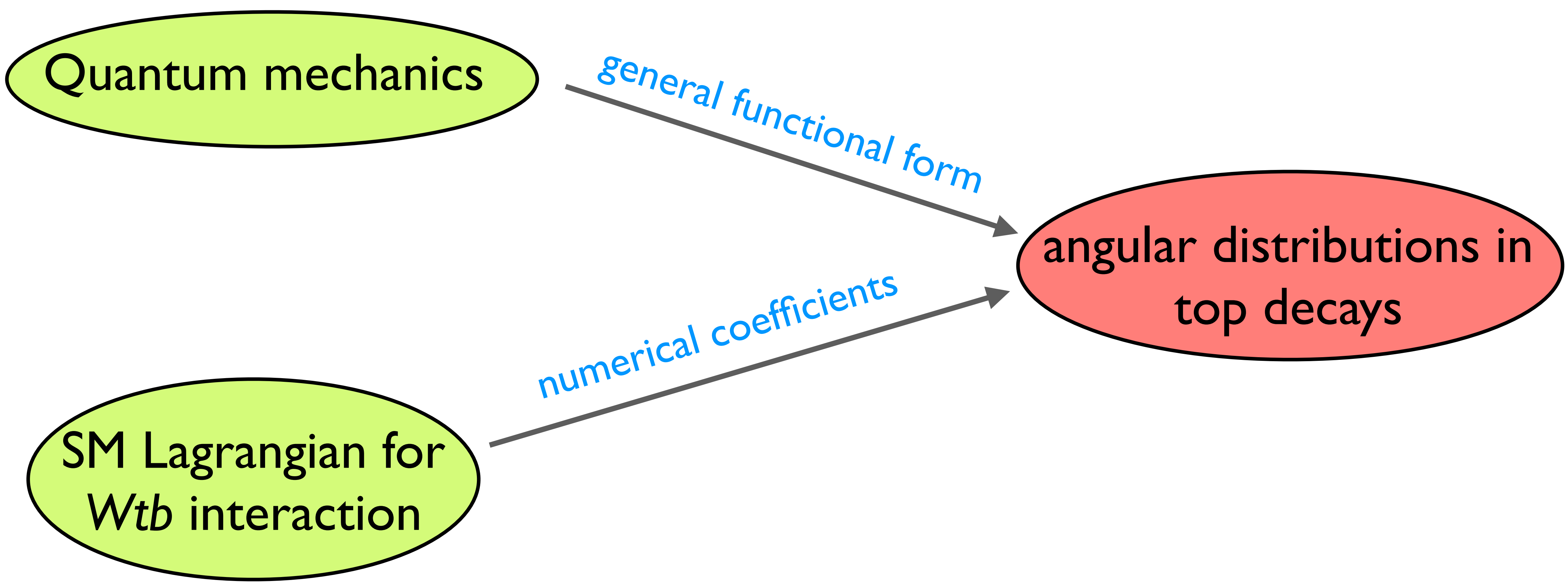
- top decays
  - corrections to SM decay $t \to W^+ b$
  - enhanced decays $t \to W^+ d$, $t \to W^+ s$
  - new decays $t \to Zc$, $t \to \gamma c$, … that are very rare in the SM
- top production
  - corrections to SM mechanisms
  - new production processes

We first discuss top decays and then single and pair production, in the SM as well as including some BSM possibilities.

Top quark decay $t \to W^+b$

The top quark is a spin-1/2 particle decaying into a spin-1 plus a spin-1/2 particle.

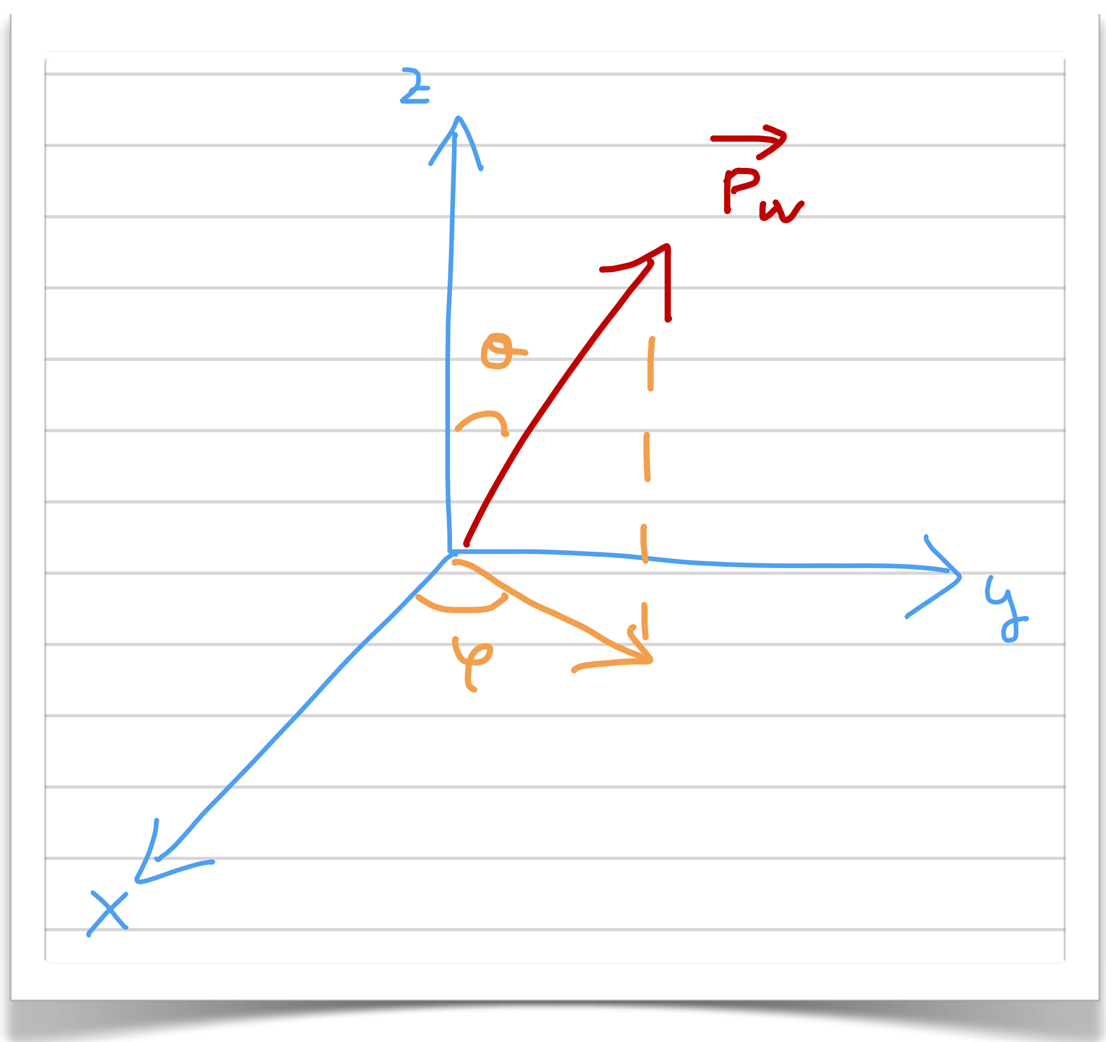




Let us assume we have an ensemble of polarised top quarks, with the spin along some direction, which we choose as our *z* axis. The *x* and *y* axes are not specified - for the moment.

initial (spin) state: $|\hat{z}, M\rangle\,, M = \frac{1}{2}$

Let θ, φ be the spherical coordinates of the *W* 3-momentum $\vec{p}_W$ in this reference system. The *b* quark moves in the opposite direction.



To quantise the spins of the *W* and *b*, we choose the helicity directions, $\vec{p}_W$ and $\vec{p}_b = -\vec{p}_W$, respectively [for the top the direction is arbitrary].

final (spin) state: $|\hat{p}_W, \lambda_1\rangle \otimes |\hat{p}_b, \lambda_2\rangle$

rotational invariance implies that the amplitude for the "polarised" decay

$$\overset{M}{t} \to \overset{\lambda_1}{W^+}\overset{\lambda_2}{b}$$

can be written as

$$A_{M\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} D^{\frac{1}{2}*}_{M\Lambda}(\phi, \theta, 0)$$

with $a_{\lambda_1\lambda_2}$ a number that depends on λ_1, λ_2, $\Lambda = \lambda_1 - \lambda_2$ and $D^{\frac{1}{2}}_{M\Lambda}$ the so-called Wigner functions

rotation parameterised by Euler angles

$$D^j_{m'm}(\alpha, \beta, \gamma) \equiv \langle jm' | e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z} | jm \rangle$$

The numbers $a_{\lambda_1\lambda_2}$ are given by the dynamics of the decay. Still, quantum mechanics has a lot to say.

Total angular momentum in the direction of $\vec{p}_W$

$\lambda_1 \backslash \lambda_2$	1/2	-1/2
1	1/2	3/2
0	-1/2	1/2
-1	-3/2	-1/2

the relative orbital angular momentum of *W* and *b* is zero in this direction since

$$\vec{L} = \vec{r} \times \vec{p}$$

impossible because initial state has j=1/2

So, quantum mechanics tells us that $a_{1\,-\frac{1}{2}} = a_{-1\,\frac{1}{2}} = 0$

In addition, the SM Lagrangian [$\bar{t}_L \gamma^\mu b_L$] tells us that $a_{\lambda_1\,\frac{1}{2}} = 0$ in the limit $m_b = 0$

⇒ W bosons seldom produced with λ_1 = 1 in top decays !

[Experimentally confirmed]

The picture is not yet complete because:

- in general, the top quarks are not produced in a definite spin state

  ⇨ must use a spin density matrix

- the *W* decays and its helicity is not measured

  ⇨ must include *W* decay too

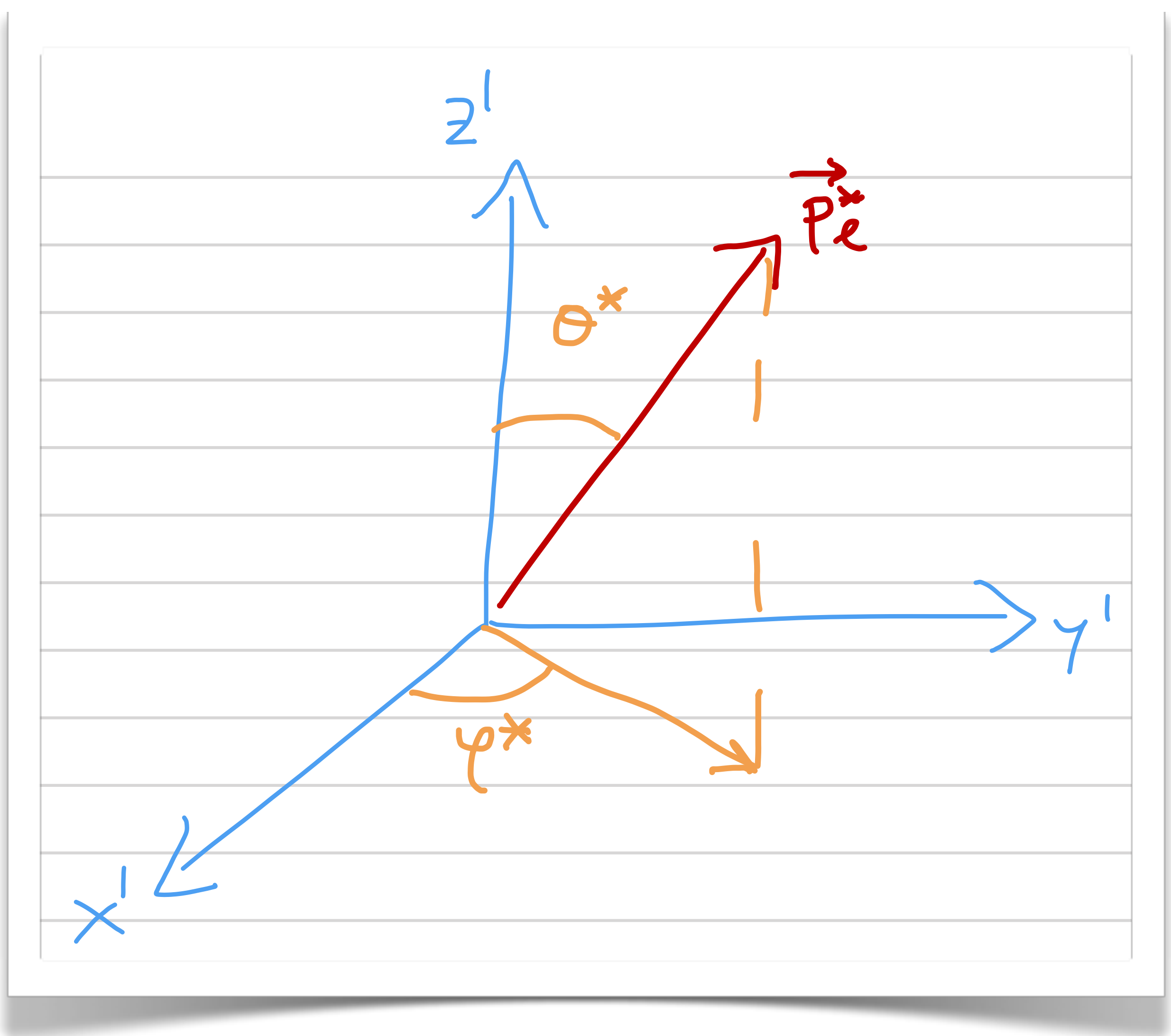
- the helicity of the *b* is not measured either

  ⇨ must sum over *b* helicities

Let us implement all these, step by step.

The [leptonic] decay of the *W* can be described in a similar fashion introducing a (x', y', z') coordinate system in the *W* rest frame

$$\overset{m}{W^+} \to \overset{\lambda_3}{\ell^+}\overset{\lambda_4}{\nu}$$



$$A_{m\lambda_3\lambda_4} = b_{\lambda_3\lambda_4} D^{1*}_{m\lambda}(\phi^*, \theta^*, 0)\,, \qquad \lambda = \lambda_3 - \lambda_4$$

Quantum mechanics does not tell us anything about $b_{\lambda_3 \lambda_4}$: all combinations are allowed in principle.

Total angular momentum
in the direction of $\vec{p}_\ell^{\,*}$

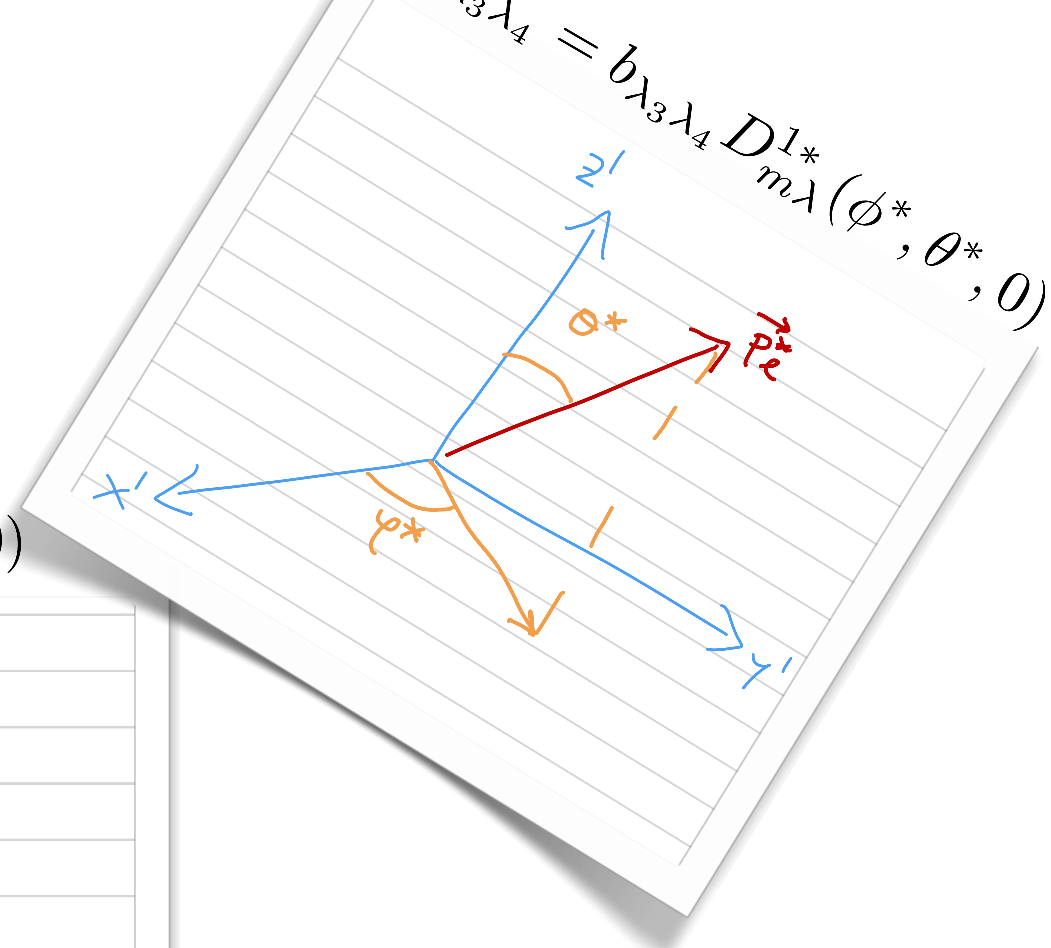
$\lambda_3 \setminus \lambda_4$	1/2	-1/2
1/2	0	1
-1/2	-1	0

all combinations allowed by quantum mechanics

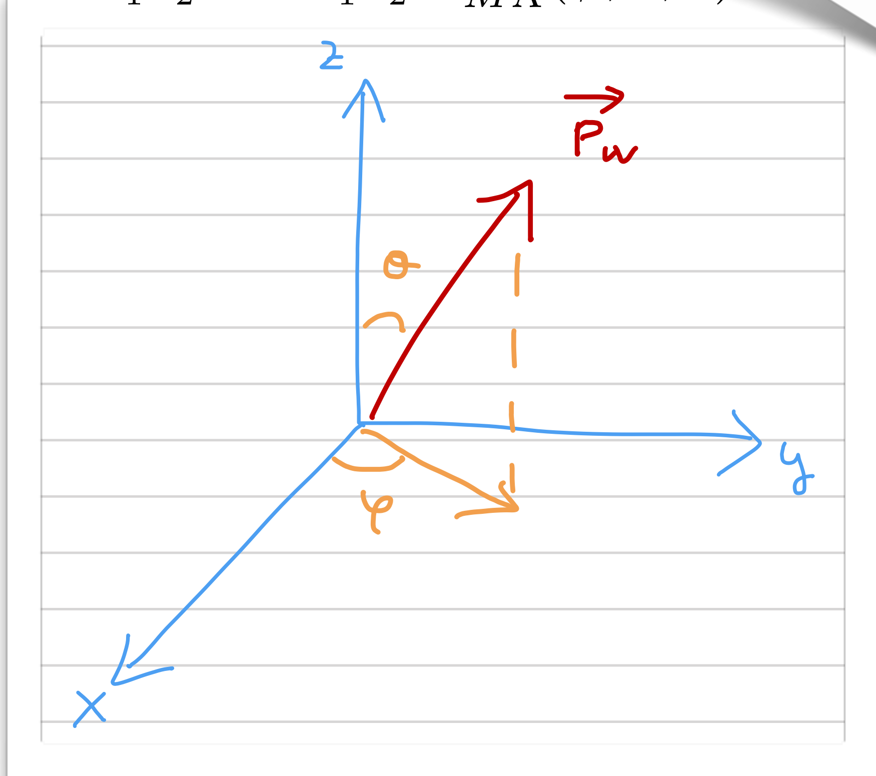
But the SM predicts [and it is confirmed by high-precision measurements] that the *W*l*ν* interaction is left-handed [$\bar{\ell}_L \gamma^\mu \nu_L$], so all coefficients are zero except $b_{\frac{1}{2}\,-\frac{1}{2}}$

Now, the decay chain can be connected by choosing z' precisely in the direction of $\vec{p}_W$, so that $m = \lambda_1$

$$A_{m\lambda_3\lambda_4} = b_{\lambda_3\lambda_4} D^{1*}_{m\lambda}(\phi^*, \theta^*, 0)$$



$$A_{M\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} D^{\frac{1}{2}*}_{M\Lambda}(\phi, \theta, 0)$$



we are using here the narrow width approximation

$$A_{M\lambda_2\lambda_3\lambda_4} = \sum_{\lambda_1} a_{\lambda_1\lambda_2} b_{\lambda_3\lambda_4} D^{\frac{1}{2}*}_{M\Lambda}(\phi, \theta, 0) D^{1*}_{\lambda_1\lambda}(\phi^*, \theta^*, 0)$$

Top quarks are not generally produced in a definite spin state. The most general (spin) state of an ensemble of top quarks can be described by a density matrix [hermitian and with unit trace]

$$\rho = \begin{pmatrix} \rho_{\frac{1}{2}\,\frac{1}{2}} & \rho_{\frac{1}{2}\,-\frac{1}{2}} \\ \rho_{-\frac{1}{2}\,\frac{1}{2}} & \rho_{-\frac{1}{2}\,-\frac{1}{2}} \end{pmatrix}$$

that contains 3 independent real parameters. Since the expectation values of operators are $\langle O \rangle = \mathrm{tr}[\rho O]$, once we fix any reference system (x, y, z), the density matrix can be written in terms of the expected values of the spin operators in this reference system,

$$\rho = \frac{1}{2}\begin{pmatrix} 1+P_z & P_x+iP_y \\ P_x-iP_y & 1-P_z \end{pmatrix}, \qquad P_i \equiv 2\langle S_i \rangle$$

and the squared matrix element has the form

$$|\mathcal{M}|^2 = \sum_{MM'} \rho_{MM'} A_M A^*_{M'}$$

Then, the differential decay width looks as terrible as

global phase space factor

b helicities summed

common factor

$$\frac{d\Gamma}{d\phi\, d\cos\theta\, d\phi^* d\cos\theta^*} = C \sum_{MM'\lambda_1\lambda_1'\lambda_2} \rho_{MM'} a_{\lambda_1\lambda_2} a^*_{\lambda_1'\lambda_2} |b_{\lambda_3\lambda_4}|^2$$

$$\times D^{\frac{1}{2}*}_{M\lambda}(\phi,\theta,0) D^{\frac{1}{2}}_{M'\lambda'}(\phi,\theta,0)$$

$$\times D^{1*}_{\lambda_1\lambda}(\phi^*,\theta^*,0) D^{1}_{\lambda_1'\lambda}(\phi^*,\theta^*,0)$$

Since this is really frightening, let us integrate azimuthal angles.

The integration is really easy, once we remember that $J_z|jm\rangle = m|jm\rangle$

$$D^j_{m'm}(\alpha,\beta,\gamma) = \langle jm'|e^{-i\alpha J_z}e^{-i\beta J_y}e^{-i\gamma J_z}|jm\rangle = e^{-i\alpha m'}e^{-i\gamma m}\langle jm'|e^{-i\beta J_y}|jm\rangle$$
$$\equiv e^{-i\alpha m'}e^{-i\gamma m}d^j_{m'm}(\beta)$$

$$\Rightarrow \quad \int d\phi = 2\pi\delta_{MM'} \qquad \int d\phi^* = 2\pi\delta_{\lambda_1\lambda_1'}$$

kills off-diagonal density matrix contributions

kills interference of different *W* polarisations

By integrating over ϕ, ϕ^* we have erased all quantum interference effects!

And the result is

$$\frac{d\Gamma}{d\cos\theta\, d\cos\theta^*} = 4\pi^2 C|b_{\lambda_3\lambda_4}|^2 \sum_{M\lambda_1\lambda_2} \rho_{MM}|a_{\lambda_1\lambda_2}|^2 \left[d^{\frac{1}{2}}_{M\lambda}(\theta)d^1_{\lambda_1\lambda}(\theta^*)\right]^2$$

Note: this is the motivation for apparently quantum-mechanics-unaware calculations that assume that *t*, *W* have definite spins.

And where can we get these *d*'s?

40. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J M	J M	…
m_1	m_2	Coefficients		
m_1	m_2			
⋮	⋮			

1/2×1/2

m_1 m_2	1 +1
+1/2 +1/2	1

m_1 m_2	1 0	0 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	1 −1
−1/2 −1/2	1

1×1/2

m_1 m_2	3/2 +3/2
+1 +1/2	1

m_1 m_2	3/2 +1/2	1/2 +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2 −1/2	1/2 −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2 −3/2
−1 −1/2	1

2×1

m_1 m_2	3 +3
+2 +1	1

m_1 m_2	3 +2	2 +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3 +1	2 +1	1 +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3 0	2 0	1 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3 −1	2 −1	1 −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3 −2	2 −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3 −3
−2 −1	1

1×1

m_1 m_2	2 +2
+1 +1	1

m_1 m_2	2 +1	1 +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2 0	1 0	0 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2 −1	1 −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2 −2
−1 −1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

3/2×1

m_1 m_2	5/2 +5/2
+3/2 +1	1

m_1 m_2	5/2 +3/2	3/2 +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2 −3/2	3/2 −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2 −5/2
−3/2 −1	1

$$d^{\,\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

2×1/2

m_1 m_2	5/2 +5/2
+2 +1/2	1

m_1 m_2	5/2 +3/2	3/2 +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2 +1/2	3/2 +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2 −1/2	3/2 −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2 −3/2	3/2 −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2 −5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	2 +2
+3/2 +1/2	1

m_1 m_2	2 +1	1 +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2 0	1 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2 −1	1 −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2 −2
−3/2 −1/2	1

$$\begin{aligned}&\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle \\ &= (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle\end{aligned}$$

here!

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

3/2×3/2

m_1 m_2	3 +3
+3/2 +3/2	1

m_1 m_2	3 +2	2 +2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

m_1 m_2	3 +1	2 +1	1 +1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

m_1 m_2	3 0	2 0	1 0	0 0

2×3/2

m_1 m_2	7/2 +7/2
+2 +3/2	1

m_1 m_2	7/2 +5/2	5/2 +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

m_1 m_2	7/2 +3/2	5/2 +3/2	3/2 +3/2
+2 −1/2	1/7	16/35	2/5

$$d^{\,1}_{0,0} = \cos\theta \qquad d^{\,1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{\,1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{\,1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{\,1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{\,1}_{1,-1} = \frac{1-\cos\theta}{2}$$

We now have all the tools to calculate a couple of simple distributions that can be measured at the Tevatron and the LHC:

- the distribution of the *W* decay products with respect to $\vec{p}_W$

→ it allows to measure the *W* helicity in top decays

- the distribution of the top decay products with respect to a fixed axis

→ it allows to measure the top polarisation along this axis

First, we have to normalise to the total width. Integrating $\dfrac{d\Gamma}{d\cos\theta \, d\cos\theta^*}$ over θ and θ*,

$$\Gamma = \frac{8\pi^2}{3} C |b_{\lambda_3\lambda_4}|^2 \left\{ |a_{-1\,-\frac{1}{2}}|^2 + |a_{0\,-\frac{1}{2}}|^2 + |a_{0\,\frac{1}{2}}|^2 + |a_{1\,\frac{1}{2}}|^2 \right\}$$

sum of non-zero $|a|^2$
as expected

1. Integrating $\dfrac{d\Gamma}{d\cos\theta\, d\cos\theta^*}$ over θ we get a well-known distribution

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{3}{8}(1+\cos\theta^*)^2 F_+ + \frac{3}{8}(1-\cos\theta^*)^2 F_- + \frac{3}{4}\sin^2\theta^* F_0$$

with

$$F_+ = \frac{|a_{1\,\frac{1}{2}}|^2}{\sum |a|^2}$$

fraction of W's with λ_l=1

$$F_- = \frac{|a_{-1\,-\frac{1}{2}}|^2}{\sum |a|^2}$$

fraction of W's with λ_l=-1

$$F_0 = \frac{|a_{0\,-\frac{1}{2}}|^2 + |a_{0\,\frac{1}{2}}|^2}{\sum |a|^2}$$

fraction of W's with λ_l=0

Experimentally [CMS 2013]

$$F_+ = 0.008 \pm 0.018$$
$$F_- = 0.310 \pm 0.031$$
$$F_0 = 0.682 \pm 0.045$$

Prediction for left-handed b [for example SM]

$$F_+ \simeq 0$$
$$F_- = \frac{|a_{-1\,-\frac{1}{2}}|^2}{\sum |a|^2}, \quad F_0 = \frac{|a_{0\,-\frac{1}{2}}|^2}{\sum |a|^2}$$

To obtain the values of F_- and F_0 we need an explicit calculation

Helicity fractions in the SM

Leading order (LO)

$F_+ = 0.0004$
$F_- = 0.302$
$F_0 = 0.697$

difference

0.07σ
0.29σ
0.22σ

Next-to-next-to LO (NNLO)

$F_+ = 0.0017$
$F_- = 0.311$
$F_0 = 0.687$

Experimentally [CMS 2013]

$F_+ = 0.008 \pm 0.018$
$F_- = 0.310 \pm 0.031$
$F_0 = 0.683 \pm 0.045$

Therefore, the tree-level calculation provides a more than acceptable approximation given the current and forthcoming experimental precision.

2. Integrating $\dfrac{d\Gamma}{d\cos\theta\, d\cos\theta^*}$ over θ^* we get the distribution

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{1}{2}\left(1 + P_z \alpha_W \cos\theta\right)$$

with $\alpha_W = \dfrac{|a_{1\,\frac{1}{2}}|^2 + |a_{0\,-\frac{1}{2}}|^2 - |a_{0\,\frac{1}{2}}|^2 - |a_{-1\,-\frac{1}{2}}|^2}{\sum |a|^2}$ ← \`spin analysing power´ of the *W*

Q&A mini-session

1. What does distribution mean?

   If we choose <u>any</u> \`*z*´ axis, the distribution of *W* momenta with respect to it follows that equation, with P_z the top polarisation [$2\langle S_z\rangle$] along <u>that</u> axis [which may be zero].

2. What can be it used for?

   To measure the top polarisation P_z along any given axis [with the implicit assumption that the spin analysing power α_W takes its SM value].

3. Why is α_W called \`spin analysing power´?

The larger is $|\alpha_W|$, the larger is the correlation between the W momentum direction and the top spin. And the better it allows to determine P_z. Obviously, $|\alpha_W| \leq 1$.

4. Could be calculate α_W in the SM right now without writing Feynman diagrams, etc.?

Sure.

For a left-handed *Wtb* interaction we saw that $a_{\lambda_1\, \frac{1}{2}} = 0$ in the [good] approximation of massless *b*. Then,

$$\alpha_W = \frac{|a_{0\,-\frac{1}{2}}|^2 - |a_{-1\,-\frac{1}{2}}|^2}{\sum |a|^2} = F_0 - F_- = 0.395$$

Of course, we *had* to write Feynman diagrams to calculate the *F* 's.

5. Are there analogous distributions for top decay products other than *W* and *b*?

Sure. For example, if (θ_l,φ_l) are the spherical coordinates of the charged lepton 3-momentum in the top quark rest frame $\vec{p}_\ell$, we have the distribution

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{1}{2}\left(1 + P_z \alpha_\ell \cos\theta_\ell\right)$$

[do not confuse with (θ^*,φ^*), which correspond to the charged lepton 3-momentum in the *W* boson rest frame $\vec{p}^{\,*}_\ell$]

In the SM α_l = 1

the charged lepton distribution has the largest possible correlation with the top polarisation and is the best suited to determine P_z.

[With the implicit assumption α_l = 1]

In general, α_ℓ is a function of $a_{\lambda_1\lambda_2}$ and not only their moduli. The interference between *a*'s is essential.

What about anti-top decays?

The helicity fractions ($\bar{F}$) are exchanged:

$$\bar{F}_0 = F_0$$
$$\bar{F}_+ = F_-$$
$$\bar{F}_- = F_+$$

The spin analysing powers ($\alpha_{\bar{X}}$) change sign:

$$\alpha_{\bar{X}} = -\alpha_X$$

Top decays beyond the SM

New physics may induce tree-level or radiative corrections to the top interactions. Some of these corrections may manifest in top decays [and some in top production].

- corrections to the *Wtb* vertex ⇒ modification of $t \to W^+b \to l^+\nu b$ angular distributions
- enhanced V_{td} / V_{ts} ⇒ decays $t \to W^+d$, $t \to W^+s$
- enhanced *t-u* / *t-c* interactions with Z, γ, g, H ⇒ flavour-changing neutral decays

Also, new particles lighter than the top may induce new channels, such as $t \to H^+b$

Corrections to the *Wtb* vertex

As we have seen, the angular distributions in $t \to W^+b \to l^+\nu b$ are determined by angular momentum conservation and the specific *Wtb* interaction $[\bar{t}_L\gamma^\mu b_L]$ of the SM.

The first always holds, but the latter can be changed with new physics. The most general *Wtb* interaction is

$V_{tb}^* \simeq 1$ in the SM

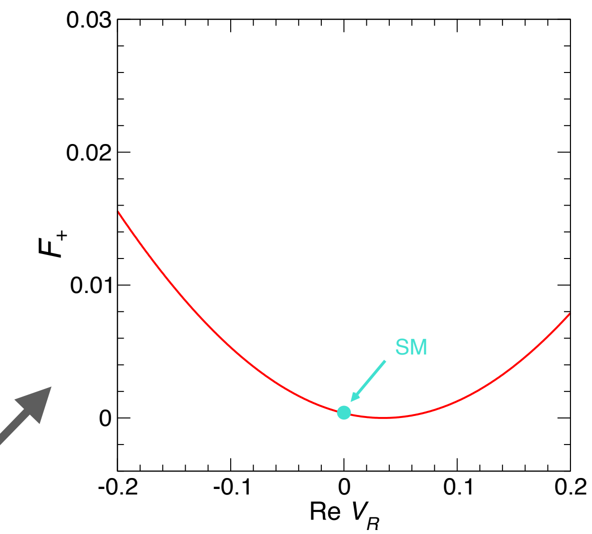
0 at tree level

$$\mathcal{L}_{Wtb} = -\frac{g}{\sqrt{2}}\bar{b}\gamma^\mu(V_L P_L + V_R P_R)t\,W_\mu^- - \frac{g}{\sqrt{2}}\bar{b}\frac{i\sigma^{\mu\nu}q_\nu}{M_W}(g_L P_L + g_R P_R)\,t\,W_\mu^- + \text{h.c.}$$

0 at tree level

Prominent effects of *anomalous Wtb couplings* in distributions

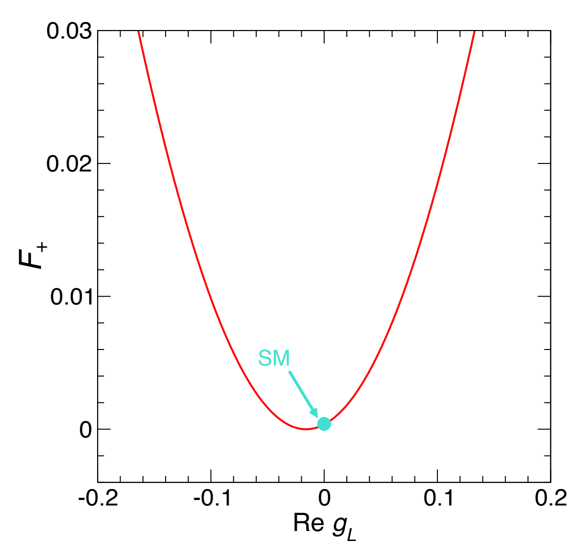
no effect as long as
$V_L \gg V_R, g_L, g_R$

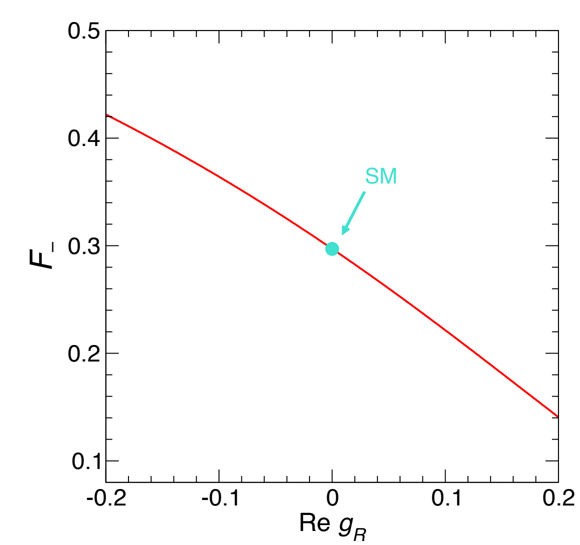


non-zero F_+

$$\mathcal{L}_{Wtb} = -\frac{g}{\sqrt{2}}\bar{b}\gamma^{\mu}(V_L P_L + V_R P_R)t\, W_{\mu}^{-}$$
$$-\frac{g}{\sqrt{2}}\bar{b}\frac{i\sigma^{\mu\nu}q_{\nu}}{M_W}\left(g_L P_L + g_R P_R\right)t\, W_{\mu}^{-} + \text{h.c.}$$

non-zero F_+





deviations in
F_- and F_0

Enhanced V_{td} / V_{ts}

The direct measurement of CKM matrix elements of the first two rows leaves little room for significant values of V_{td} or V_{ts}.

$$|V| = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 & \cdots \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 & \cdots \\ |V_{td}| & |V_{ts}| & |V_{tb}| & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

small mixings

$$|V_{td}|^2 \leq 0.008 + \sin^2\theta_d$$
$$|V_{ts}|^2 \leq 0.028 + \sin^2\theta_s$$

$$\xrightarrow{\sin\theta_{d,s} \lesssim 0.1} \quad \mathrm{Br}(t \to W^+ d, W^+ s) \lesssim 0.05$$

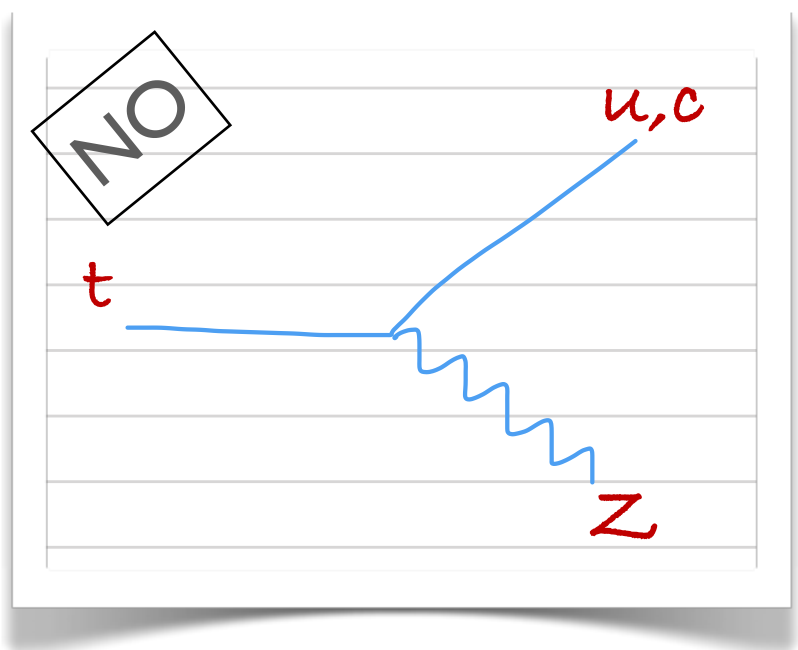
These decays are investigated by measuring the ratio [data agrees with SM]

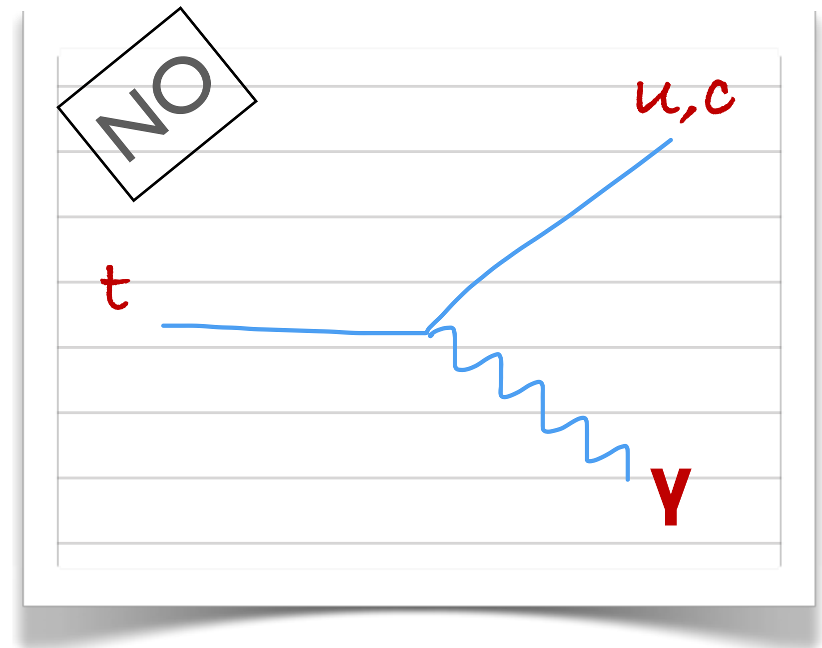
$$R = \frac{\mathrm{Br}(t \to W^+ b)}{\sum_{q=d,s,b} \mathrm{Br}(t \to W^+ q)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$$

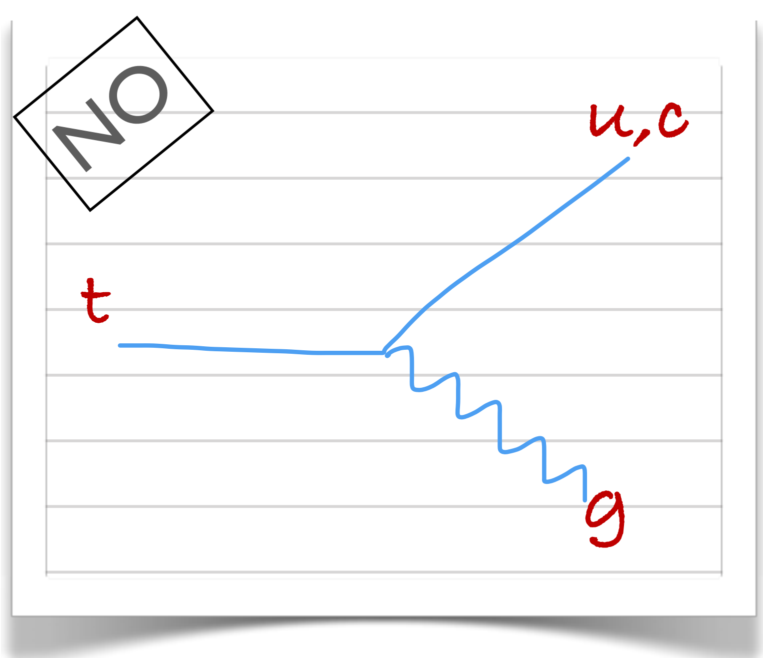
More in chapter 4

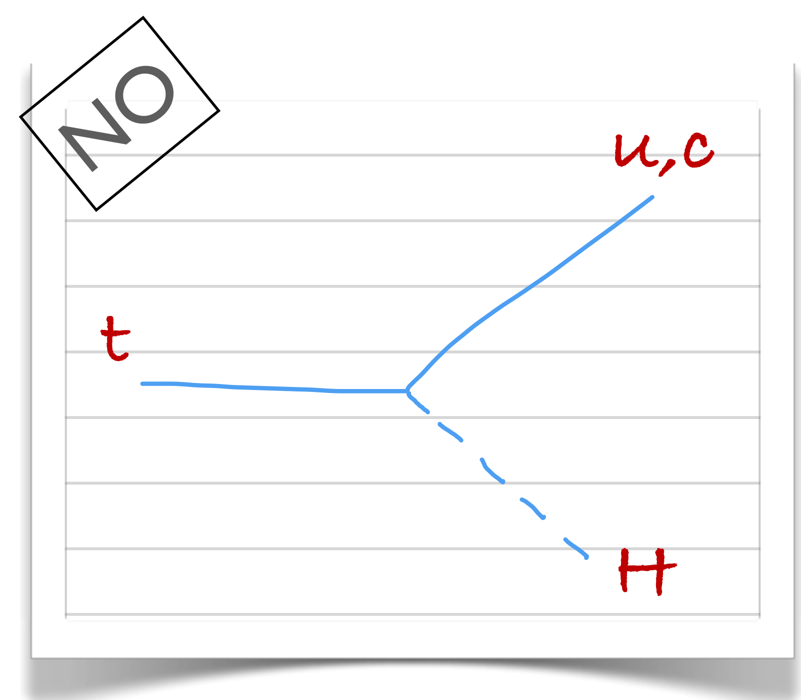
Top flavour-changing neutral decays

Top FCN interactions vanish at the tree level in the SM, as for any other quark.



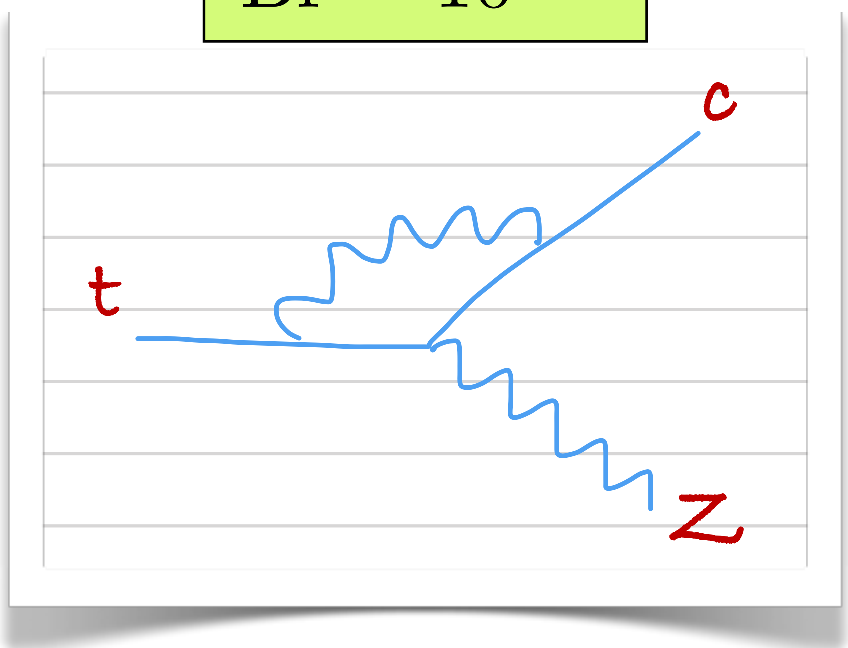




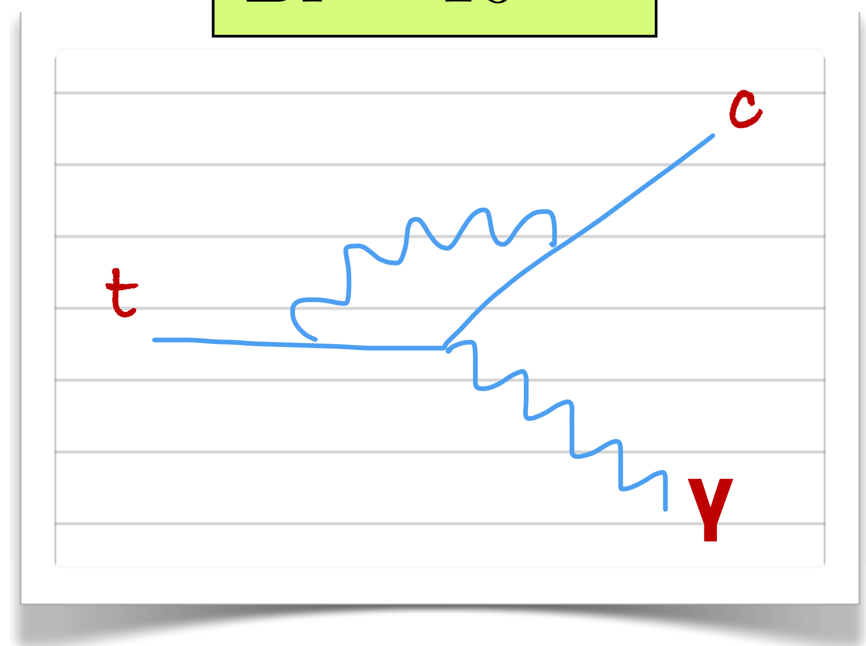


Top FCN decays can occur radiatively. But, in contrast with the lighter quarks, the branching ratios are tiny.

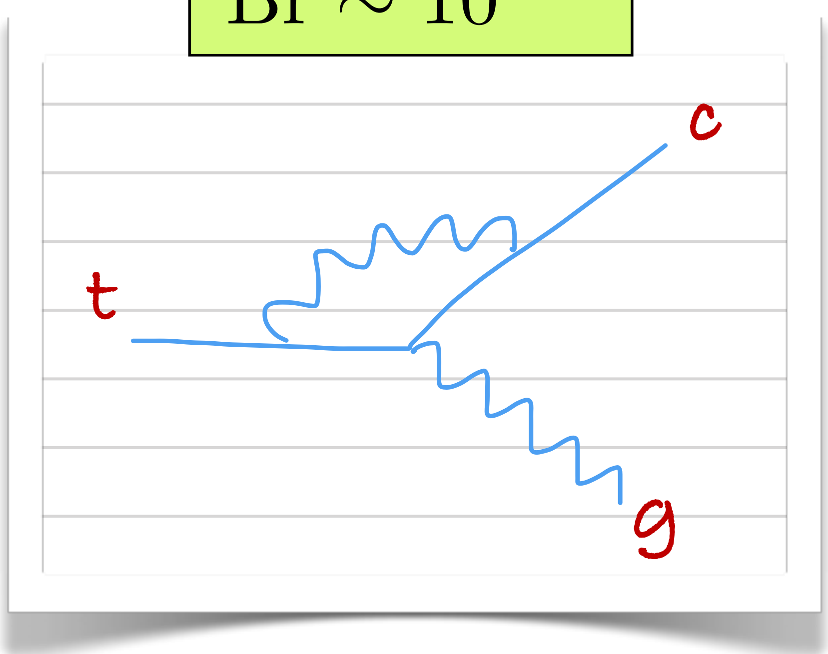
$\mathrm{Br} \sim 10^{-14}$



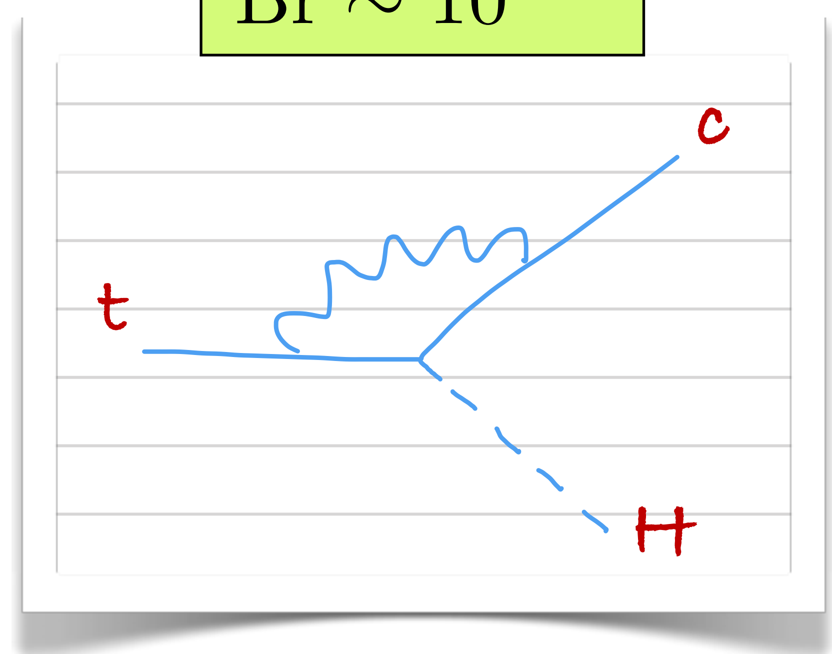
$\mathrm{Br} \sim 10^{-14}$



$\mathrm{Br} \sim 10^{-12}$



$\mathrm{Br} \sim 10^{-15}$

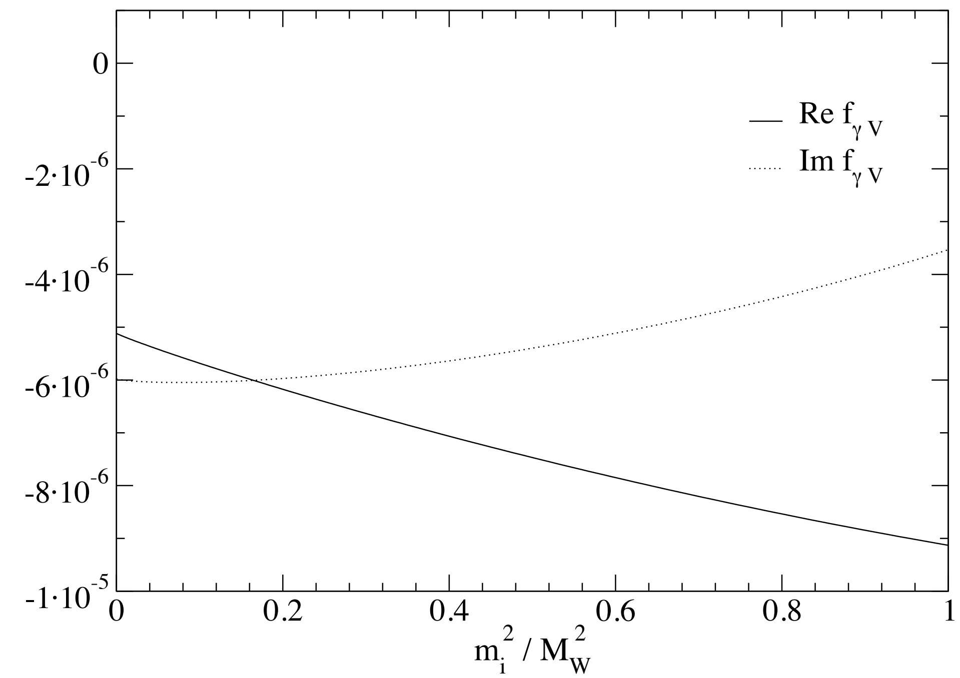


But why so small? Because amplitudes are proportional to sums

$$\sum_{q=d,s,b} f\left(\frac{m_q^2}{M_W^2}\right) V_{cq} V_{tq}^*$$

tcγ / tcg

$$f(x) = (-5.1 - 6.0\,i) + (-7.6 - 3.9\,i)\,x + \mathcal{O}(x^2)$$



[the three terms correspond to quarks *d*, *s*, *b* in the loop]

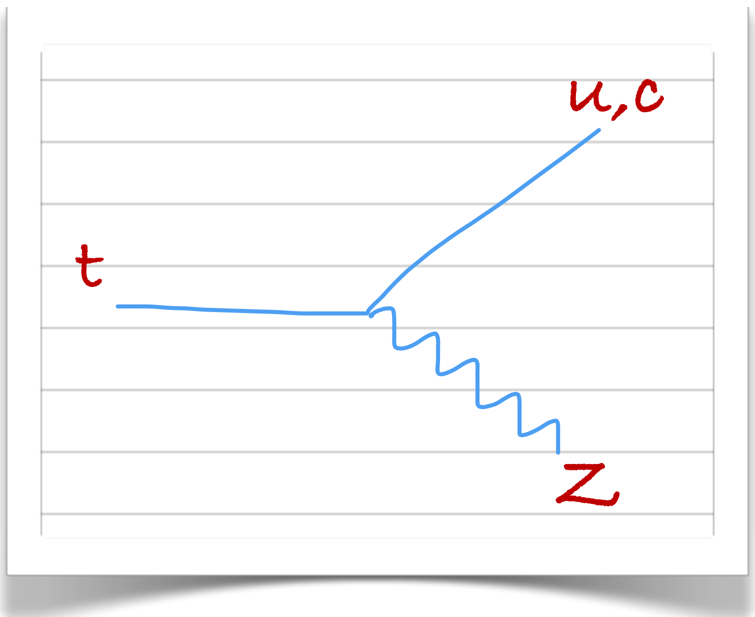
The constant term cancels due to the unitarity of the CKM matrix, and the linear term is suppressed by $m_b^2/M_W^2 \simeq 1.2 \times 10^{-3}$.

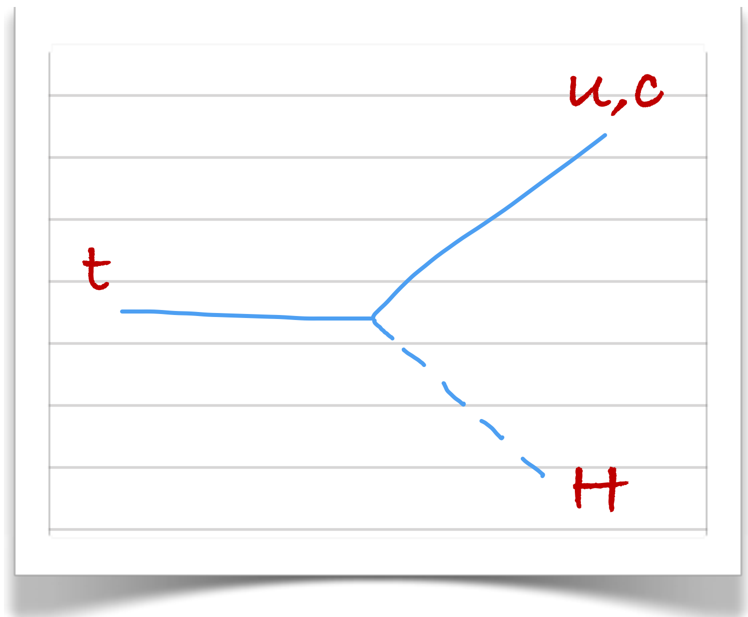
suppression factor of 10^{-6} in the decay width!

In addition, there is a suppression due to CKM mixings, which is stronger for $t \to u$.

How to overcome this suppression?

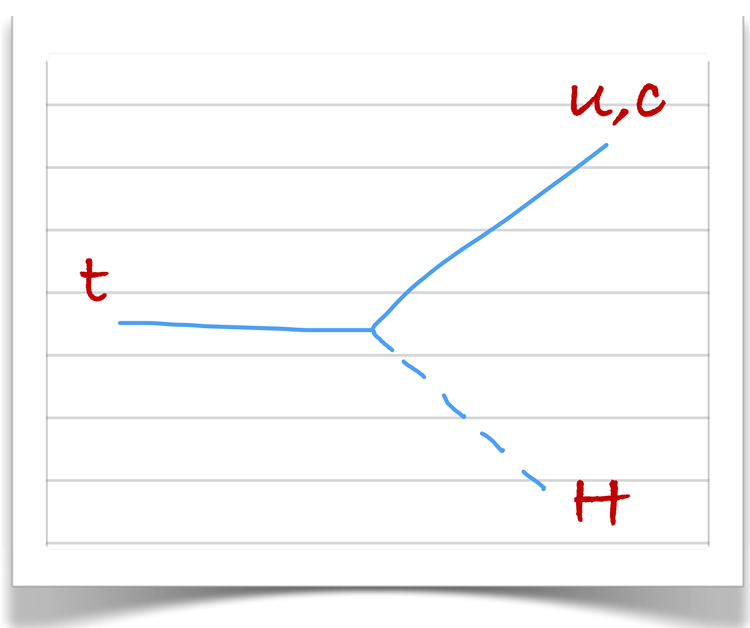
- Tree-level FCN couplings to Z / H [couplings to γ, g protected by gauge symmetry]
  - Extra vector-like quarks: breaking of GIM mechanism





+ enhanced *tc*γ and *tcg* at one loop

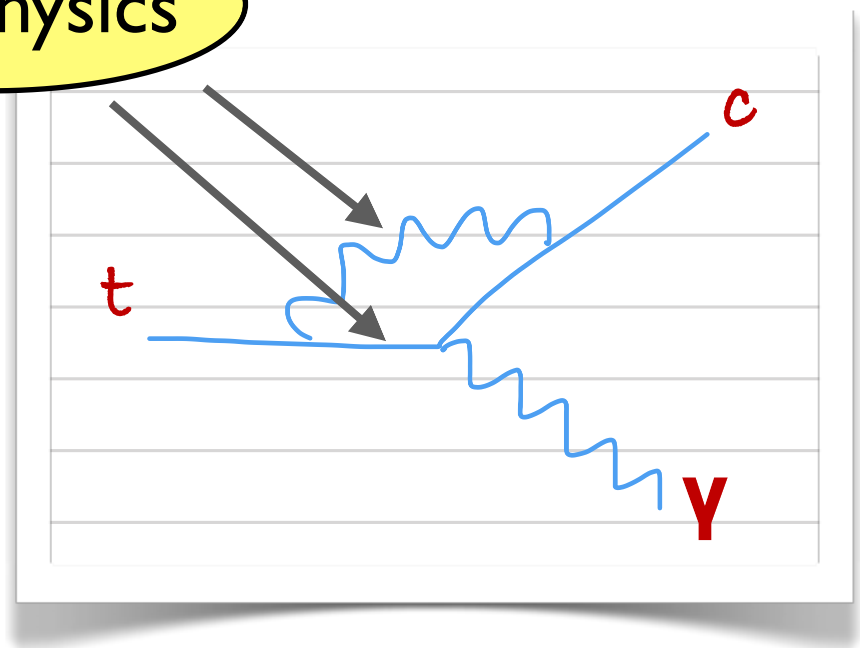
  - Extra scalar doublets: Yukawa matrices not generally aligned



+ enhanced *tc*γ and *tcg* at one loop

New radiative contributions to *effective vertices*

new physics



If the flavour couplings of the new physics do not follow the CKM pattern, the GIM suppression is not present.

Maximum branching ratios

	Extra quarks	Extra scalars
$t \to Zu$	10^{-4}	?
$t \to \gamma u$	10^{-8}	?
$t \to gu$	10^{-7}	?
$t \to Hu$	10^{-5}	10^{-6}

	Extra quarks	Extra scalars
$t \to Zc$	10^{-4}	10^{-7}
$t \to \gamma c$	10^{-8}	10^{-6}
$t \to gc$	10^{-7}	10^{-4}
$t \to Hc$	10^{-5}	10^{-3}

LHC future reach: ~10^{-6} [no positive signals found yet]

Extended quark sector and top mixing

The SM predictions for top mixing are based on the unitarity of the 3 x 3 CKM matrix and the absence of RH charged currents.

These predictions can change substantially - at the tree level - <u>only</u> if there are new heavy quarks.

New chiral quarks (for example 4th family) are now excluded [except for contrived model building with extra scalars].

But new quarks can also be vector-like, which means that the *L* and *R* parts transform under the same $\mathrm{SU}(2)_L$ irreducible representation.

$$\left(\cdot \right)_L , \left(\cdot \right)_R \qquad \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}_L , \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}_R \qquad \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}_L , \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}_R$$

Vector-like quarks coupling to SM quarks can appear in 7 possible multiplets [assuming the scalar sector only contains doublets]:

Singlets

charge 2/3 charge -1/3

$$T_{L,R} \qquad\qquad B_{L,R}$$

Doublets

charge 5/3

$$\begin{pmatrix} X \\ T \end{pmatrix}_{L,R} \qquad \begin{pmatrix} T \\ B \end{pmatrix}_{L,R} \qquad \begin{pmatrix} B \\ Y \end{pmatrix}_{L,R}$$

charge -4/3

Triplets

$$\begin{pmatrix} X \\ T \\ B \end{pmatrix}_{L,R} \qquad \begin{pmatrix} T \\ B \\ Y \end{pmatrix}_{L,R}$$

⇒ These are all the possibilities, no matter how one wants to name them (Little Higgs, composite top, …)

But why only these?

New quarks couple to SM ones is through Yukawa interactions. The SM has singlet and doublet quark fields.

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad , \quad u_R \ , \ d_R$$

Assuming the scalar sector comprises only doublets, as in the SM

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

the possible $\mathrm{SU}(2)_L$ representations are obtained from group theory:

$$2 \otimes 2 = 3 \oplus 1$$
$$2 \otimes 1 = 2$$

and the hypercharges of the new fields are determined by the SM ones.

Mixing with heavy quarks

In the SM, the mass eigenstates (for example $u_{L,R}$, $c_{L,R}$, $t_{L,R}$ in the up quark sector) are linear combinations of interaction eigenstates with the same charge ($u^0_{L,R}$, $c^0_{L,R}$, $t^0_{L,R}$).

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} u_L^0 \\ c_L^0 \\ t_L^0 \end{pmatrix} \qquad + \quad L \to R$$

When new electroweak eigenstates $T^0_{L,R}$ are added to the SM, the resulting mass eigenstates $u_{L,R}$, $c_{L,R}$, $t_{L,R}$, $T_{L,R}$ are linear combinations of all of them.

$$\begin{pmatrix} u_L \\ c_L \\ t_L \\ T_L \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} u_L^0 \\ c_L^0 \\ t_L^0 \\ T_L^0 \end{pmatrix} \quad + \quad L \to R$$

The same applies to the down sector, of course.

The mixing of new quarks is *expected* largest with the 3rd generation:

$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \simeq \begin{pmatrix} \cdot & \cdot & \varepsilon_{13} & \varepsilon_{14} \\ \cdot & \cdot & \varepsilon_{23} & \varepsilon_{24} \\ \varepsilon_{31} & \varepsilon_{32} & \cos\theta & -\sin\theta e^{i\phi} \\ \varepsilon_{41} & \varepsilon_{42} & \sin\theta e^{-i\phi} & \cos\theta \end{pmatrix}$$

ε_{ij} small

Therefore, to a good approximation

mass eigenstates

weak eigenstates

$$\begin{pmatrix} t_L \\ T_L \end{pmatrix} = \begin{pmatrix} \cos\theta^u_L & -\sin\theta^u_L e^{i\phi_u} \\ \sin\theta^u_L e^{-i\phi_u} & \cos\theta^u_L \end{pmatrix} \begin{pmatrix} t^0_L \\ T^0_L \end{pmatrix}$$

$$\begin{pmatrix} t_R \\ T_R \end{pmatrix} = \begin{pmatrix} \cos\theta^u_R & -\sin\theta^u_R e^{i\phi_u} \\ \sin\theta^u_R e^{-i\phi_u} & \cos\theta^u_R \end{pmatrix} \begin{pmatrix} t^0_R \\ T^0_R \end{pmatrix}$$

mass eigenstates

weak eigenstates

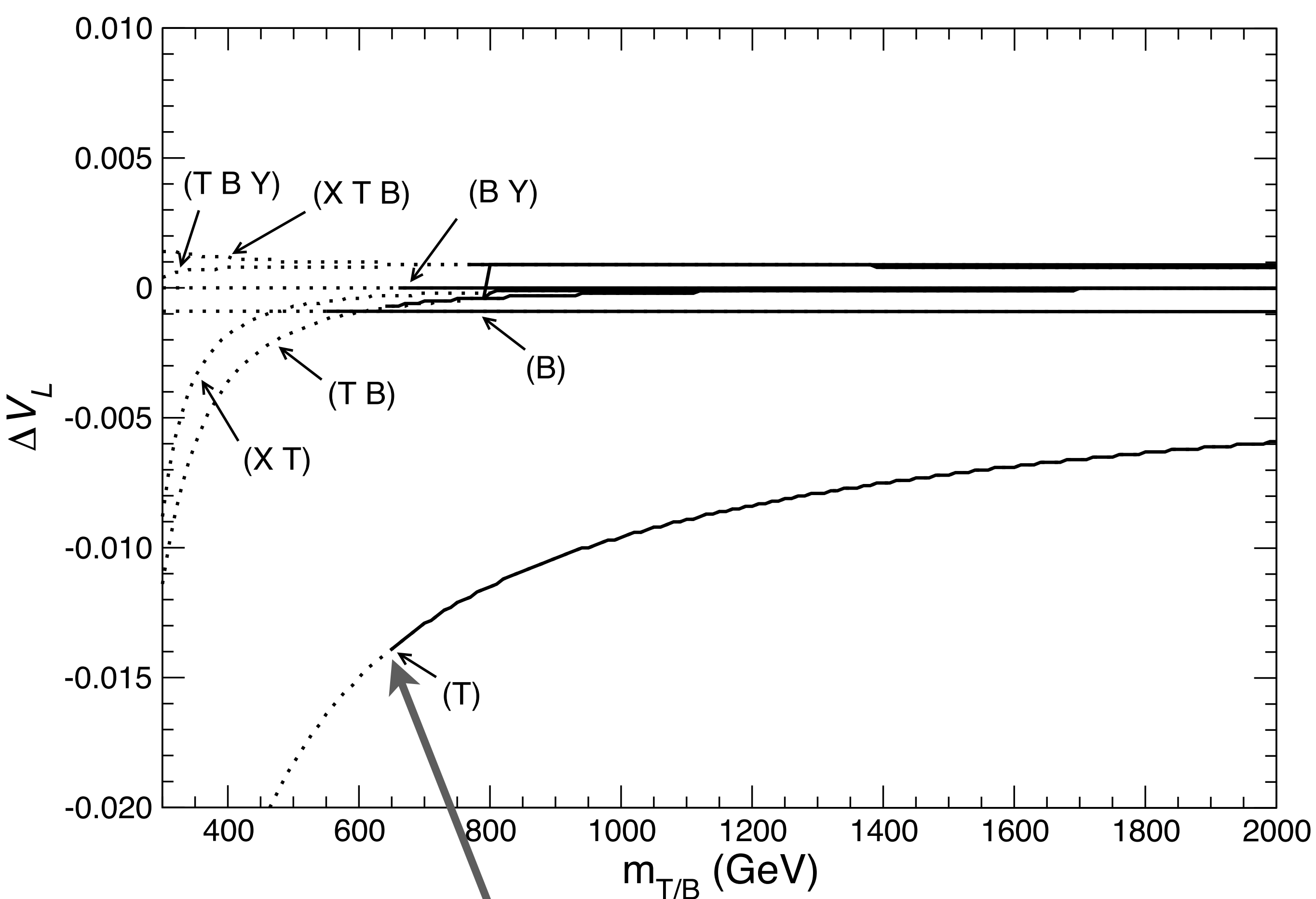
$$\begin{pmatrix} b_L \\ B_L \end{pmatrix} = \begin{pmatrix} \cos\theta^d_L & -\sin\theta^d_L e^{i\phi_d} \\ \sin\theta^d_L e^{-i\phi_d} & \cos\theta^d_L \end{pmatrix} \begin{pmatrix} b^0_L \\ B^0_L \end{pmatrix}$$

$$\begin{pmatrix} b_R \\ B_R \end{pmatrix} = \begin{pmatrix} \cos\theta^d_R & -\sin\theta^d_R e^{i\phi_d} \\ \sin\theta^d_R e^{-i\phi_d} & \cos\theta^d_R \end{pmatrix} \begin{pmatrix} b^0_R \\ B^0_R \end{pmatrix}$$

this mixing induces deviations in top & bottom couplings to *W*, *Z*, *H*

Effects in V_L

If new quarks mix with the top quark, $V_L = V_{tb}^*$ can be larger or smaller than its SM prediction [$V_{tb} = 0.999$].



maximum deviation
$\Delta V_L \sim -0.01$

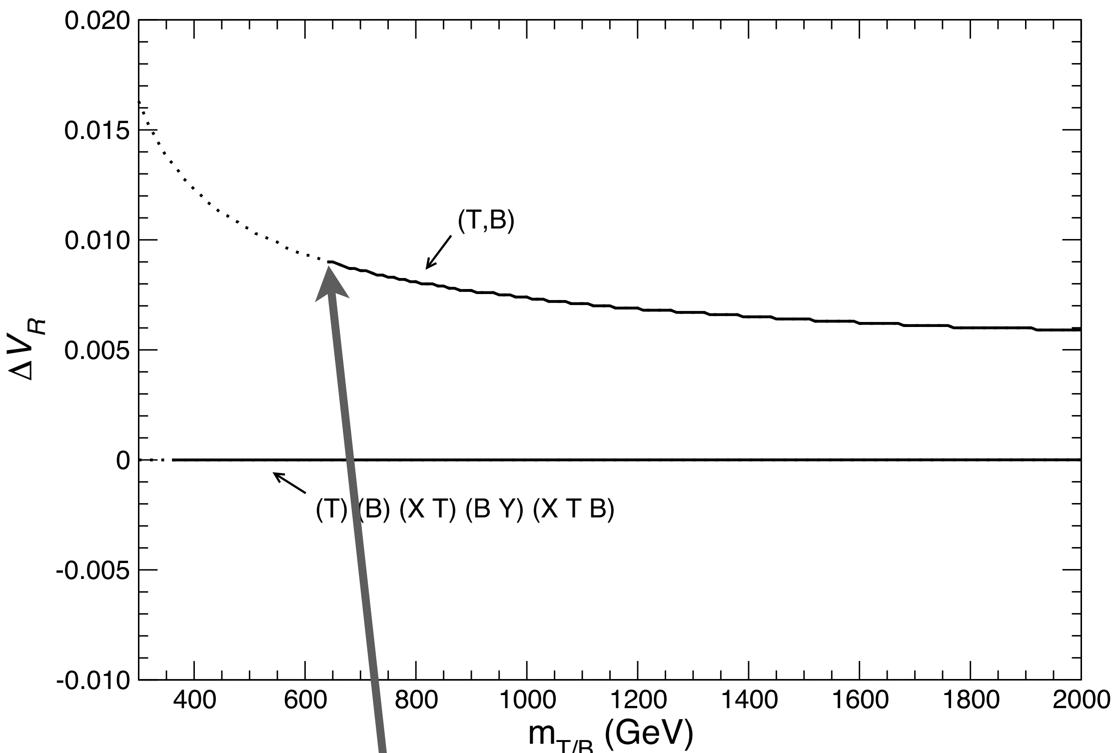
The possible deviations are subject to indirect constraints that depend on the masses of the new heavy quarks.

The constraints may be relaxed in non-minimal models.

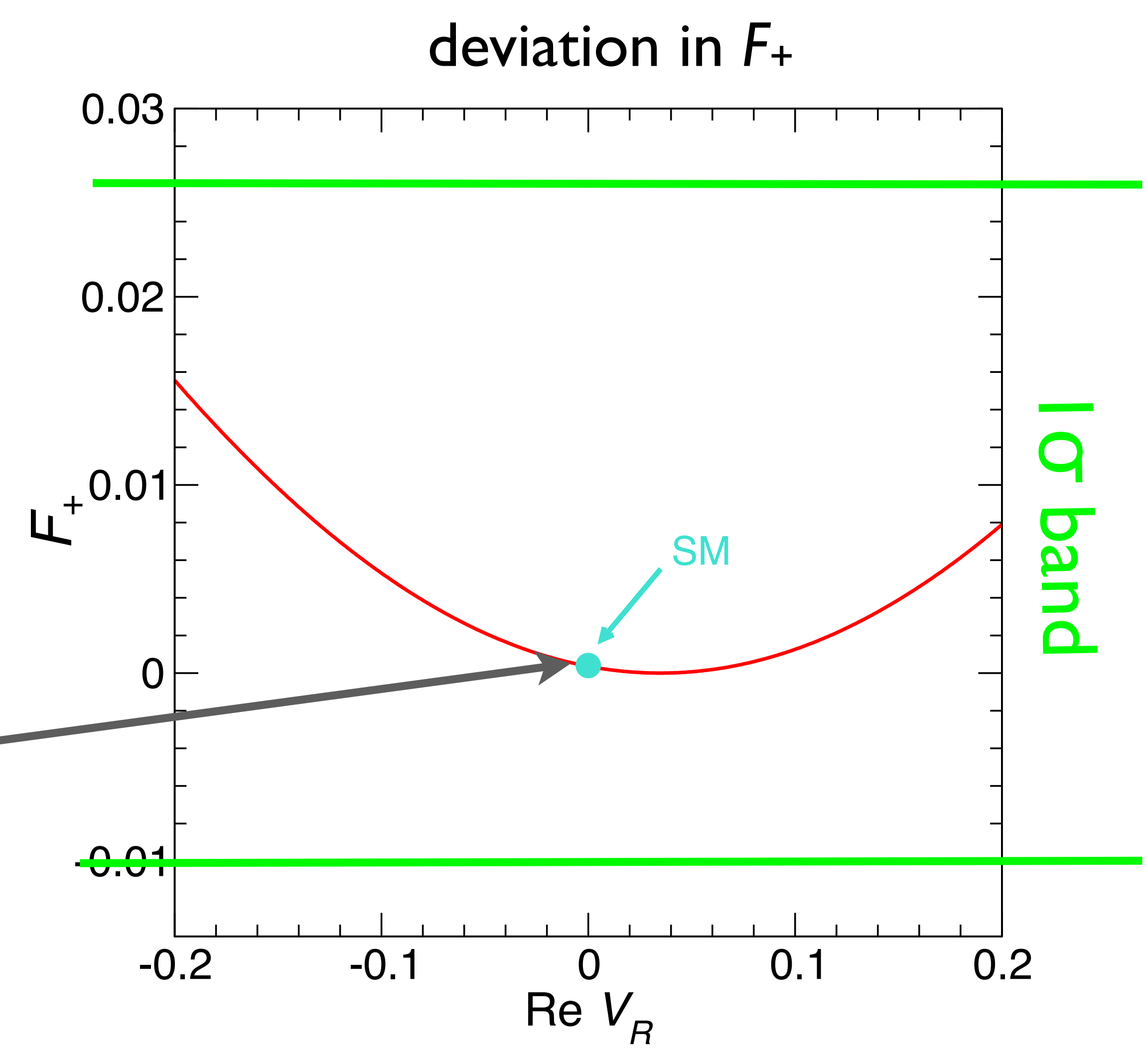
Deviations not visible in top decays

Effects in V_R

New multiplets that are not RH singlets introduced RH charged currents that communicate to SM quarks via mixing.



maximum value $V_R \sim$ 0.01



unobservable with current precision

Enhanced V_{td} / V_{ts}

The size of V_{td} and V_{ts} is constrained by unitarity and the measurements of the first two rows of the CKM matrix: In the absence of quark triplets, the sum of $|V|^2$ in a column must not exceed one:

$$\sum_{i=1}^{n} |V_{ij}|^2 \leq 1$$

If there exist triplets, the upper bound is one plus the square of the mixing with triplets:

$$\sum_{i=1}^{n} |V_{ij}|^2 \leq 1 + \sin^2\theta_j \leq 2$$

Still, this mixing modifies the couplings of the light quarks *u*,*d* / *c*,*s* to the *Z* and is somewhat constrained [apart from B physics constraints].

Likely, V_{td} and V_{ts} must be close to their SM values.

GIM breaking

We have seen that in the SM the neutral currents are diagonal in the mass eigenstate basis. For example, in the up-left sector

$$\mathcal{U}^{uL}\begin{pmatrix} 1-\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & 1-\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & 1-\frac{4}{3}s_W^2 \end{pmatrix}\mathcal{U}^{uL\dagger} = \text{diagonal}$$

This feature holds no longer if we introduce a new charge 2/3 field with a different isospin assignment, e.g. a singlet $T^0{}_{L,R}$

$$\mathcal{U}^{uL}\begin{pmatrix} 1-\frac{4}{3}s_W^2 & 0 & 0 & 0 \\ 0 & 1-\frac{4}{3}s_W^2 & 0 & 0 \\ 0 & 0 & 1-\frac{4}{3}s_W^2 & 0 \\ 0 & 0 & 0 & -\frac{4}{3}s_W^2 \end{pmatrix}\mathcal{U}^{uL\dagger} \neq \text{diagonal}$$

$T_{3L} = 0$

How much non-diagonal?

The mixing of the new fields (in this example the $T^0_{L,R}$ singlet) with the first two generations is small:

$$\mathcal{U}^{uL} = \begin{pmatrix} \cdot & \cdot & \varepsilon_{13} & \varepsilon_{14} \\ \cdot & \cdot & \varepsilon_{23} & \varepsilon_{24} \\ \varepsilon_{31} & \varepsilon_{32} & \cos\theta_L & -\sin\theta_L e^{i\phi} \\ \varepsilon_{41} & \varepsilon_{42} & \sin\theta_L e^{-i\phi} & \cos\theta_L \end{pmatrix}$$

Therefore, the tree-level *Ztc* / *Ztu* couplings are suppressed by small ε_{ij} entries.

$$\mathcal{L}_{Ztc} = -\frac{g}{2c_W} \varepsilon_{24} \sin\theta_L e^{i\phi} \bar{t}_L \gamma^\mu c_L \, Z_\mu + \mathrm{h.c.}$$

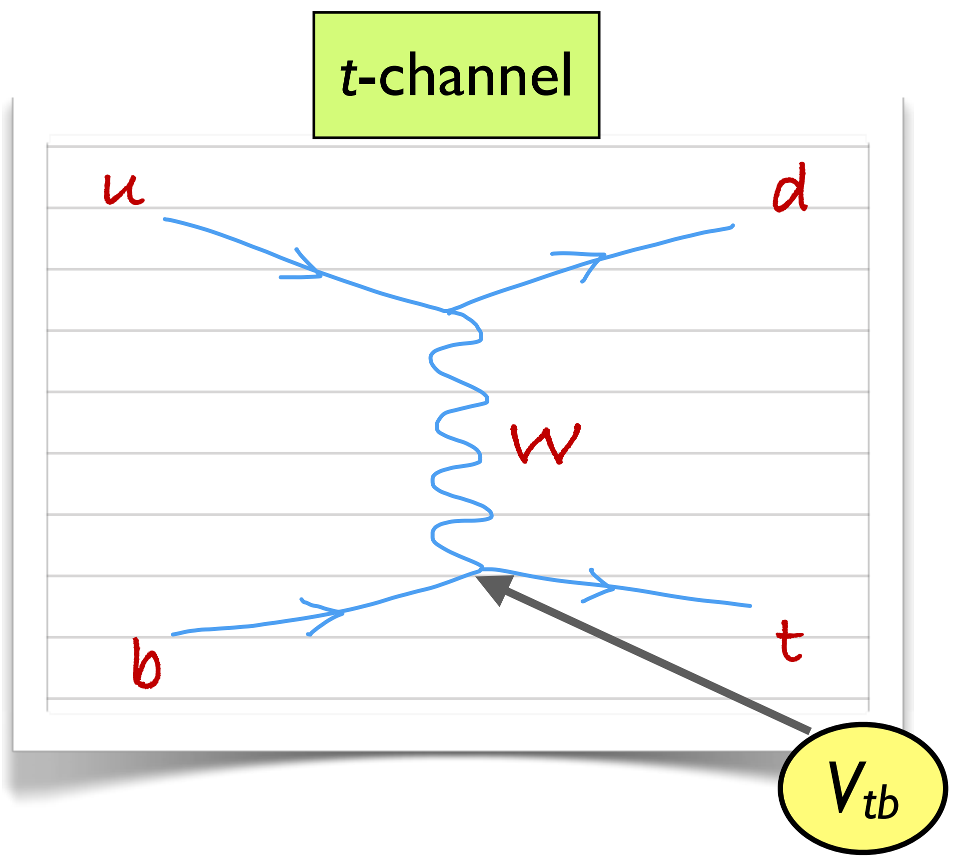
Still, they can lead to observable decays $t \to Zc$ or $t \to Zu$

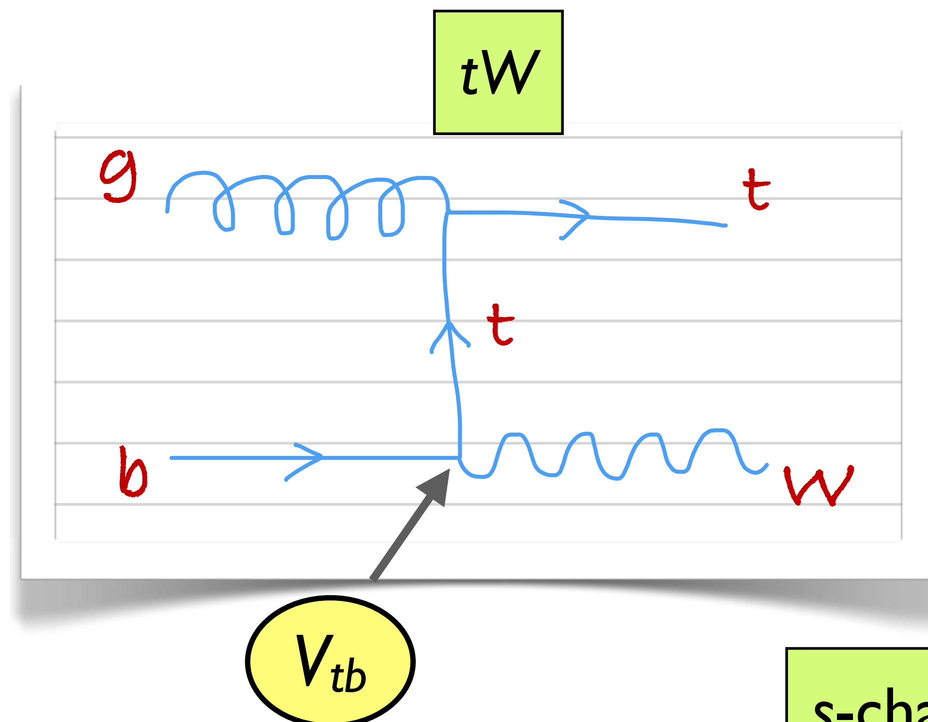
[Not simultaneously.]

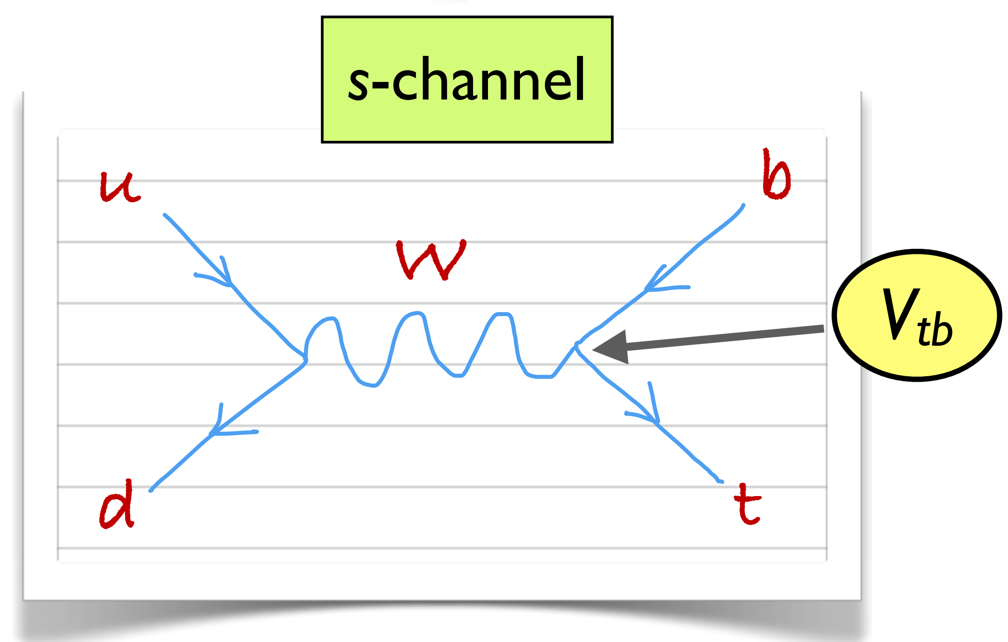
Single top production

Because neutral interactions are flavour-diagonal, single top quarks can only be produced mediated by charged interactions. There are three processes in hadron collisions, named as `*t*-channel´, `*s*-channel´ and `*tW*´.

Sample diagrams:

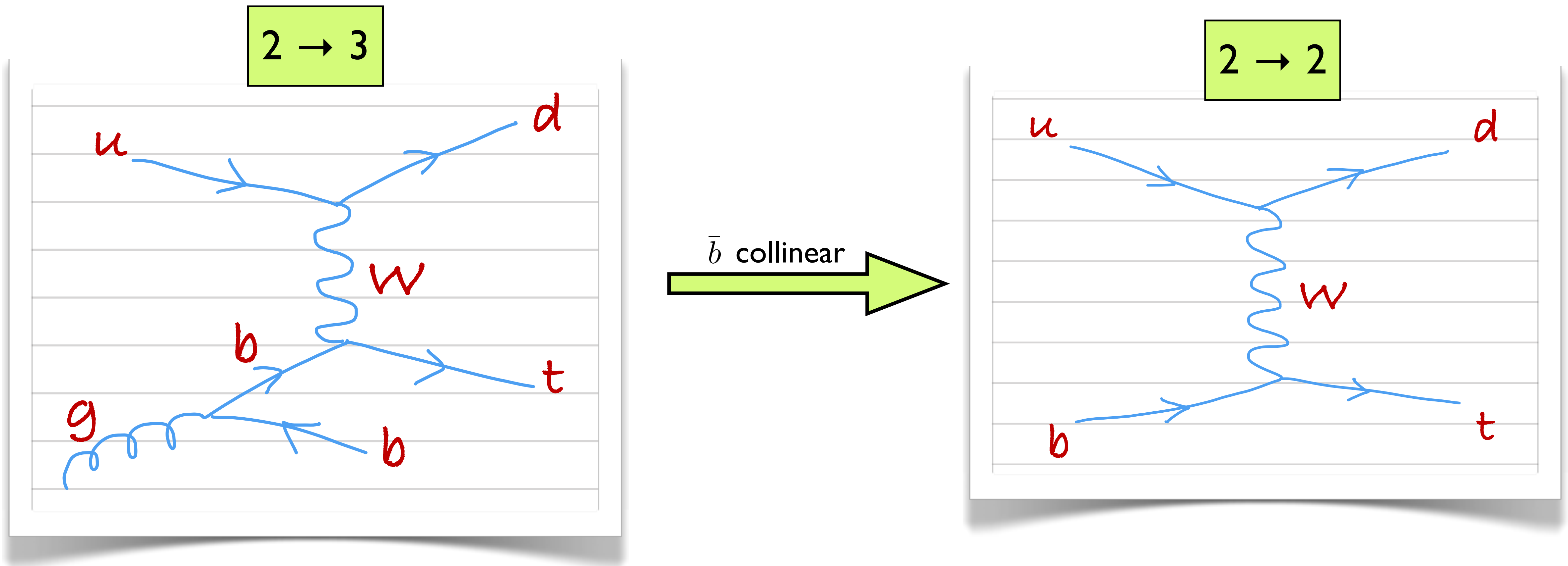






t-channel matching

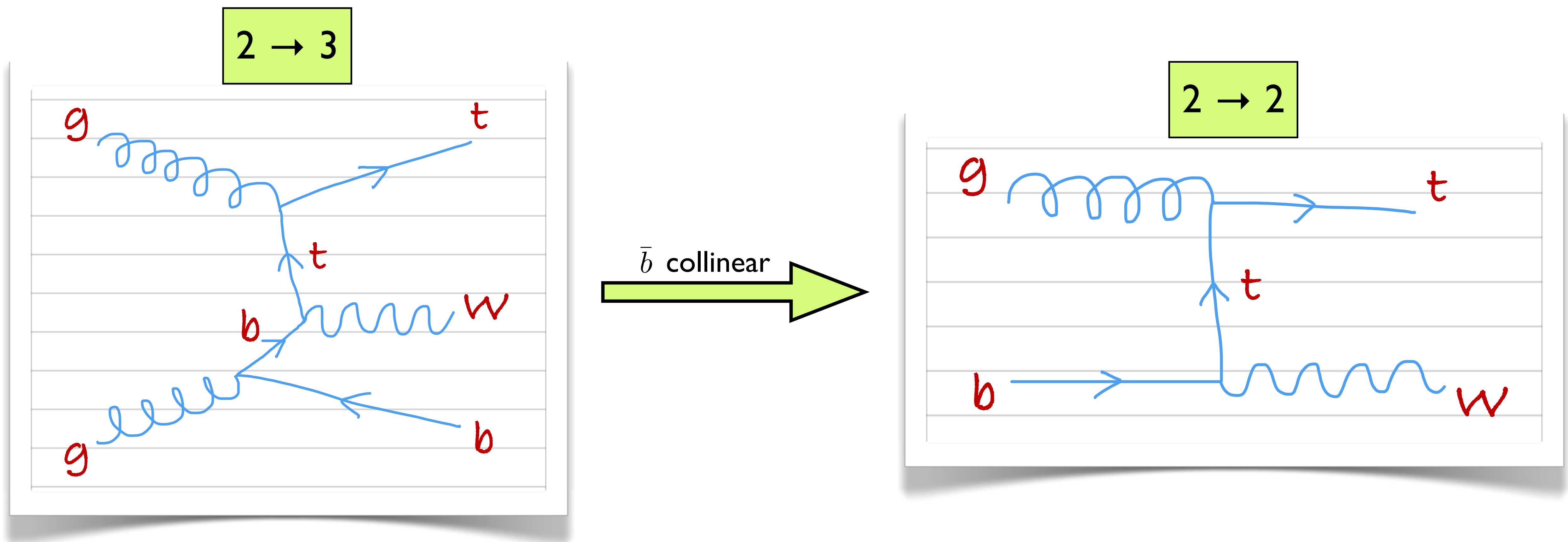
The process that actually takes place is 2 → 3: initial *b* quarks come from splitting $g \to b\bar{b}$. But the kinematical region where g and $\bar{b}$ are collinear is better described by introducing a *b* quark PDF and considering a 2 → 2 process.



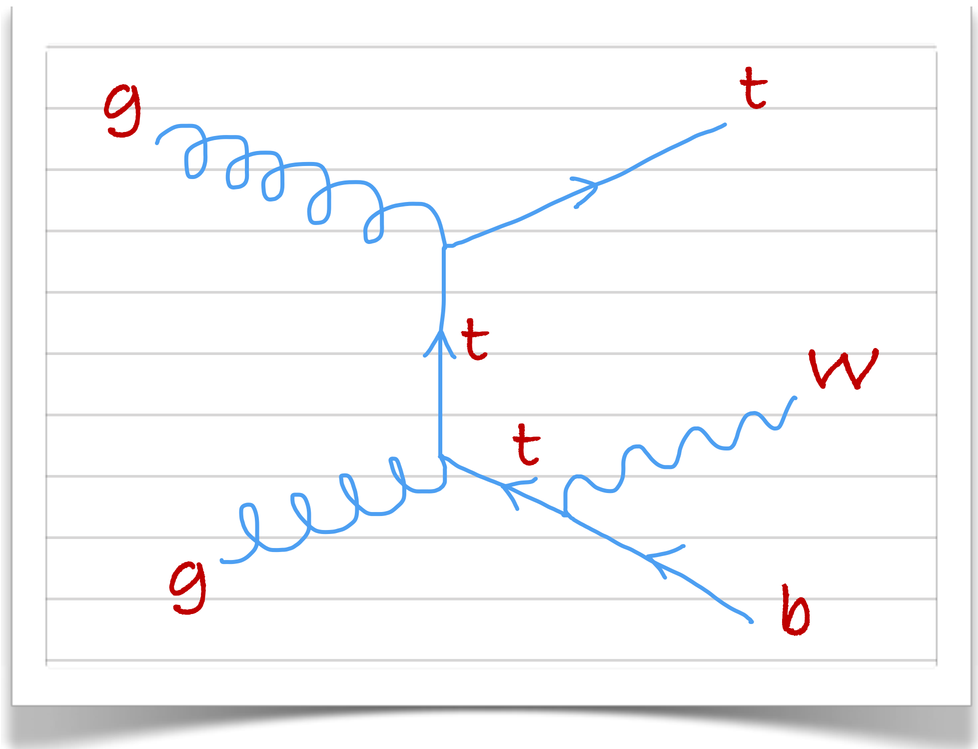
A good kinematical description is achieved by using both and performing some matching [there are several options] to remove the overlapping kinematical regions.

tW matching

The same happens in tW production: initial b quarks actually result from splitting $g \to b\bar{b}$



But in this case, the gauge-invariant set of diagrams for *gg* → *tWb* also includes several ones that correspond to on-shell $t\bar{t}$ production



For bookkeeping purposes [the $t\bar{t}$ cross section does not depend on V_{tb}, for example] it is better to consider $t\bar{t}$ as a separate process. Then, some *subtraction* has to be made on *gg* → *tWb* to *remove* $t\bar{t}$. There are several options for that.

Cross sections

similar size and difficult to separate

unobservable

	t-channel	s-channel	tW
Tevatron	2.08 pb	1.05 pb	0.01 pb
LHC7	66 pb	4.6 pb	15.6 pb
LHC8	87 pb	5.6 pb	22.2 pb

t-channel dominant

hard to separate from t-channel

All these cross sections assume V_{tb} = 1 [and no anomalous couplings].

This coupling is not measured elsewhere, so single top production provides its unique measurement.

[measurements agree with SM]

Polarisation

Single top quarks are produced with non-zero polarisation along suitably chosen axes.

the P_z-dependent top decay distributions can be measured

Notice that the charged current interaction produces t_L but not t_R.

	t-channel		*s*-channel		*tW*
z axis	helicity	spectator jet	helicity	proton	helicity
Tevatron	-0.70	0.92	-0.62	-0.90	-0.25
LHC7	-0.69	0.90	-0.62	0	-0.26
LHC8	-0.68	0.89	-0.62	0	-0.26

large σ
large P_z

not useful because the signals are not clean

of little use

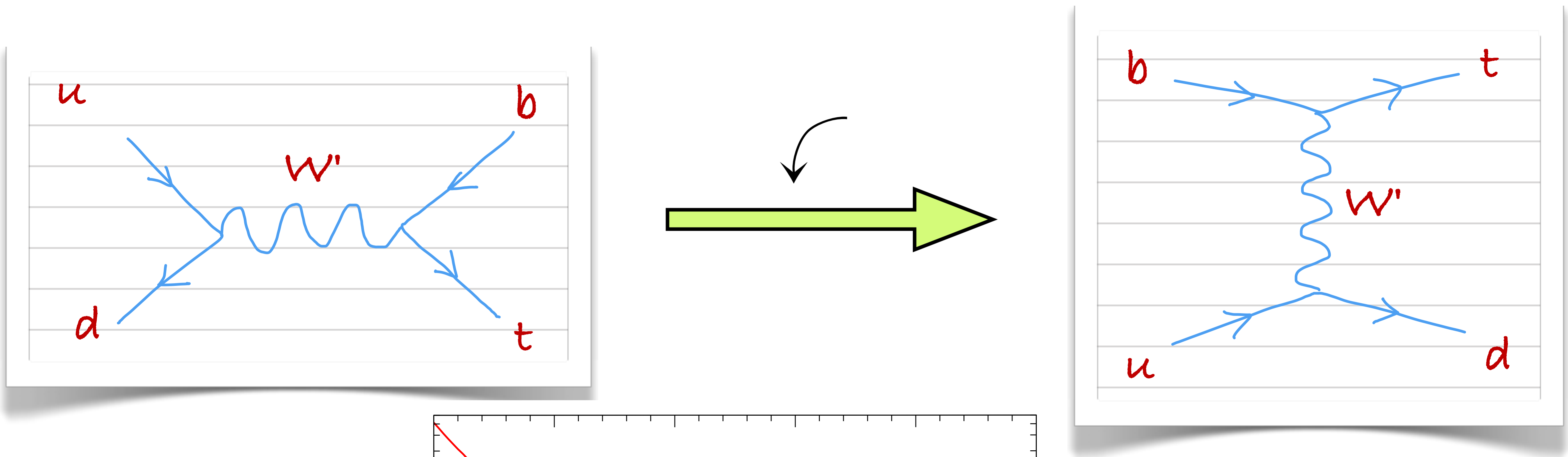
Single top beyond the SM

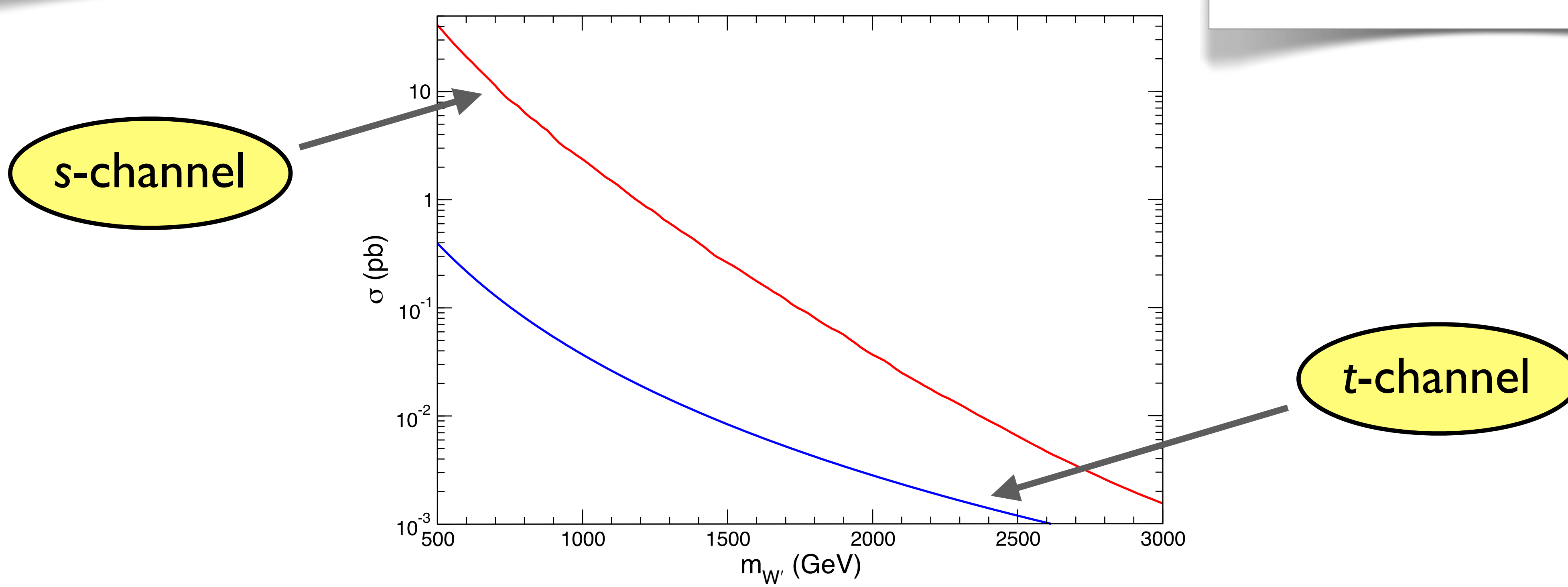
There are several possible sources of single top production beyond the SM processes. We will focus on few of them.

- New charged bosons
- Flavour-changing neutral processes
- Anomalous *Wtb* couplings

New charged bosons

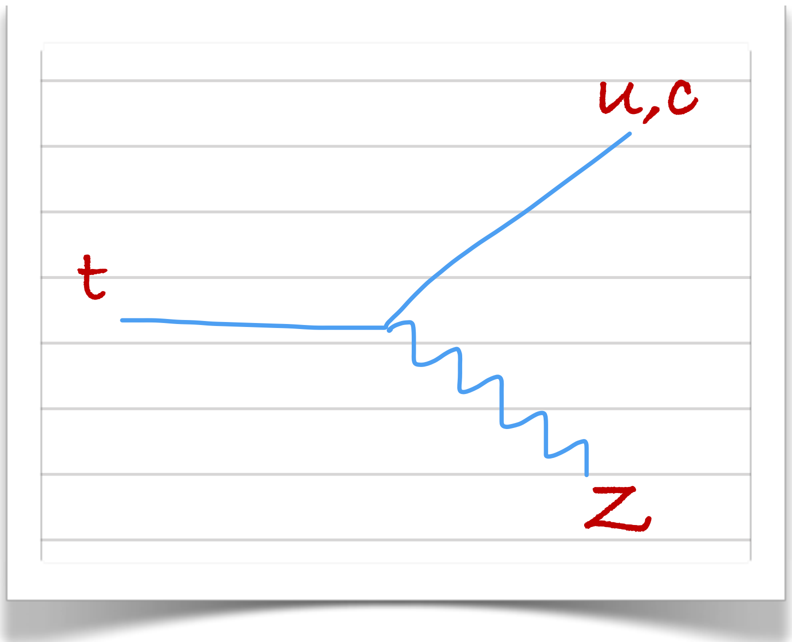
A new charged boson W' can mediate single top production both in the *s* and *t* channel. The former has a much larger cross section and is easier to separate from the backgrounds due to the $t\bar{b}$ resonant structure.



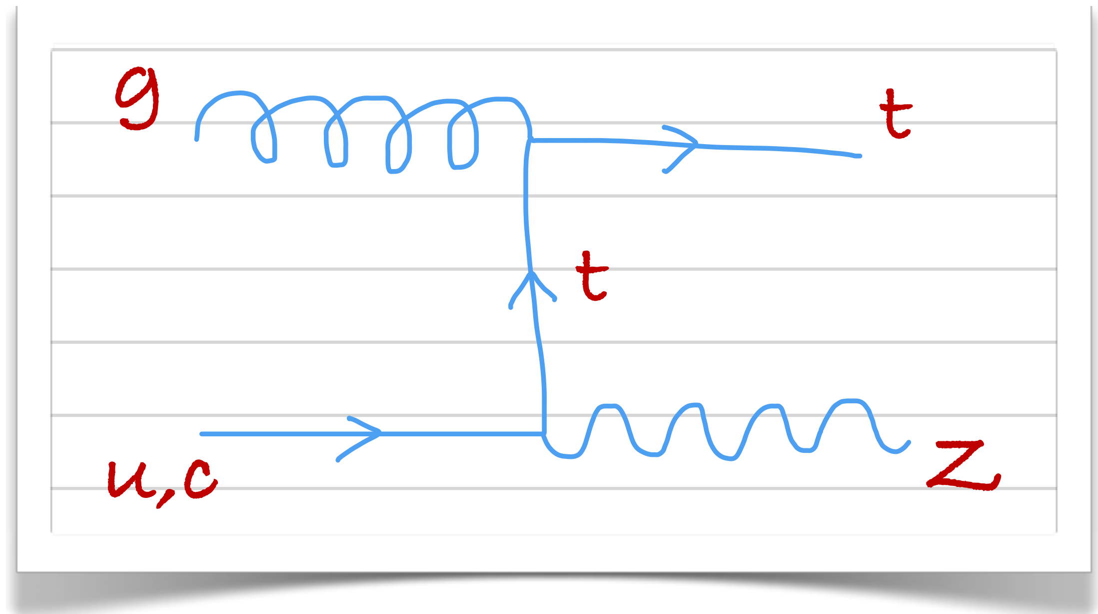


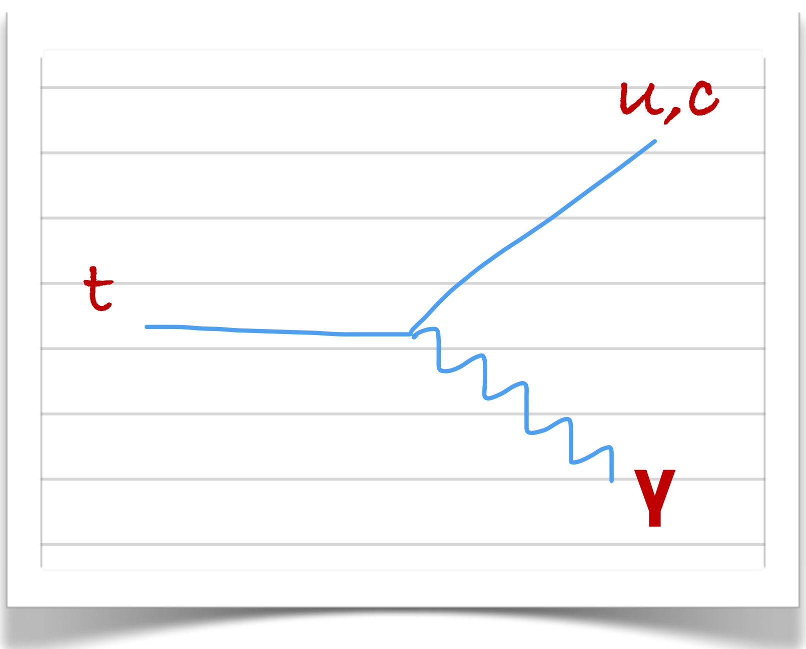
Flavour-changing neutral processes

Top FCN decays have single production counterparts

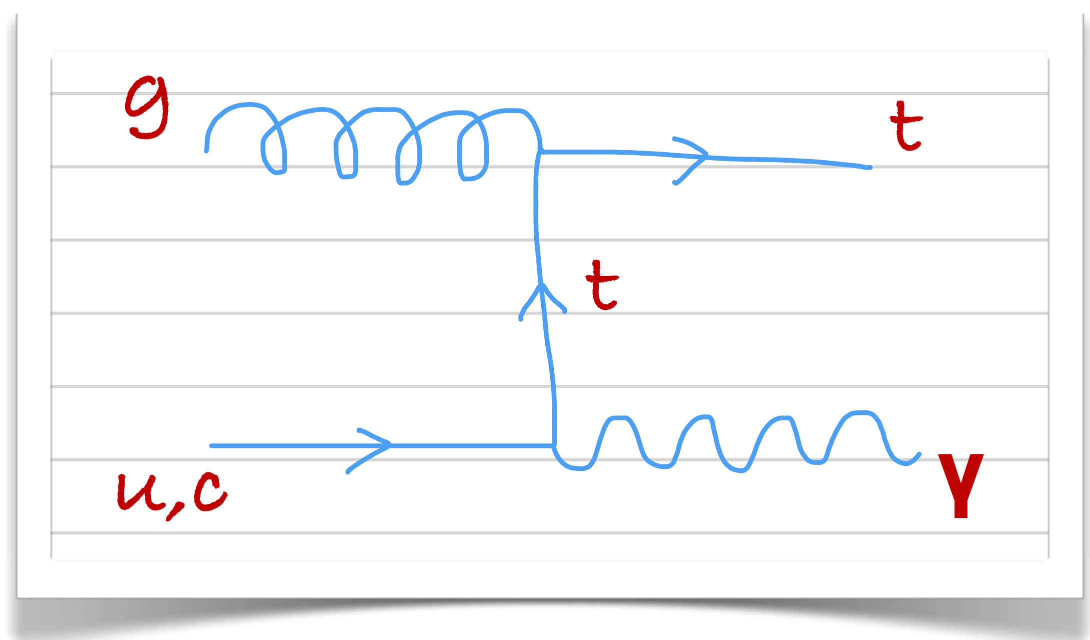


Ztu / Ztc



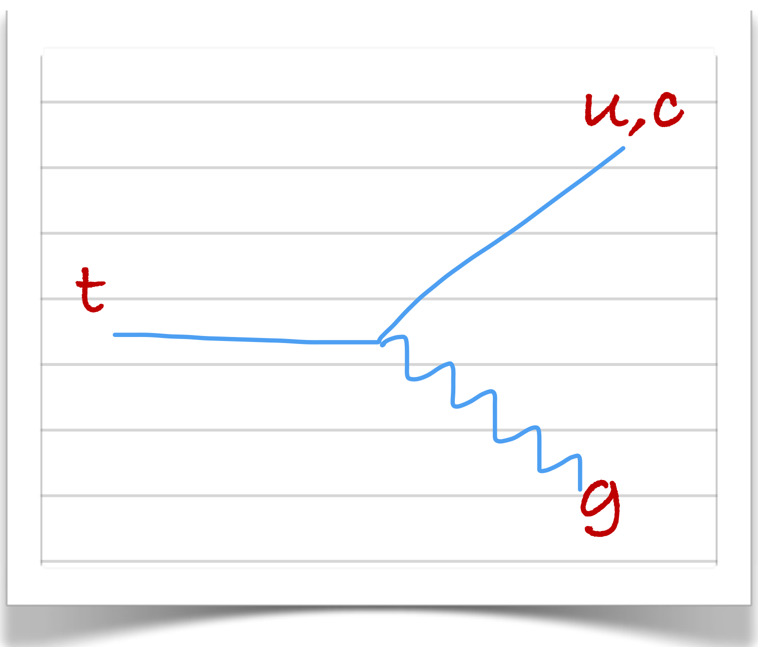


γtu / γtc

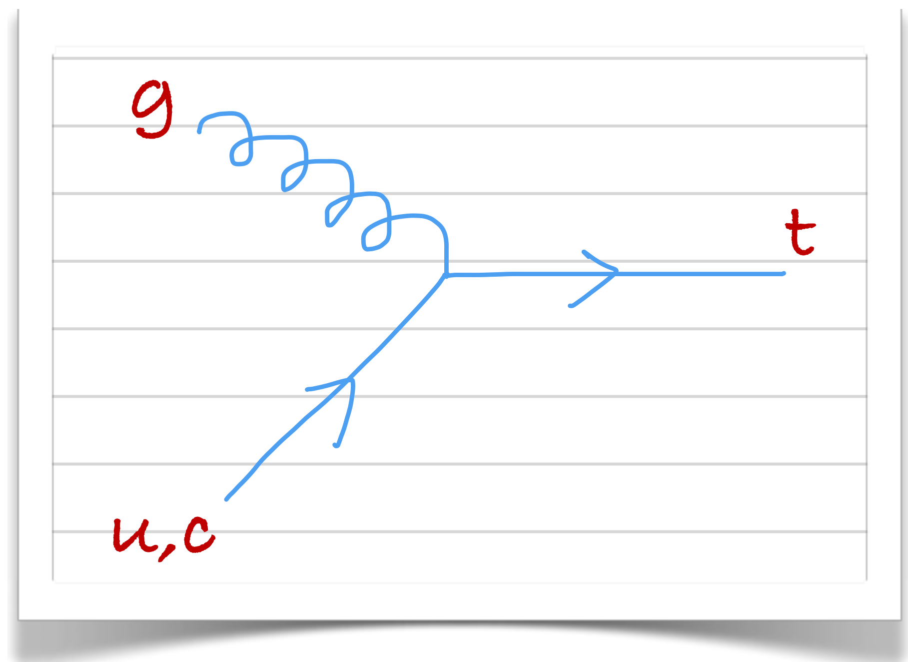


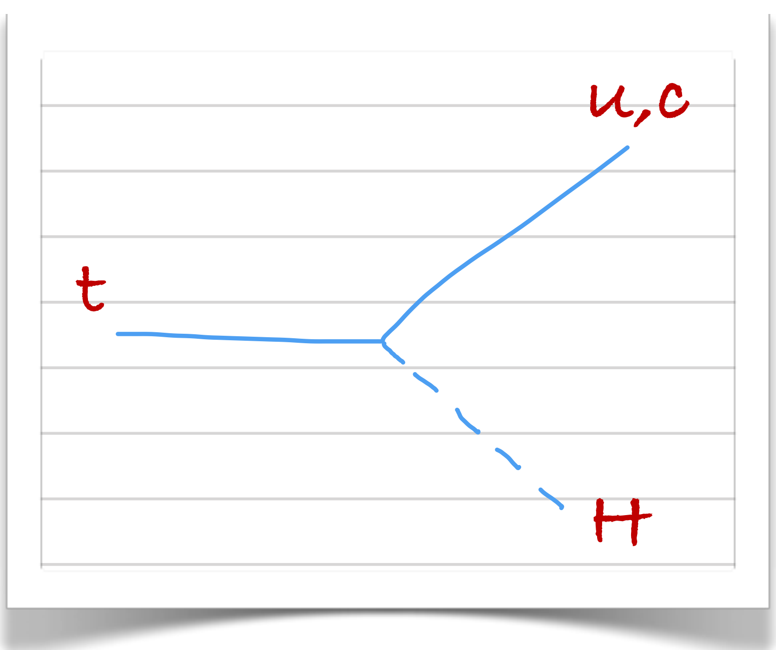
Flavour-changing neutral processes

Top FCN decays have single production counterparts

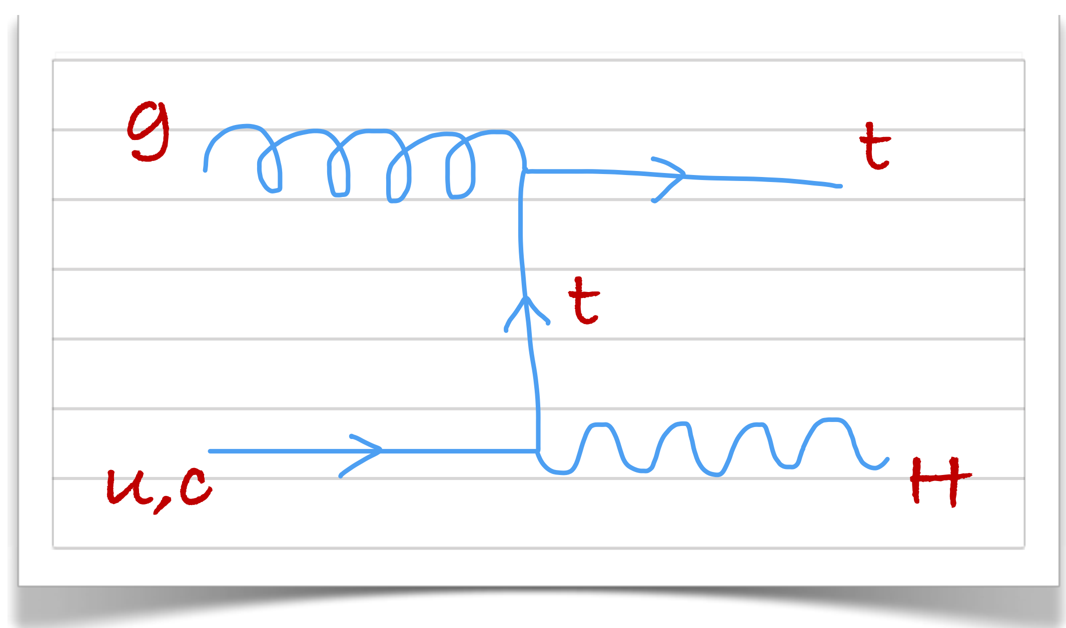


gtu / gtc





Htu / Htc



Flavour-changing neutral processes

The sensitivity of single production versus top decays depends not only on the signal cross sections but on the backgrounds.

Estimated LHC sensitivity with 100 fb^{-1} [in terms of Br]

	Top decay	Single production
tuZ	10^{-5}	10^{-5}
$tu\gamma$	10^{-5}	10^{-6}
tug	10^{-4}	10^{-6}
tuH	10^{-5}	10^{-4}

	Top decay	Single production
tcZ	10^{-5}	10^{-4}
$tc\gamma$	10^{-5}	10^{-5}
tcg	10^{-4}	10^{-5}
tcH	10^{-5}	10^{-3}

Anomalous *Wtb* couplings

Single top production involves a *Wtb* interaction [$\bar{b}_L \gamma^\mu t_L$ in the SM]. The presence of anomalous *Wtb* couplings changes:

- The total cross section
- The kinematical distributions
- The top polarisation

Changes in the total cross section are easy to parameterise and allow to obtain limits on anomalous *Wtb* couplings. We take again the Lagrangian

$$
\begin{aligned}
\mathcal{L}_{Wtb} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t \, W_\mu^- \\
& - \frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) \, t \, W_\mu^- + \text{h.c.}
\end{aligned}
$$

Then, one can write the cross sections as

$$\sigma = \sigma_{\rm SM} \left(|V_L|^2 + \kappa_{V_R}|V_R|^2 + \kappa_{g_L}|g_L|^2 + \kappa_{g_R}|g_R|^2 + \kappa_{V_L g_R}{\rm Re}V_L g_R^* + \dots\right)$$

We are assuming here that no other new physics contributes to single top production

Example: LHC 7 TeV

	κ_{V_R}	κ_{g_L}	κ_{g_R}	$\kappa_{V_L g_R}$
t-channel (t)	0.9	1.4	2.3	-0.6
t-channel ($\bar{t}$)	1.1	2.4	1.5	-0.1
s-channel (t)	1	11.5	11.5	-5.4
s-channel ($\bar{t}$)	1	10.7	10.7	-5.4
tW (t)	1	2.9	2.9	1
tW ($\bar{t}$)	1	2.9	2.9	1

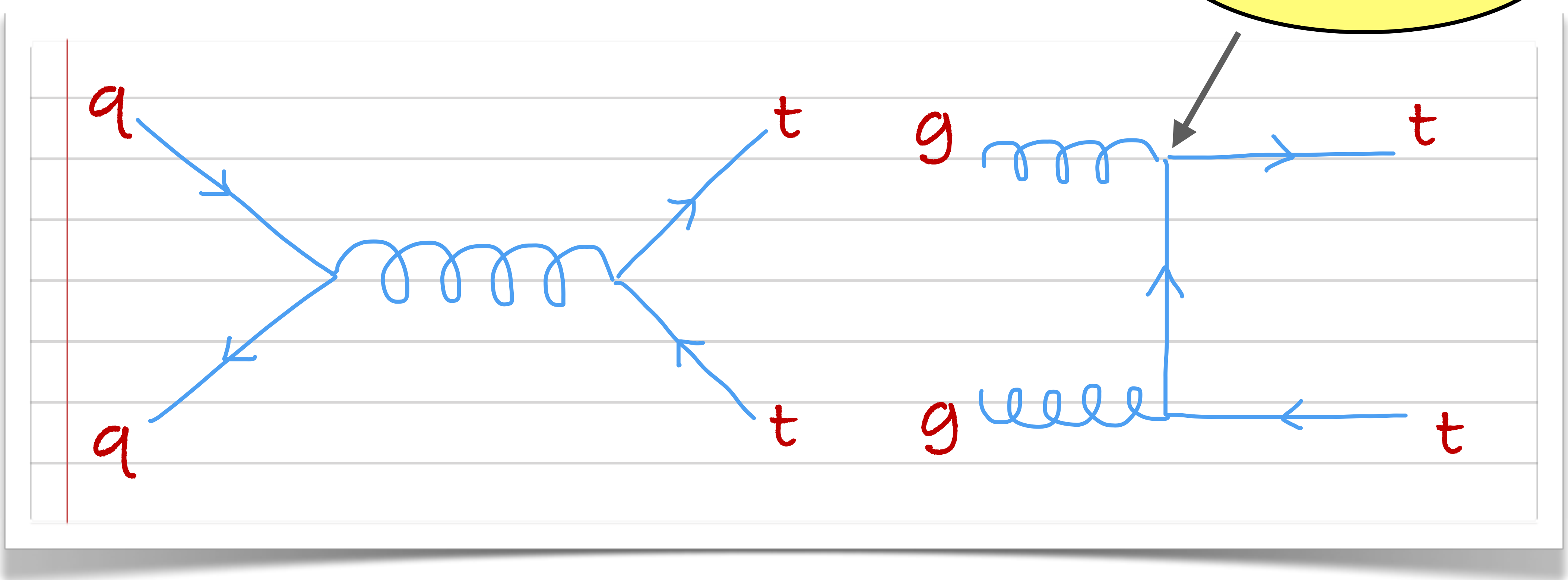
stringent limits on anomalous couplings

Top pair production

The top quarks was discovered in $p\bar{p}$ collisions at the Tevatron, produced through hard interactions of partons $q\,(= u, d, s, \dots), g\,.$

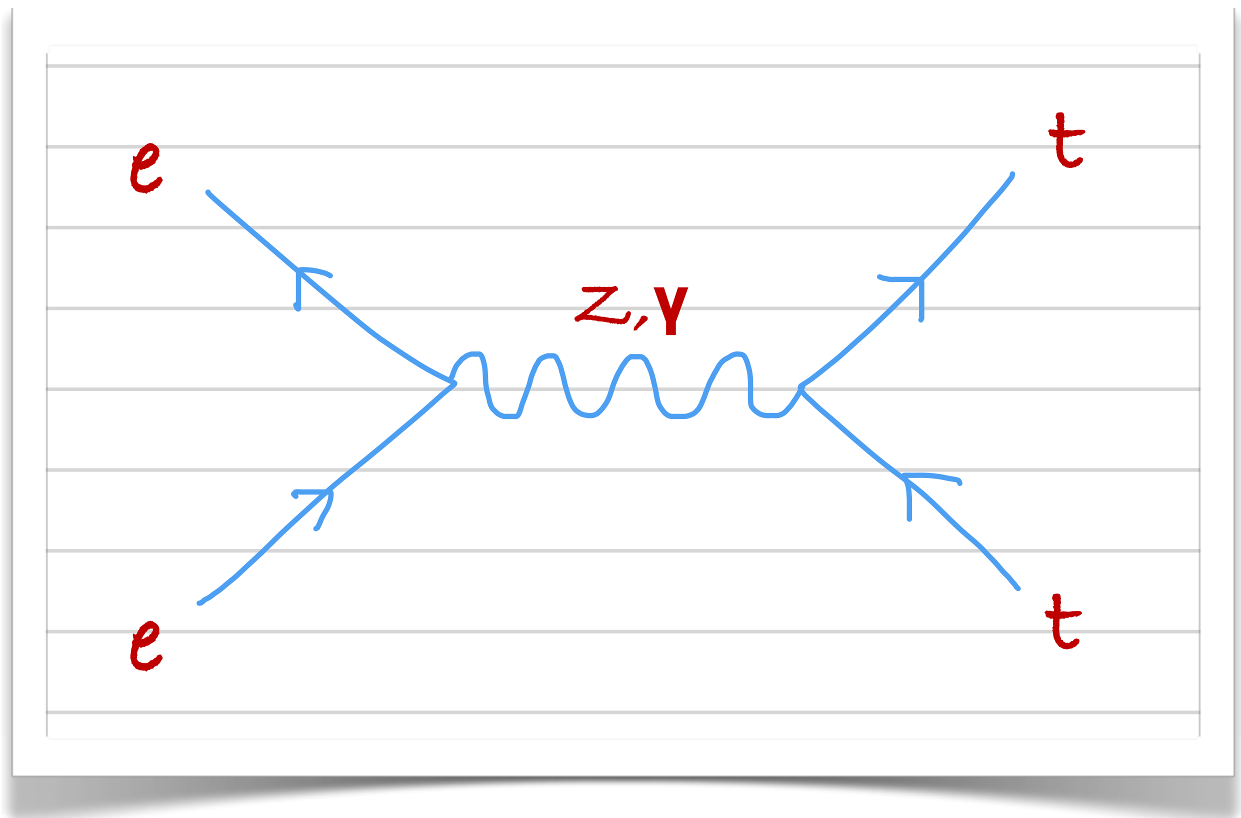
Top quarks are also produced in pairs at the LHC.

gtt interaction determined by gauge symmetry



			σ
Tevatron (2 TeV)	4/5	1/5	7.16 pb
LHC (7 TeV)	1/5	4/5	172 pb
LHC (8 TeV)	1/5	4/5	246 pb

Top quark pairs can also be produced in e$^+$e$^-$ collisions, but no lepton collider has reached the required energy $\sqrt{s} = 2m_t \simeq 350\ \mathrm{GeV}$



As it is well known from collision theory, plane waves (states with definite momentum) *contain* all possible orbital angular momenta.

$$e^{i\vec{k}\cdot\vec{r}} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta)$$

Therefore, the top pairs are produced in a superposition of states with definite orbital angular momentum *l*.

However, in two useful limits the situation is simpler:

- The threshold

  → $l = 0$ because the top pair is produced at rest.

- The high-energy regime

  → the top helicity and chirality coincide because m_t effects are small.

Example: $t\bar{t}$ production at the Tevatron

- dominated by $q\bar{q}, q = u, d$ ⇨ ignore gg.
- moderate CM energy ⇨ bulk of $t\bar{t}$ production close to threshold.
- $p\bar{p}$ collisions ⇨ we know where q and $\bar{q}$ come from with a high degree of confidence (p and $\bar{p}$, respectively).

QCD interactions $[\bar{q}\gamma^\mu q\,]$ are vectorial and therefore involve same-chirality (anti-)quarks: $\bar{q}_L q_L\,, \bar{q}_R q_R$.

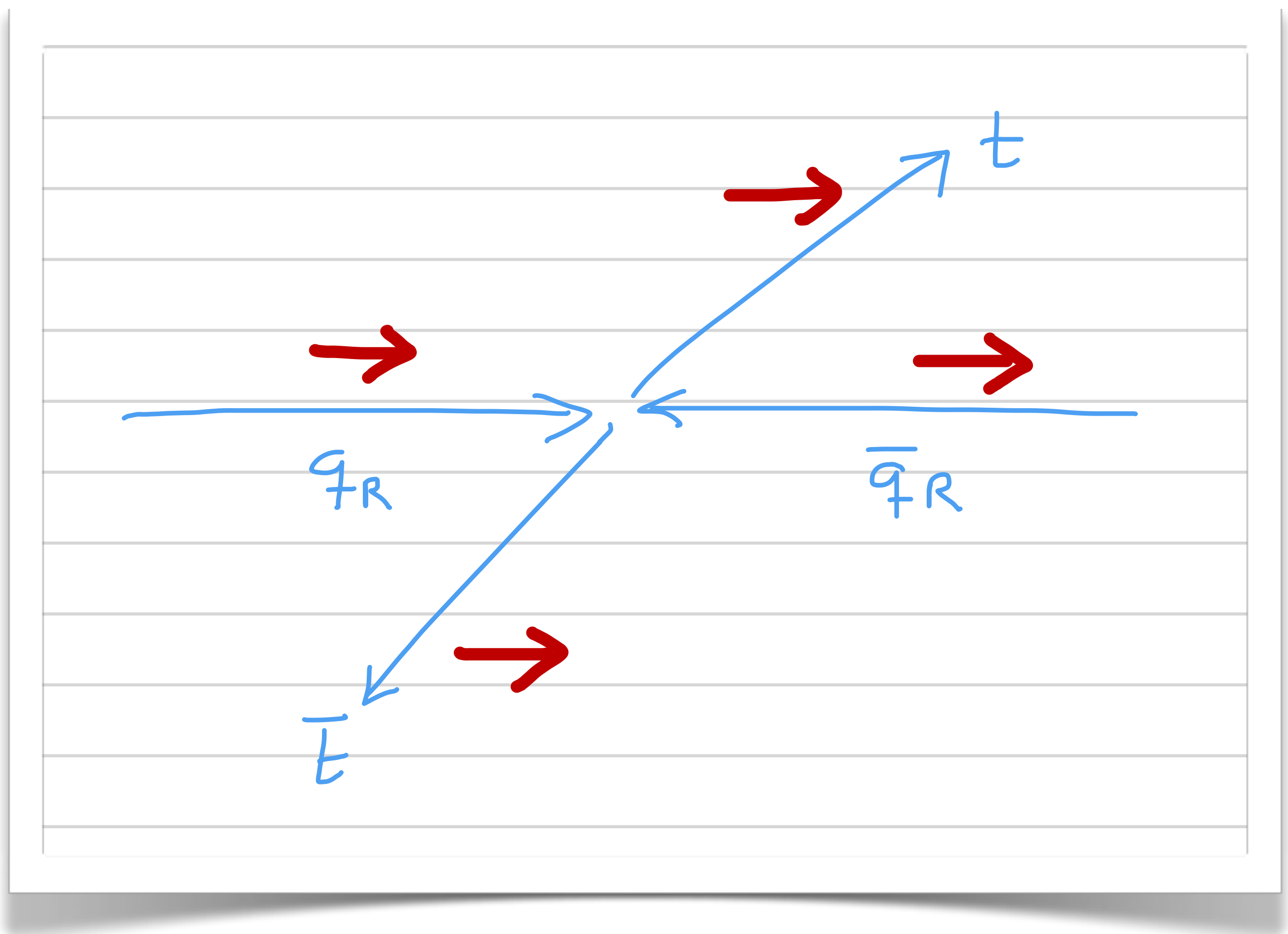
We can assume that $q = u, d$ are massless. Therefore:

for q : helicity = chirality
for $\bar{q}$: helicity = - chirality

For $\bar{q}_R q_R$ the initial spin state is

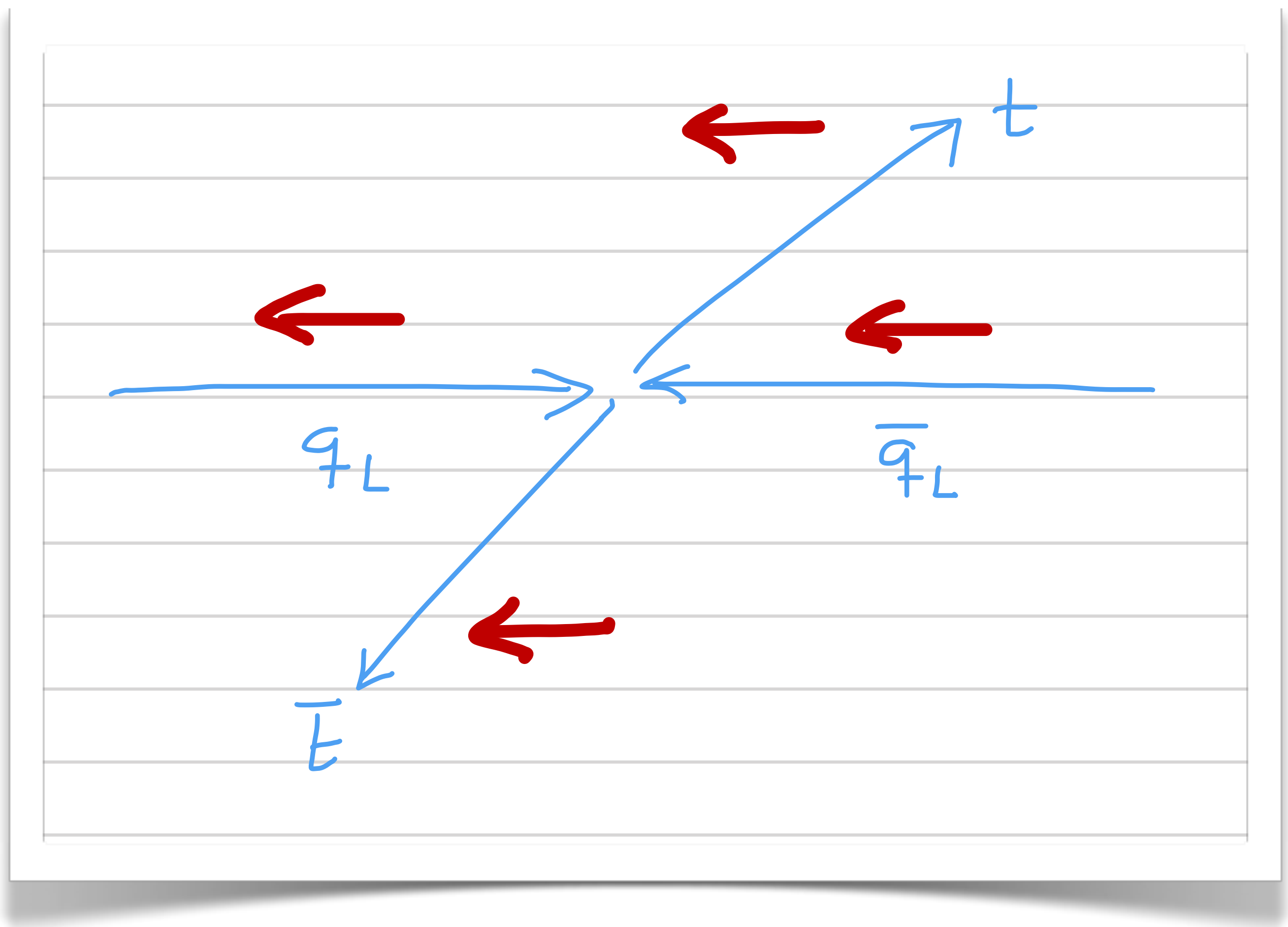
$$|\tfrac{1}{2}\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\tfrac{1}{2}\rangle = |11\rangle$$

taking the z axis in the direction of the proton. Moreover, the relative orbital angular momentum is L_z = 0 $[\vec{L} = \vec{r} \times \vec{p}]$ ⇒ total J_z = 1



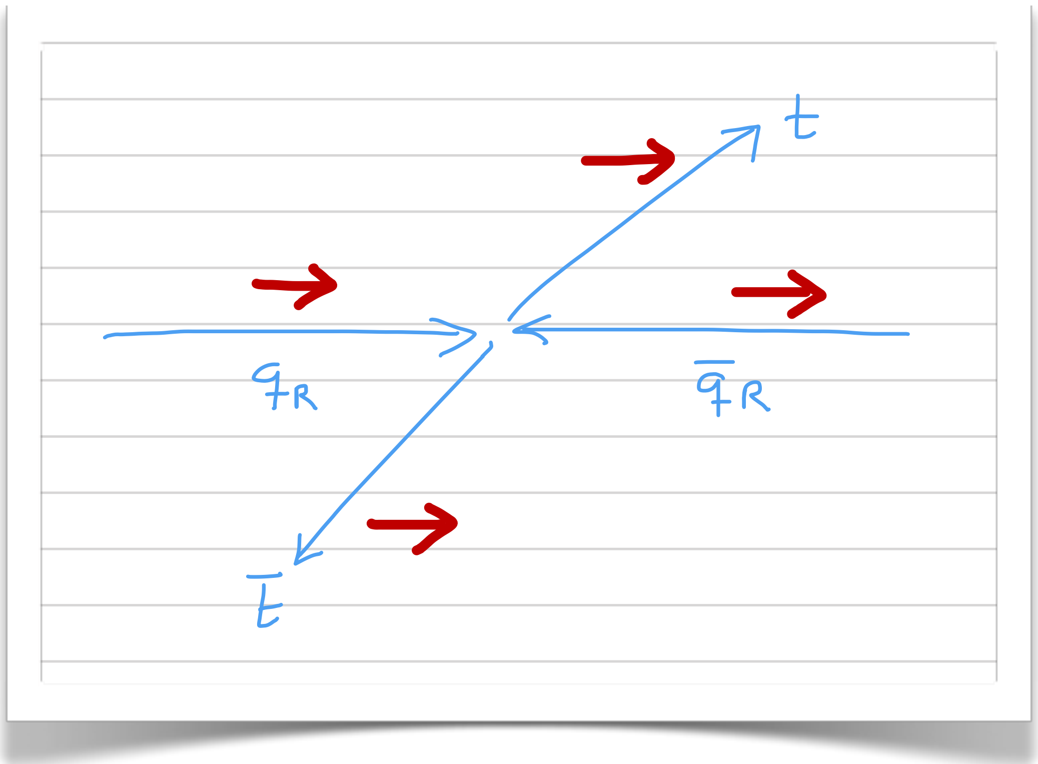
Since at threshold the final state has *l = 0*, this implies that both t and $\bar{t}$ have the spin in the positive z direction. An interesting consequence!

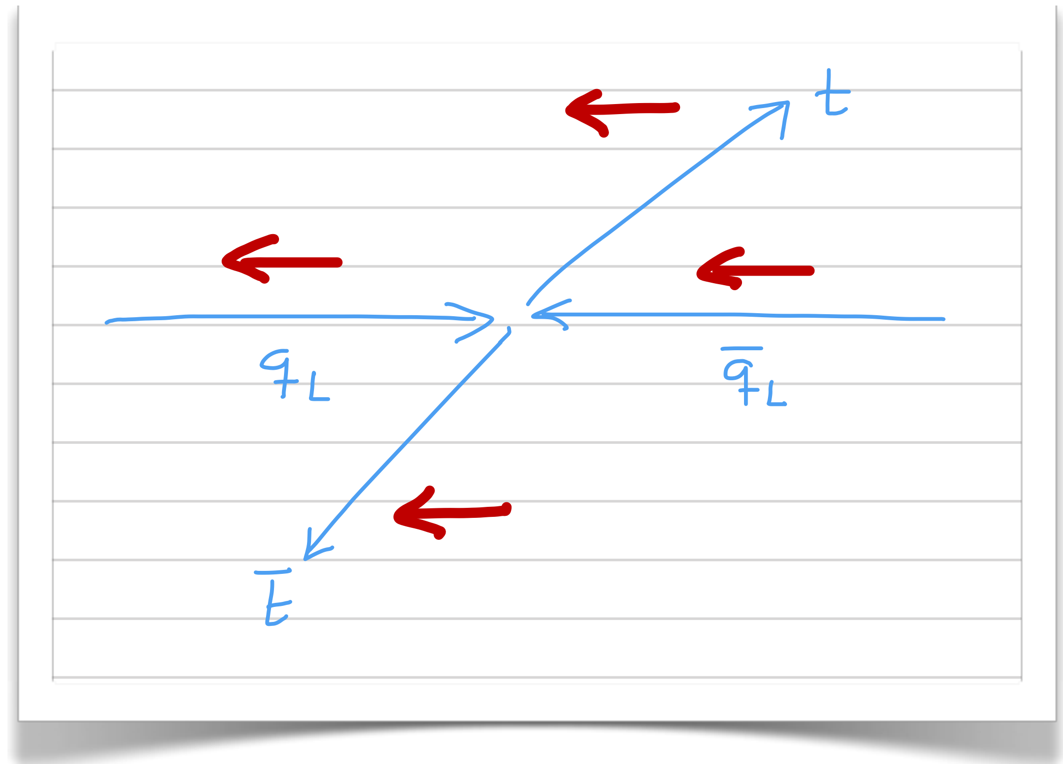
For $\bar{q}_L q_L$ the picture is the opposite:



Therefore, since $\bar{q}_R q_R$ and $\bar{q}_L q_L$ initial states have the same weight, the top (anti-)quarks are produced with *P*$_z$ = *0*.

However, the t and $\bar{t}$ spins are correlated!





Let us define a spin correlation parameter

$$C = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}$$

same direction minus opposite direction

total

With the approximations used, C = 1. An exact (tree-level) calculation including gg gives C = 0.928 (!) and P_z = 0.

Spin correlations in $t\bar{t}$ production - General

Let us define a (*x*, *y*, *z*) coordinate system in the top rest frame, and a (*x*´, *y*´, *z*´) system [which may be the same] in the antitop rest frame.

The spin correlation parameter can be defined as in the previous example:

$$C = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}$$

but $\uparrow$ and $\downarrow$ refer to the z and z´axes, respectively, for t and $\bar{t}$.

We are here considering the top and antitop as stable particles that are produced in definite spin states - we have shown this is correct under certain conditions.

sizeable corrections because NLO is 2 → 3

	LO	NLO
Tevatron "beamline basis"	0.928	0.777
Tevatron "helicity basis"	-0.471	-0.352
LHC7 "helicity basis"	0.228	0.310

Measurement $\Rightarrow$ from analysis of $t\bar{t}$ decay distributions.

Example: dilepton decay channel $t\bar{t} \to \ell^+ \nu b\, \ell^- \bar{\nu} \bar{b}$. We choose as *spin analysers* the two charged leptons.

$\vec{p}_{\ell+}$ $\Rightarrow$ 3-momentum of ℓ^+ in the t rest frame), with spherical coordinates $(\theta_{\ell+}, \phi_{\ell+})$ in the (*x*, *y*, *z*) system

$\vec{p}_{\ell-}$ $\Rightarrow$ 3-momentum of ℓ^- in the $\bar{t}$ rest frame, with spherical coordinates $(\theta_{\ell-}, \phi_{\ell-})$ in the (*x*´, *y*´, *z*´) system

Then, the double differential distribution in $\vec{p}_{\ell+}, \vec{p}_{\ell-}$ polar angles is

top $P_z \simeq 0$

antitop $P_{z'} \simeq 0$

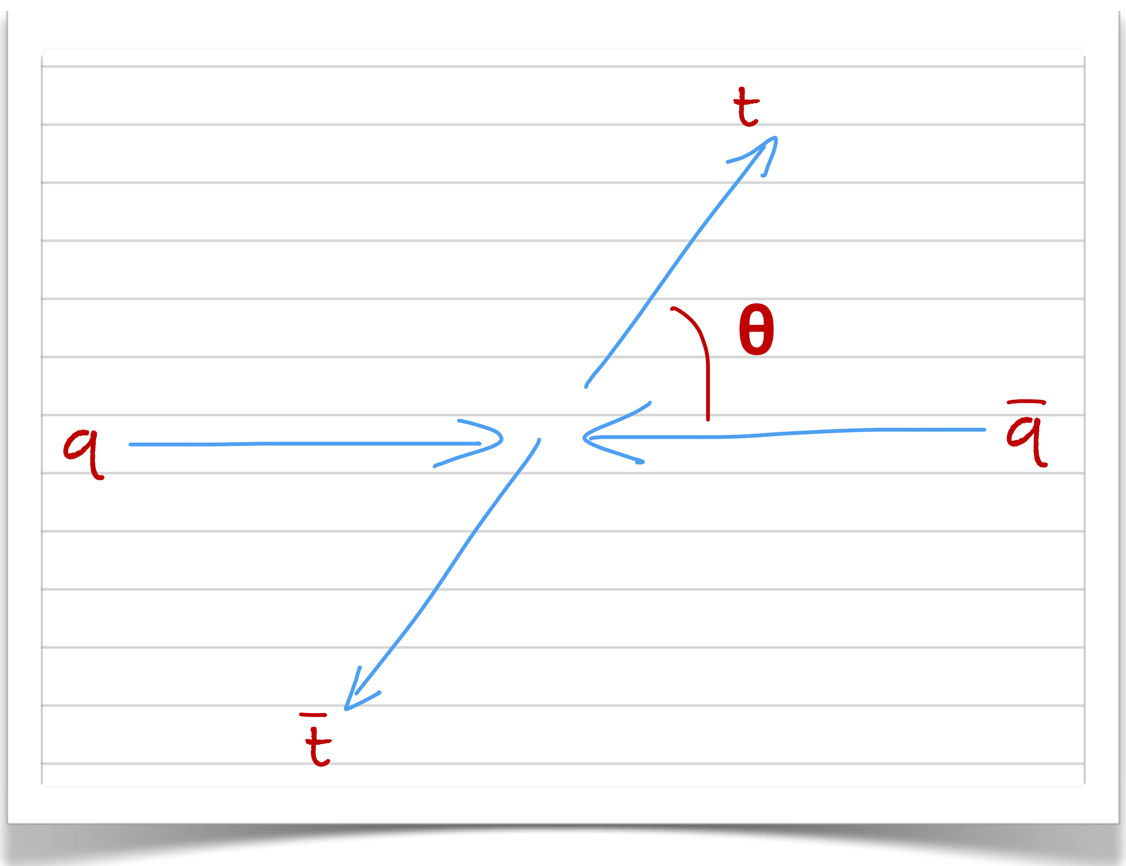
$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_{\ell+}\, d\cos\theta_{\ell-}} = \frac{1}{4}\Big[1 + P_z\, \alpha_{\ell+} \cos\theta_{\ell+} + \bar{P}_{z'}\, \alpha_{\ell-} \cos\theta_{\ell-} + C\, \alpha_{\ell+}\alpha_{\ell-} \cos\theta_{\ell+} \cos\theta_{\ell-}\Big]$$

spin correlation

[measurements agree with SM predictions]

Opening angle distribution

In the $q\bar{q} \to t\bar{t}$ subprocesses (q = u,d) , a variable of interest is the angle between the top and the initial quark in the CM frame.



In pp collisions the initial quark comes from either proton with equal probability but in $p\bar{p}$ collisions it comes from the proton with probability very close to 1.

⇨ this distribution can be measured at the Tevatron

A simple observable to test this distribution is the forward-backward asymmetry

$$A_{\rm FB} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta > 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta > 0)}$$

Since:

- in the CM frame the top and antitop have opposite rapidities $y_{\bar t} = -y_t$
- the rapidity difference $\Delta y = y_t - y_{\bar t}$ is invariant under boosts in the beam direction

this asymmetry is equivalent to $A_{\rm FB} = \dfrac{\sigma(\Delta y > 0) - \sigma(\Delta y < 0)}{\sigma(\Delta y > 0) + \sigma(\Delta y < 0)}$

$A^{\rm th}_{\rm FB} = 0.088$ (NLO)

$A^{\rm exp}_{\rm FB} = 0.187 \pm 0.036$

naive average of CDF and D0

~2.8σ deviation

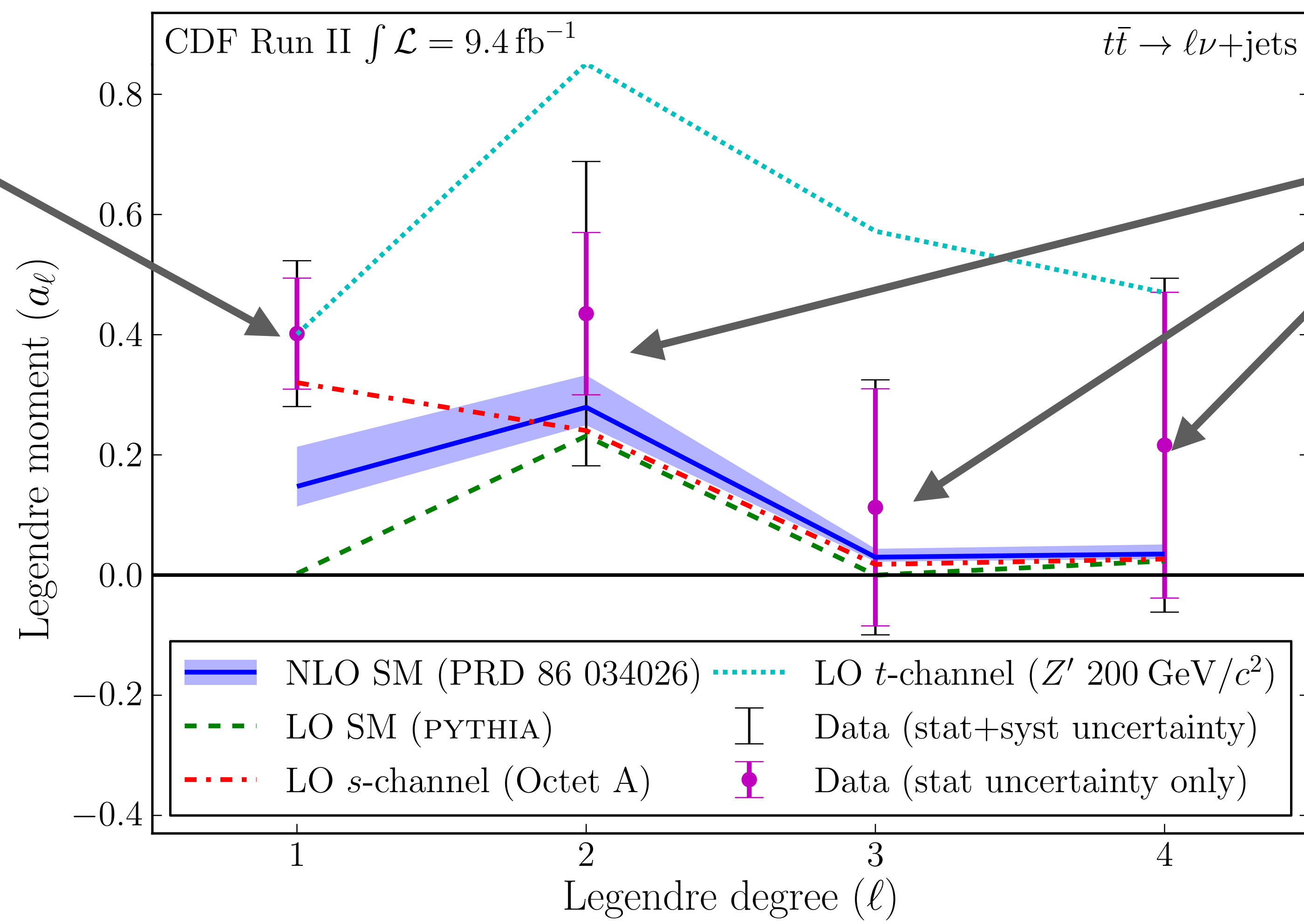
Also, the cos θ distribution can be measured. Setting our z axis in the proton direction and recalling the plane wave expansion

$$e^{i\vec{k}\cdot\vec{r}} = \sum_{l=0}^{\infty} \underbrace{i^l(2l+1)j_l(kr)}_{a_l} P_l(\cos\theta)$$

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \tfrac{1}{2}(3x^2 - 1)$$
$$\ldots$$

the distribution can be expanded in terms of Legendre polynomials and the coefficients a_l can be measured from data.

2.1σ deviation in a_1



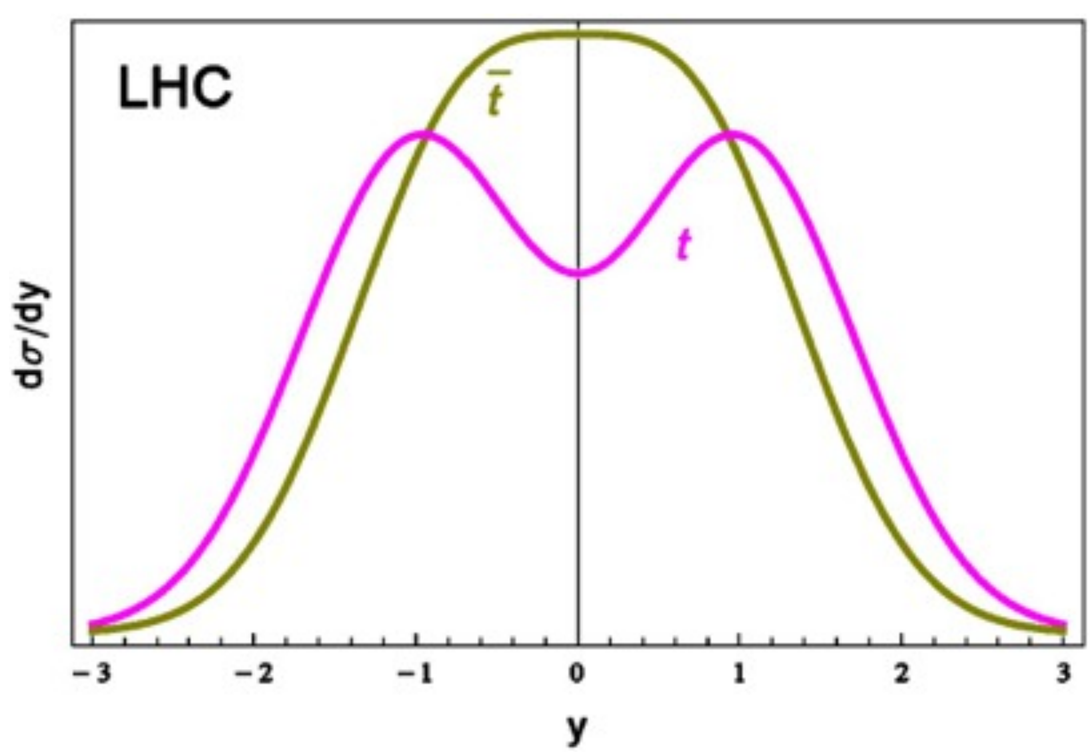
the rest are compatible with SM

LHC charge asymmetry

At the LHC the initial state has no preferred *fixed* direction to define "forward" and "backward". A suitable observable to test asymmetric $t\bar{t}$ production is

$$A_{\mathrm{C}} = \frac{\sigma(\Delta|y| > 0) - \sigma(\Delta|y| < 0)}{\sigma(\Delta|y| > 0) - \sigma(\Delta|y| < 0)}$$

[measurements agree with SM]



Valence quarks have on average larger momentum than antiquarks.

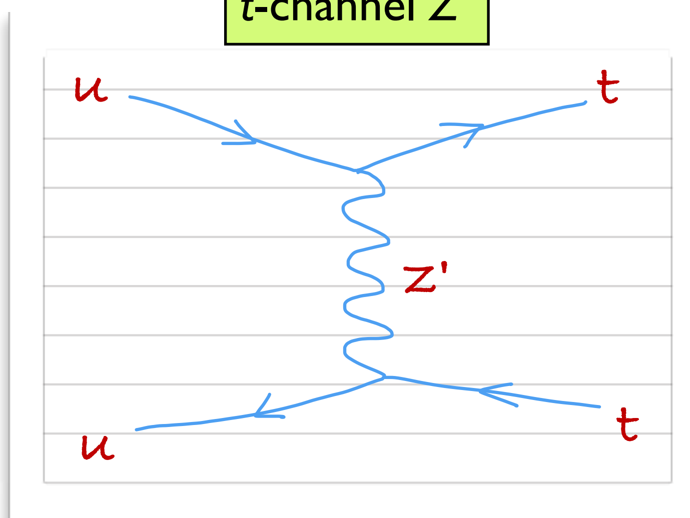
The CM system is boosted in the initial quark direction, on average.

Tops that are forward in the CM system have larger $|y|$ than backward antitops asymmetry in $\Delta|y|$

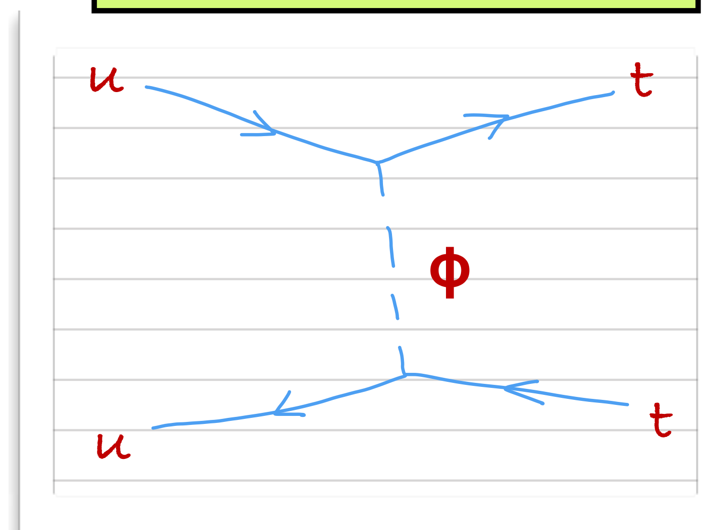
Top pair production beyond the SM

While there are several possible new physics contributions to $t\bar{t}$ production, those that can explain the Tevatron A_{FB} excess have received most attention.

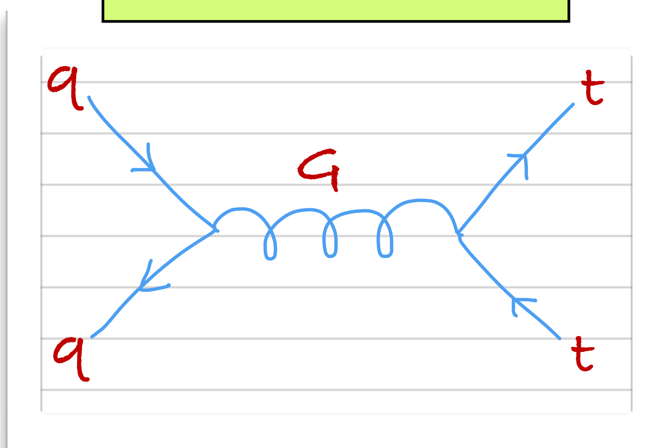
t-channel *Z′*



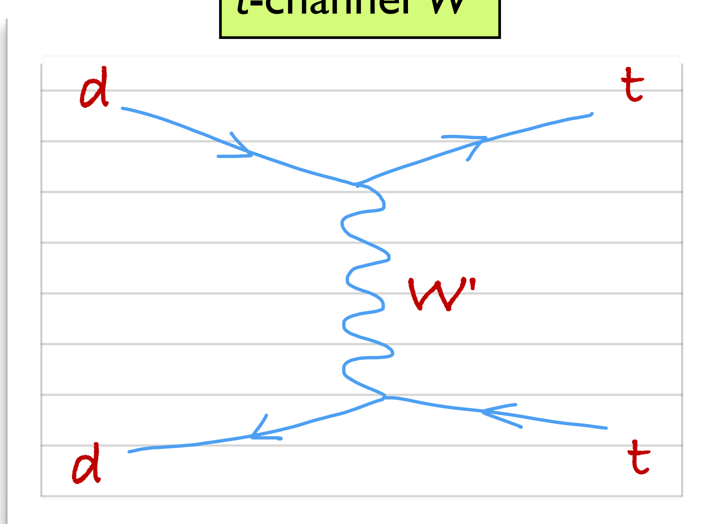
t-channel weak doublet scalar



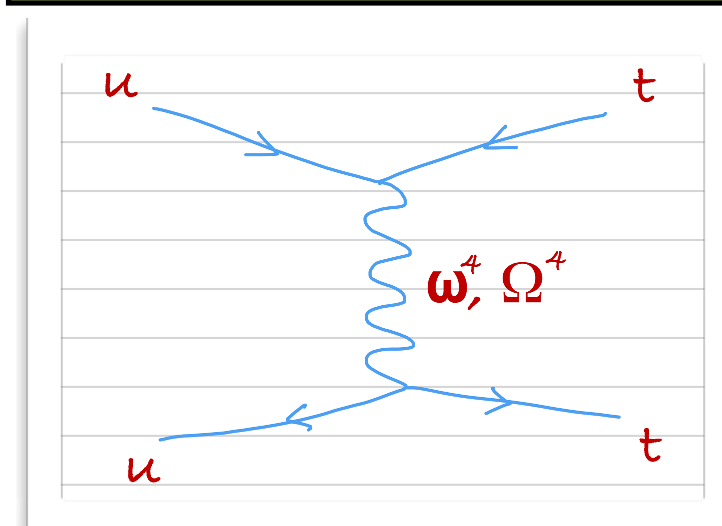
s-channel colour octet



t-channel *W′*



u-channel colour triplet/sextet scalar



In any case, the possibilities of tree-level new physics are determined by group theory and have been thorougly explored.

Colour

$$3 \otimes \bar{3} = 8 \oplus 1$$

$$3 \otimes 3 = 6 \oplus \bar{3}$$

Isospin

$$2 \otimes 2 = 3 \oplus 1$$

$$2 \otimes 1 = 2$$

$$1 \otimes 1 = 1$$

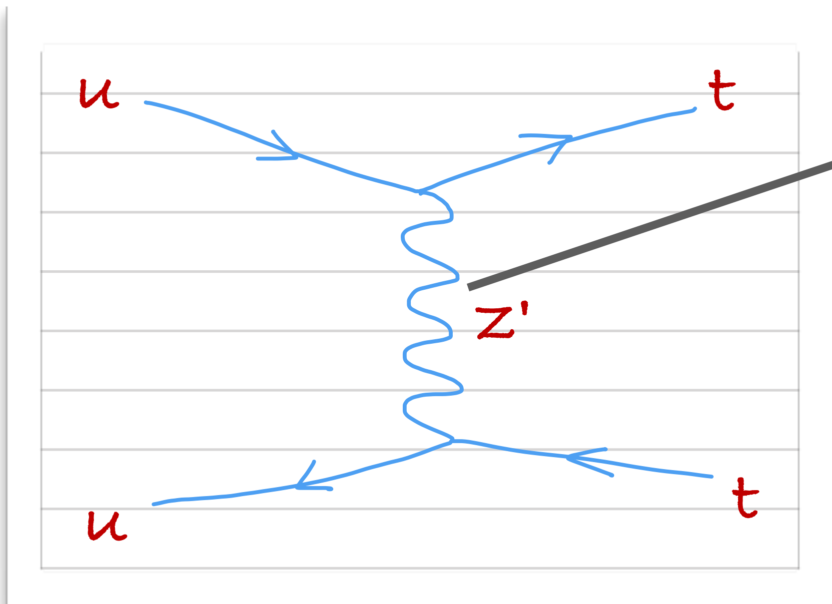
Hypercharge

$$\sum Y = 0$$

Vector bosons		Scalars	
label	rep	label	rep
Z′ → $\mathcal{B}$	$(1,1)_0$	φ	$(1,2)_{-1/2}$
$\mathcal{W}$	$(1,3)_0$	Φ	$(8,2)_{-1/2}$
W′ → $\mathcal{B}^1$	$(1,1)_1$	ω^1	$(3,1)_{-1/3}$
$\mathcal{G}$	$(8,1)_0$	Ω^1	$(6,1)_{-1/3}$
$\mathcal{H}$	$(8,3)_0$	ω^4	$(3,1)_{-4/3}$
$\mathcal{G}^1$	$(8,1)_1$	Ω^4	$(6,1)_{-4/3}$
$\mathcal{Q}^1$	$(3,2)_{1/6}$	σ	$(3,3)_{-1/3}$
$\mathcal{Q}^5$	$(3,2)_{-5/6}$	Σ	$(6,3)_{-1/3}$
$\mathcal{Y}^1$	$(6,2)_{1/6}$		
$\mathcal{Y}^5$	$(6,2)_{-5/6}$		

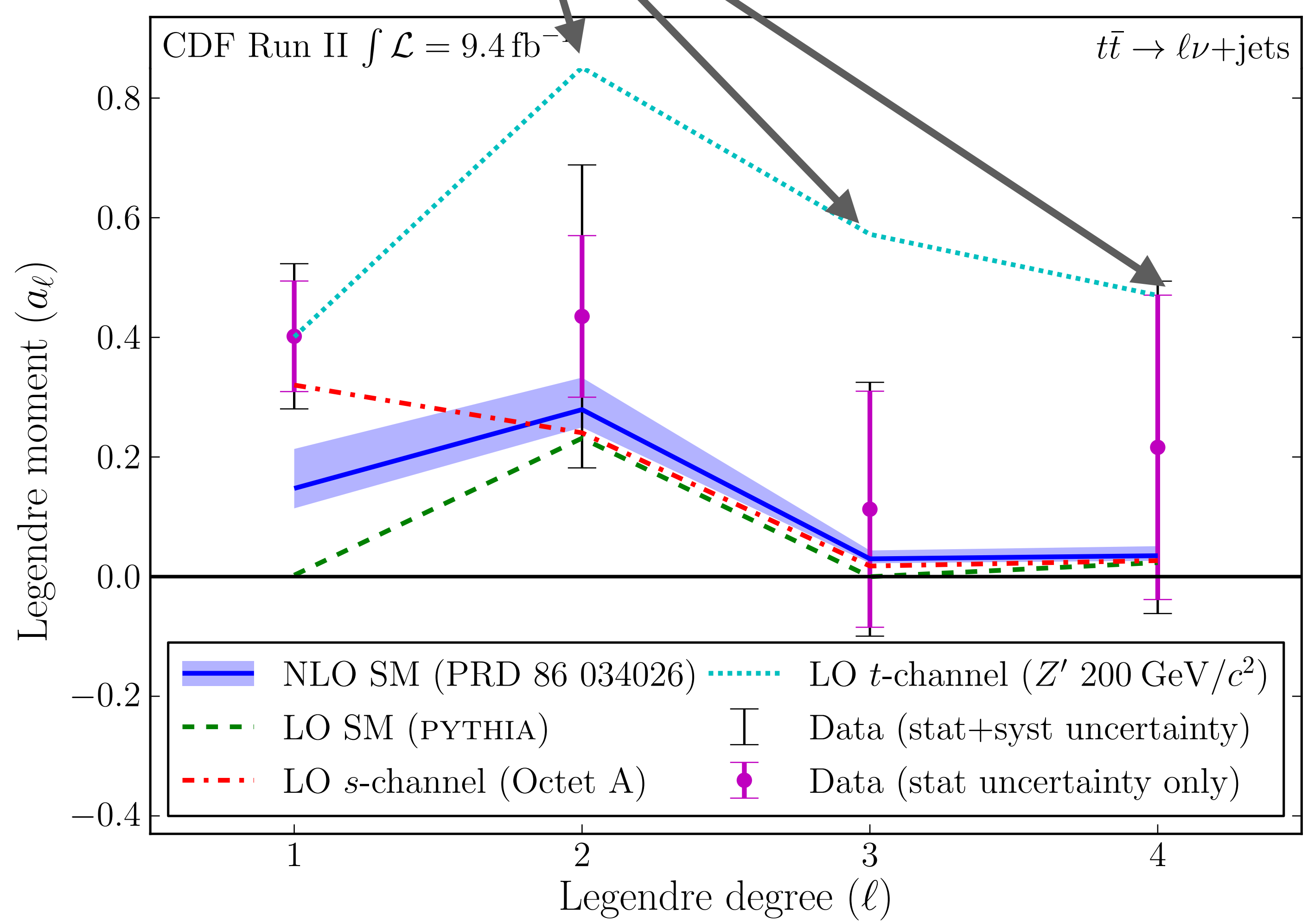
While all these models can accommodate the inclusive Tevatron A_{FB}, discrepancies with data arise when looking at the details.

Remarkably, *t*-channel exchange of light particles also enhances Legendre momenta with $l \geq 2$.

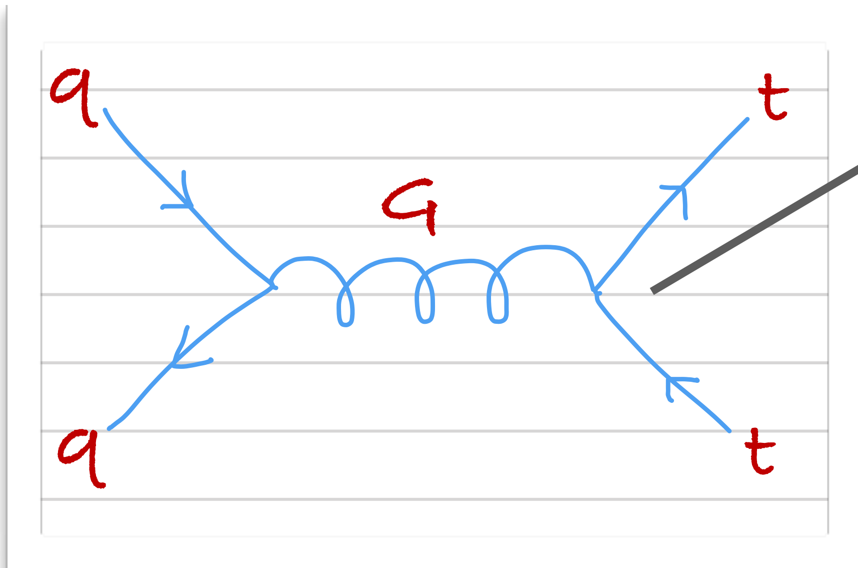


$$\frac{1}{\hat{t} - M_{Z'}^2} f(\hat{s}, \hat{t})$$

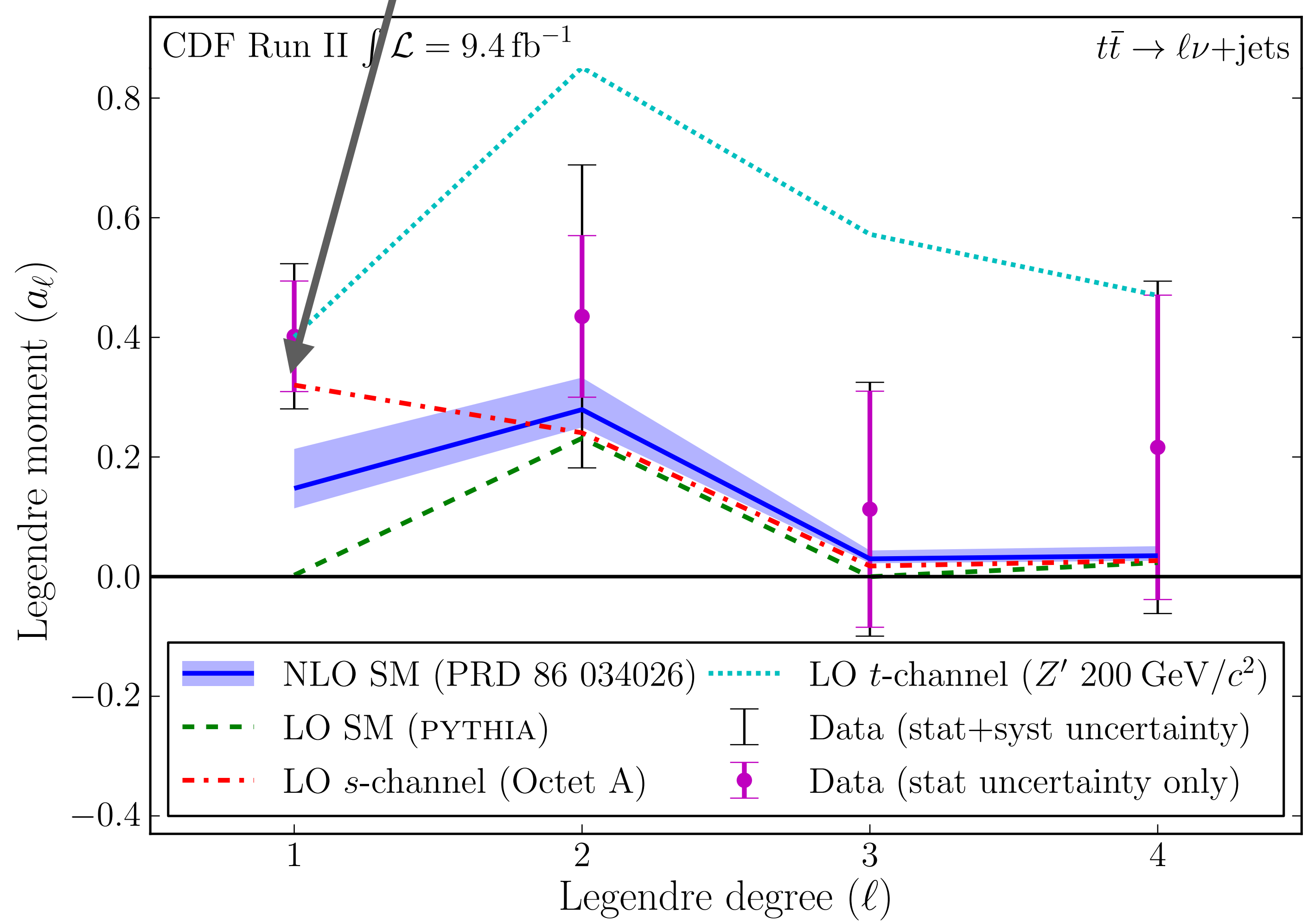
$$\hat{t} = \frac{\hat{s}}{4}(1 - \beta \cos\theta)$$



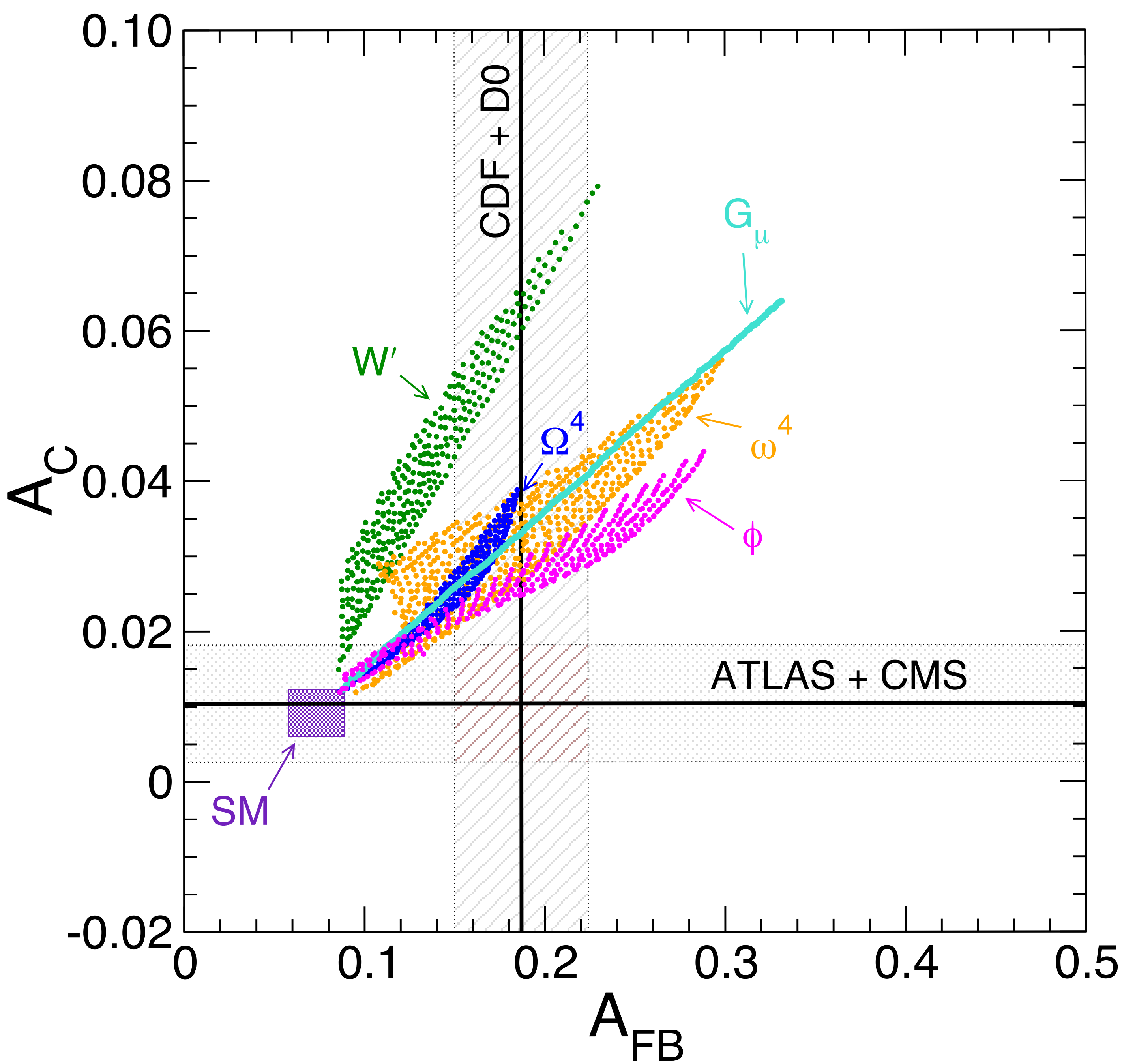
Whereas, s-channel exchange only modifies a_1, precisely the one that exhibits discrepancies (!)



$$\frac{1}{\hat{s} - M_G^2} f(\hat{s}, \hat{t})$$



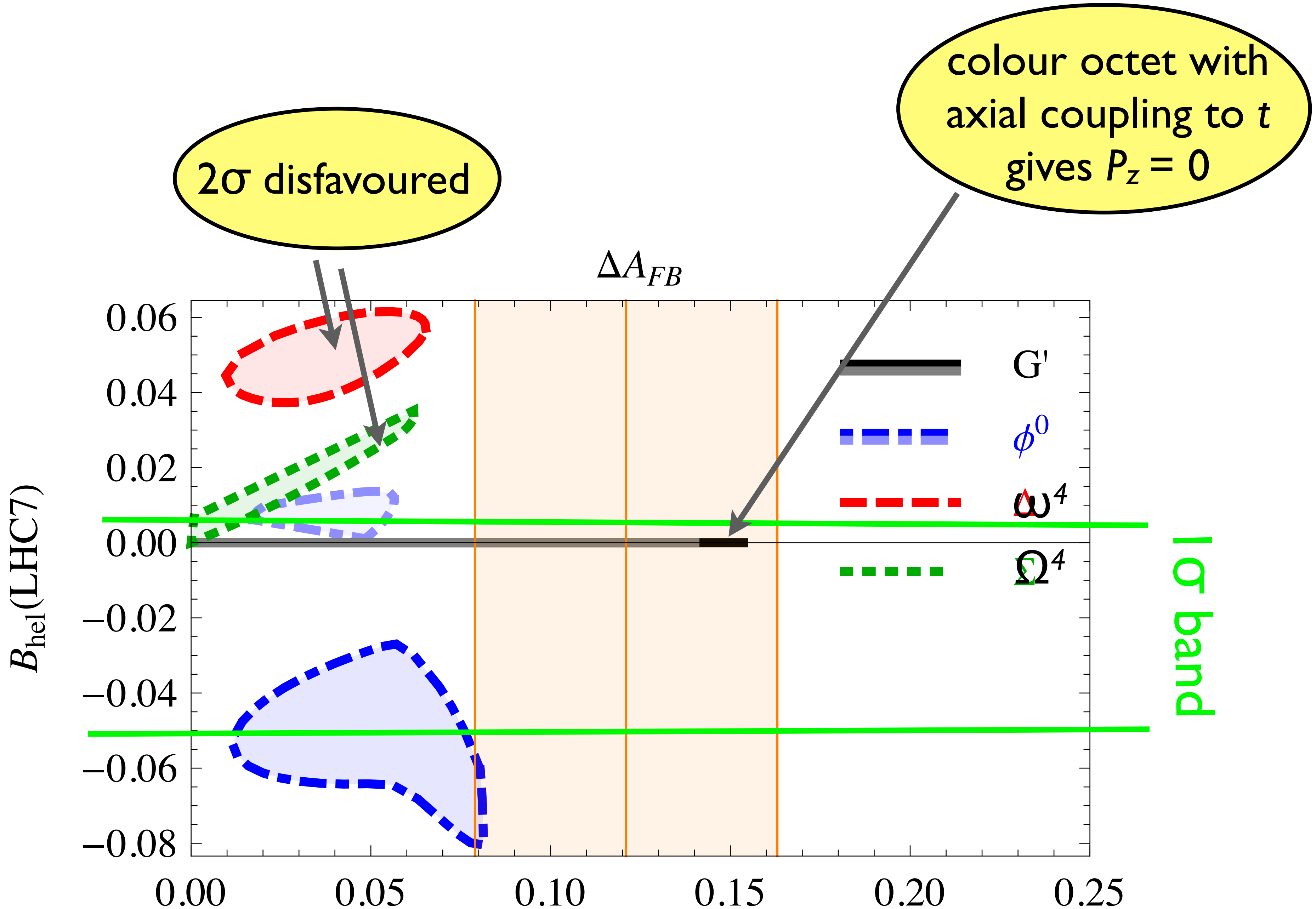
Moreover, the predictions for A_{FB} at the Tevatron and A_C at the LHC are related in *simple* models



A_{FB} and A_C are different observables. This plot only has implications on models, not on experimental measurements.

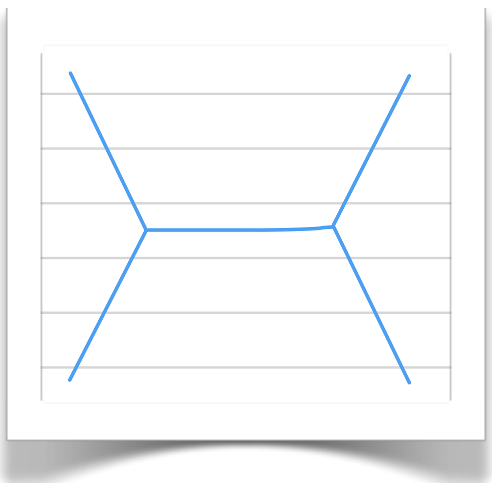
In other words, A_C consistent with the SM does <u>not</u> say anything about A_{FB}.

Even more, P_z measurements [in the helicity basis] at LHC disfavour large $t_R t_R$ new contributions - as predicted by colour sextet and triplet scalars.



Summary: models that once were popular

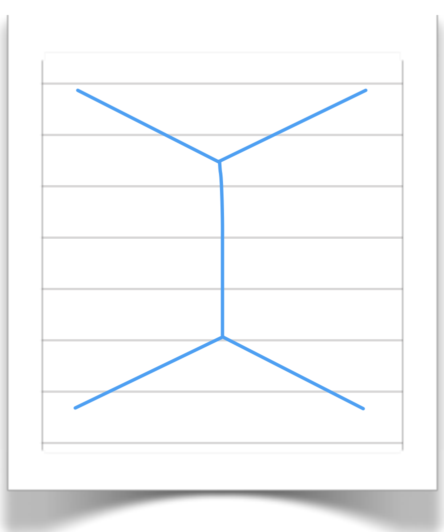
	status	cause of disease



s channel

$G \sim (8,1)_0$

- LHC resonance searches
- dijet pair searches



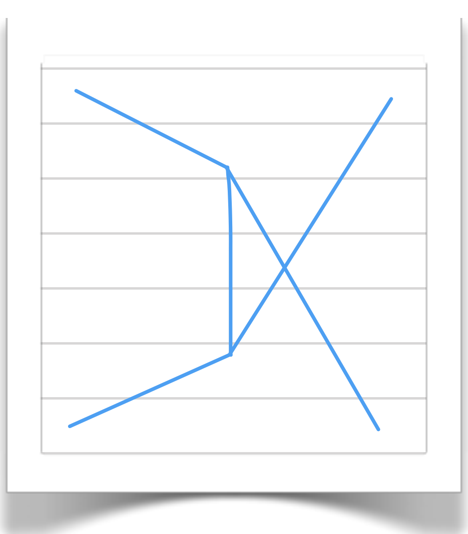
t channel

$Z' \sim (1,1)_0$

$W' \sim (1,1)_1$

$\varphi \sim (1,2)_{-1/2}$

- Z' overpredicts A_{FB} at high m_{tt}
- W' overpredicts A_C at LHC
- not consistent with measured Legendre coefficients
- Z', W' overpredict high m_{tt} tail at LHC



u channel

$\omega^4 \sim (3,1)_{-4/3}$

$\Omega^4 \sim (6,1)_{-4/3}$

- overpredict A_C at LHC
- not consistent with P_z at LHC

So, what?

The A_{FB} puzzle is far from being solved. And there are still hopes that new physics is hiding in the top sector.

This new physics might also be visible indirectly in top pair production, in measurements of (*i*) high m_{tt} tail; (*ii*) A_C; (*iii*) P_z.

Or not. There are examples (*light s*-channel octet with ~ axial coupling to top and different couplings to *u*, *d*) that preserve the three of them and agree with all LHC data.

The actual problem is on models [there aren't really appealing candidates], rather than on the consistency of experimental data.