A few Higgs questions

Problem 1 : Vacuum stability

In the Standard Model the Higgs boson mass is a free parameter. However from the evolution of the quartic coupling of the Higgs boson λ, meaningful constraints on the Higgs boson mass can be derived. The quartic couplings appears in the simplest for of the Higgs potential :

$$V(\varphi) = \mu^2\varphi^*\varphi + \lambda(\varphi^*\varphi)^2$$

Given the following equation for the running of the quartic coupling.

$$32\pi^2\frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

where t is defined as : $t \equiv \ln\frac{Q^2}{Q_0^2}$ where Q_0 is a reference low scale which will take to be equal to the vacuum expectation value v.

1.a.- Which terms can be neglected in the low Higgs boson mass approximation? The Higgs boson mass is related to the quartic coupling by $m_H^2=2\lambda v^2$ where v is the vacuum expectation value.

1.b – Solve the previous equation and derive the lower bound on the Higgs boson mass as a function of a given cutoff scale Q = Λ, assuming that the quartic coupling is never negative up to =Λ. The latter condition protects the vacuum to become unstable.

1.c – Given that $y_t \sim 1$, $g= 2m_W/v$, g' is such that $m_Z^2 = (g^2+g'^2).v/2$ and that v = 246 GeV what is the lower limit on the Higgs boson mass if the cutoff is at the planck scale i.e. $\Lambda = 10^{19}$ GeV?

Corollary: the limit obtained does not really correspond to the limit presented in the lecture, a more involved calculation with higher orders is necessary. Further reading on this subject see M. Sher, Phys. Rept. **179**, 273 (1989).

Problem 2 : Triviality

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where t is defined as : $t \equiv \ln \frac{Q^2}{Q_0^2}$ where Q_0 is a reference low scale which will take to be equal to the vacuum expectation value v = 246 GeV.

2.a.- Knowing that the Higgs boson mass is related to the quartic coupling by $m_H^2 = 2\lambda v^2$ where v is the vacuum expectation value, how does the above equation simplify in the large Higgs boson mass limit?

2.b- Solve the simplified equation. Hint : further simplify the differential equation as a derivative of $1/\lambda(Q^2)$.

2.c- The presence of a Landau pole is due to the fact for $1/\lambda(Q^2)$ to transition from positive to negative values necessarily implies that there is a scale at which the quartic coupling is infinite. What simple condition can be imposed on $1/\lambda(Q^2)$ for this to never happen? On what type of constraint on the Higgs boson mass does this translate.

2.d- Assuming that the Standard Model is valid up to the Planck scale i.e. $\Lambda = 10^{19}$ GeV what is the constraint on the higgs boson mass? What if the Standard Model was valid up to the grand unification scale scale i.e. $\Lambda = 10^{16}$ GeV?

Corollary : Following the solution found in (2.b), this condition is called triviality because in the limit when Q^2 tends to 0, $\lambda(Q^2)$ will also tend to 0 and the theory will become trivial.

Problem 3 : The Discovery of a New Particle at LHC

On July 4th the ATLAS and CMS spokespersons, and the CERN director general have announced the discovery of a new particle with mass close to 125 GeV compatible with the Higgs boson.

3.a- What are the arguments that lead the experiments to claim that the observed new particle is neutral?

3.b- What are the convincing arguments that lead the experiments to claim that the observed new particle is a boson?

3.c- What are the convincing arguments that lead the experiments to think that it is not a spin-1 particle?

3.d- What are the convincing arguments that lead the experiments to think that its charge conjugation is +1?

3.e- The experiments claim that the observation is compatible with the production of a Standard Model Higgs boson. Are there any observations that, if confirmed with more data, could contradict this statement? Would it give an indication for which BSM physics it could be?

Problem 4 : Categories and combinations

Imagine an inclusive selection (event counting) that has 30 events of signal and 100 events of background. What is the expected significance of an excess?

What if we can further divide the inclusive selection in two categories C1 with 12 events of signal and 60 of background and C2 with 18 events of signal and 40 of background. What is the combined sensitivity of the two channels? Can you generalize the combination formulae?

What is the expected limit on a signal for the inclusive selection? Can you also compute the green and yellow bands?

Problem 5 : Coupling fit.

The fit model yields for N_c categories n_s^c signal events. These numbers of signal events can be parameterized in terms of signal strength by:

$$n_s^c = \left(\sum_{i \in \{ggF, VBF, VH, ttH\}} \mu^i \sigma_{SM}^i \times A^{ic} \times \varepsilon^{ic} \right) \times \mu^f Br^f \times L^c$$

Can the parameters μ_i and μ_f be all fitted simultaneously?

We now want to measure deviations from the standard couplings of the Higgs boson to the particles spectrum. The parameterization will be defined as:

$$\begin{aligned} \mathcal{L} \;=\; & \kappa_W \frac{2 m_W{}^2}{v} W_\mu^+ W_\mu^- H + \kappa_Z \frac{m_Z{}^2}{v} Z_\mu Z_\mu H - \sum_f \kappa_f \frac{m_f}{v} f\bar{f}H \\ & + c_g \frac{\alpha_s}{12\pi v} G_{\mu\nu}^a G_{\mu\nu}^a H + c_\gamma \frac{\alpha}{\pi v} A_{\mu\nu} A_{\mu\nu} H \end{aligned}$$

Can you express the μ_i and μ_f parameters as a function of the kappa parameters in the following case:

a.- The main contribution to the gluon fusion channel with subsequent decay to the WW boson in the model with coupling modifiers of fermions and vector bosons only.

b.- The diphton channel in the VBF case in the the model with coupling modifiers of fermions and vector bosons only. What is the dominant term in the gluon fusion dominated channel?

The expression for the total width and the partial widths for the loops are given in the talk