Proposed Problems on Flavour Physics

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Problem 1

a) The electromagnetic interaction of a fermion with electric charge Q is governed by the on-shell matrix element of the electromagnetic current $[q^\mu = (k-k')^\mu)]$:

$$\begin{aligned}\langle f(k')|J^\mu_{\rm em}(x)|f(k)\rangle &= \langle f(k')|\mathrm{e}^{iPx}\, J^\mu_{\rm em}(0)\,\mathrm{e}^{-iPx}|f(k)\rangle \\ &= \mathrm{e}^{-iqx}\, Q\;\; \bar{u}_f(k')\left[F_1(q^2)\gamma^\mu - i\, F_2(q^2)\,\sigma^{\mu\nu}q_\nu\right] u_f(k)\,.\end{aligned}$$

Using the conservation of the electromagnetic current, show that $F_1(0)=1$ to all orders in perturbation theory.

b) The QCD vector current $V^\mu_{ij} = \bar{q}_j\gamma^\mu q_i$ satisfies $\partial_\mu V^\mu_{ij} = i\,(m_{q_j} - m_{q_i})\,\bar{q}_j q_i$. Show that in the isospin limit $\langle p|\bar{u}\gamma^\mu d|n\rangle = \bar{p}\gamma^\mu n$ at $q^2=0$.

c) The transition $\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$ is governed by the hadronic matrix element

$$\langle \pi^+(k')|\bar{u}\gamma^\mu s|\bar{K}^0(k)\rangle \;=\; f_+(q^2)\,(k+k')^\mu + f_-(q^2)\,(k-k')^\mu\,.$$

Show that the contribution to the decay amplitude from the form factor $f_-(q^2)$ is proportional to m_e. Show also that $f_+(0)=1$ in the limit $m_s = m_u$.

Problem 2

Consider the mixing between a neutral meson P^0 and its antiparticle $\bar{P}^0$, with mass eigenstates

$$|P_\mp\rangle \;=\; \frac{1}{\sqrt{|p|^2+|q|^2}}\left[p\,|P^0\rangle \,\mp\, q\,|\bar{P}^0\rangle\right].$$

Show that the time evolution of a state which was originally produced as a P^0 or a $\bar{P}^0$ is given by

$$\begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p}\, g_2(t) \\ \frac{p}{q}\, g_2(t) & g_1(t)\end{pmatrix}\begin{pmatrix} |P^0\rangle \\ |\bar{P}^0\rangle\end{pmatrix},$$

where

$$\begin{pmatrix} g_1(t) \\ g_2(t)\end{pmatrix} = \mathrm{e}^{-iMt}\,\mathrm{e}^{-\Gamma t/2}\begin{pmatrix} \cos\left[(\Delta M - \frac{i}{2}\Delta\Gamma)t/2\right] \\ -i\sin\left[(\Delta M - \frac{i}{2}\Delta\Gamma)t/2\right]\end{pmatrix},$$

with $\Delta M \equiv M_{P_+} - M_{P_-}$, $\Delta\Gamma \equiv \Gamma_{P_+} - \Gamma_{P_-}$.

Problem 3

Consider the most general Yukawa Lagrangian with two Higgs doublets:

$$\mathcal{L}_Y \;=\; -\sum_{a=1}^{2}\;\left\{\bar{Q}_L'\left(\mathcal{Y}_d^{(a)'}\,\phi_a\,d_R' + \mathcal{Y}_u^{(a)'}\,\tilde{\phi}_a\,u_R'\right) + \bar{L}_L'\,\mathcal{Y}_l^{(a)'}\,\phi_a\,l_R'\right\} + \text{h.c.}\,,$$

where $\phi_a(x)$ are the $Y = \frac{1}{2}$ scalar doublets, $\tilde{\phi}_a(x) \equiv i\tau_2\,\phi_a^*$ their charge-conjugate fields, Q_L' and L_L' denote the left-handed quark and lepton doublets and d_R', u_R' and l_R' the corresponding right-handed fermion singlets. All fermionic fields are written as N_G-dimensional flavour vectors, with N_G the number of fermion generations; the couplings $\mathcal{Y}_f^{(a)'}$ $(f = d, u, l)$ are $N_G \times N_G$ complex matrices in flavour space.

a) Show that this Lagrangian leads to flavour-changing neutral current (FCNC) interactions of the physical scalars (for simplicity, work in the so-called 'Higgs basis' where only ϕ_1 acquires a vacuum expectation value).

b) Impose the alignment in flavour space of the Yukawa matrices $\mathcal{Y}_f^{(a)}$; i.e., the conditions $\mathcal{Y}_f^{(2)} = \zeta_f\,\mathcal{Y}_f^{(1)}$ with ζ_f $(f = u, d, s)$ 3 arbitrary complex constants. Check that FCNC interactions are absent at tree level. Work out the form of the Yukawa Lagrangian in the fermion mass-eigenstate basis.