

# QCD & Jets & MC Modeling

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## Lecture I

Many thanks to Guenther Dissertori, Rikkert Frederix, Fabio Maltoni, Paolo Nason, Gavin Salam, Gregory Soyez, Maria Ubiali, and probably others, from whose talks/lectures I have drawn inspiration, as well as extracted many slides

# Misplaced fears

Before the LHC started, I was afraid of two things:

- ▶ That the machine would be often stopped because of technical issues → low integrated luminosity
- ▶ That simulations and data would differ significantly → long time before enough confidence for physics results

**Both fears proved to be totally misplaced (luckily)**

- ▶ the LHC has accumulated more than  $20 \text{ fb}^{-1}$  of data (enough for producing about half a million Higgs bosons...)
- ▶ The vast majority of predictions and simulations was in very good agreement with the experimental data

The LHC Collaborations have so far published more than 600 physics papers

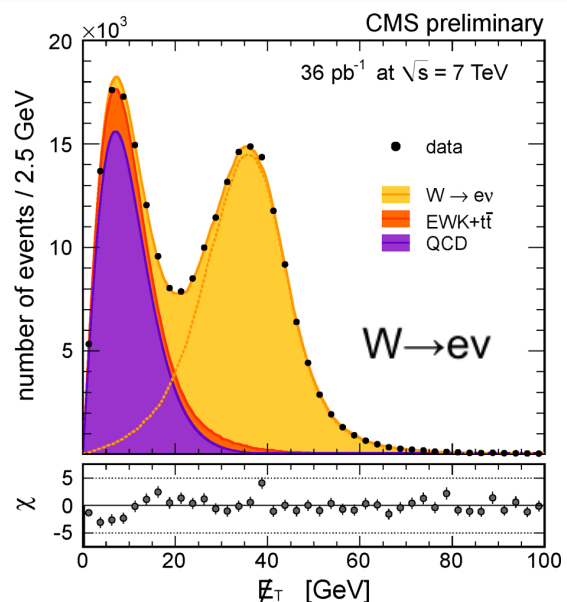
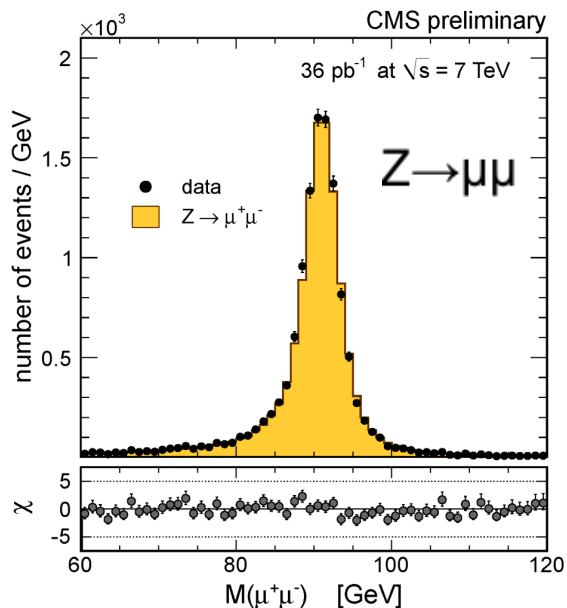
Some examples of their results, also highlighting the accuracy of theoretical predictions, follow

(Note that many plots are now outdated, and could be replaced by even better ones)

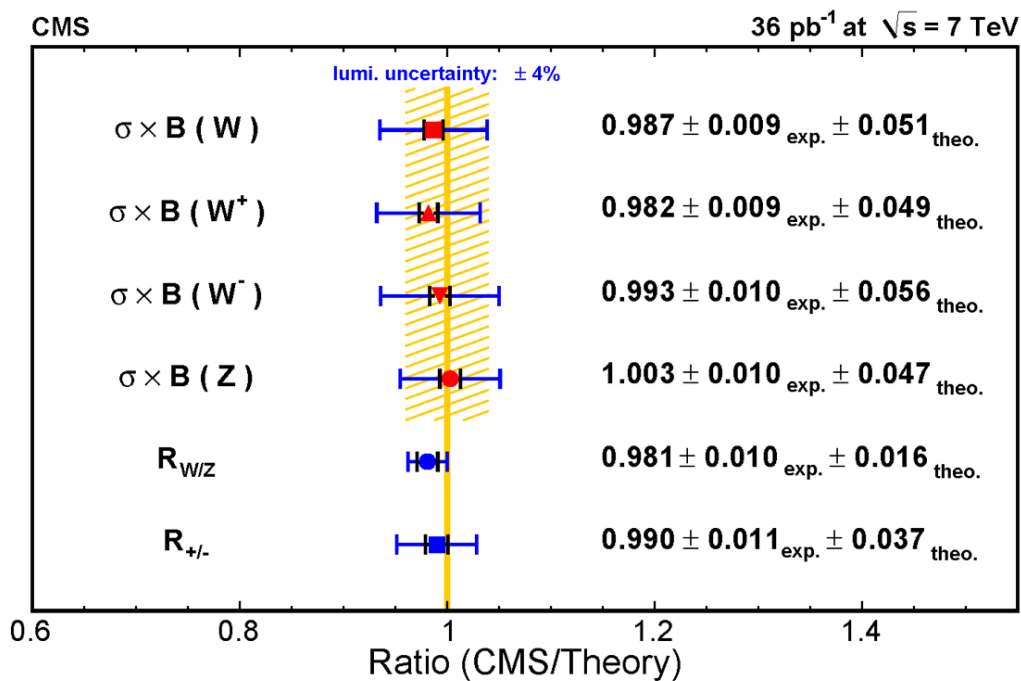


LHCphenNet

## Inclusive W and Z production



- Z important tool : data-driven methods for controlling lepton eff, scale, resolution,  $E_{Tmiss}$  (hadronic recoil).
- In general excellent data-MC agreement



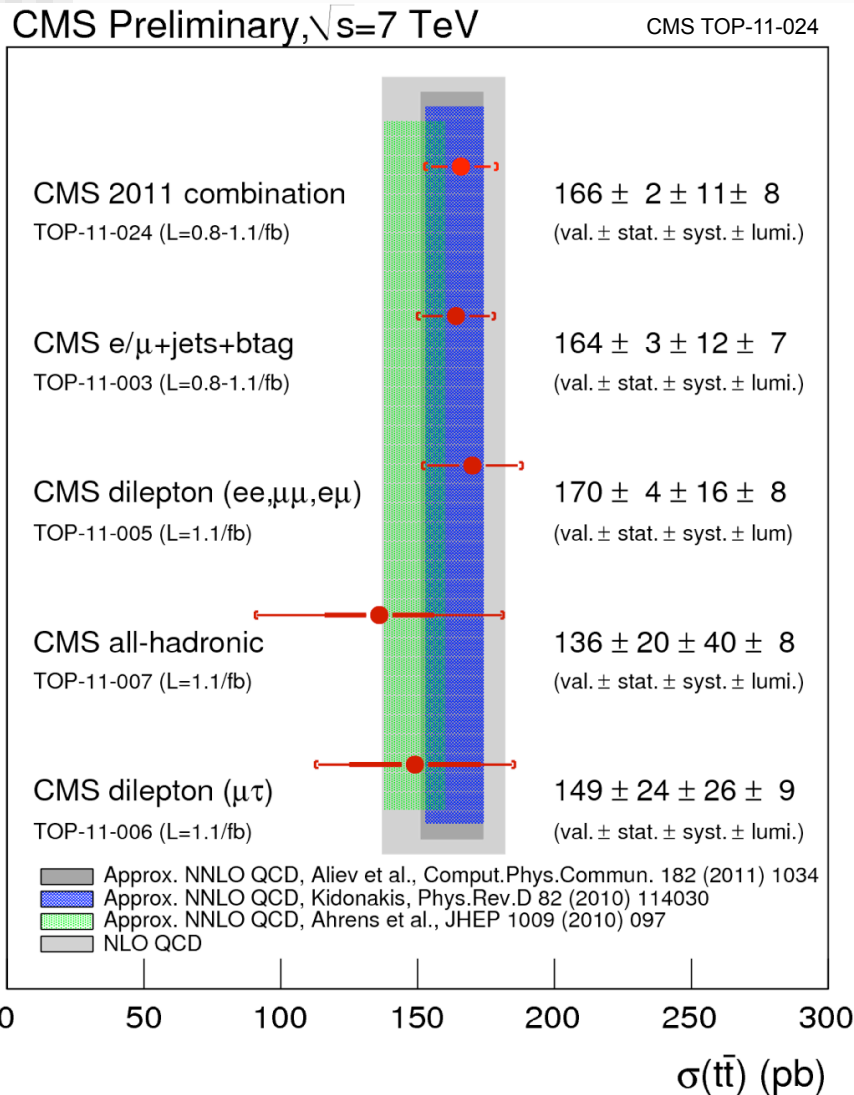
Amazing precision reached ( ~1% experimental ! )  
Start to put important constraints on theory (NNLO, PDFs)

CMS arXiv:1107.4789



LHCphenonet

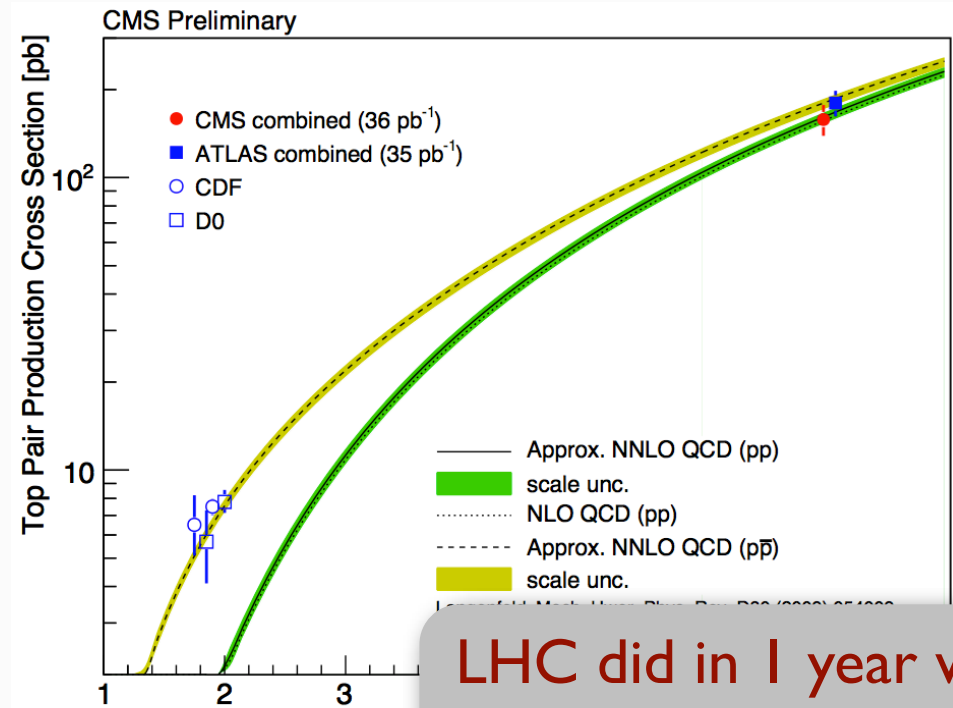
## Top cross section



also tau channels included by now!

Similar results by ATLAS.

Excellent agreement with theoretical calculations so far...



### Other measurements

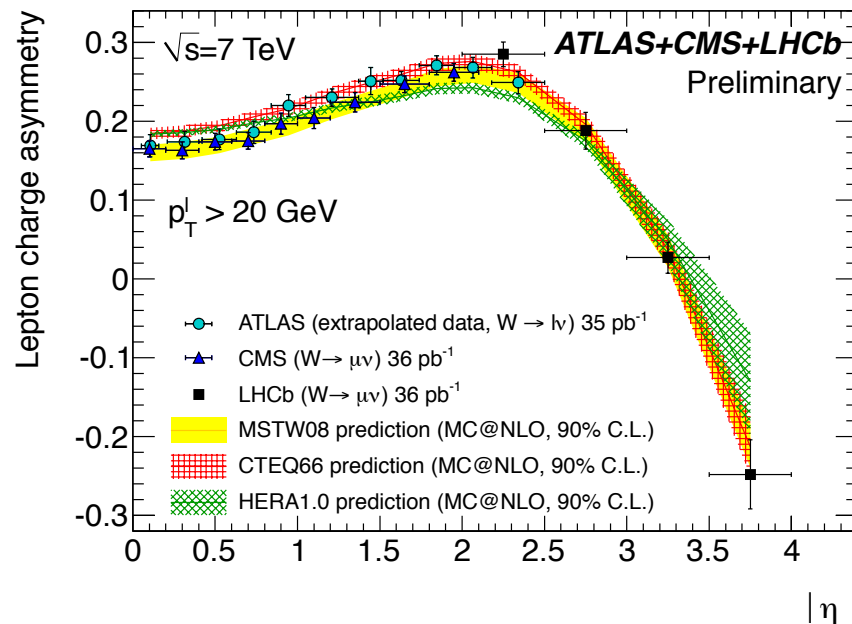
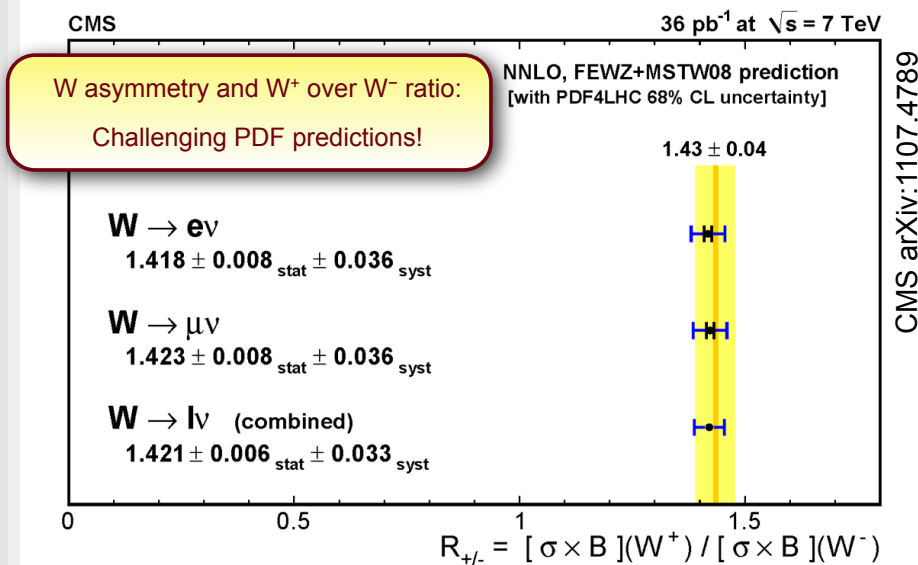
- Top Charge asymmetry at ppbar collider,
- Top-AntiTop non-resonant production,
- single top production

LHC did in 1 year what the Tevatron took 10 to do. Theory keeping up: full NNLO calculation just completed

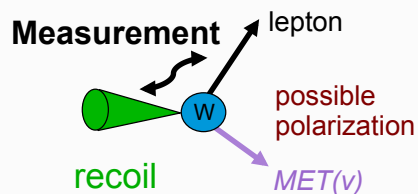


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## W properties, constraining PDFs

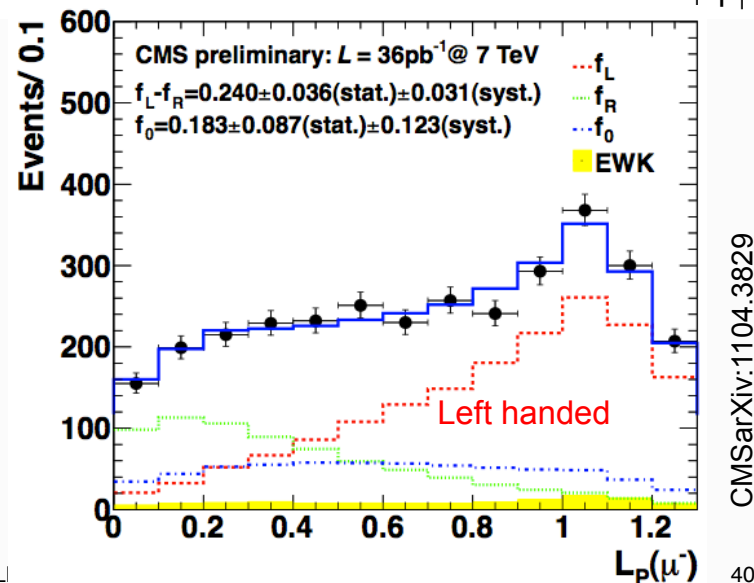


Measurement of W polarization:  
both W<sup>+</sup> and W<sup>-</sup> preferred left-handed



$$LP = \frac{\vec{p}_T(\ell) \cdot \vec{p}_T(W)}{|\vec{p}_T(W)|^2}$$

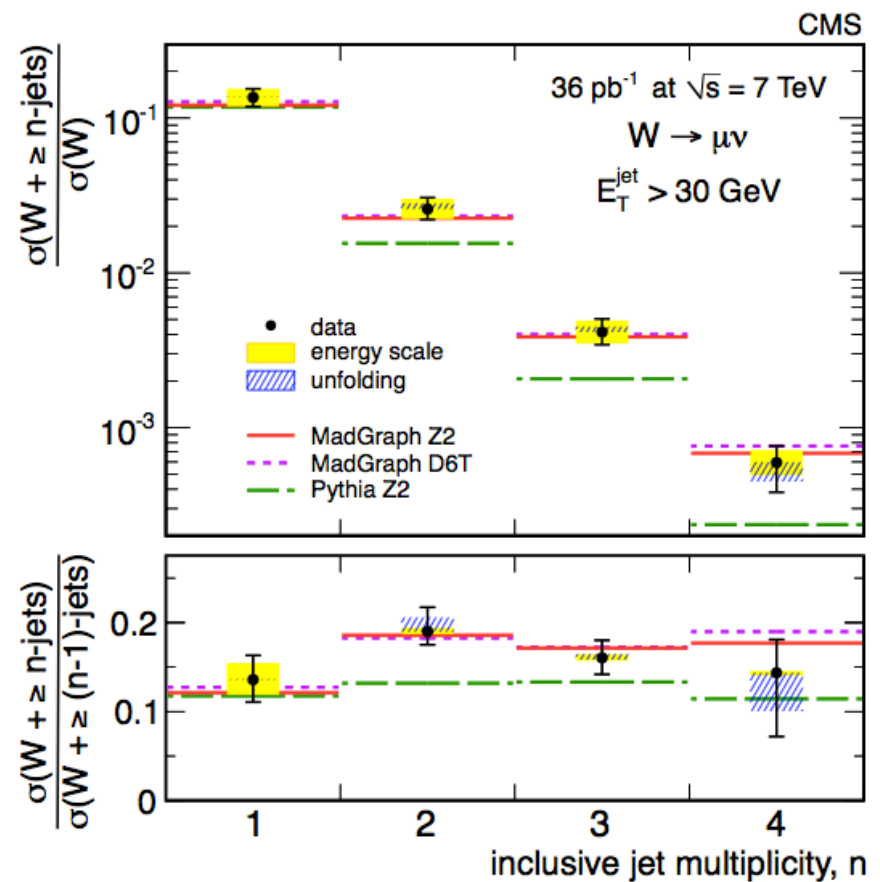
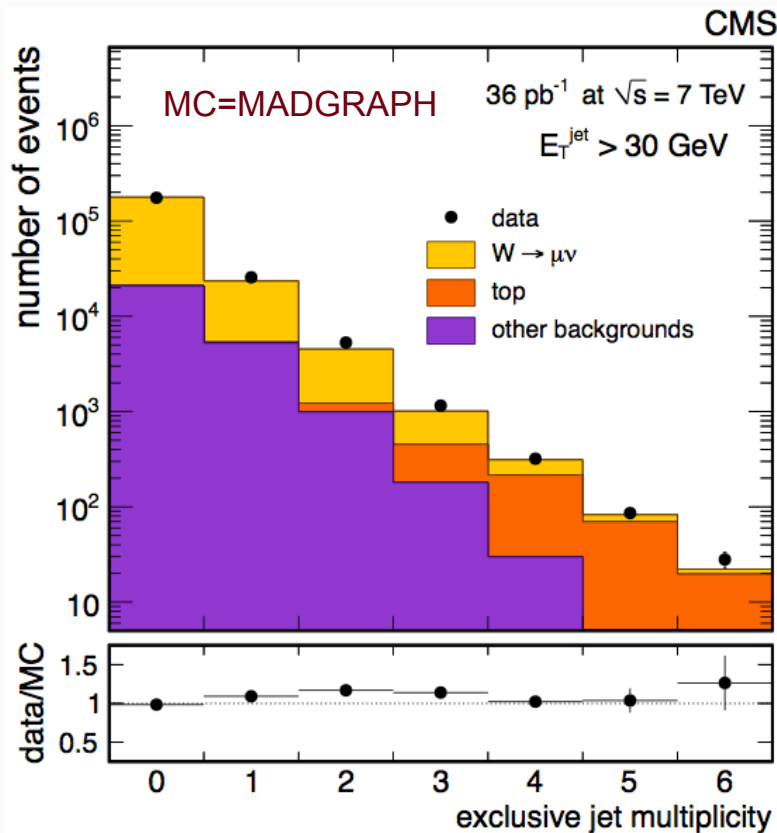
$$p_T(W) > 50 \text{ GeV}$$





LHCphenOnet

## W+jets

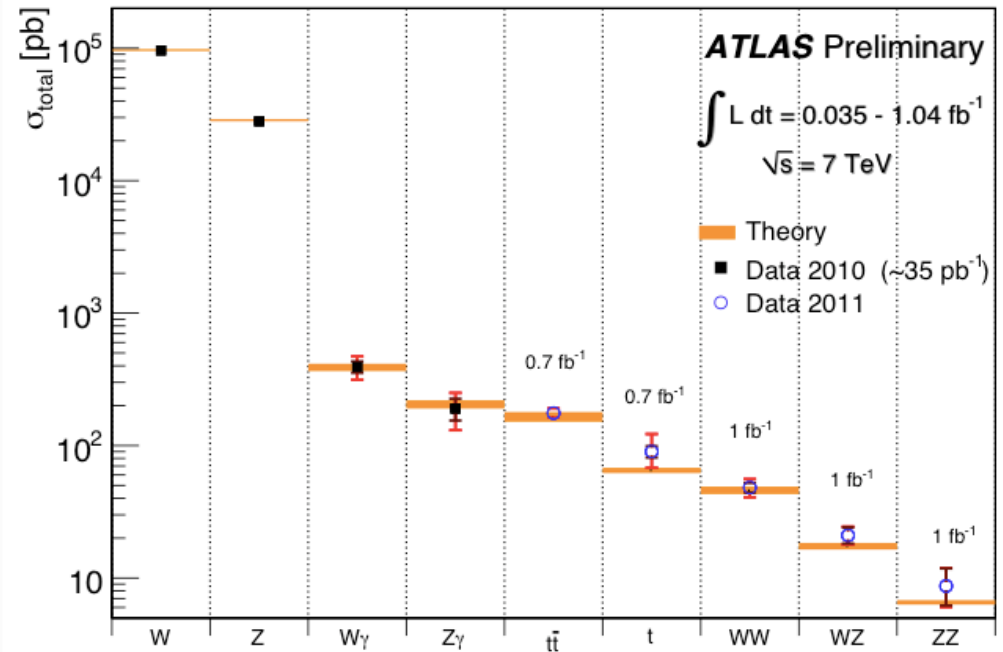
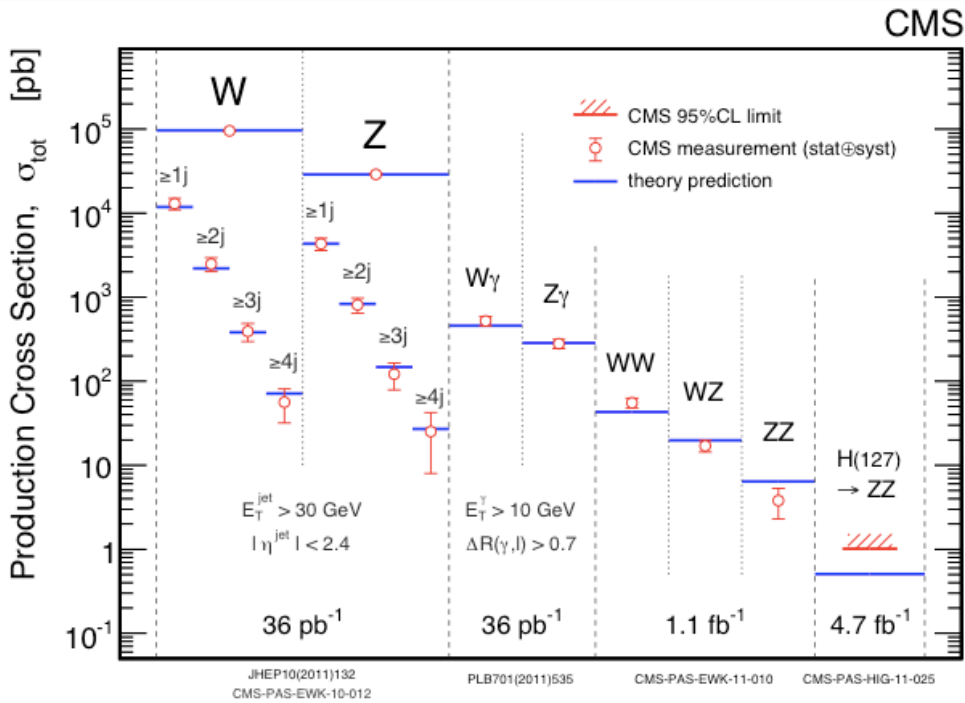


- **simultaneous** extraction of W signal and top background
- final distributions: **unfolded to particle level**
- presented for experimental lepton and jet acceptance, eg.  $p_{T\text{jet}} > 30$  GeV

An additional jets “costs”  $\sim 1 \alpha_s$   
 Excellent agreement with ME+PS matched Monte Carlo model.

**Great predictivity up to large multiplicities**

## The mother of all data/theory comparisons



Mostly excellent agreement



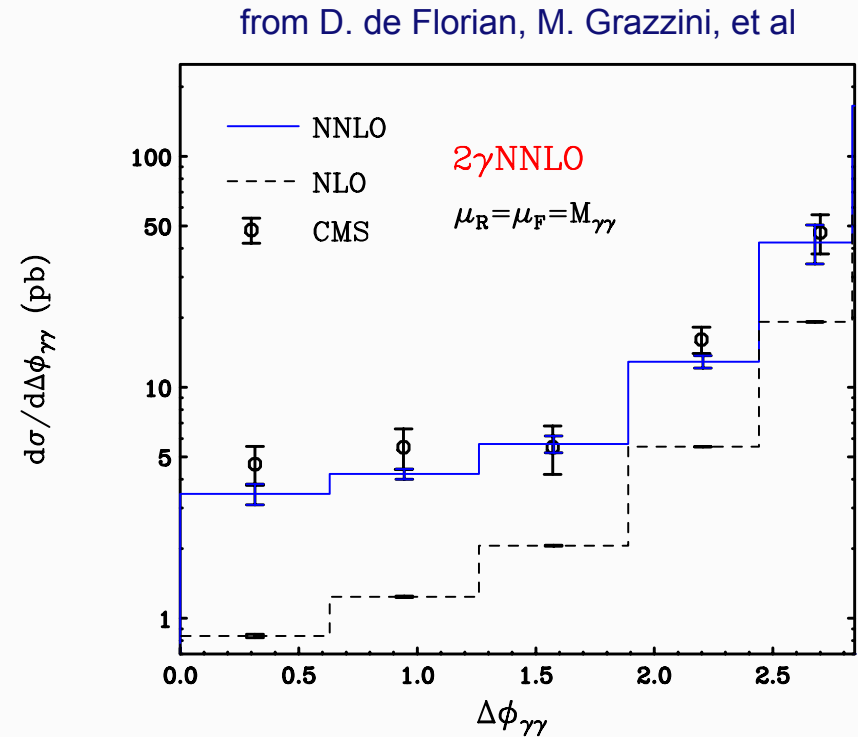
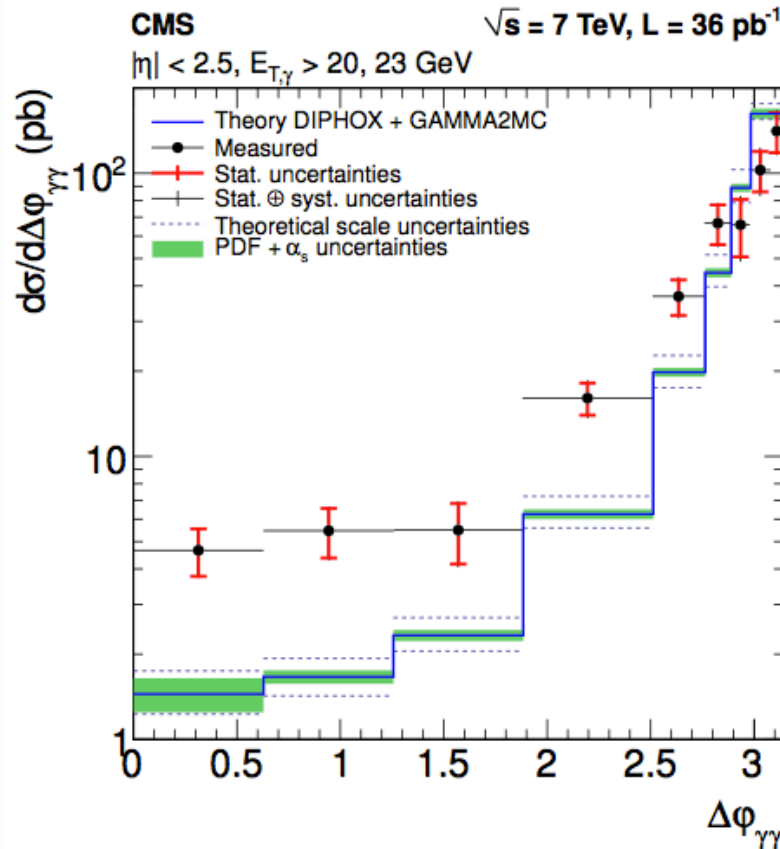
It is worth noting that the data/theory comparison does not **always** work perfectly.

On the other hand, theoretical progress continues to be made, and often wrongs are righted



LHCphenonnet

## Di-Photon Production: Results



● **Big discrepancy at small angles???**

- But note: at very small angles, the NLO calculation is
- confirmed by very recent calculation (see plot on the right)

Very recent NNLO calculation seems to eliminate the discrepancy

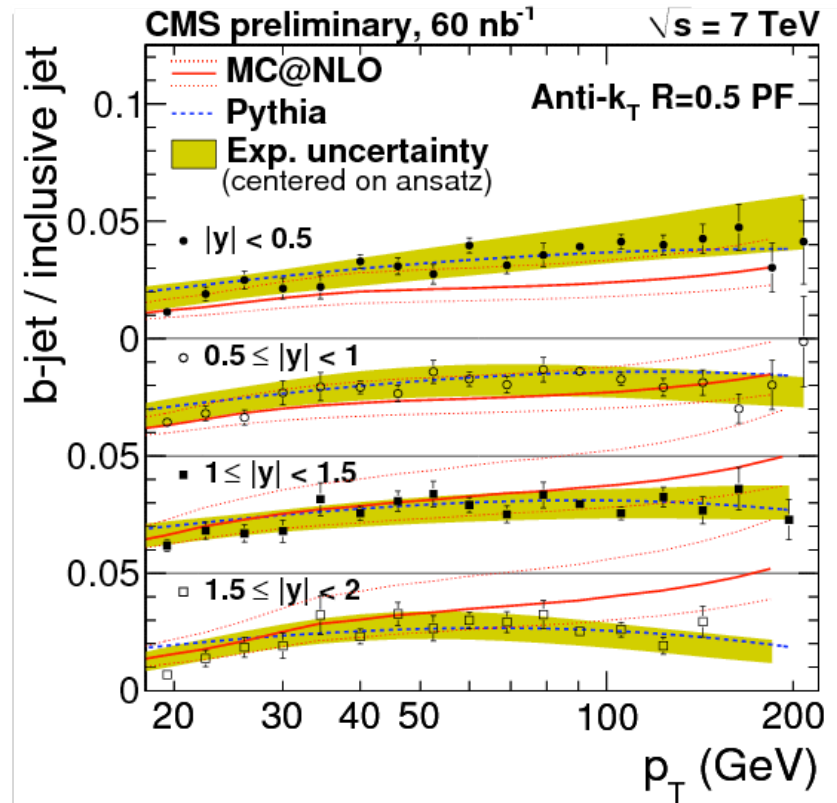
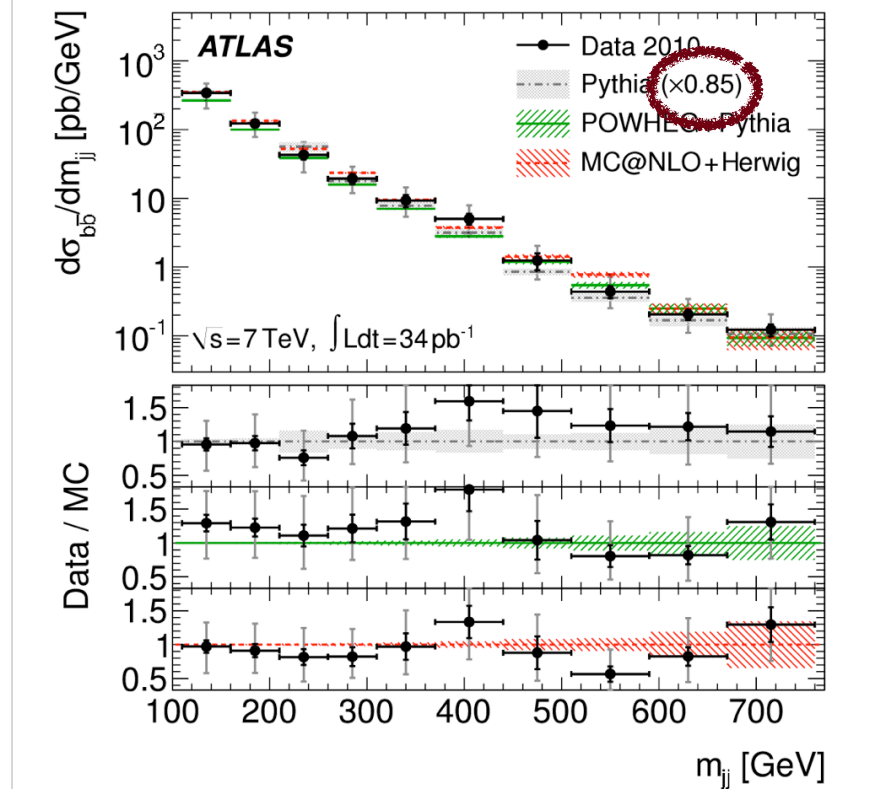


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## b-jet production: Results



ATLAS arXiv:1109.6833



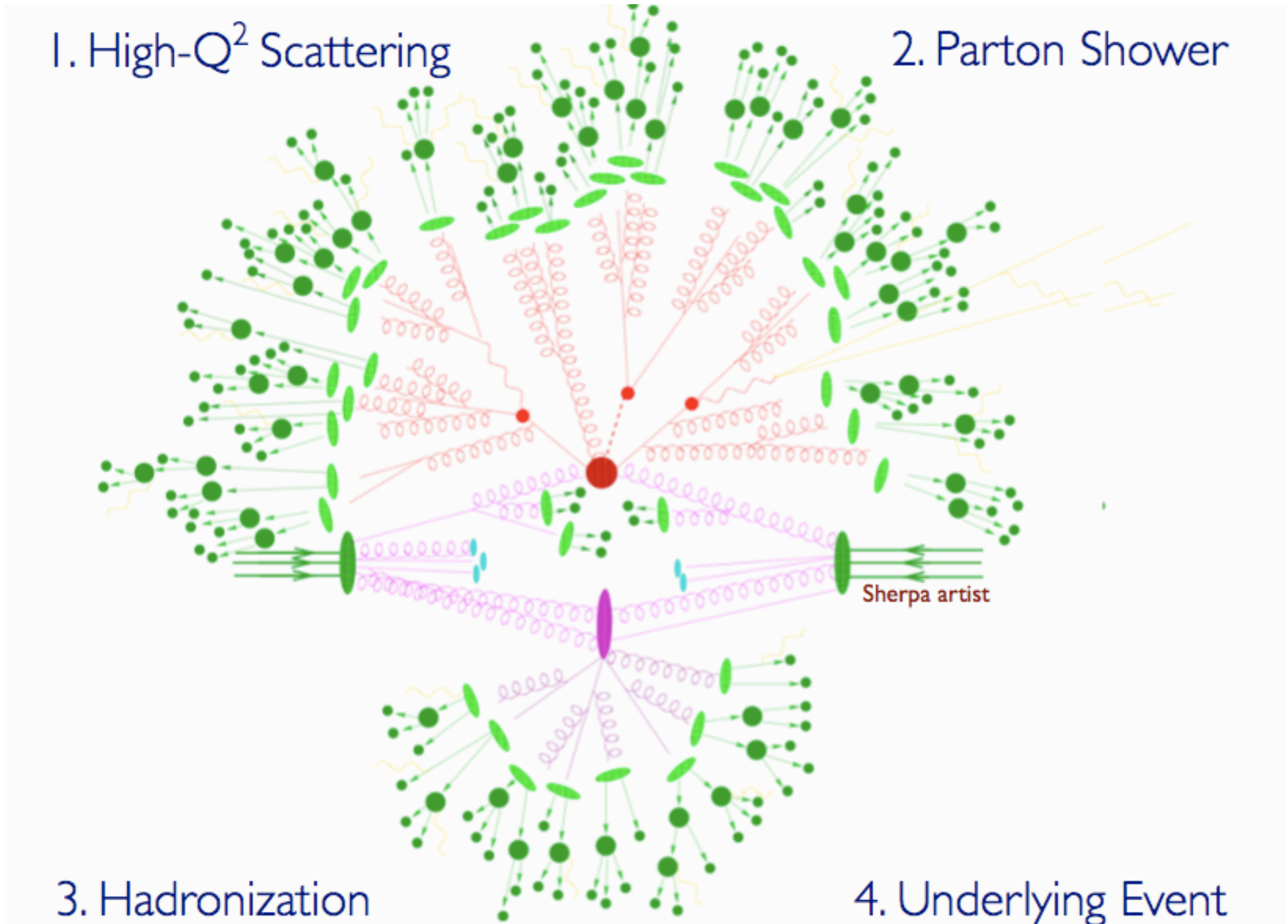
CMS PAS BPH-10-009

- Also discrepancies seen with MC@NLO, for inclusive cross-section
- ratio to inclusive jet cross section helps to eliminate some of the

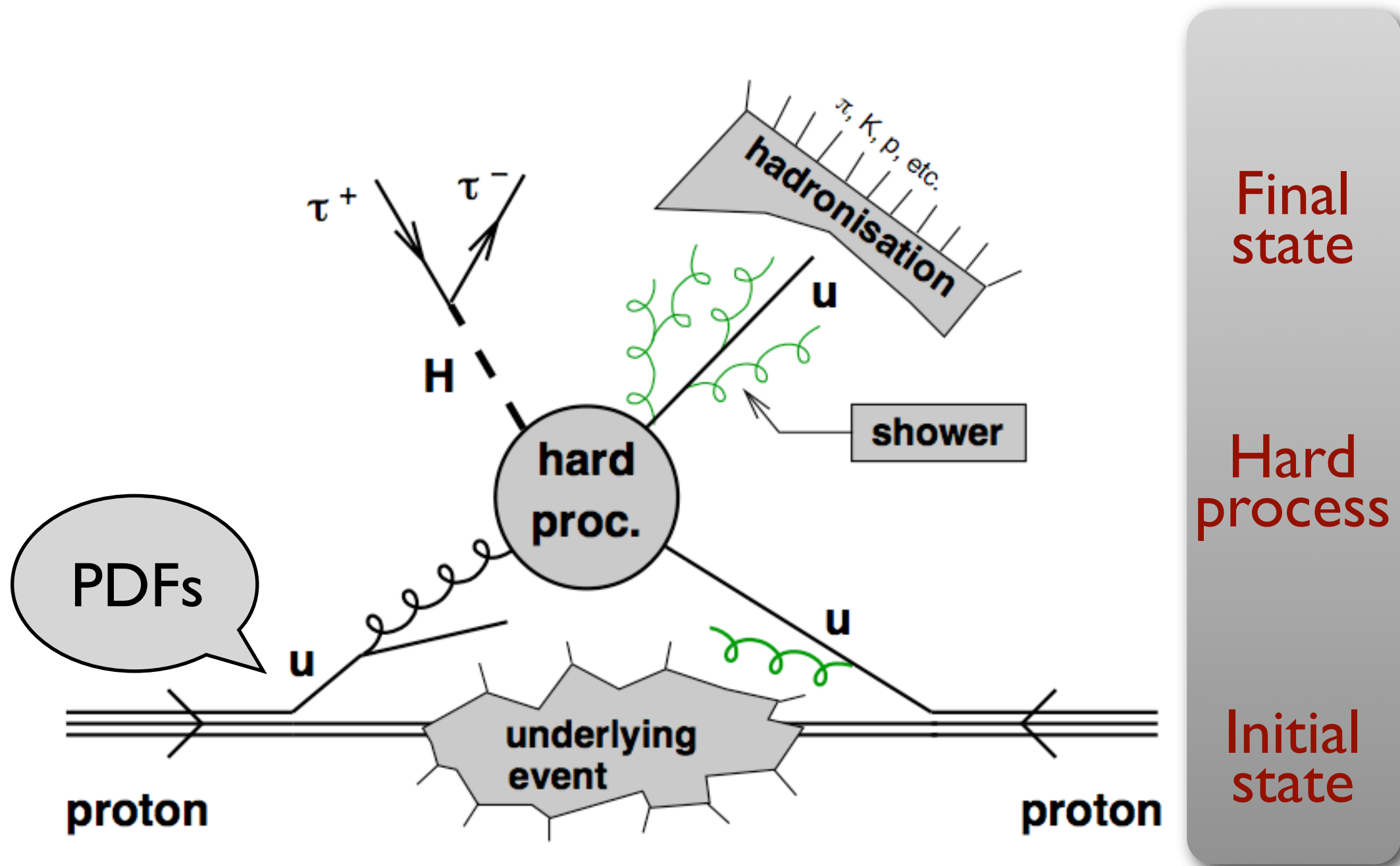
Something still wrong at very large  $p_T$ ?

**In what kind of environment have these measurements and calculations taken place?**

# A hadronic process



# A hadronic process



# Describing complexity

A large part of the success of LHC physics (and the speed with which it has come) must be due to the excellent quality of the simulation tools for detectors and physics employed there.

Tevatron did not have such good tools, especially at the beginning of its 25 years run. It took a lot longer to understand the detector and to extract physics.

[I think it was at LEP that the need/usefulness of high-precision simulations/predictions became evident]

# Evolution of (physics) tools

## ▶ 10 years ago we had

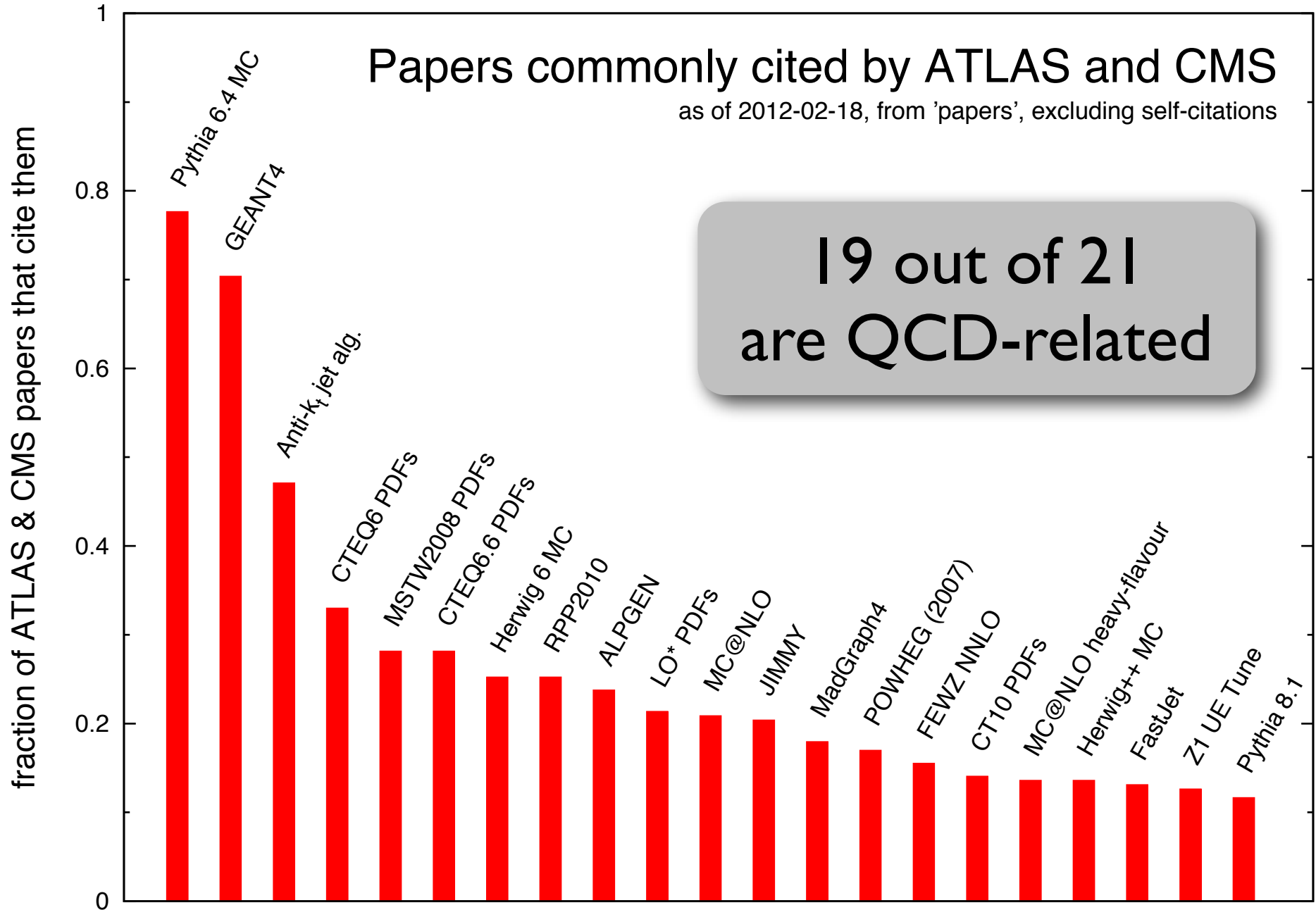
- ▶ PYTHIA, HERWIG (parton shower MCs)
- ▶ GRV, CTEQ, MRST (NLL PDFs)
- ▶ first automated tools for tree level (CompHEP,...)
- ▶ dedicated NLO codes, for fairly simple processes

## ▶ now we also have

- ▶ PYTHIA8, HERWIG++, SHERPA
- ▶ MC@NLO, POWHEG (matching of NLO with PS)
- ▶ matching of PS with matrix elements (CKKW, MLM)
- ▶ more PDFs sets, some at NNLL (NNPDF, HERAPDF, ABKM, JR,...)
- ▶ many more NLO calculations, including for complex processes
- ▶ automated tools for LO and NLO (MadGraph, aMC@NLO,...)
- ▶ dedicated NNLO codes, for fairly simple processes



# Role of tools in ATLAS and CMS



*“We are driven to the conclusion that the Hamiltonian method for strong interactions is dead and must be buried, although of course with deserved honor”*

Lev Landau

*“The correct theory [of strong interactions] will not be found in the next hundred years”*

Freeman Dyson

**We have come a long way towards  
disproving these predictions**

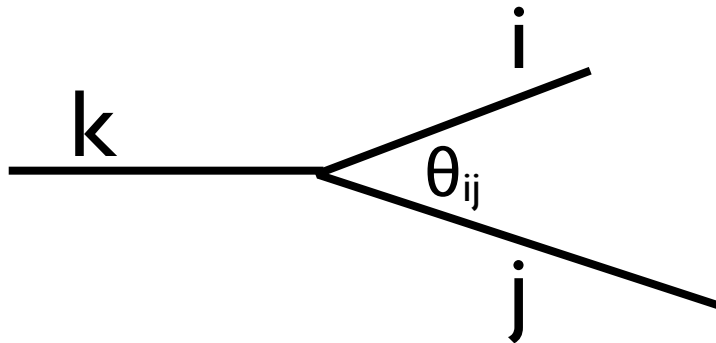
- ▶ **Some basics of QCD**
- ▶ **Initial state**
  - ▶ PDFs
- ▶ **Hard scattering (and more)**
  - ▶ higher order calculations and generators
    - ▶ Parton shower MCs
    - ▶ Merging
- ▶ **Final state**
  - ▶ Jets algorithms and jet areas
  - ▶ Jets as tools (jet substructure)

Lecture 1

Lectures 2 and 3

[Subdivision in parts actually quite unreliable. Length/depth of descriptions varies quite a lot]

# QCD emission probability



$$\frac{dP_{k \rightarrow ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j) \theta_{ij}}$$

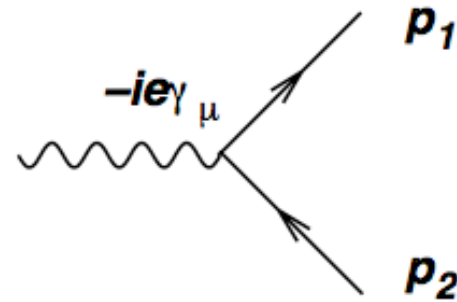
Divergent in the **soft** ( $E_{i,j} \rightarrow 0$ ) and  
in the **collinear** ( $\theta_{ij} \rightarrow 0$ ) limits

The divergences can be cured by the addition of virtual corrections  
and/or **if** the definition of an observable is appropriate

# QCD emission : more details

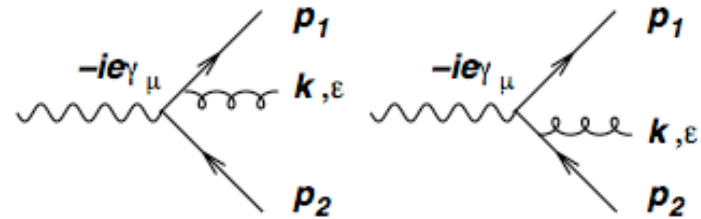
Start with  $\gamma^* \rightarrow q\bar{q}$ :

$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1)ie_q\gamma_\mu v(p_2)$$



Emit a gluon:

$$\begin{aligned} \mathcal{M}_{q\bar{q}g} = & \bar{u}(p_1)ig_s\not{\epsilon}t^A \frac{i}{\not{p}_1 + \not{k}} ie_q\gamma_\mu v(p_2) \\ & - \bar{u}(p_1)ie_q\gamma_\mu \frac{i}{\not{p}_2 + \not{k}} ig_s\not{\epsilon}t^A v(p_2) \end{aligned}$$



In the **soft** limit ,  $k \ll p_{1,2}$

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}(p_1)ie_q\gamma_\mu t^A v(p_2) g_s \left( \frac{p_1 \cdot \epsilon}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon}{p_2 \cdot k} \right)$$

# QCD emission: more details

Squared amplitude, including phase space

$$d\Phi_{q\bar{q}g} |M_{q\bar{q}g}^2| \simeq (d\Phi_{q\bar{q}} |M_{q\bar{q}}^2|) \frac{d^3\vec{k}}{2E(2\pi)^3} C_F g_s^2 \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$

**Factorisation:** Born × radiation

Changing variables (use energy of gluon  $E$  and emission angle  $\theta$ ) we get for the radiation part

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

## Bremsstrahlung spectrum: $1/E$ and $1/\theta$

- ▶ It *diverges* for  $E \rightarrow 0$  — *infrared (or soft) divergence*
- ▶ It *diverges* for  $\theta \rightarrow 0$  and  $\theta \rightarrow \pi$  — *collinear divergence*

[NB. If the quark is massive, the collinear divergence is absent ,  
it is 'screened' by the finite quark mass]

# Altarelli-Parisi kernel

Using the variables  $E=(1-z)p$  and  $k_t = E\theta$  we can rewrite

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi} \rightarrow \frac{\alpha_s C_F}{\pi} \frac{1}{1-z} dz \frac{dk_t^2}{k_t^2} \frac{d\phi}{2\pi}$$

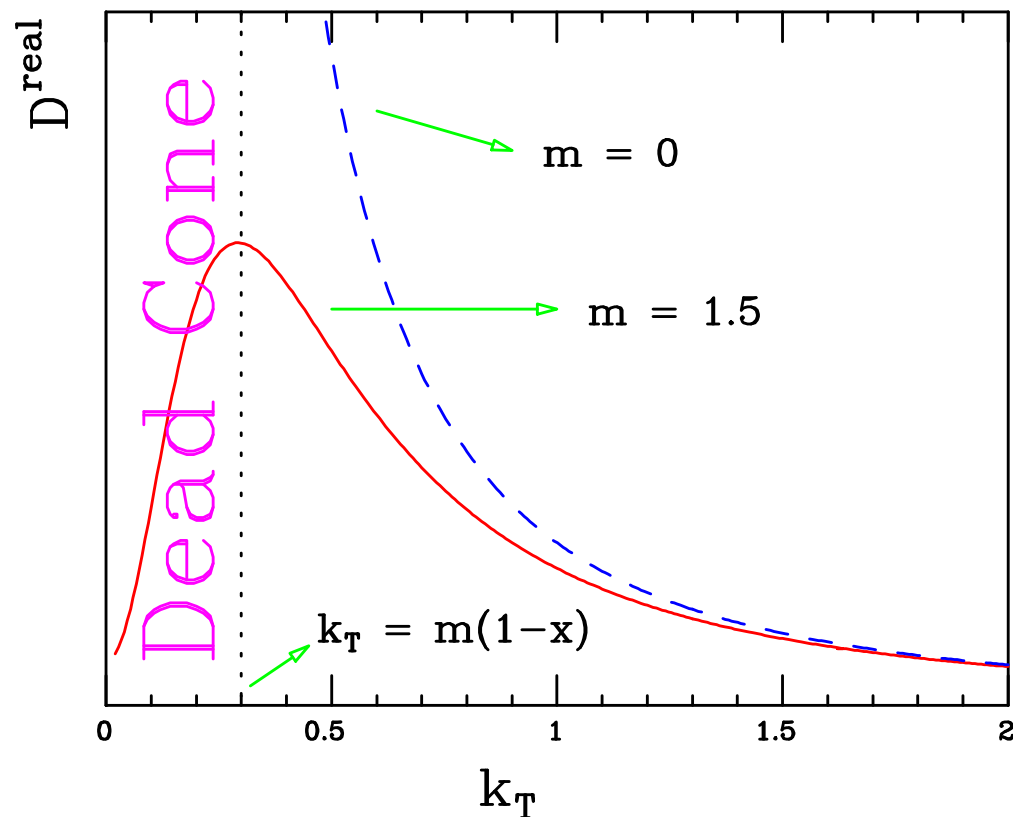
‘almost’ the Altarelli-Parisi  
splitting function  $P_{qq}$



# Massive quarks

If the quark is massive the collinear singularity is **screened**

$$\frac{\alpha_s C_F}{\pi} \frac{1}{1-z} dz \frac{dk_t^2}{k_t^2} \frac{d\phi}{2\pi} \rightarrow \frac{\alpha_s C_F}{\pi} \frac{1}{1-z} dz \frac{dk_t^2}{k_t^2 + (1-z)^2 m^2} \frac{d\phi}{2\pi} + \dots$$



# The template for an hadronic process

$$H_1 H_2 \rightarrow H_3 + X$$

$$\frac{d\sigma}{d^3p}(Q) \sim F(\mu_F) \times F(\mu_F) \times \frac{d\hat{\sigma}}{d^3\hat{p}}(\mu_f, \mu_R, \alpha_s(\mu_R)) \times D(\mu_F) + O\left(\frac{\Lambda}{Q}\right)^p$$

fit from data,  
use in other predictions

short-distance,  
calculable  
in pQCD

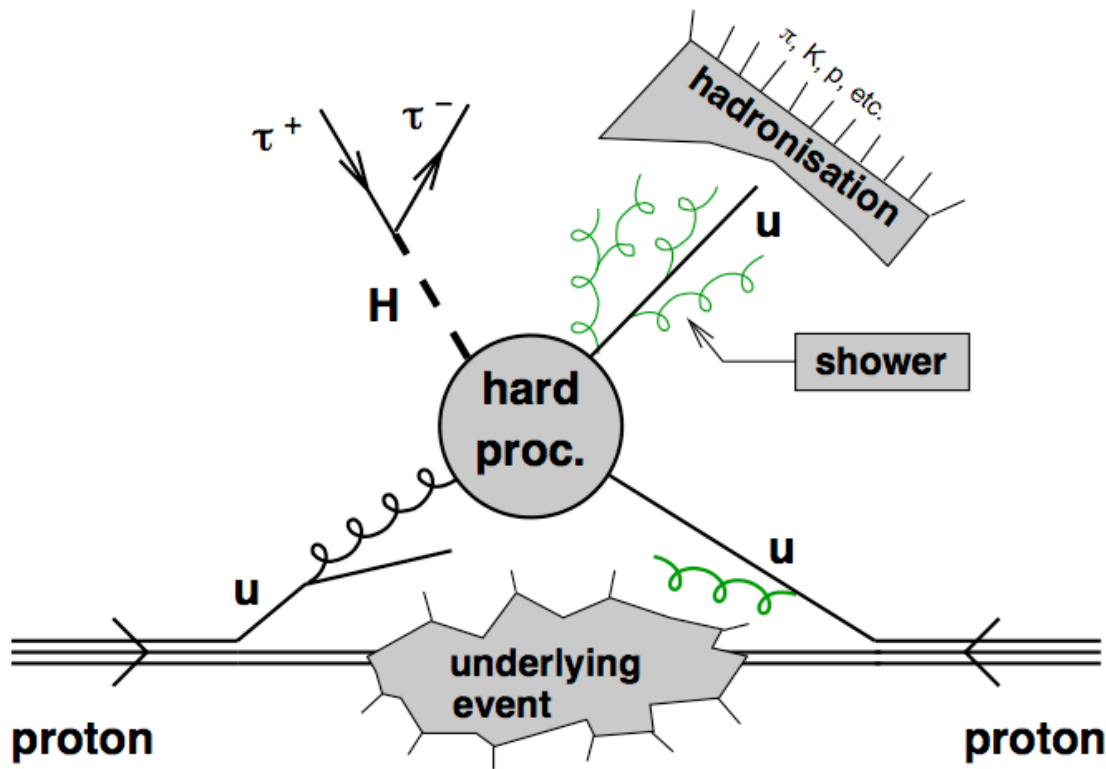
fit from data,  
use in other predictions

‘leading twist’ long-distance non-  
perturbative contributions

‘higher twist’ non-  
perturbative power  
corrections. Can be  
neglected to some extent

Testing (and using) QCD is essentially an iterative procedure which amounts to running an equation like this one through many sets of data, extracting ingredients and using them for predictions, always checking for consistency

# Ingredients and tools



- ▶ PDFs
- ▶ Hard scattering
- ▶ Final state tools

## Extracting PDFs from data has become a favourite pastime

- ▶ Then: CTEQ, MRST, GRV, ...
- ▶ Today: CTEQ, MSTW, NNPDF, HERAPDF, ABKM, GJR, ...

pdfs	authors	arXiv
<b>ABKM</b>	S. Alekhin, J. Blümlein, S. Klein, S. Moch	1105.5349, 1101.5261, 1107.3657, 0908.3128, 0908.2766, ...
<b>CTEQ/TEA</b>	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1108.5112, 1101.0561, 1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, ...
<b>GJR</b>	M. Glück, P. Jimenez-Delgado, E. Reya	1003.3168, 0909.1711, 0810.4274, ...
<b>HERAPDF</b>	H1 and ZEUS collaborations	1107.4193, 1006.4471, 0906.1108, ...
<b>MSTW</b>	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1107.2624, 1006.2753, 0905.3531, 0901.0002, ...
<b>NNPDF</b>	R. Ball, V. Bertone, F. Cerutti, L. Del Debbio, S. Forte, A. Guffanti, N. Hartland, J. I. Latorre, J. Rojo, M. Ubiali	1110.2483, 1108.2758, 1107.2652, 1103.2369, 1102.3182, 1101.1300, 1005.0397, 1002.4407, 0912.2276, 0906.1958, ...

M. Ubiali

## Is the abundance of PDF sets redundant?

Only up to a point, since many different choices can be made

- ▶ What data to fit? Everything? A more limited and more consistent set?
- ▶ What technique to use to describe the PDFs? Parametric form? Neural network?
- ▶ Fit  $\alpha_s$  with PDFs, or use external value?
- ▶ What treatment for heavy quark masses?
- ▶ How to exploit higher order calculations? K-factors or exact results?
- ▶ .....

There is value in having (a reasonable number of) independently obtained PDF sets

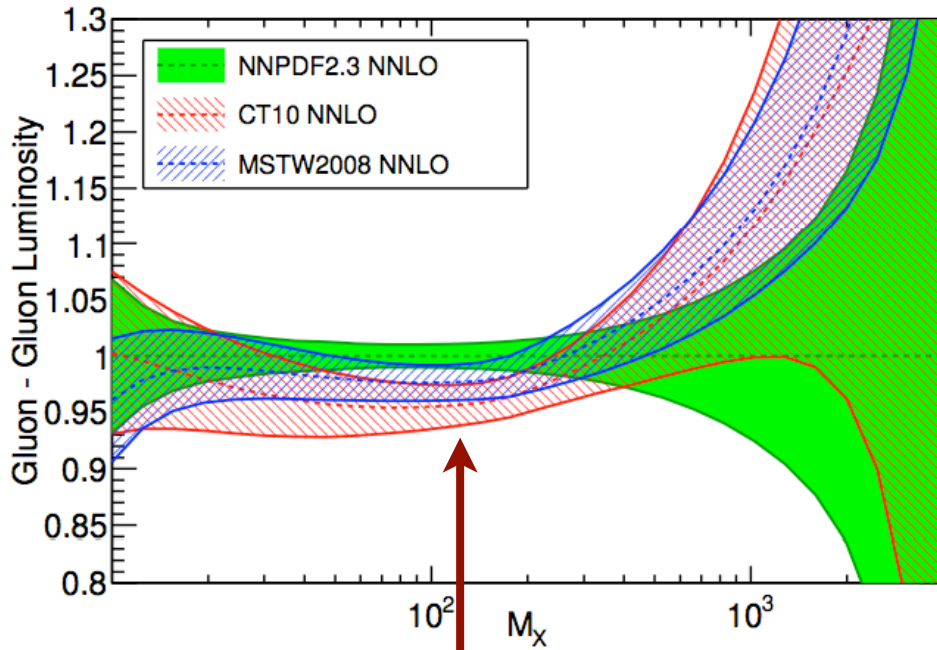
## PDFs: the most recent sets

as on LHAPDF v5.8.6

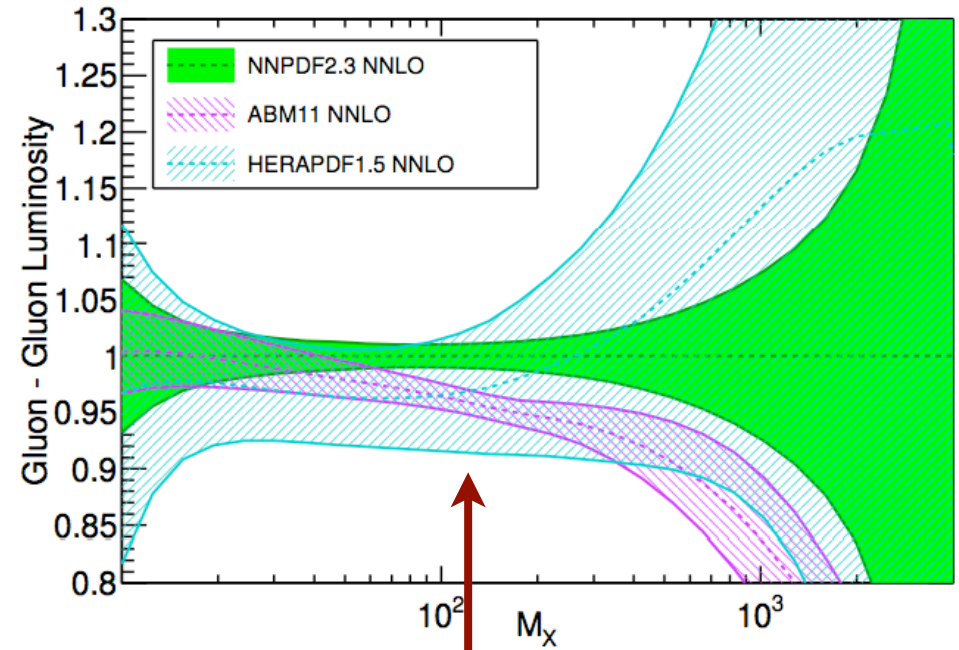
	Data	Parametrization	Stat. treatment	Pert. Order	HQ scheme	$\alpha_S$
<b>CT10(w)</b>	<b>global</b> DIS (FT + HERA) DY (FT + TeV) Inclusive Jets	* 6 independent $f_i$ * Polynomial par (26 pars)	Hessian with fixed tolerance $\Delta\chi^2 = 100$	NLO	S-ACOT- $\chi$	external parameter - several $\alpha_S$ values
<b>MSTW08</b>	<b>global</b> DIS (FT + HERA) DY (FT + TeV) Inclusive Jets	* 7 independent $f_i$ * Polynomial par (20 pars)	Hessian with dynamic tolerance $\Delta\chi^2 \sim 25$	LO NLO NNLO	ACOT + TR'	external parameter - several $\alpha_S$ values + fitted
<b>NNPDF2.1 (NNPDF2.2)</b>	<b>global</b> DIS (FT + HERA) DY (FT + TeV) Inclusive Jets (+ LHC data)	* 7 independent $f_i$ * Neural Networks (259 pars)	Monte Carlo sampling + Cross validation	LO NLO NNLO	FONLL-A	external parameter - several $\alpha_S$ values
<b>HERAPDF1.5</b>	only DIS HERA-I + prel. HERA-II	* 5 independent $f_i$ * Polynomial par (14 pars)	Hessian with $\Delta\chi^2 = 1$	NLO NNLO	ACOT + TR'	external parameter
<b>ABKM09</b>	only DIS + Fixed-Target DY	* 6 independent $f_i$ * Polynomial par (25 pars)	Hessian with $\Delta\chi^2 = 1$	NLO NNLO	FFNS $n_f=3,4,5$	fitted, not external parameter
<b>JR09</b>	only DIS + Fixed-Target DY+(Jets)	* 5 independent $f_i$ * Valence-like assumptions (15 pars)	Hessian with $\Delta\chi^2 = 1$	NLO NNLO	FFN, $n_f=3,4,5$ and VFN	fitted, not external parameter

# NNLO: gg luminosity

LHC 8 TeV - Ratio to NNPDF2.3 NNLO -  $\alpha_s = 0.118$



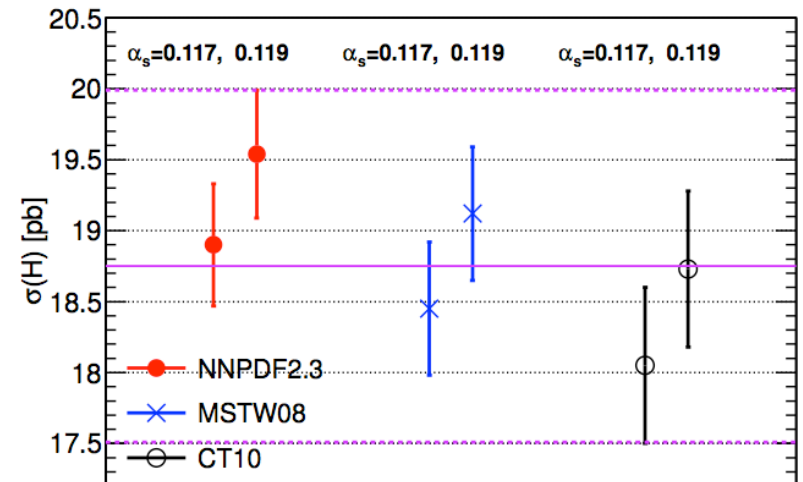
LHC 8 TeV - Ratio to NNPDF2.3 NNLO -  $\alpha_s = 0.118$



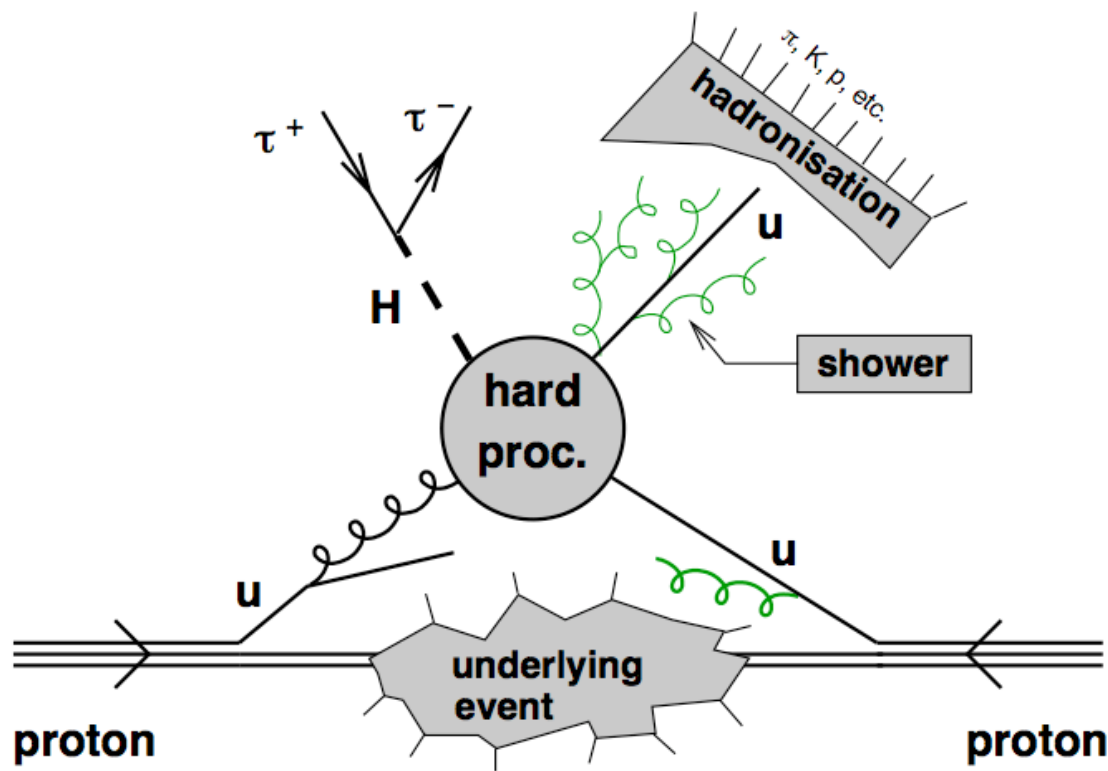
Higgs mass

It is rather unfortunate that the Higgs mass corresponds to the position of **largest difference** between the PDF sets  
 $\Rightarrow$  significant increase of the 'PDF uncertainty'

LHC 8 TeV - iHixs 1.3 NNLO - PDF+ $\alpha_s$  uncertainties



# Ingredients and tools



► PDFs

► Hard scattering

► Final state tools



# Tools for the hard scattering

Can be divided in

## ▶ **Integrators**

- ▶ evaluate the (differential) cross section by integrating the calculation over the phase space, yielding (partly) inclusive quantities
- ▶ Produce weighted events (the weight being the value of the cross section)
- ▶ Calculations exist at LO, NLO, NNLO

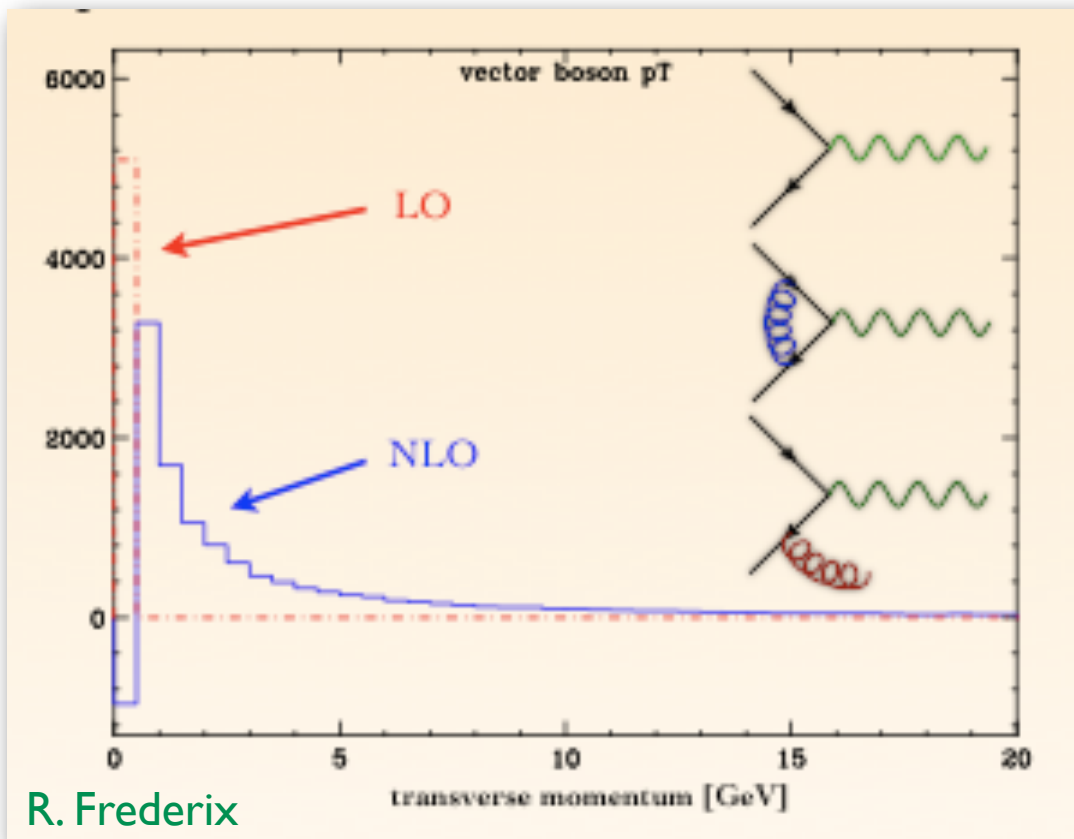
## ▶ **Generators**

- ▶ generate fully exclusive configurations
- ▶ Events are unweighted (i.e. produced with the frequency nature would produce them)
- ▶ Easy at LO, get complicated when dealing with higher orders

# It's easy to say 'NLO'...

Even if a calculation yields an NLO-accurate result for a quantity, not all distributions that can be returned by the same code have necessarily NLO accuracy

## Example: vector boson production in Drell-Yan



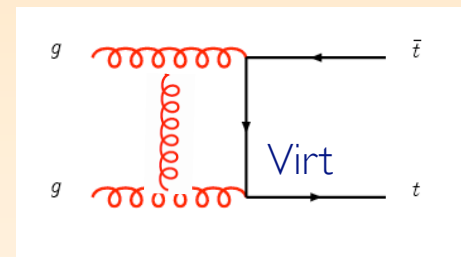
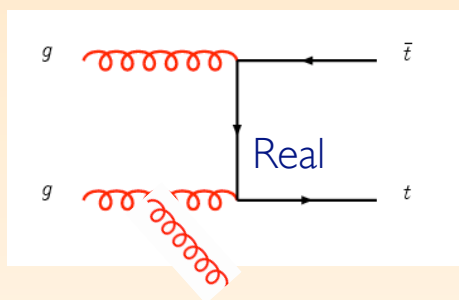
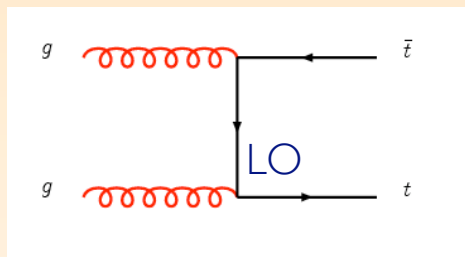
- ▶ At  $O(\alpha_s^0)$ , the total rate is LO, the  $p_T$  is always zero
- ▶ at  $O(\alpha_s^1)$  (1 gluon emission + virtual) the total rate is NLO, but the  $p_T$  distribution is only LO

You only get NLO when you calculate something that was not trivially zero at the lower order



# NLO...?

- Another example: we have a NLO code for  $pp \rightarrow t\bar{t}$



NLO?

- Total cross section ✓
- Transverse momentum of the top quark ✓
- Transverse momentum of the top-antitop pair ✗
- Transverse momentum of the jet ✗
- Top-antitop invariant mass ✓
- Azimuthal distance between the top and anti-top ✗

# Fixed order calculation

## Born

$$d\sigma^{Born} = B(\Phi_B) d\Phi_B$$

## NLO

$$d\sigma^{NLO} = [B(\Phi_B) + V(\Phi_B)] d\Phi_B + R(\Phi_R) d\Phi_R$$

$$d\Phi_R = d\Phi_B d\Phi_{rad}$$

$$d\Phi_{rad} = d\cos\theta dE d\phi$$

**Problem:**  
 $V(\Phi_B)$  and  $\int R d\Phi_R$  are divergent

# Subtraction terms

An observable  $O$  is **infrared and collinear safe** if

$$O(\Phi_R(\Phi_B, \Phi_{\text{rad}})) \xrightarrow{\text{Soft or collinear limit}} O(\Phi_B)$$

One can then write, with  $C \rightarrow R$  in the soft/coll limit,

$$\langle O \rangle = \int \left[ B(\Phi_B) + V(\Phi_B) + \int C(\Phi_R) d\Phi_{\text{rad}} \right] O(\Phi_B) d\Phi_B$$

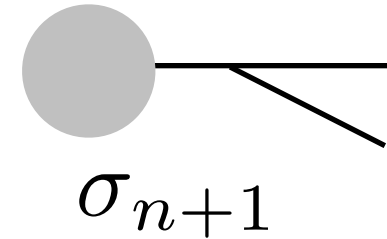
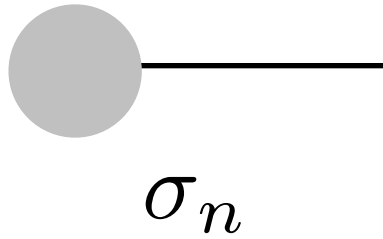
This integration performed analytically

$$+ [R(\Phi_R)O(\phi_R) - C(\Phi_R)O(\Phi_B)] d\Phi_R$$

Separately finite

This (or a similar) cancellation will always be implicit in all subsequent equations

# Sudakov form factor



Factorisation

$$d\sigma_{n+1}(\Phi_{n+1}) = \mathcal{P}(\Phi_{\text{rad}}) d\sigma_n(\Phi_n) d\Phi_{\text{rad}}$$

Emission probability

$$\mathcal{P}(\Phi_{\text{rad}}) d\Phi_{\text{rad}} \approx \frac{\alpha_S(q)}{\pi} \frac{dq}{q} P(z, \phi) dz \frac{d\phi}{2\pi}$$

**Sudakov form factor** = probability of **no emission**  
from large scale  $q_1$  to smaller scale  $q_2$

$$\Delta_S(q_1, q_2) = \exp \left[ - \int_{q_2}^{q_1} \frac{\alpha_S(q)}{\pi} \frac{dq}{q} \int_{z_0}^1 P(z) dz \right]$$

# Conventions for Sudakov form factor

$$\Delta_S(q_1, q_2) = \exp \left[ - \int_{q_2}^{q_1} \frac{\alpha_S(q)}{\pi} \frac{dq}{q} \int_{z_0}^1 P(z) dz \right]$$

Full expression, with details of soft-collinear radiation probability

$$\Delta(p_T) = \exp \left[ - \int_{p_T}^Q \frac{\frac{d\sigma^{(MC)}}{dy dp'_T}}{\frac{d\sigma^{(B)}}{dy}} dp'_T \right]$$

Dropped upper limit, taken implicitly to be the hard scale  $Q$

$$\Delta_R(p_T) = \exp \left[ - \int \frac{R}{B} \Theta(k_T(\Phi_R) - p_T) d\Phi_{rad} \right]$$

Introduced suffix (R in this case) to indicate expression used to describe radiation

$$\Delta_R(p_T) = \exp \left[ - \int_{p_T} \frac{R}{B} d\Phi_{rad} \right]$$

Integration boundaries only implicitly indicated

Based on the **iterative emission of radiation**  
described in the **soft-collinear limit**

$$d\sigma^{(MC)}(\Phi_R)d\Phi_R = B(\Phi_B)d\Phi_B\mathcal{P}(\Phi_{rad})d\Phi_{rad}$$

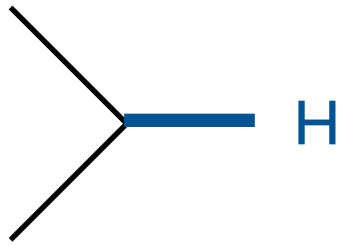
**Pros:** soft-collinear radiation is resummed to all orders in pQCD

**Cons:** hard large-angle radiation is missing

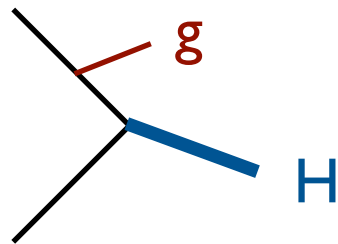
Overall accuracy will be leading log (LL) for the radiation,  
and leading order (i.e. Born) for the integrated cross sections



# PS example: Higgs plus radiation



Leading order.  
No radiation, Higgs  $p_T = 0$



With emission of radiation  
Higgs  $p_T \neq 0$

Description of hardest emission in PS MC (either event is generated)

$$\frac{d\sigma^{(\text{MC})}}{dy dp_T} = \frac{d\sigma^{(\text{B})}}{dy} \delta(p_T) \Delta(Q_0) + \Delta(p_T) \frac{d\sigma^{(\text{MC})}}{dy dp_T}$$

$$\Delta(p_T) = \exp \left[ - \int_{p_T}^Q \frac{\frac{d\sigma^{(\text{MC})}}{dy dp'_T}}{\frac{d\sigma^{(\text{B})}}{dy}} dp'_T \right]$$

Sudakov form factor

x-sect for  
no emission

prob. of  
**no emission**  
(down to the  
PS cutoff)

prob. of  
no emission  
down to  $p_T$

x-sect for  
**emission at  $p_T$** ,  
as described by the MC

# Shower unitarity

It holds

$$\int_0^Q \left[ \delta(p_T) \Delta(Q_0) + \frac{\Delta(p_T) \frac{d\sigma^{(MC)}}{dy dp_T}}{\frac{d\sigma^{(B)}}{dy}} \right] dp_T = \Delta(Q_0) + \int_{Q_0}^Q \frac{d\Delta(p_T)}{dp_T} dp_T = \Delta(Q) = 1$$

**Shower  
unitarity**

so that

$$\int_0^Q dp_T \frac{d\sigma^{(MC)}}{dy dp_T} = \int_0^Q \left[ \delta(p_T) \Delta(Q_0) + \frac{\Delta(p_T) \frac{d\sigma^{(MC)}}{dy dp_T}}{\frac{d\sigma^{(B)}}{dy}} \right] dp_T = \frac{d\sigma^{(B)}}{dy}$$

A parton shower MC correctly reproduces the Born cross section for integrated quantities

# PS MC in different notation

Writing the real cross section as described by the Monte Carlo (i.e. with the parton shower) simply as  $R^{MC}$ , we can rewrite

$$d\sigma^{MC} = Bd\Phi_B \left[ \Delta_{MC}(Q_0) + \Delta_{MC}(p_T) \frac{R^{MC}}{B} d\Phi_{rad} \right]$$

**with** 
$$\Delta_{MC}(p_T) = \exp \left[ - \int_{p_T} \frac{R^{MC}}{B} d\Phi_{rad} \right]$$

as our Master Formula for a Parton Shower Monte Carlo.

Thanks to the shower unitarity, it holds

$$\Rightarrow \int d\sigma^{MC} = \int Bd\Phi_B = \sigma^{LO}$$

# Matrix Element corrections

In a PS Monte Carlo  $R^{(MC)}(\Phi_R) = B(\Phi_B)\mathcal{P}(\Phi_{rad})$

soft-collinear  
approximation

Replace the MC description of radiation with the **correct** one:

$$\mathcal{P}(\Phi_{rad}) \rightarrow \frac{R}{B}$$

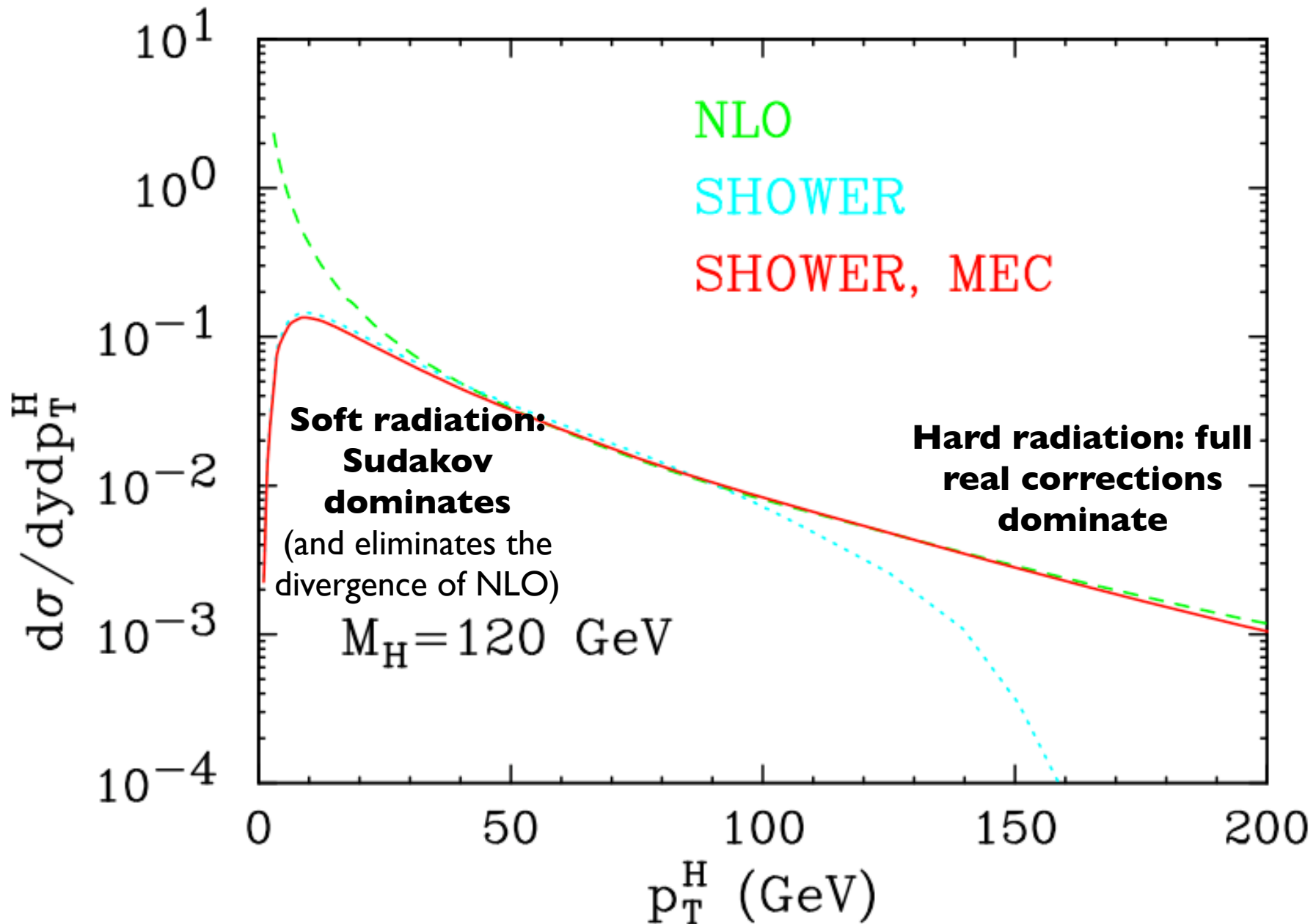
The Sudakov becomes

$$\Delta(p_T) = \exp \left[ - \int_{p_T}^Q \frac{\frac{d\sigma^{(MC)}}{dy dp'_T}}{\frac{d\sigma^{(B)}}{dy}} dp'_T \right] \longrightarrow \Delta_R(p_T) = \exp \left[ - \int \frac{R}{B} \Theta(k_T(\Phi_R) - p_T) d\Phi_{rad} \right]$$

and the x-sect formula for the hardest emission

$$d\sigma^{MEC} = B d\Phi_B \left[ \Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

# Matrix Element corrections



We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- ▶ we can successfully interface **matrix elements for multi-parton production with a parton shower**
- ▶ we can successfully interface a **parton shower with a NLO calculation**

## The quest for exactness

**E** exact      **PS**

G. Salam, ICHEP10

	1	2	3	4	5	...	n. of radiated QCD particles
1	<b>E</b>						
2	PS	PS					
3	PS	PS	PS				
4	PS	PS	PS	PS			
5	PS	PS	PS	PS	PS		
...							

Powers of coupling

Parton shower (PS+MEC)  
Montecarlo (PYTHIA, HERWIG...)

## The quest for exactness

**E** exact      **PS**

G. Salam, ICHEP10

	1	2	3	4	5	...	n. of radiated QCD particles
1	<b>E</b>						1
2	PS	<b>E</b>					2
3	PS	PS	<b>E</b>				3
4	PS	PS	PS	<b>E</b>			4
5	PS	PS	PS	PS	<b>E</b>		5
...							

Powers of coupling

Parton shower (PS+MEC)  
Montecarlo (PYTHIA, HERWIG...)

**PS + Matrix Element (ME)**  
(using CKKW/MLM)



## The quest for exactness

**E** exact      **PS**

G. Salam, ICHEP10

	1	2	3	4	5	...	n. of radiated QCD particles
1	<b>E</b>						1
2	<b>E</b>	<b>E</b>					2
3	PS	PS	PS				3
4	PS	PS	PS	PS			4
5	PS	PS	PS	PS	PS		5
...							...

Powers of coupling

Parton shower (PS+MEC)  
Montecarlo (PYTHIA, HERWIG...)
















PS + Matrix Element (ME)  
(using CKKW/MLM)

**PS + NLO**  
(MC@NLO, POWHEG)

## The quest for exactness

 exact       PS

G. Salam, ICHEP10

	1	2	3	4	5 ...	n. of radiated QCD particles
1						1
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4						4
5						5
...						...

Powers of coupling

Parton shower (PS+MEC)  
Montecarlo (PYTHIA, HERWIG...)

PS + Matrix Element (ME)  
(using CKKW/MLM)
















PS + NLO  
(MC@NLO, POWHEG)

PS + NLO + ME  
(MENLOPS)

[Hamilton, Nason '10]

## The quest for exactness

G. Salam, ICHEP10

	1	2	3	4	5	...	n. of radiated QCD particles
1							1
2							2
3							3
4							4
5							5
...							...

Powers of coupling

 exact       PS

Parton shower (PS+MEC)  
Montecarlo (PYTHIA, HERWIG...)

PS + Matrix Element (ME)  
(using CKKW/MLM)

PS + NLO  
(MC@NLO, POWHEG)

PS + NLO + ME  
(MENLOPS)

[Hamilton, Nason '10]

**The future**  
PS + NLO + ME<sub>NLO</sub>  
(aMC@NLO)

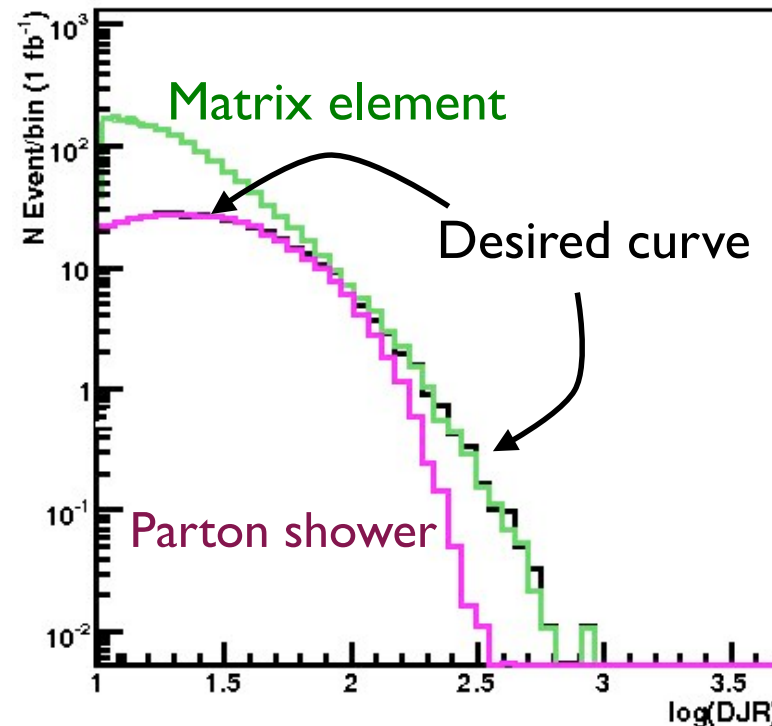
We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- ▶ we can successfully interface **matrix elements for multi-parton production with a parton shower**
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## GOAL FOR ME-PS MERGING/MATCHING

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in  
top pair production at  
the LHC

# ME+PS matching methods

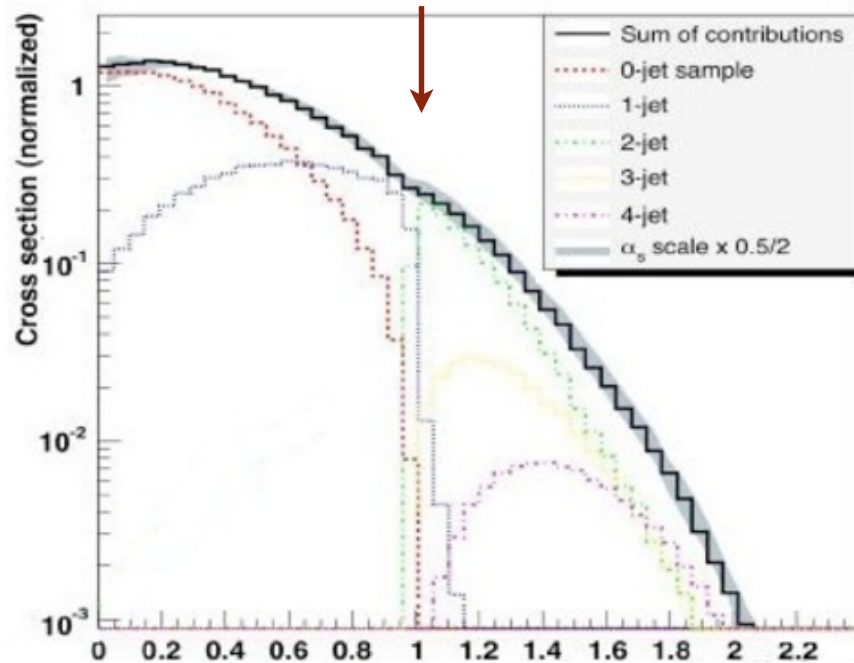
- ▶ **CKKW** [Catani, Krauss, Kuhn, Webber, 2001]
- ▶ **CKKW-L** [Lonnblad, 2002]
- ▶ **MLM** [Mangano, 2002]



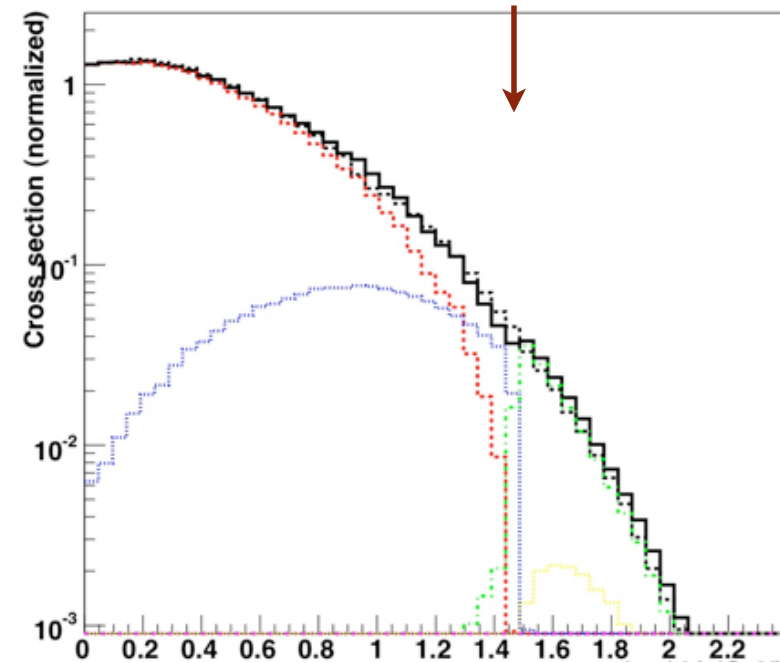
## MATCHING RESULTS

W+jets production at the Tevatron for MadGraph+Pythia  
(kT-jet MLM scheme)

$Q^{\text{match}} = 10 \text{ GeV}$



$Q^{\text{match}} = 30 \text{ GeV}$

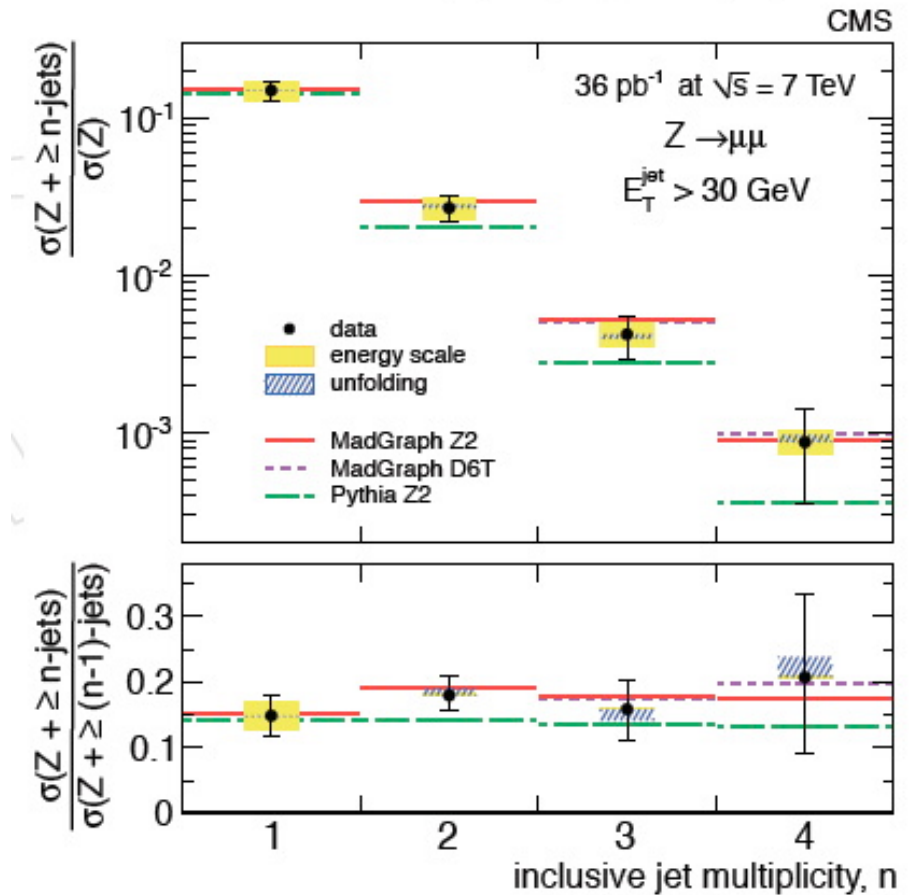
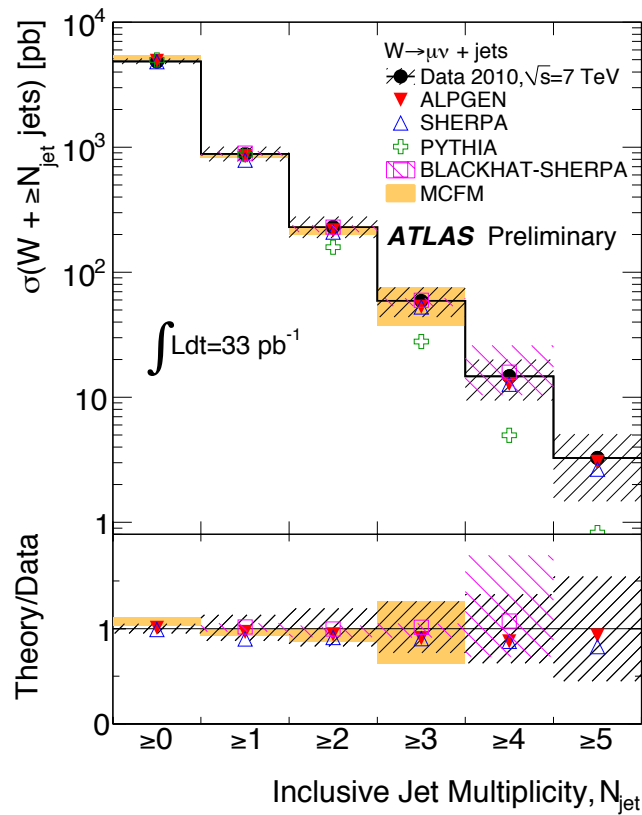


$\log(\text{Differential jet rate for } 1 \rightarrow 2 \text{ radiated jets} \sim p_T(2\text{nd jet}))$

Jet distributions smooth, and stable when we vary the matching scale!



## TH/EXP COMPARISON AT THE LHC





We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- ▶ we can successfully interface **matrix elements for multi-parton production with a parton shower**

- ▶ we can successfully interface a **parton shower with a NLO calculation**

## Existing 'MonteCarlos at NLO':

▶ **MC@NLO** [Frixione and Webber, 2002]

▶ **POWHEG** [Nason, 2004]

NB. MC@NLO is a **code**, POWHEG is a **method**

## Evolving into (semi)automated forms:

▶ **The POWHEG BOX** [powhegbox.mib.infn.it 2010]

▶ **aMC@NLO** [amcatnlo.cern.ch 2011]

# MC@NLO v. POWHEG

The two methods are largely equivalent.  
They do, however, have separate **pros** and **cons**.

## MC@NLO

- ▶ can have negative weights
- ▶ needs specific implementation for each PS MonteCarlo (but now exists for both HERWIG and PYTHIA)
- ▶ ‘rapidity dip’ in some distributions
- ▶ Distributions from NLO codes rigorously reproduced
- ▶ fully automated in aMC@NLO

## POWHEG

- ▶ weights always positive
- ▶ interfaces naturally to any PS MonteCarlo
- ▶ can generate large (NNLO) K-factors in some distributions (but a practical solution is available)
- ▶ not yet fully automated (but the POWEG BOX is a step in this direction)

# Backup slides

Matrix-element corrected shower Monte Carlos still have leading order accuracy for the total rates

$$d\sigma^{MEC} = B d\Phi_B \left[ \Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right] \quad \text{and} \quad \Delta_R(Q_0) + \int \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} = 1$$
$$\Rightarrow \int d\sigma^{MEC} = \int B d\Phi_B = \sigma^{LO}$$

We want to do better, and **merge** PS and NLO, so that

$$\int d\sigma^{PS+NLO} = \int (B + V) d\Phi_B + \int R d\Phi_R = \sigma^{NLO}$$

Idea: remove from the NLO the terms that are already generated by the parton shower (NB. MC-specific)

$$d\sigma^{MC@NLO} = \bar{B}_{MC} d\Phi_B \left[ \Delta_{MC}(Q_0) + \Delta_{MC}(p_T) \frac{R^{MC}}{B} d\Phi_{rad} \right] + \frac{[R - R^{MC}] d\Phi_R}{1}$$

$$\bar{B}_{MC} = B + \left[ V + \int R^{MC} d\Phi_{rad} \right]$$

‘soft’ event
MC shower
‘hard’ event

It is easy to see that, as desired,

$$\int d\sigma^{MC@NLO} = \int d\sigma^{NLO}$$

Idea: generated hardest radiation first, then pass event to MC for generation of subsequent, softer radiation

$$d\sigma^{POWHEG} = \bar{B} d\Phi_B \left[ \Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

$$\bar{B} = B + \left[ V + \int R d\Phi_{rad} \right]$$

NLO x-sect
MC shower

It is easy to see that, as desired,

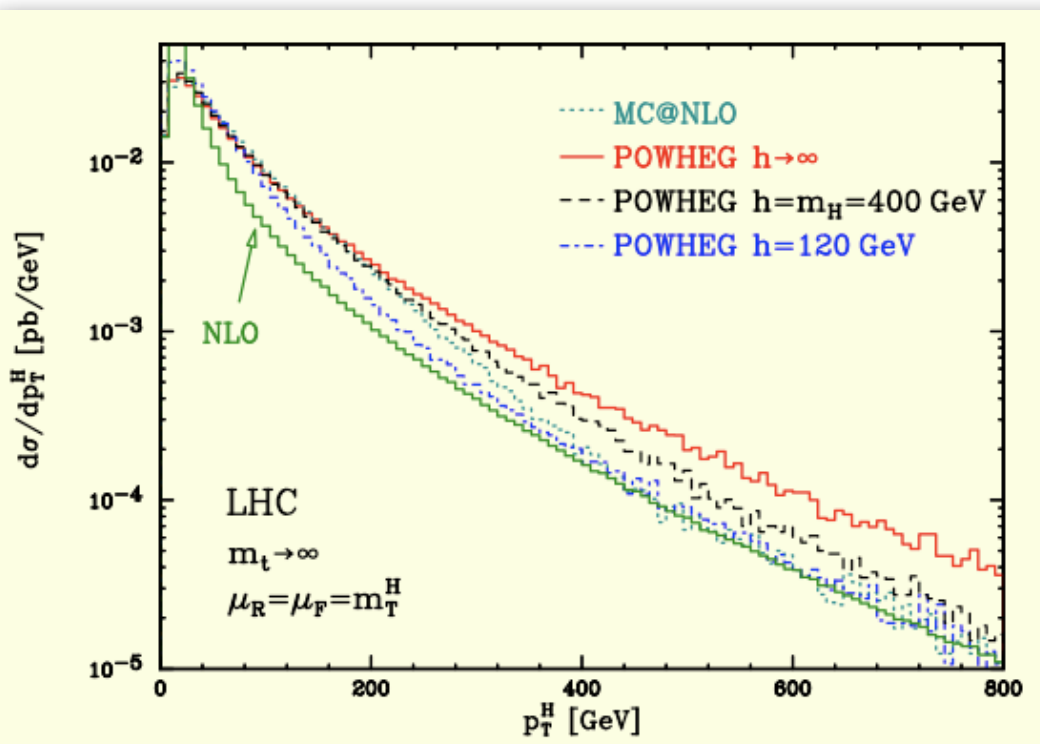
$$\int d\sigma^{POWHEG} = \int d\sigma^{NLO}$$

# Large $p_T$ enhancement in POWHEG

The 'naive' formulation for POWHEG is

$$d\sigma^{POWHEG} = \bar{B}d\Phi_B \left[ \Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

In this form  $\bar{B}d\Phi_B$  provides the NLO K-factor (order  $1 + \mathcal{O}(\alpha_s)$ ), but also associates it to large  $p_T$  radiation, where the calculation is already  $\mathcal{O}(\alpha_s)$  (but only LO accuracy).



This generates an effective (but not necessarily correct)  $\mathcal{O}(\alpha_s^2)$  term (i.e. NNLO for the total cross section)

OK because beyond nominal accuracy, but one may feel uncomfortable with such large numerical factors



# Modified POWHEG

The ‘problem’ with the naive POWHEG comes from the hard radiation being enhanced by spurious higher orders. In order to suppress this effect, we split

$$R = R^S + R^F \quad R^S \equiv \frac{h^2}{h^2 + p_T^2} R \quad R^F \equiv \frac{p_T^2}{h^2 + p_T^2} R$$

Contains  
singularities

Regular in  
small  $p_T$  region

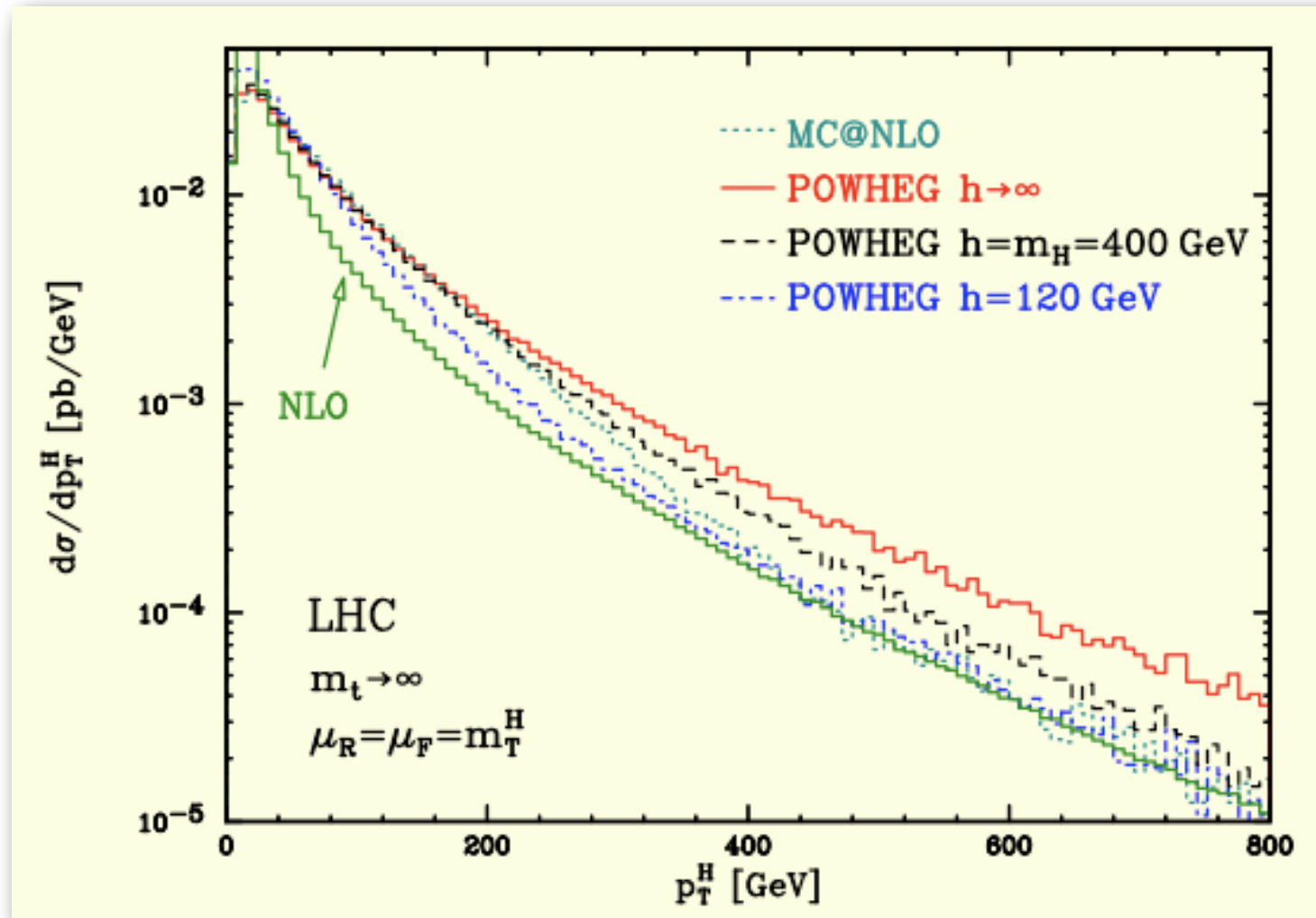
$$d\sigma^{POWHEG} = \bar{B}^S d\Phi_B \left[ \Delta_S(Q_0) + \Delta_S(p_T) \frac{R^S}{B} d\Phi_{rad} \right] + R^F d\Phi_R$$

$$\bar{B}^S = B + \left[ V + \int R^S d\Phi_{rad} \right]$$

$$\Delta_S(p_T) = \exp \left[ - \int_{p_T} \frac{R^S}{B} d\Phi_{rad} \right]$$

# Modified POWHEG

In the  $h \rightarrow \infty$  limit the exact NLO result is recovered



# Comparisons

$$d\sigma^{MC} = Bd\Phi_B \left[ \Delta(Q_0) + \Delta(p_T) \frac{R^{MC}}{B} d\Phi_{rad} \right]$$

$$d\sigma^{MEC} = Bd\Phi_B \left[ \Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

$$d\sigma^{NLO} = [B + V] d\Phi_B + Rd\Phi_R$$

$$d\sigma^{MC@NLO} = \bar{B}_{MC} d\Phi_B \left[ \Delta_{MC}(Q_0) + \Delta_{MC}(p_T) \frac{R^{MC}}{B} d\Phi_{rad} \right] + [R - R^{MC}] d\Phi_R$$

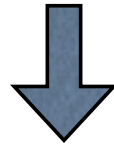
$$d\sigma^{POWHEG} = \bar{B}^S d\Phi_B \left[ \Delta_S(Q_0) + \Delta_S(p_T) \frac{R^S}{B} d\Phi_{rad} \right] + R^F d\Phi_R$$

POWHEG approaches MC@NLO if  $R^S \rightarrow R^{MC}$



## MATRIX ELEMENTS VS. PARTON SHOWERS

ME

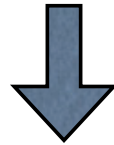


1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are **hard and well separated**
5. Quantum interference correct
6. Needed for multi-jet description



## MATRIX ELEMENTS VS. PARTON SHOWERS

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Shower MC

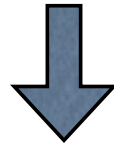


1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are **collinear and/or soft**
5. Partial interference through angular ordering
6. Needed for hadronization



## MATRIX ELEMENTS VS. PARTON SHOWERS

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**Approaches are complementary: merge them!**

**Difficulty: avoid double counting, ensure smooth distributions**