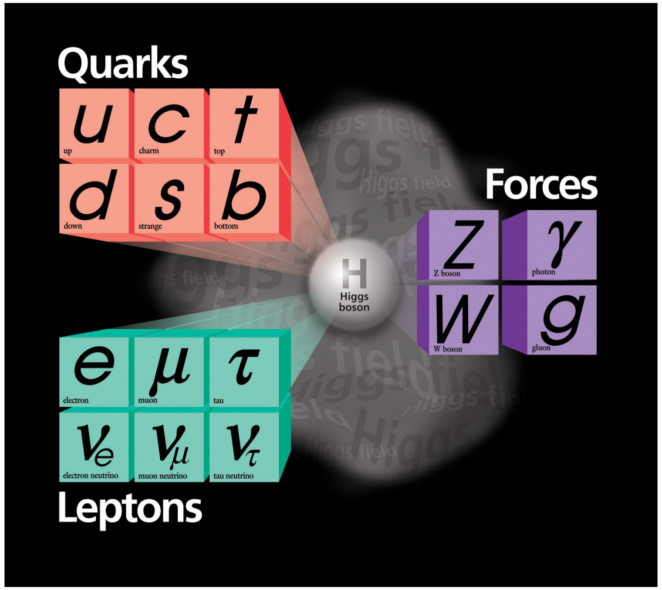
Quantum Field Theory: Standard Model and Electroweak Symmetry Breaking



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Outline

1. Quantum Field Theory: Gauge Theories
   - ▷ The symmetry principle
   - ▷ Quantization of gauge theories
   - ▷ Spontaneous Symmetry Breaking
2. The Standard Model
   - ▷ Gauge group and particle representations
   - ▷ The SM with one family: electroweak interactions
   - ▷ Electroweak SSB: Higgs sector, gauge boson and fermion masses
   - ▷ Additional generations: fermion mixings
   - ▷ Complete Lagrangian and Feynman rules
3. Phenomenology of the Electroweak Standard Model
   - ▷ Input parameters, experiments, observables, precise predictions
   - ▷ Global fits

1. Gauge Theories

The symmetry principle | free Lagrangian

- Lagrangian of a free fermion field $\psi(x)$:

$$\text{(Dirac)} \qquad \boxed{\mathcal{L}_0 = \overline{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi} \quad \partial\!\!\!/ \equiv \gamma^\mu \partial_\mu \,, \quad \overline{\psi} = \psi^\dagger \gamma^0$$

⇒ Invariant under global U(1) phase transformations:

$$\psi(x) \mapsto \psi'(x) = \mathrm{e}^{-\mathrm{i}q\theta}\psi(x) \,, \quad q,\ \theta \text{ (constants)} \in \mathbb{R}$$

⇒ By Noether's theorem there is a conserved current:

$$j^\mu = q\,\overline{\psi}\gamma^\mu\psi \,, \quad \partial_\mu j^\mu = 0$$

and a Noether charge:

$$Q = \int \mathrm{d}^3x\, j^0, \quad \partial_t Q = 0$$

The symmetry principle | free Lagrangian

- A quantized free fermion field:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\boldsymbol{p}}}} \sum_{s=1,2} \left(a_{\boldsymbol{p},s} u^{(s)}(\boldsymbol{p}) e^{-ipx} + b^\dagger_{\boldsymbol{p},s} v^{(s)}(\boldsymbol{p}) e^{ipx} \right)$$

– is a solution of the Dirac equation (Euler-Lagrange):

$$(i\partial\!\!\!/ - m)\psi(x) = 0\,, \quad (p\!\!\!/ - m)u(\boldsymbol{p}) = 0\,, \quad (p\!\!\!/ + m)v(\boldsymbol{p}) = 0\,,$$

– is an operator from the canonical quantization rules (anticommutation):

$$\{a_{\boldsymbol{p},r}, a^\dagger_{\boldsymbol{k},s}\} = \{b_{\boldsymbol{p},r}, b^\dagger_{\boldsymbol{k},s}\} = (2\pi)^3\delta^3(\boldsymbol{p}-\boldsymbol{k})\delta_{rs}\,, \quad \{a_{\boldsymbol{p},r}, a_{\boldsymbol{k},s}\} = \cdots = 0\,,$$

that annihilates/creates particles/antiparticles on the Fock space of fermions

The symmetry principle | free Lagrangian

- For a quantized free fermion field:

  $\Rightarrow$ Normal ordering for fermionic operators (H spectrum bounded from below):

$$: a_{\boldsymbol{p},r} a^\dagger_{\boldsymbol{q},s} : \equiv -a^\dagger_{\boldsymbol{q},s} a_{\boldsymbol{p},r} \; , \qquad : b_{\boldsymbol{p},r} b^\dagger_{\boldsymbol{q},s} : \equiv -b^\dagger_{\boldsymbol{q},s} b_{\boldsymbol{p},r}$$

  $\Rightarrow$ The Noether charge is an operator:

$$: Q : = q \int \mathrm{d}^3x \; : \overline{\psi} \gamma^0 \psi : = q \int \frac{\mathrm{d}^3p}{(2\pi)^3} \sum_{s=1,2} \left(a^\dagger_{\boldsymbol{p},s} a_{\boldsymbol{p},s} - b^\dagger_{\boldsymbol{p},s} b_{\boldsymbol{p},s} \right)$$

$$Q \; a^\dagger_{\boldsymbol{k},s} |0\rangle = +q \; a^\dagger_{\boldsymbol{k},s} |0\rangle \;\; (\text{particle}) \; , \qquad Q \; b^\dagger_{\boldsymbol{k},s} |0\rangle = -q \; b^\dagger_{\boldsymbol{k},s} |0\rangle \;\; (\text{antiparticle})$$

The symmetry principle — gauge symmetry dictates interactions

- To make $\mathcal{L}_0$ invariant under local $\equiv$ gauge transformations of U(1):

$$\psi(x) \mapsto \psi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\psi(x)\ , \quad \theta = \theta(x) \in \mathbb{R}$$

perform the minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu + \mathrm{i}eqA_\mu \quad \text{(covariant derivative)}$$

where a gauge field $A_\mu(x)$ is introduced transforming as:

$$A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x) \quad \Leftarrow \quad \boxed{D_\mu\psi \mapsto \mathrm{e}^{-\mathrm{i}q\theta(x)}D_\mu\psi} \quad \overline{\psi}\not{D}\psi \text{ inv.}$$

$\Rightarrow$ The new Lagrangian contains interactions between ψ and A_μ:

$$\boxed{\mathcal{L}_{\text{int}} = -eq\,\overline{\psi}\gamma^\mu\psi A_\mu} \quad \propto \begin{cases} \text{coupling} & e \\ \text{charge} & q \end{cases}$$

$$(= -e\,j^\mu A_\mu)$$

The symmetry principle | gauge invariance dictates interactions

- Dynamics for the gauge field ⇒ add gauge invariant kinetic term:

$$\text{(Maxwell)} \qquad \boxed{\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}} \qquad \Leftarrow \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \mapsto F_{\mu\nu}$$

- The full U(1) gauge invariant Lagrangian for a fermion field $\psi(x)$ reads:

$$\mathcal{L}_{\text{sym}} = \overline{\psi}(\mathrm{i}\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad (= \mathcal{L}_0 + \mathcal{L}_{\text{int}} + \mathcal{L}_1)$$

- The same applies to a complex scalar field $\phi(x)$:

$$\mathcal{L}_{\text{sym}} = (D_\mu\phi)^\dagger D^\mu\phi - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

The symmetry principle | non-Abelian gauge theories

- A general gauge symmetry group G is an N-dimensional compact Lie group

$$g \in G\ , \quad g(\boldsymbol{\theta}) = e^{-iT_a\theta^a}\ , \quad a = 1,\ldots,N$$

$$\theta^a = \theta^a(x) \in \mathbb{R}\ , \quad T_a = \text{Hermitian generators}\ , \quad [T_a, T_b] = \mathrm{i} f_{abc} T_c \quad \text{(Lie algebra)}$$

$$\mathrm{Tr}\{T_a T_b\} \equiv \tfrac{1}{2}\delta_{ab}\ , \quad \text{structure constants:} \quad f_{abc} = 0 \quad \text{Abelian}$$

$$f_{abc} \neq 0 \quad \text{non-Abelian}$$

$\Rightarrow$ Finite-dimensional irreducible representations are unitary:

$$d\text{-multiplet}: \quad \Psi(x) \mapsto \Psi'(x) = U(\boldsymbol{\theta})\Psi(x)\ , \quad \Psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_d \end{pmatrix}$$

$$d \times d \text{ matrices}: \quad U(\boldsymbol{\theta}) \text{ [given by } \{T_a\} \text{ algebra representation]}$$

The symmetry principle | **non-Abelian gauge theories**

- Examples:

G	N	Abelian	
U(1)	1	Yes	
SU(n)	$n^2 - 1$	No	($n \times n$ matrices with det $= 1$)

  - U(1): 1 generator (q), one-dimensional irreps only
  - SU(2): 3 generators
    $f_{abc} = \epsilon_{abc}$ (Levi-Civita symbol)
    * Fundamental irrep ($d = 2$): $T_a = \frac{1}{2}\sigma_a$ (3 Pauli matrices)
    * Adjoint irrep ($d = N = 3$): $(T_a^{\text{adj}})_{bc} = -\mathrm{i} f_{abc}$
  - SU(3): 8 generators
    $f^{123} = 1,\ f^{458} = f^{678} = \frac{\sqrt{3}}{2},\ f^{147} = f^{156} = f^{246} = f^{247} = f^{345} = -f^{367} = \frac{1}{2}$
    * Fundamental irrep ($d = 3$): $T_a = \frac{1}{2}\lambda_a$ (8 Gell-Mann matrices)
    * Adjoint irrep ($d = N = 8$): $(T_a^{\text{adj}})_{bc} = -\mathrm{i} f_{abc}$

(for SU(n): f_{abc} totally antisymmetric)

The symmetry principle | non-Abelian gauge theories

- To make $\mathcal{L}_0$ invariant under local $\equiv$ gauge transformations of G:

$$\Psi(x) \mapsto \Psi'(x) = U(\boldsymbol{\theta})\Psi(x)\ , \quad \boldsymbol{\theta} = \boldsymbol{\theta}(x) \in \mathbb{R}$$

substitute the covariant derivative:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g\widetilde{W}_\mu\ , \quad \widetilde{W}_\mu \equiv T_a W_\mu^a$$

where a gauge field $A_\mu^a(x)$ per generator is introduced, transforming as:

$$\widetilde{W}_\mu(x) \mapsto \widetilde{W}'_\mu(x) = U\widetilde{W}_\mu(x)U^\dagger - \frac{\mathrm{i}}{g}(\partial_\mu U)U^\dagger \quad \Leftarrow \quad \boxed{D_\mu\Psi \mapsto UD_\mu\Psi} \quad \overline{\Psi}\not{D}\Psi \text{ inv.}$$

$\Rightarrow$ The new Lagrangian contains interactions between Ψ and W_μ^a:

$$\boxed{\mathcal{L}_{\text{int}} = g\,\overline{\Psi}\gamma^\mu T_a \Psi W_\mu^a} \quad \propto \begin{cases} \text{coupling} & g \\ \text{charge} & T_a \end{cases}$$

$$(= g\, j_a^\mu W_\mu^a)$$

The symmetry principle | non-Abelian gauge theories

- Dynamics for the gauge fields $\Rightarrow$ add gauge invariant kinetic terms:

$$\text{(Yang-Mills)} \quad \boxed{\mathcal{L}_{\rm YM} = -\frac{1}{2}\mathrm{Tr}\left\{\widetilde{W}_{\mu\nu}\widetilde{W}^{\mu\nu}\right\} = -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}} \quad \Leftarrow \quad \widetilde{W}_{\mu\nu} \mapsto U\widetilde{W}_{\mu\nu}U^\dagger$$

$$\widetilde{W}_{\mu\nu} \equiv D_\mu\widetilde{W}_\nu - D_\nu\widetilde{W}_\mu = \partial_\mu\widetilde{W}_\nu - \partial_\nu\widetilde{W}_\mu - ig[\widetilde{W}_\mu, \widetilde{W}_\nu]$$

$$\Rightarrow \quad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f_{abc} W^b_\mu W^c_\nu$$

$\Rightarrow$ $\mathcal{L}_{\rm YM}$ contains cubic and quartic self-interactions of the gauge fields W^a_μ:

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)(\partial^\mu W^{a,\nu} - \partial^\nu W^{a,\mu})$$

$$\mathcal{L}_{\rm cubic} = -\frac{1}{2} g f_{abc}\,(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu) W^{b,\mu} W^{c,\nu}$$

$$\mathcal{L}_{\rm quartic} = -\frac{1}{4} g^2 f_{abe} f_{cde}\, W^a_\mu W^b_\nu W^{c,\mu} W^{d,\nu}$$

- The (Feynman) propagator of a scalar field:

$$D(x-y) = \langle 0|\, T\{\phi(x)\phi^\dagger(y)\}\,|0\rangle = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\,\frac{\mathrm{i}}{p^2-m^2+\mathrm{i}\epsilon}\,\mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

is a Green's function of the Klein-Gordon operator:

$$(\Box_x+m^2)D(x-y) = -\mathrm{i}\delta^4(x-y) \quad\Leftrightarrow\quad \widetilde{D}(p) = \frac{\mathrm{i}}{p^2-m^2+\mathrm{i}\epsilon}$$

- The propagator of a fermion field:

$$S(x-y) = \langle 0|\, T\{\psi(x)\overline{\psi}(y)\}\,|0\rangle = (\mathrm{i}\partial\!\!\!/_x+m)\int \frac{\mathrm{d}^4p}{(2\pi)^4}\,\frac{\mathrm{i}}{p^2-m^2+\mathrm{i}\epsilon}\,\mathrm{e}^{-\mathrm{i}p\cdot(x-y)}$$

is a Green's function of the Dirac operator:

$$(\mathrm{i}\partial\!\!\!/_x-m)S(x-y) = \mathrm{i}\delta^4(x-y) \quad\Leftrightarrow\quad \widetilde{S}(p) = \frac{\mathrm{i}}{p\!\!\!/-m+\mathrm{i}\epsilon}$$

Quantization of gauge theories | propagators

- BUT the propagator of a gauge field cannot be defined unless $\mathcal{L}$ is modified:

(e.g. modified Maxwell) $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

Euler-Lagrange: $$\frac{\partial\mathcal{L}}{\partial A_\nu} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0 \quad \Rightarrow \quad \left[g^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu\right] A_\mu = 0$$

– In momentum space the propagator is the inverse of:

$$-k^2 g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right) k^\mu k^\nu \quad \Rightarrow \quad \widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]$$

$\Rightarrow$ Note that $(-k^2 g^{\mu\nu} + k^\mu k^\nu)$ is singular!

$\Rightarrow$ One may argue that $\mathcal{L}$ above will not lead to Maxwell equations …
unless we fix a (Lorentz) gauge where:

$$\partial^\mu A_\mu = 0 \quad \Leftarrow \quad A_\mu \mapsto A'_\mu = A_\mu + \partial_\mu \Lambda \;\text{ with }\; \partial^\mu\partial_\mu\Lambda \equiv -\partial^\mu A_\mu$$

Quantization of gauge theories | gauge fixing (Abelian case)

- The extra term is called Gauge Fixing:

$$\boxed{\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2}$$

$\Rightarrow$ modified $\mathcal{L}$ equivalent to Maxwell Lagrangian just in the gauge $\partial^\mu A_\mu = 0$

$\Rightarrow$ the ξ-dependence always cancels out in physical amplitudes

- Several choices for the gauge fixing term (simplify calculations): R_ξ gauges

('t Hooft-Feynman gauge) $\quad \xi = 1: \quad \widetilde{D}_{\mu\nu}(k) = -\dfrac{\mathrm{i}g_{\mu\nu}}{k^2 + \mathrm{i}\epsilon}$

(Landau gauge) $\quad \xi = 0: \quad \widetilde{D}_{\mu\nu}(k) = \dfrac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + \dfrac{k_\mu k_\nu}{k^2}\right]$

Quantization of gauge theories | gauge fixing (non-Abelian case)

- For a non-Abelian gauge theory, the gauge fixing terms:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}\sum_a (\partial^\mu W_\mu^a)^2$$

allow to define the propagators:

$$\widetilde{D}_{\mu\nu}^{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2+\mathrm{i}\epsilon}\left[-g_{\mu\nu}+(1-\xi)\frac{k_\mu k_\nu}{k^2}\right]$$

BUT, unlike the Abelian case, this is not the end of the story . . .

Quantization of gauge theories | Faddeev-Popov ghosts

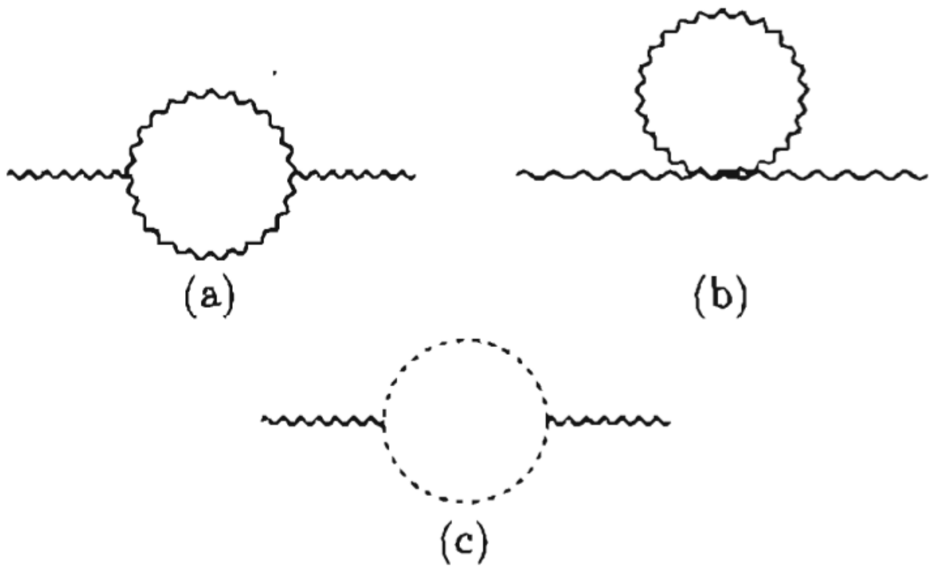
- Add Faddeev-Popov ghost fields $c_a(x)$ in the adjoint irrep:

$$\boxed{\mathcal{L}_{\rm FP} = (\partial^\mu \bar{c}_a)(D_\mu^{\rm adj})_{ab} c_b = (\partial^\mu \bar{c}_a)(\partial_\mu c_a - g {\color{blue} f_{abc}} c_b W_\mu^c)} \quad \Leftarrow \quad D_\mu^{\rm adj} = \partial_\mu - {\rm i} g T_c^{\rm adj} W_\mu^c$$

Computational trick: anticommuting scalar fields, just in loops as virtual particles

$$\widetilde{D}_{ab}(k) = \frac{{\rm i}\delta_{ab}}{k^2 + {\rm i}\epsilon} \qquad [(-1) \text{ sign for closed loops! (like fermions)}]$$

$\Rightarrow$ Faddeev-Popov ghosts needed to preserve gauge symmetry:



$$= (g_{\mu\nu} k^2 - k_\mu k_\nu)\Pi(k^2)$$

Quantization of gauge theories | complete Lagrangian

- Then the complete quantum Lagrangian is

$$\boxed{\mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}}$$

⇒ Note that in the case of a massive vector field

$$\text{(Proca)} \qquad \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A_\mu A^\mu$$

it is not gauge invariant

– The propagator is:

$$\widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M^2 + \mathrm{i}\epsilon} \left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{M^2} \right)$$

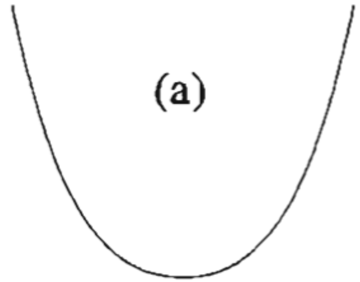
Spontaneous Symmetry Breaking | discrete symmetry

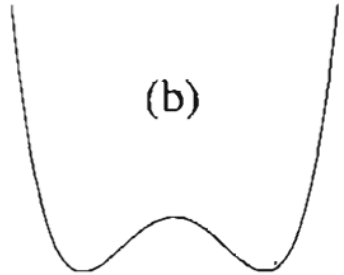
- Consider a real scalar field $\phi(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4 \quad \text{invariant under} \quad \phi \mapsto -\phi$$

$$\Rightarrow \quad \mathcal{H} = \frac{1}{2}(\dot{\phi}^2 + (\nabla\phi)^2) + V(\phi)$$

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$





$\mu^2,\ \lambda \in \mathbb{R}$ (Real/Hermitian Hamiltonian) and $\lambda > 0$ (existence of a ground state)

(a) $\mu^2 > 0$: min of $V(\phi)$ at $\phi_{\text{cl}} = 0$

(b) $\mu^2 < 0$: min of $V(\phi)$ at $\phi_{\text{cl}} = v \equiv \pm\sqrt{\dfrac{-\mu^2}{\lambda}}$, in QFT $\langle 0|\,\phi\,|0\rangle = v \neq 0$ (VEV)

– A quantum field must have $v = 0$

$$a\,|0\rangle = 0$$

$$\Rightarrow \qquad \phi(x) \equiv v + \eta(x)\,, \quad \langle 0|\,\eta\,|0\rangle = 0$$

Spontaneous Symmetry Breaking — discrete symmetry

- At the quantum level, the same system is described by $\eta(x)$ with Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4 \quad \text{not invariant under} \quad \eta \mapsto -\eta$$

$$(m_\eta = \sqrt{2\lambda}\, v)$$

⇒ Lesson:

$\mathcal{L}(\phi)$ had the symmetry but the parameters can be such that the ground state of the Hamiltonian is not symmetric (Spontaneous Symmetry Breaking)

⇒ Note:

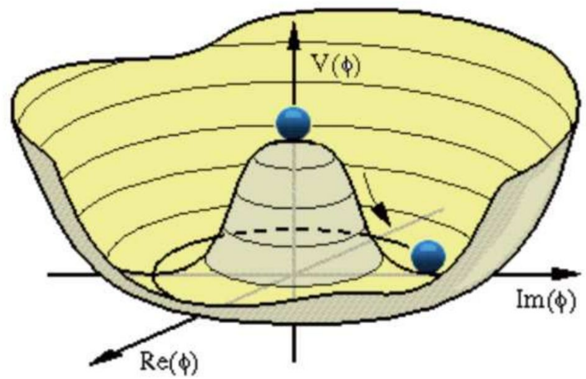
One may argue that $\mathcal{L}(\eta)$ exhibits an explicit breaking of the symmetry. However this is not the case since the coefficients of terms η^2, η^3 and η^4 are determined by just two parameters, λ and v (remnant of the original symmetry)

Spontaneous Symmetry Breaking | continuous symmetry

- Consider a complex scalar field $\phi(x)$ with Lagrangian:

$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 \quad \text{invariant under U(1):} \quad \phi \mapsto e^{-iq\theta}\phi$$

$$\lambda > 0,\ \mu^2 < 0: \quad \langle 0|\,\phi\,|0\rangle \equiv \frac{v}{\sqrt{2}}, \quad |v| = \sqrt{\frac{-\mu^2}{\lambda}}$$



Take $v \in \mathbb{R}^+$. In terms of quantum fields:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + i\chi(x)], \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\chi\,|0\rangle = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - \lambda v^2 \eta^2 - \lambda v \eta(\eta^2 + \chi^2) - \frac{\lambda}{4}(\eta^2 + \chi^2)^2 + \frac{1}{4}\lambda v^4$$

Note: if $ve^{i\alpha}$ (complex) replace η by $(\eta \cos\alpha - \chi \sin\alpha)$ and χ by $(\eta \sin\alpha + \chi \cos\alpha)$

$\Rightarrow$ The actual quantum Lagrangian $\mathcal{L}(\eta, \chi)$ is not invariant under U(1)

U(1) broken $\Rightarrow$ one scalar field remains massless: $m_\eta = \sqrt{2\lambda}\, v$, $m_\chi = 0$

Spontaneous Symmetry Breaking	continuous symmetry

- Another example: consider a real scalar SU(2) triplet $\Phi(x)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^\mathsf{T})(\partial^\mu \Phi) - \frac{1}{2}\mu^2 \Phi^\mathsf{T}\Phi - \frac{\lambda}{4}(\Phi^\mathsf{T}\Phi)^2 \quad \text{inv. under SU(2):} \quad \Phi \mapsto \mathrm{e}^{-\mathrm{i}T_a\theta^a}\Phi$$

that for $\lambda > 0,\ \mu^2 < 0$ acquires a VEV $\langle 0|\,\Phi^\mathsf{T}\Phi\,|0\rangle = v^2 \qquad (\mu^2 = -\lambda v^2)$

Assume $\Phi(x) = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ v + \varphi_3(x) \end{pmatrix}$ and define $\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_1 + \mathrm{i}\varphi_2)$

$$\mathcal{L} = (\partial_\mu \varphi^\dagger)(\partial^\mu \varphi) + \frac{1}{2}(\partial_\mu \varphi_3)(\partial^\mu \varphi_3) - \lambda v^2 \varphi_3^2 - \lambda v(2\varphi^\dagger\varphi + \varphi_3^2)\varphi_3 - \frac{\lambda}{4}(2\varphi^\dagger\varphi + \varphi_3^2)^2 + \frac{1}{4}\lambda v^4$$

$\Rightarrow$ Not symmetric under SU(2) but invariant under U(1):

$$\varphi \mapsto \mathrm{e}^{-\mathrm{i}q\theta}\varphi \quad (q = \text{arbitrary}) \qquad\qquad \varphi_3 \mapsto \varphi_3 \quad (q = 0)$$

SU(2) broken to U(1) $\Rightarrow 3 - 1 = 2$ broken generators

$\Rightarrow$ 2 (real) scalar fields (= 1 complex) remain massless: $m_\varphi = 0$, $m_{\varphi_3} = \sqrt{2\lambda}\,v$

Spontaneous Symmetry Breaking | continuous symmetry

⇒ Goldstone's theorem: [Nambu '60; Goldstone '61]

The number of massless particles (Nambu-Goldstone bosons) is equal to the number of spontaneously broken generators of the symmetry

$$\text{Hamiltonian symmetric under group } G \quad \Rightarrow \quad [T_a, H] = 0\,, \quad a = 1, \ldots, N$$

$$\text{By definition: } H\,|0\rangle = 0 \quad \Rightarrow \quad H(T_a\,|0\rangle) = T_a H\,|0\rangle = 0$$

– If $|0\rangle$ is such that $T_a\,|0\rangle = 0$ for all generators

⇒ non-degenerate minimum: *the* vacuum

– If $|0\rangle$ is such that $T_{a'}\,|0\rangle \neq 0$ for some (broken) generators a'

⇒ degenerate minimum: chose one (*true* vacuum) and $\mathrm{e}^{-\mathrm{i}T_{a'}\theta^{a'}}\,|0\rangle \neq |0\rangle$

⇒ excitations (particles) from $|0\rangle$ to $\mathrm{e}^{-\mathrm{i}T_{a'}\theta^{a'}}\,|0\rangle$ cost no energy: massless!

Spontaneous Symmetry Breaking | gauge symmetry

- Consider a U(1) gauge invariant Lagrangian for a complex scalar field $\phi(x)$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 , \quad D_\mu = \partial_\mu + \mathrm{i}eqA_\mu$$

inv. under $\phi(x) \mapsto \phi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\phi(x) , \quad A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x)$

If $\lambda > 0$, $\mu^2 < 0$, the $\mathcal{L}$ in terms of quantum fields η and χ with null VEVs:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + \mathrm{i}\chi(x)] , \quad \mu^2 = -\lambda v^2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi)$$
$$- \lambda v^2\eta^2 - \lambda v\eta(\eta^2+\chi^2) - \frac{\lambda}{4}(\eta^2+\chi^2)^2 + \frac{1}{4}\lambda v^4$$
$$\boxed{+\, eqvA_\mu\partial^\mu\chi} + eqA_\mu(\eta\partial^\mu\chi - \chi\partial^\mu\eta)$$
$$\boxed{+\, \frac{1}{2}(eqv)^2A_\mu A^\mu} + \frac{1}{2}(eq)^2A_\mu A^\mu(\eta^2 + 2v\eta + \chi^2)$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\, v$
$m_\chi = 0$

(ii) $M_A = |eqv|$ (!)

(iii) Term $A_\mu\partial^\mu\chi$ (?)

(iv) Add $\mathcal{L}_{\mathrm{GF}}$

Spontaneous Symmetry Breaking | gauge symmetry

- Removing the cross term and the (new) gauge fixing Lagrangian:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu - \xi M_A \chi)^2$$

$$\Rightarrow \quad \mathcal{L} + \mathcal{L}_{\rm GF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2 A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \overbrace{M_A \partial_\mu (A^\mu \chi)}^{\text{total deriv.}}$$
$$+ \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - \frac{1}{2}\xi M_A^2 \chi^2 + \dots$$

and the propagators of A_μ and χ are:

$$\widetilde{D}_{\mu\nu}(k) = \frac{\rm i}{k^2 - M_A^2 + {\rm i}\epsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi M_A^2}\right]$$
$$\widetilde{D}(k) = \frac{\rm i}{k^2 - \xi M_A^2}$$

$\Rightarrow \chi$ has a gauge-dependent mass: actually it is not a physical field!

Spontaneous Symmetry Breaking | gauge symmetry

- A more transparent parameterization of the quantum field ϕ is

$$\phi(x) \equiv \mathrm{e}^{\mathrm{i}q\zeta(x)/v} \frac{1}{\sqrt{2}}[v + \eta(x)]\,, \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\zeta\,|0\rangle = 0$$

$$\phi(x) \mapsto \mathrm{e}^{-\mathrm{i}q\zeta(x)/v}\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x)] \quad \Rightarrow \quad \zeta \text{ gauged away!}$$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \\ &-\lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{1}{4}\lambda v^4 \\ &+\frac{1}{2}(eqv)^2 A_\mu A^\mu + \frac{1}{2}(eq)^2 A_\mu A^\mu (2v\eta + \eta^2)\end{aligned}$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\,v$

(ii) $M_A = |eqv|$

(iii) No need for $\mathcal{L}_{\mathrm{GF}}$

$\Rightarrow$ This is the unitary gauge ($\xi \to \infty$): just physical fields

Spontaneous Symmetry Breaking | gauge symmetry

⇒ Higgs mechanism: [Anderson '62; Higgs '64; Englert, Brout '64; Guralnik, Hagen, Kibble '64]

The gauge bosons associated with the spontaneously broken generators become massive, the corresponding would-be Goldstone bosons are unphysical and can be absorbed, the remaining massive scalars (Higgs bosons) are physical (the smoking gun!)

- The would-be Goldstone bosons are 'eaten up' by the gauge bosons ('get fat') and disappear (gauge away) in the unitary gauge ($\xi \to \infty$)

  ⇒ Degrees of freedom are preserved

Before SSB:	2 (massless gauge boson)	+ 1 (Goldstone boson)
After SSB:	3 (massive gauge boson)	+ 0 (absorbed would-be Goldstone)

- For loops calculations, 't Hooft-Feynman gauge ($\xi = 1$) is more convenient:

  ⇒ Gauge boson propagators are simpler, but

  ⇒ Goldstone bosons must be included in internal lines

Spontaneous Symmetry Breaking | gauge symmetry

- Comments:

  - After SSB the FP ghost fields (unphysical) acquire a gauge-dependent mass, due to interactions with the scalar field(s):

$$\widetilde{D}_{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 - \xi M_A^2 + \mathrm{i}\epsilon}$$

  - Gauge theories with SSB are renormalizable ['t Hooft, Veltman '72]

    UV divergences appearing at loop level can be removed by renormalization of parameters and fields of the classical Lagrangian ⇒ predictive!

2. The Standard Model

Gauge group and particle representations

[Glashow '61; Weinberg '67; Salam '68]
[Gell-Mann '64; Zweig '64]

- The Standard Model is a gauge theory based on the local symmetry group:

$$\underbrace{SU(3)_c}_{\text{strong}} \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{electroweak}} \to SU(3)_c \otimes \underbrace{U(1)_Q}_{\text{em}}$$

  with the electroweak symmetry spontaneously broken to the electromagnetic $U(1)_Q$ symmetry by the Higgs mechanism

- The particle (field) content: (ingredients: 12 *flavors* + 12 gauge bosons + H)

Fermions			I	II	III	Q
spin $\frac{1}{2}$	Quarks	f	uuu	ccc	ttt	$\frac{2}{3}$
		f'	ddd	sss	bbb	$-\frac{1}{3}$
	Leptons	f	ν_e	ν_μ	ν_τ	0
		f'	e	μ	τ	-1

Bosons		
spin 1	8 gluons	strong interaction
	$W^\pm$, Z	weak interaction
	γ	em interaction
spin 0	Higgs	origin of mass

$$Q_f = Q_{f'} + 1$$

Gauge group and particle representations

- The fields lay in the following representations (color, weak isospin, hypercharge):

Multiplets	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	I	II	III	$Q = T_3 + Y$
Quarks	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$; $-\frac{1}{3} = -\frac{1}{2} + \frac{1}{6}$
	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$	u_R	c_R	t_R	$\frac{2}{3} = 0 + \frac{2}{3}$
	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	d_R	s_R	b_R	$-\frac{1}{3} = 0 - \frac{1}{3}$
Leptons	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$	$0 = \frac{1}{2} - \frac{1}{2}$; $-1 = -\frac{1}{2} - \frac{1}{2}$
	$(\mathbf{1}, \mathbf{1}, -1)$	e_R	μ_R	τ_R	$-1 = 0 - 1$
	$(\mathbf{1}, \mathbf{1}, 0)$	ν_{e_R}	ν_{μ_R}	ν_{τ_R}	$0 = 0 + 0$
Higgs	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	(3 families of quarks & leptons)			

$\Rightarrow$ From now on just the electroweak part (EWSM): $SU(2)_L \otimes U(1)_Y$

The EWSM with one family (of quarks or leptons)

- Consider two massless fermion fields $f(x)$ and $f'(x)$ with electric charges $Q_f = Q_{f'} + 1$ in three irreps of SU(2)$_L \otimes$U(1)$_Y$:

$$\mathcal{L}_F^0 = \mathrm{i}\overline{f}\partial\!\!\!/ f + \mathrm{i}\overline{f}'\partial\!\!\!/ f' \qquad f_{R,L} = \frac{1}{2}(1 \pm \gamma_5) f\ , \quad f'_{R,L} = \frac{1}{2}(1 \pm \gamma_5) f'$$

$$= \mathrm{i}\overline{\Psi}_1 \partial\!\!\!/ \Psi_1 + \mathrm{i}\overline{\psi}_2 \partial\!\!\!/ \psi_2 + \mathrm{i}\overline{\psi}_3 \partial\!\!\!/ \psi_3 \quad ; \quad \Psi_1 = \underbrace{\begin{pmatrix} f_L \\ f'_L \end{pmatrix}}_{(\mathbf{2}, y_1)}, \quad \psi_2 = \underbrace{f_R}_{(\mathbf{1}, y_2)}, \quad \psi_3 = \underbrace{f'_R}_{(\mathbf{1}, y_3)}$$

- To get a Langrangian invariant under gauge transformations:

$$\Psi_1(x) \mapsto U_L(x) \mathrm{e}^{-\mathrm{i} y_1 \beta(x)} \Psi_1(x), \quad U_L(x) = \mathrm{e}^{-\mathrm{i} T_i \alpha^i(x)}, \quad T_i = \frac{\sigma_i}{2} \quad \text{(weak isospin gen.)}$$

$$\psi_2(x) \mapsto \mathrm{e}^{-\mathrm{i} y_2 \beta(x)} \psi_2(x)$$

$$\psi_3(x) \mapsto \mathrm{e}^{-\mathrm{i} y_3 \beta(x)} \psi_3(x)$$

The EWSM with one family | covariant derivatives

⇒ Introduce gauge fields $W_\mu^i(x)$ $(i=1,2,3)$ and $B_\mu(x)$ through covariant derivatives:

$$\left.\begin{aligned} D_\mu \Psi_1 &= (\partial_\mu - \mathrm{i} g \widetilde{W}_\mu + \mathrm{i} g' y_1 B_\mu)\Psi_1 \,, \quad \widetilde{W}_\mu \equiv \frac{\sigma_i}{2} W_\mu^i \\ D_\mu \psi_2 &= (\partial_\mu + \mathrm{i} g' y_2 B_\mu)\psi_2 \\ D_\mu \psi_3 &= (\partial_\mu + \mathrm{i} g' y_3 B_\mu)\psi_3 \end{aligned}\right\} \Rightarrow \quad \mathcal{L}_F$$

where two couplings g and g' have been introduced and

$$\widetilde{W}_\mu(x) \mapsto U_L(x)\widetilde{W}_\mu(x)U_L^\dagger(x) - \frac{\mathrm{i}}{g}(\partial_\mu U_L(x))U_L^\dagger(x)$$

$$B_\mu(x) \mapsto B_\mu(x) + \frac{1}{g'}\partial_\mu \beta(x)$$

⇒ Add gauge invariant kinetic terms for the gauge fields

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} W_{\mu\nu}^i W^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \,, \quad W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon_{ijk} W_\mu^j W_\nu^k$$

(include self-interactions of the SU(2) gauge fields) and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

The EWSM with one family | mass terms forbidden

⇒ Note that mass terms are not invariant under $SU(2)_L \otimes U(1)_Y$, since LH and RH components do not transform the same:

$$m\overline{f}f = m(\overline{f_L}f_R + \overline{f_R}f_L)$$

⇒ Mass terms for the gauge bosons are not allowed either

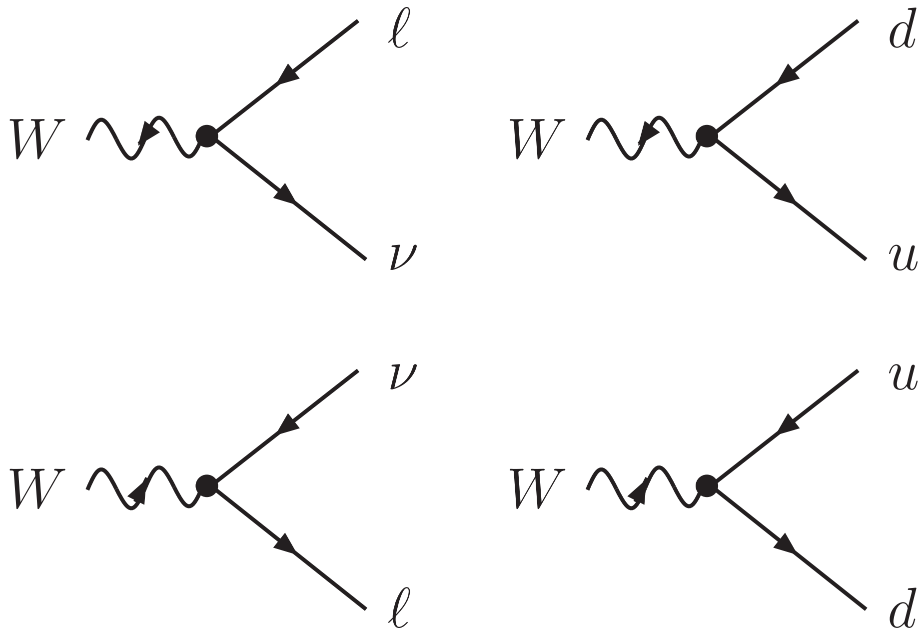
⇒ Next the different types of interactions are analyzed

The EWSM with one family | **charged current interactions**

- $$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu \widetilde{W}_\mu \Psi_1 \,, \quad \widetilde{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^\dagger \\ \sqrt{2}W_\mu & -W_\mu^3 \end{pmatrix}$$

$\Rightarrow$ charged current interactions of LH fermions with complex vector boson field W_μ:

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}}\overline{f}\gamma^\mu(1-\gamma_5)f'W_\mu^\dagger + \text{h.c.} \,, \quad W_\mu \equiv \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2)$$



- The diagonal part of

$$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu \widetilde{W}_\mu \Psi_1 - g' B_\mu (y_1 \overline{\Psi}_1 \gamma^\mu \Psi_1 + y_2 \overline{\psi}_2 \gamma^\mu \psi_2 + y_3 \overline{\psi}_3 \gamma^\mu \psi_3)$$

$\Rightarrow$ neutral current interactions with neutral vector boson fields W^3_μ and B_μ

We would like to identify B_μ with the photon field A_μ but that requires:

$$y_1 = y_2 = y_3 \quad \text{and} \quad g' y_j = eQ_j \quad \Rightarrow \quad \text{impossible!}$$

$\Rightarrow$ Since they are both neutral, try a combination:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \qquad s_W \equiv \sin\theta_W \,, \quad c_W \equiv \cos\theta_W$$

$\theta_W =$ weak mixing angle

$$\mathcal{L}_{\text{NC}} = \sum_{j=1}^{3} \overline{\psi}_j \gamma^\mu \left\{ - \left[g T_3 s_W + g' y_j c_W \right] A_\mu + \left[g T_3 c_W - g' y_j s_W \right] Z_\mu \right\} \psi_j$$

with $T_3 = \dfrac{\sigma_3}{2}$ (0) the third weak isospin component of the doublet (singlet)

- To make A_μ the photon field:

$$\boxed{e = g s_W = g' c_W} \qquad \boxed{Q = T_3 + Y}$$

where the electric charge operator is: $Q_1 = \begin{pmatrix} Q_f & 0 \\ 0 & Q_{f'} \end{pmatrix}\,, \quad Q_2 = Q_f\,, \quad Q_3 = Q_{f'}$

$\Rightarrow$ Electroweak unification: g of SU(2) and g' of U(1) are related

$\Rightarrow$ The hyperchages are fixed in terms of electric charges and weak isospin:

$$y_1 = Q_f - \frac{1}{2} = Q_{f'} + \frac{1}{2}\,, \qquad y_2 = Q_f\,, \qquad y_3 = Q_{f'}$$

$$\mathcal{L}_{\text{QED}} = -e\, Q_f \bar{f} \gamma^\mu f\, A_\mu \quad + (f \to f')$$

$\Rightarrow$ RH neutrinos are sterile: $y_2 = Q_f = 0$

The EWSM with one family | neutral current interactions

- The Z_μ is the neutral weak boson field:

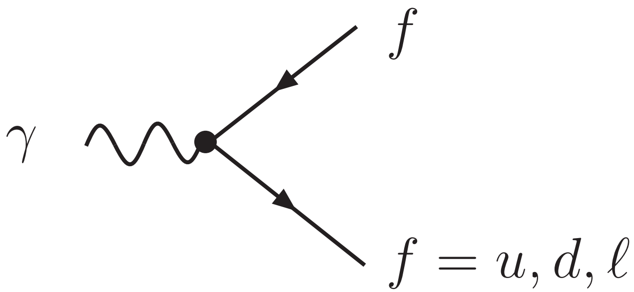
$$\mathcal{L}_{\rm NC}^{Z} = e\,\overline{f}\gamma^\mu(v_f - a_f\gamma_5)f\;Z_\mu \quad + (f \to f')$$

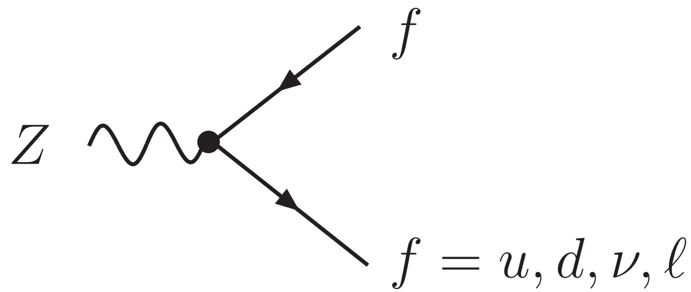
with

$$v_f = \frac{T_3^{f_L} - 2Q_f s_W^2}{2s_W c_W}\,, \quad a_f = \frac{T_3^{f_L}}{2s_W c_W}$$

- The complete neutral current Lagrangian reads:

$$\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm NC}^{Z}$$





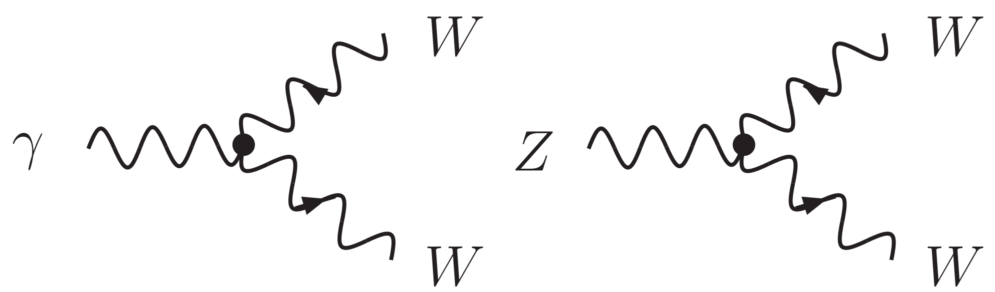
The EWSM with one family | gauge boson self-interactions

- Cubic:

$$\mathcal{L}_{\text{YM}} \supset \mathcal{L}_3 = -\frac{\mathrm{i}ec_W}{s_W}\left\{W^{\mu\nu}W^\dagger_\mu Z_\nu - W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\dagger_\mu W_\nu Z^{\mu\nu}\right\}$$
$$+\,\mathrm{i}e\left\{W^{\mu\nu}W^\dagger_\mu A_\nu - W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu W_\nu F^{\mu\nu}\right\}$$

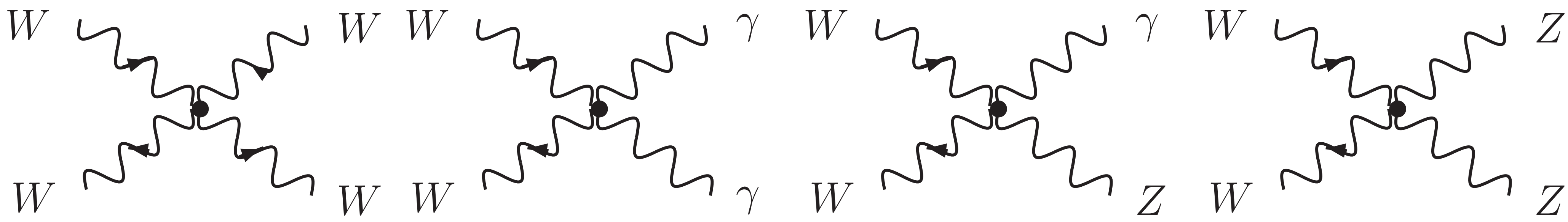
with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \qquad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$



- Quartic:

$$\mathcal{L}_{\rm YM} \supset \mathcal{L}_4 = -\frac{e^2}{2s_W^2}\left\{\left(W_\mu^\dagger W^\mu\right)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu\right\}$$
$$-\frac{e^2 c_W^2}{s_W^2}\left\{W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu\right\}$$
$$+\frac{e^2 c_W}{s_W}\left\{2W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu\right\}$$
$$-e^2\left\{W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu\right\}$$



Note: even number of W and no vertex with just γ or Z

Electroweak symmetry breaking — setup

- Out of the 4 gauge bosons of $SU(2)_L \otimes U(1)_Y$ with generators $T_1,\ T_2,\ T_3,\ Y$ we need all to be broken except the combination $Q = T_3 + Y$ so that A_μ remains massless and the other three gauge bosons get massive after SSB

  $\Rightarrow$ Introduce a complex SU(2) Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \langle 0 | \Phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

  with gauge invariant Lagrangian ($\mu^2 = -\lambda v^2$):

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 , \qquad D_\mu \Phi = (\partial_\mu - \mathrm{i} g \widetilde{W}_\mu + \mathrm{i} g' y_\Phi B_\mu) \Phi$$

$$\text{take } y_\Phi = \frac{1}{2} \quad \Rightarrow \quad (T_3 + Y) |0\rangle = Q \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$\{T_1, T_2, T_3 - Y\} |0\rangle \neq 0$$

Electroweak symmetry breaking | gauge boson masses

- Quantum fields in the unitary gauge:

$$\Phi(x) \equiv \exp\left\{\mathrm{i}\frac{\sigma_i}{2v}\theta^i(x)\right\}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$\Phi(x) \mapsto \exp\left\{-\mathrm{i}\frac{\sigma_i}{2v}\theta^i(x)\right\}\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \Rightarrow$$

1 physical Higgs field $H(x)$

3 would-be Goldstones $\theta^i(x)$ gauged away

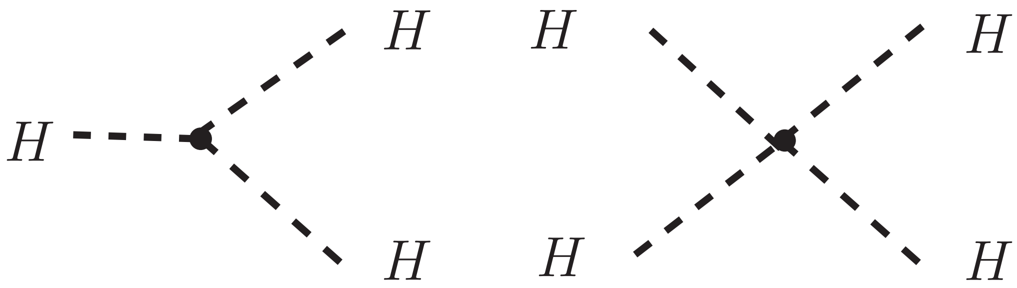
– The 3 dof apparently lost become the longitudinal polarizations of $W^\pm$ and Z that get massive after SSB:

$$\mathcal{L}_\Phi \supset \mathcal{L}_M = \underbrace{\frac{g^2v^2}{4}}_{M_W^2} W_\mu^\dagger W^\mu + \underbrace{\frac{g^2v^2}{8c_W^2}}_{\frac{1}{2}M_Z^2} Z_\mu Z^\mu \quad \Rightarrow \quad M_W = M_Z c_W = \frac{1}{2}gv$$

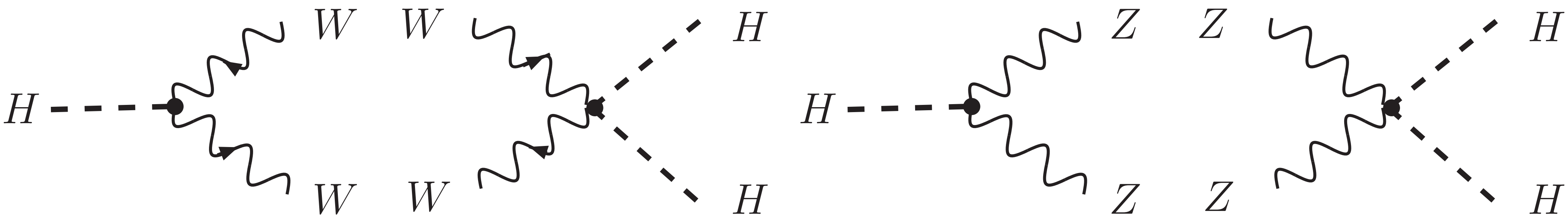
Electroweak symmetry breaking | Higgs sector

⇒ In the unitary gauge (just physical fields): $\mathcal{L}_\Phi = \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 - \frac{M_H^2}{2v}H^3 - \frac{M_H^2}{8v^2}H^4 \,, \quad M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$$



$$\mathcal{L}_M + \mathcal{L}_{HV^2} = M_W^2 W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}$$



Electroweak symmetry breaking | Higgs sector

- Quantum fields in the R_ξ gauges:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix} , \quad \phi^-(x) = [\phi^+(x)]^*$$

$$\begin{aligned}
\mathcal{L}_\Phi = {} & \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4 \\
& + (\partial_\mu \phi^+)(\partial^\mu \phi^-) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) \\
& + \mathrm{i}M_W \, (W_\mu \partial^\mu \phi^+ - W_\mu^\dagger \partial^\mu \phi^-) + \, M_Z \, Z_\mu \partial^\mu \chi \\
& + \text{trilinear interactions [SSS, SSV, SVV]} \\
& + \text{quadrilinear interactions [SSSS, SSVV]}
\end{aligned}$$

Electroweak symmetry breaking — gauge fixing

- To remove the cross terms $W_\mu \partial^\mu \phi^+$, $W^\dagger_\mu \partial^\mu \phi^-$, $Z_\mu \partial^\mu \chi$ and define propagators add:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi_\gamma}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z \chi)^2 - \frac{1}{\xi_W}|\partial_\mu W^\mu + \mathrm{i}\xi_W M_W \phi^-|^2$$

$\Rightarrow$ Massive propagators for gauge and (unphysical) would-be Goldstone fields:

$$\widetilde{D}^\gamma_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_\gamma)\frac{k_\mu k_\nu}{k^2}\right]$$

$$\widetilde{D}^Z_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_Z^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_Z)\frac{k_\mu k_\nu}{k^2 - \xi_Z M_Z^2}\right] \quad ; \quad \widetilde{D}^\chi(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\epsilon}$$

$$\widetilde{D}^W_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_W^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right] \quad ; \quad \widetilde{D}^\phi(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\epsilon}$$

('t Hooft-Feynman gauge: $\xi_\gamma = \xi_Z = \xi_W = 1$)

Electroweak symmetry breaking | Faddeev-Popov ghosts

- The SM is a non-Abelian theory $\Rightarrow$ add Faddeev-Popov ghosts $c_i(x)$ $(i = 1,2,3)$

$$c_1 \equiv \frac{1}{\sqrt{2}}(u_+ + u_-)\ , \quad c_2 \equiv \frac{\mathrm{i}}{\sqrt{2}}(u_+ - u_-)\ , \quad c_3 \equiv c_W\, u_Z - s_W\, u_\gamma$$

$$\mathcal{L}_{\mathrm{FP}} = \underbrace{(\partial^\mu \bar{c}_i)(\partial_\mu c_i - g\epsilon_{ijk} c_j W^k_\mu)}_{\text{U kinetic + [UUV]}} + \underbrace{\text{interactions with } \Phi}_{\text{U masses + [SUU]}}$$

$\Rightarrow$ Massive propagators for (unphysical) FP ghost fields:

$$\widetilde{D}^{u_\gamma}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\ , \quad \widetilde{D}^{u_Z}(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\epsilon}\ , \quad \widetilde{D}^{u_\pm}(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\epsilon}$$

('t Hooft-Feynman gauge: $\xi_Z = \xi_W = 1$)

Electroweak symmetry breaking | **Faddeev-Popov ghosts**

$$
\begin{aligned}
\mathcal{L}_{\text{FP}} = \quad & (\partial_\mu \bar{u}_\gamma)(\partial^\mu u_\gamma) + (\partial_\mu \bar{u}_Z)(\partial^\mu u_Z) + (\partial_\mu \bar{u}_+)(\partial^\mu u_+) + (\partial_\mu \bar{u}_-)(\partial^\mu u_-) \\
[\text{UUV}] \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & + \mathrm{i}e[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]A_\mu - \frac{\mathrm{i}ec_W}{s_W}[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]Z_\mu \\
& - \mathrm{i}e[(\partial^\mu \bar{u}_+)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_-]W^\dagger_\mu + \frac{\mathrm{i}ec_W}{s_W}[(\partial^\mu \bar{u}_+)u_Z - (\partial^\mu \bar{u}_Z)u_-]W^\dagger_\mu \\
& + \mathrm{i}e[(\partial^\mu \bar{u}_-)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_+]W_\mu - \frac{\mathrm{i}ec_W}{s_W}[(\partial^\mu \bar{u}_-)u_Z - (\partial^\mu \bar{u}_Z)u_+]W_\mu \\
& - \xi_Z M_Z^2\, \bar{u}_Z u_Z - \xi_W M_W^2\, \bar{u}_+ u_+ - \xi_W M_W^2\, \bar{u}_- u_- \\
[\text{SUU}] \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & - e\xi_Z M_Z\, \bar{u}_Z \left[\frac{1}{2s_W c_W} H u_Z - \frac{1}{2s_W}\left(\phi^+ u_- + \phi^- u_+\right) \right] \\
& - e\xi_W M_W\, \bar{u}_+ \left[\frac{1}{2s_W}(H + \mathrm{i}\chi)u_+ - \phi^+ \left(u_\gamma - \frac{c_W^2 - s_W^2}{2s_W c_W} u_Z \right) \right] \\
& - e\xi_W M_W\, \bar{u}_- \left[\frac{1}{2s_W}(H - \mathrm{i}\chi)u_- - \phi^- \left(u_\gamma - \frac{c_W^2 - s_W^2}{2s_W c_W} u_Z \right) \right]
\end{aligned}
$$

Electroweak symmetry breaking | fermion masses

- We need masses for quarks and leptons without breaking gauge symmetry

  $\Rightarrow$ Introduce Yukawa interactions:

$$\mathcal{L}_{\rm Y} = -\lambda_d \begin{pmatrix} \overline{u}_L & \overline{d}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - \lambda_u \begin{pmatrix} \overline{u}_L & \overline{d}_L \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R$$

$$- \lambda_\ell \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \ell_R - \lambda_\nu \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \nu_R \quad + \text{h.c.}$$

where $\Phi^c \equiv i\sigma_2\Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ transforms under SU(2) like $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$\Rightarrow$ After EW SSB, fermions acquire masses:

$$\mathcal{L}_{\rm Y} \supset -\frac{1}{\sqrt{2}}(v+H)\left\{\lambda_d\,\overline{d}d + \lambda_u\,\overline{u}u + \lambda_\ell\,\overline{\ell}\ell + \lambda_\nu\,\overline{\nu}\nu\right\} \quad \Rightarrow \quad m_f = \lambda_f \frac{v}{\sqrt{2}}$$

Additional generations **Yukawa matrices**

- There are 3 generations of quarks and leptons in Nature. They are identical copies with the same properties under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ differing only in their masses

⇒ Take a general case of n_G generations and let $u_j^I, d_j^I, \nu_j^I, \ell_j^I$ be the members of family j ($j = 1, \ldots, n_G$). Superindex I (interaction basis) was omitted so far

⇒ General gauge invariant Yukawa Lagrangian:

$$\mathcal{L}_\mathrm{Y} = -\sum_{jk} \left\{ \begin{pmatrix} \overline{u}_{jL}^I & \overline{d}_{jL}^I \end{pmatrix} \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{jk}^{(d)}\, d_{kR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{jk}^{(u)}\, u_{kR}^I \right] \right.$$
$$\left. + \begin{pmatrix} \overline{\nu}_{jL}^I & \overline{\ell}_{jL}^I \end{pmatrix} \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{jk}^{(\ell)}\, \ell_{kR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{jk}^{(\nu)}\, \nu_{kR}^I \right] \right\} + \text{h.c.}$$

where $\lambda_{jk}^{(d)}, \lambda_{jk}^{(u)}, \lambda_{jk}^{(\ell)}, \lambda_{jk}^{(\nu)}$ are arbitrary Yukawa matrices

Additional generations | mass matrices

- After EW SSB, in n_G-dimensional matrix form:

$$\mathcal{L}_{\rm Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}_L^I\,\mathbf{M}_d\,\mathbf{d}_R^I \;+\; \overline{\mathbf{u}}_L^I\,\mathbf{M}_u\,\mathbf{u}_R^I \;+\; \overline{\mathbf{l}}_L^I\,\mathbf{M}_\ell\,\mathbf{l}_R^I \;+\; \overline{\boldsymbol{\nu}}_L^I\,\mathbf{M}_\nu\,\boldsymbol{\nu}_R^I \;+\; \text{h.c.}\right\}$$

with mass matrices

$$(\mathbf{M}_d)_{ij} \equiv \lambda_{ij}^{(d)}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_u)_{ij} \equiv \lambda_{ij}^{(u)}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\ell)_{ij} \equiv \lambda_{ij}^{(\ell)}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\nu)_{ij} \equiv \lambda_{ij}^{(\nu)}\frac{v}{\sqrt{2}}$$

$\Rightarrow$ Diagonalization determines mass eigenstates d_j, u_j, ℓ_j, ν_j in terms of interaction states d_j^I, u_j^I, ℓ_j^I, ν_j^I, respectively

$\Rightarrow$ Each $\mathbf{M}_f$ can be written as

$$\mathbf{M}_f = \mathbf{H}_f\,\mathcal{U}_f = \mathbf{S}_f^\dagger\,\mathcal{M}_f\,\mathbf{S}_f\,\mathcal{U}_f \quad\Longleftrightarrow\quad \mathbf{M}_f\mathbf{M}_f^\dagger = \mathbf{H}_f^2 = \mathbf{S}_f^\dagger\,\mathcal{M}_f^2\,\mathbf{S}_f$$

with $\mathbf{H}_f \equiv \sqrt{\mathbf{M}_f\mathbf{M}_f^\dagger}$ a Hermitian positive definite matrix and $\mathcal{U}_f$ unitary

– Every $\mathbf{H}_f$ can be diagonalized by a unitary matrix $\mathbf{S}_f$

– The resulting $\mathcal{M}_f$ is diagonal and positive definite

Additional generations | fermion masses and mixings

- In terms of diagonal mass matrices (mass eigenstate basis):

$$\mathcal{M}_d = \text{diag}(m_d, m_s, m_b, \ldots)\,, \quad \mathcal{M}_u = \text{diag}(m_u, m_c, m_t, \ldots)$$
$$\mathcal{M}_\ell = \text{diag}(m_e, m_\mu, m_\tau, \ldots)\,, \quad \mathcal{M}_\nu = \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, \ldots)$$

$$\mathcal{L}_\text{Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}\,\mathcal{M}_d\,\mathbf{d} + \overline{\mathbf{u}}\,\mathcal{M}_u\,\mathbf{u} + \overline{\mathbf{l}}\,\mathcal{M}_\ell\,\mathbf{l} + \overline{\boldsymbol{\nu}}\,\mathcal{M}_\nu\,\boldsymbol{\nu}\right\}$$

where fermion couplings to Higgs are proportional to masses and

$$\mathbf{d}_L \equiv \mathbf{S}_d\,\mathbf{d}_L^I \qquad \mathbf{u}_L \equiv \mathbf{S}_u\,\mathbf{u}_L^I \qquad \mathbf{l}_L \equiv \mathbf{S}_\ell\,\mathbf{l}_L^I \qquad \boldsymbol{\nu}_L \equiv \mathbf{S}_\nu\,\boldsymbol{\nu}_L^I$$
$$\mathbf{d}_R \equiv \mathbf{S}_d\mathcal{U}_d\,\mathbf{d}_R^I \qquad \mathbf{u}_R \equiv \mathbf{S}_u\mathcal{U}_u\,\mathbf{u}_R^I \qquad \mathbf{l}_R \equiv \mathbf{S}_\ell\mathcal{U}_\ell\,\mathbf{l}_R^I \qquad \boldsymbol{\nu}_R \equiv \mathbf{S}_\nu\mathcal{U}_\nu\,\boldsymbol{\nu}_R^I$$

Neutral Currents preserve chirality
$\overline{\mathbf{f}}_L^I\,\mathbf{f}_L^I = \overline{\mathbf{f}}_L\,\mathbf{f}_L$ and $\overline{\mathbf{f}}_R^I\,\mathbf{f}_R^I = \overline{\mathbf{f}}_R\,\mathbf{f}_R$
$\Rightarrow \mathcal{L}_\text{NC}$ does not change flavor

$\Rightarrow$ GIM mechanism [Glashow, Iliopoulos, Maiani '70]

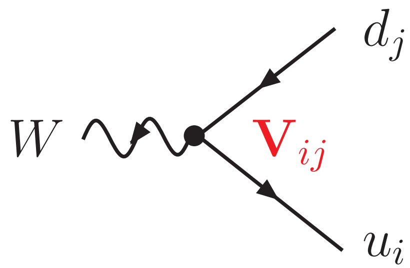
Additional generations — quark sector

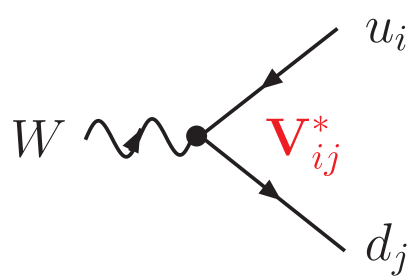
- However, in Charged Currents (also chirality preserving and only LH):

$$\overline{\mathbf{u}}_L^I\,\mathbf{d}_L^I = \overline{\mathbf{u}}_L\,\mathbf{S}_u\,\mathbf{S}_d^\dagger\,\mathbf{d}_L = \overline{\mathbf{u}}_L\mathbf{V}\mathbf{d}_L$$

with $\mathbf{V} \equiv \mathbf{S}_u\,\mathbf{S}_d^\dagger$ the (unitary) CKM mixing matrix [Cabibbo '63; Kobayashi, Maskawa '73]

$$\Rightarrow \quad \mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}}\sum_{ij}\overline{u}_i\gamma^\mu(1-\gamma_5)\,\mathbf{V}_{ij}\,d_j\,W_\mu^\dagger + \text{h.c.}$$





$\Rightarrow$ If u_i or d_j had degenerate masses one could choose $\mathbf{S}_u = \mathbf{S}_d$ (field redefinition) and flavor would be conserved in the quark sector. But they are not degenerate

$\Rightarrow$ $\mathbf{S}_u$ and $\mathbf{S}_d$ are not observable. Just masses and CKM mixings are observable

Additional generations | quark sector

- How many physical parameters in this sector?

  – Quark masses and CKM mixings determined by mass (or Yukawa) matrices

  – A general $n_G \times n_G$ unitary matrix, like the CKM, is given by

$$n_G^2 \text{ real parameters} = n_G(n_G-1)/2 \text{ moduli} + n_G(n_G+1)/2 \text{ phases}$$

  Some phases are unphysical since they can be absorbed by field redefinitions:

$$u_i \to \mathrm{e}^{\mathrm{i}\phi_i}\, u_i\ , \quad d_j \to \mathrm{e}^{\mathrm{i}\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to \mathbf{V}_{ij}\, \mathrm{e}^{\mathrm{i}(\theta_j-\phi_i)}$$

  Therefore $2n_G - 1$ unphysical phases and the physical parameters are:

$$\boxed{(n_G-1)^2 = n_G(n_G-1)/2 \text{ moduli } + (n_G-1)(n_G-2)/2 \text{ phases}}$$

Additional generations **quark sector**

⇒ Case of $n_G = 2$ generations: 1 parameter, the Cabibbo angle θ_C:

$$\mathbf{V} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

⇒ Case of $n_G = 3$ generations: 3 angles + 1 phase. In the standard parameterization:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ub} \\ \mathbf{V}_{cd} & \mathbf{V}_{cs} & \mathbf{V}_{cb} \\ \mathbf{V}_{td} & \mathbf{V}_{ts} & \mathbf{V}_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{-\mathrm{i}\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

⇒ δ_{13} only source of CP violation in the SM !

with $c_{ij} \equiv \cos\theta_{ij} \geq 0, \quad s_{ij} \equiv \sin\theta_{ij} \geq 0 \quad (i < j = 1, 2, 3)$ and $0 \leq \delta_{13} \leq 2\pi$

Additional generations | lepton sector

- If neutrinos were massless we could redefine the (LH) fields $\Rightarrow$ no lepton mixing
  But they have (tiny) masses because there are neutrino oscillations!

- Neutrinos are special:
  they *may* be their own antiparticle (Majorana) since they are neutral

- *If* they are Majorana:
  - Mass terms are different to Dirac case
    (neutrino and antineutrino *may* mix)
  - Intergenerational mixings are richer (more CP phases)

⋆ lepton sector

- About Majorana fermions

  – A Dirac fermion field is a spinor with 4 independent components: 2 LH+2 RH (left/right-handed particles and antiparticles)

  $$\psi_L = P_L\psi \,, \quad \psi_R = P_R\psi \,, \quad \psi_L^c \equiv (\psi_L)^c = P_R\psi^c \,, \quad \psi_R^c \equiv (\psi_R)^c = P_L\psi^c$$

  where $\psi^c \equiv C\overline{\psi}^\mathsf{T} = \mathrm{i}\gamma^2\psi^*$ (charge conjugate) with $C = \mathrm{i}\gamma^2\gamma^0$, $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$

  – A Majorana fermion field has just 2 independent components since $\psi^c \equiv \eta^*\psi$:

  $$\psi_L = \eta\psi_R^c \,, \quad \psi_R = \eta\psi_L^c$$

  where $\eta = -\mathrm{i}\eta_{\mathrm{CP}}$ (CP parity) with $|\eta|^2 = 1$. Only possible if neutral

⋆ lepton sector

- About mass terms

$$\begin{array}{lll} \overline{\psi_R}\psi_L = \overline{\psi_L^c}\psi_R^c & , \ \overline{\psi_L}\psi_R = \overline{\psi_R^c}\psi_L^c & (\Delta F = 0) \\ \overline{\psi_L^c}\psi_L & , \ \overline{\psi_L}\psi_L^c & \multirow{2}{*}{$\Bigg\} \quad (|\Delta F| = 2)$} \\ \overline{\psi_R}\psi_R^c & , \ \overline{\psi_R^c}\psi_R & \end{array}$$

$$\Rightarrow \quad -\mathcal{L}_m = \underbrace{m_D\,\overline{\psi_R}\psi_L}_{\text{Dirac term}} + \underbrace{\frac{1}{2}m_L\,\overline{\psi_L^c}\psi_L + \frac{1}{2}m_R\,\overline{\psi_R}\psi_R^c}_{\text{Majorana terms}} + \text{h.c.}$$

  - A Dirac fermion can only have Dirac mass term
  - A Majorana fermion can have both Dirac and Majorana mass terms

$\Rightarrow$ In the SM: ∗ m_D from Yukawa coupling after EW SSB $\quad (m_D = \lambda_\nu\, v/\sqrt{2})$

∗ m_L forbidden by gauge symmetry

∗ m_R compatible with gauge symmetry!

⋆ lepton sector

- About mass terms (a more transparent parameterization)

  Rewrite previous mass terms introducing a doublet of Majorana fermions:

$$\chi^0 = \chi^{0c} = \chi^0_L + \chi^{0c}_L \equiv \begin{pmatrix} \chi^0_1 \\ \chi^0_2 \end{pmatrix} , \qquad \begin{matrix} \chi^0_1 = \chi^{0c}_1 = \chi^0_{1L} + \chi^{0c}_{1L} \equiv \psi_L + \psi^c_L \\ \chi^0_2 = \chi^{0c}_2 = \chi^0_{2L} + \chi^{0c}_{2L} \equiv \psi^c_R + \psi_R \end{matrix}$$

$$\Rightarrow \quad -\mathcal{L}_m = \frac{1}{2}\,\overline{\chi^{0c}_L}\,\mathbf{M}\,\chi^0_L + \text{h.c.} \quad \text{with} \quad \mathbf{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

  $\mathbf{M}$ is a square symmetric matrix $\Rightarrow$ diagonalizable by a unitary matrix $\widetilde{\mathcal{U}}$:

$$\widetilde{\mathcal{U}}^{\mathsf{T}}\mathbf{M}\,\widetilde{\mathcal{U}} = \mathcal{M} = \text{diag}(m'_1, m'_2) \ , \quad \chi^0_L = \widetilde{\mathcal{U}}\chi_L \quad (\chi^{0c}_L = \widetilde{\mathcal{U}}^*\chi^c_L)$$

  To get real and positive eigenvalues $m_i = \eta_i m'_i$ (physical masses) take $\chi^0_L = \mathcal{U}\xi_L$:

$$\mathcal{U} = \widetilde{\mathcal{U}}\text{diag}(\sqrt{\eta_1}, \sqrt{\eta_2}) \ , \qquad \begin{matrix} \xi_1 = \chi_{1L} + \eta_1\chi^c_{1L} \\ \xi_2 = \chi_{2L} + \eta_2\chi^c_{2L} \end{matrix} \quad \text{(physical fields)} \quad \eta_i = \text{CP parities}$$

⋆ lepton sector

- About mass terms (a more transparent parameterization)
  - Case of only Dirac term ($m_L = m_R = 0$)

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix} \quad \Rightarrow \quad \widetilde{\mathcal{U}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} , \quad m_1' = -m_D , \quad m_2' = m_D$$

Eigenstates $\Rightarrow$ Physical states

$$\chi_{1L} = \frac{1}{\sqrt{2}}(\chi^0_{1L} - \chi^0_{2L}) = \frac{1}{\sqrt{2}}(\psi_L - \psi^c_R)$$

$$\chi_{2L} = \frac{1}{\sqrt{2}}(\chi^0_{1L} + \chi^0_{2L}) = \frac{1}{\sqrt{2}}(\psi_L + \psi^c_R)$$

$$\xi_1 = \chi_{1L} + \eta_1 \chi^c_{1L} \quad [\eta_1 = -1]$$

$$\xi_2 = \chi_{2L} + \eta_2 \chi^c_{2L} \quad [\eta_2 = +1]$$

with masses $m_1 = m_2 = m_D$

$$\Rightarrow \quad -\mathcal{L}_m = \frac{1}{2} m_D (-\overline{\chi}_1 \chi_1 + \overline{\chi}_2 \chi_2) = \frac{1}{2} m_D (\overline{\xi}_1 \xi_1 + \overline{\xi}_2 \xi_2) = m_D (\overline{\psi_L} \psi_R + \overline{\psi_R} \psi_L)$$

One Dirac fermion = two Majorana of equal mass and opposite CP parities

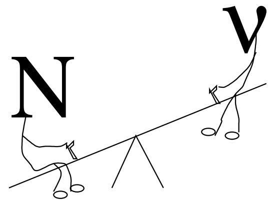
⋆ lepton sector

- About mass terms (a more transparent parameterization)
  - Case of seesaw (type I) [Yanagida '79; Gell-Mann, Ramond, Slansky '79; Mohapatra, Senjanovic '80]

    $(m_L = 0,\ m_D \ll m_R)$

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \quad \Rightarrow \quad \widetilde{\mathcal{U}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} , \quad \theta \simeq \frac{m_D}{m_R} \simeq \sqrt{\frac{m_\nu}{m_N}} \text{ (negligible)}$$

$$m_1 \equiv m_\nu \simeq \frac{m_D^2}{m_R} \ll m_2 \equiv m_N \simeq m_R$$



$$\begin{aligned} \xi_1 \equiv\ \nu &= \psi_L + \eta_1 \psi_L^c \quad [\eta_1 = -1] \\ \xi_2 \equiv N &= \psi_R^c + \eta_2 \psi_R \quad [\eta_2 = +1] \end{aligned} \quad \Rightarrow \quad -\mathcal{L}_m = \frac{1}{2} m_\nu\, \overline{\nu_L^c} \nu_L + \frac{1}{2} m_N\, \overline{N_R^c} N_R + \text{h.c.}$$

> *Perhaps* the observed neutrino ν_L is the LH component of a light Majorana ν (then $\overline{\nu}$ = RH) and light because of a very heavy Majorana neutrino N

e.g. $m_D \sim v \simeq 246$ GeV , $m_R \sim m_N \sim 10^{15}$ GeV $\Rightarrow$ $m_\nu \sim 0.1$ eV ✓

Additional generations **lepton sector**

- Lepton mixings
  - From Z lineshape: there are $n_G = 3$ generations of ν_L [ν_i $(i = 1, \ldots, n_G)$] (but we do not know (*yet*) if neutrinos are Dirac or Majorana fermions)
  - From neutrino oscillations: neutrinos are light, non degenerate and mix

$$|\nu_\alpha\rangle = \sum_i \mathbf{U}_{\alpha i} |\nu_i\rangle \qquad \Longleftrightarrow \qquad |\nu_i\rangle = \sum_\alpha \mathbf{U}^*_{\alpha i} |\nu_\alpha\rangle$$

mass eigenstates ν_i $(i = 1, 2, 3)$ / interaction states ν_α $(\alpha = e, \mu, \tau)$

⇒ **U** matrix is unitary (negligible mixing with heavy neutrinos) and analogous to $\mathbf{S}_u$, $\mathbf{S}_d$, $\mathbf{S}_\ell$ defined for quarks and charged leptons except for:

  - ν fields have both chiralities
  - *If* neutrinos are Majorana, **U** *may* contain two additional physical (Majorana) phases (irrelevant and therefore not measurable in oscillation experiments) that cannot be absorbed since then field phases are fixed by $\nu_i = \eta_i \nu_i^c$

Additional generations | lepton sector

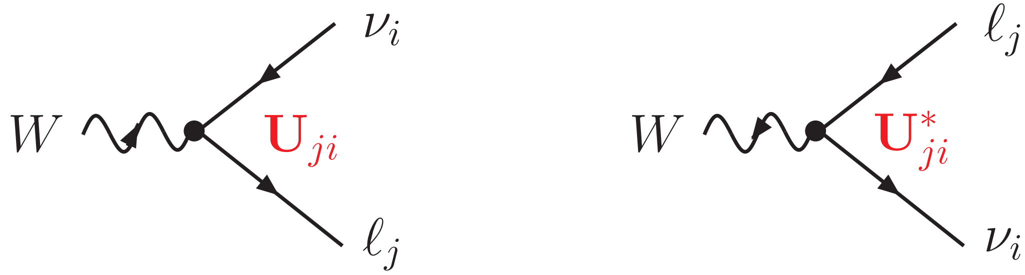
- Lepton mixings

  The so called PMNS matrix **U** [Pontecorvo '57; Maki, Nakagawa, Sakata '62; Pontecorvo '68]

  - does not change Neutral Currents (unitarity), but
  - introduces intergenerational mixings in Charged Currents:

$$\mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}} \sum_{\alpha i} \bar{\ell}_\alpha \, \gamma^\mu (1-\gamma_5) \, \mathbf{U}_{\alpha i} \, \nu_i \, W_\mu \; + \text{h.c.}$$

(basis where charged leptons are diagonal)



Additional generations **lepton sector**

⇒ Standard parameterization of the PMNS matrix:

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{e1} & \mathbf{U}_{e2} & \mathbf{U}_{e3} \\ \mathbf{U}_{\mu 1} & \mathbf{U}_{\mu 2} & \mathbf{U}_{\mu 3} \\ \mathbf{U}_{\tau 1} & \mathbf{U}_{\tau 2} & \mathbf{U}_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, \mathrm{e}^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, \mathrm{e}^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, \mathrm{e}^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, \mathrm{e}^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, \mathrm{e}^{i\delta_{13}} & c_{23}\, c_{13} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\alpha_1} & 0 & 0 \\ 0 & \mathrm{e}^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(different values than in CKM) (Majorana phases)

[$\theta_{13} \equiv \theta_{\odot}$, $\theta_{23} \equiv \theta_{\mathrm{atm}}$ and θ_{13} (not yet δ_{13}) measured in oscillations]

Complete SM Lagrangian — fields and interactions

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{\rm YM} + \mathcal{L}_\Phi + \mathcal{L}_{\rm Y} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

- Fields: [F] fermions, [S] scalars, [V] vector bosons, [U] unphysical ghosts

- Interactions: [FFV] [FFS] [SSV] [SVV] [SSVV] [VVV] [VVVV] [SSS] [SSSS] [SUU] [UUVV]

Complete SM Lagrangian | **Feynman rules**

- Feynman rules for generic couplings normalized to e (all momenta incoming):

(i$\mathcal{L}$)			
	[FFV$_\mu$]	$\mathrm{i}e\gamma^\mu(g_V - g_A\gamma_5) = \mathrm{i}e\gamma^\mu(g_L P_L + g_R P_R)$	
	[FFS]	$\mathrm{i}e(g_S - g_P\gamma_5) = \mathrm{i}e(c_L P_L + c_R P_R)$	
	[SV$_\mu$V$_\nu$]	$\mathrm{i}eKg_{\mu\nu}$	
	[S(p_1)S(p_2)V$_\mu$]	$\mathrm{i}eG(p_1 - p_2)_\mu$	
	[V$_\mu$(k_1)V$_\nu$(k_2)V$_\rho$(k_3)]	$\mathrm{i}eJ\left[g_{\mu\nu}(k_2-k_1)_\rho + g_{\nu\rho}(k_3-k_2)_\mu + g_{\mu\rho}(k_1-k_3)_\nu\right]$	
	[V$_\mu$(k_1)V$_\nu$(k_2)V$_\rho$(k_3)V$_\sigma$(k_4)]	$\mathrm{i}e^2C\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right]$	
	[SSV$_\mu$V$_\nu$]	$\mathrm{i}e^2C_2 g_{\mu\nu}$	also [UUVV]
	[SSS]	$\mathrm{i}eC_3$	also [SUU]
	[SSSS]	$\mathrm{i}e^2C_4$	

Note: $g_{L,R} = g_V \pm g_A$

$c_{L,R} = g_S \pm g_P$

Attention to symmetry factors!

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

FFV	$\bar{f}_i f_j \gamma$	$\bar{f}_i f_j Z$	$\bar{u}_i d_j W^+$	$\bar{d}_j u_i W^-$	$\bar{\nu}_i \ell_j W^+$	$\bar{\ell}_j \nu_i W^-$
g_L	$-Q_f \delta_{ij}$	$g_+^f \delta_{ij}$	$\frac{1}{\sqrt{2} s_W} \mathbf{V}_{ij}$	$\frac{1}{\sqrt{2} s_W} \mathbf{V}_{ij}^*$	$\frac{1}{\sqrt{2} s_W} \mathbf{U}_{ji}^*$	$\frac{1}{\sqrt{2} s_W} \mathbf{U}_{ji}$
g_R	$-Q_f \delta_{ij}$	$g_-^f \delta_{ij}$	0	0	0	0

$$g_\pm^f \equiv v_f \pm a_f \qquad v_f = \frac{T_3^{f_L} - 2 Q_f s_W^2}{2 s_W c_W} \qquad a_f = \frac{T_3^{f_L}}{2 s_W c_W}$$

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

FFS	$\bar{f}_i f_j H$	$\bar{f}_i f_j \chi$	$\bar{u}_i d_j \phi^+$	$\bar{d}_j u_i \phi^-$
c_L	$-\frac{1}{2s_W}\frac{m_{f_i}}{M_W}\delta_{ij}$	$-\frac{\mathrm{i}}{2s_W}2T_3^{f_L}\frac{m_{f_i}}{M_W}\delta_{ij}$	$+\frac{1}{\sqrt{2}s_W}\frac{m_{u_i}}{M_W}\mathbf{V}_{ij}$	$-\frac{1}{\sqrt{2}s_W}\frac{m_{d_j}}{M_W}\mathbf{V}^*_{ij}$
c_R	$-\frac{1}{2s_W}\frac{m_{f_i}}{M_W}\delta_{ij}$	$+\frac{\mathrm{i}}{2s_W}2T_3^{f_L}\frac{m_{f_i}}{M_W}\delta_{ij}$	$-\frac{1}{\sqrt{2}s_W}\frac{m_{d_j}}{M_W}\mathbf{V}_{ij}$	$+\frac{1}{\sqrt{2}s_W}\frac{m_{u_j}}{M_W}\mathbf{V}^*_{ij}$

FFS	$\bar{\nu}_i \ell_j \phi^+$	$\bar{\ell}_j \nu_i \phi^-$
c_L	$+\frac{1}{\sqrt{2}s_W}\frac{m_{\nu_i}}{M_W}\mathbf{U}^*_{ji}$	$-\frac{1}{\sqrt{2}s_W}\frac{m_{\ell_j}}{M_W}\mathbf{U}_{ji}$
c_R	$-\frac{1}{\sqrt{2}s_W}\frac{m_{\ell_j}}{M_W}\mathbf{U}^*_{ji}$	$+\frac{1}{\sqrt{2}s_W}\frac{m_{\nu_i}}{M_W}\mathbf{U}_{ji}$

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

SVV	HZZ	HW^+W^-	$\phi^\pm W^\mp \gamma$	$\phi^\pm W^\mp Z$
K	$M_W/s_W c_W^2$	M_W/s_W	$-M_W$	$-M_W s_W/c_W$

SSV	χHZ	$\phi^\pm \phi^\mp \gamma$	$\phi^\pm \phi^\mp Z$	$\phi^\mp H W^\pm$	$\phi^\mp \chi W^\pm$
G	$-\frac{\mathrm{i}}{2s_W c_W}$	∓ 1	$\pm\frac{c_W^2 - s_W^2}{2s_W c_W}$	$\mp\frac{1}{2s_W}$	$-\frac{\mathrm{i}}{2s_W}$

VVV	γW^+W^-	ZW^+W^-
J	-1	c_W/s_W

Complete SM Lagrangian | Feynman rules ('t Hooft-Feynman gauge)

VVVV	$W^+W^+W^-W^-$	W^+W^-ZZ	$W^+W^-\gamma Z$	$W^+W^-\gamma\gamma$
C	$\dfrac{1}{s_W^2}$	$-\dfrac{c_W^2}{s_W^2}$	$\dfrac{c_W}{s_W}$	-1

SSVV	HHW^-W^+	$HHZZ$
C_2	$\dfrac{1}{2s_W^2}$	$\dfrac{1}{2s_W^2c_W^2}$

SSS	HHH
C_3	$-\dfrac{3M_H^2}{2M_W s_W}$

SSSS	$HHHH$
C_4	$-\dfrac{3M_H^2}{4M_W^2 s_W^2}$

- Would-be Goldstone bosons in [SSVV], [SSS] and [SSSS] omitted
- Faddeev-Popov ghosts in [UUVV] and [SUU] omitted
- All Feynman rules from FeynArts (same conventions):

http://www.ugr.es/local/jillana/SM/FeynmanRulesSM.pdf

3. Phenomenology

Input parameters

- Parameters:

17 +9 =	1	1	1	1	9 +3	4	6
formal:	g	g'	v	λ	λ_f	$\mathbf{V}_{\text{CKM}}$	$\mathbf{U}_{\text{PMNS}}$
practical:	α	M_W	M_Z	M_H	m_f		

where $e = g s_W = g' c_W$ and

$$\alpha = \frac{e^2}{4\pi} \qquad M_W = \frac{1}{2} g v \qquad M_Z = \frac{M_W}{c_W} \qquad M_H = \sqrt{2\lambda}\, v \qquad m_f = \frac{v}{\sqrt{2}} \lambda_f$$

⇒ Many (more) experiments

⇒ After Higgs discovery, for the first time *all* parameters measured!

Input parameters

- Experimental values [Particle Data Group '13]
  - Fine structure constant:

    $\alpha^{-1} = 137.035\,999\,074\,(44)$ from Harvard cyclotron (g_e)

  - The SM predicts $M_W < M_Z$ in agreement with measurements:

    $M_Z = (91.1876 \pm 0.021)$ GeV from LEP1/SLD

    $M_W = (80.385 \pm 0.015)$ GeV from LEP2/Tevatron/LHC

  - Top quark mass:

    $m_t = (173.2 \pm 0.9)$ GeV from Tevatron/LHC

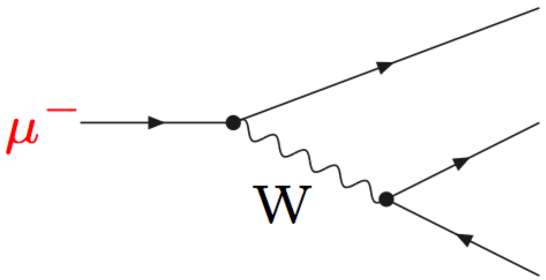
  - Higgs boson mass:

    $M_H = (125.9 \pm 0.4)$ GeV from LHC

  - ...

Observables and experiments

- Low energy observables
  - ν-nucleon (NuTeV) and νe (CERN) scattering:
    asymmetries CC/NC and $\nu/\bar{\nu}$ $\Rightarrow$ $\boxed{s_W^2}$
  - atomic parity violation (Ce, Tl, Pb):
    asymmetries $e_{R,L}N \to eX$ due to Z-exchange between e and nucleus $\Rightarrow$ $\boxed{s_W^2}$
  - muon decay (PSI):
    lifetime



$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f(m_e^2/m_\mu^2) \quad \Rightarrow \boxed{G_F}$$

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$\mathrm{i}\mathcal{M} = \left(\frac{\mathrm{i}e}{\sqrt{2}s_W}\right)^2 \bar{e}\gamma^\rho \nu_L \, \frac{-\mathrm{i}g_{\rho\delta}}{q^2 - M_W^2} \overline{\nu_L}\gamma^\delta \mu \equiv \overbrace{\mathrm{i}\frac{4G_F}{\sqrt{2}} \, (\bar{e}\gamma^\rho \nu_L)(\overline{\nu_L}\gamma_\rho \mu)}^{\text{Fermi theory } (-q^2 \ll M_W^2)} \; ; \; \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 s_W^2 M_W^2}$$

Observables and experiments

- Low energy observables

  ⇒ Fermi constant provides the Higgs VEV (electroweak scale):

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \approx 246\,\text{GeV}$$

  ⇒ Consistency checks: e.g.

  From muon lifetime:

$$G_F = 1.166\,378\,7(6) \times 10^{-5}\ \text{GeV}^{-2}$$

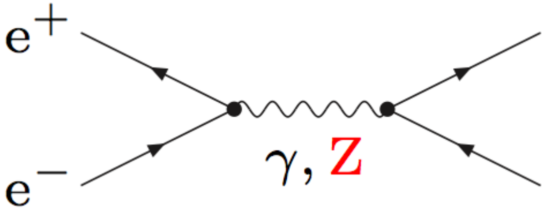
  If one compares with (tree level result)

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}$$

  using measurements of M_W, M_Z and α there is a discrepancy that disappears when quantum corrections are included

Observables and experiments

- $e^+e^- \to \bar{f}f$



$$\frac{d\sigma}{d\Omega} = N_c^f \frac{\alpha^2}{4s}\beta_f \Big\{ \left[1+\cos^2\theta + (1-\beta_f^2)\sin^2\theta\right] G_1(s) + 2(\beta_f^2-1)\; G_2(s) + 2\beta_f \cos\theta \; G_3(s) \Big\}$$

$$G_1(s) = Q_e^2 Q_f^2 + 2Q_e Q_f v_e v_f \mathrm{Re}\chi_Z(s) + (v_e^2+a_e^2)(v_f^2+a_f^2)|\chi_Z(s)|^2$$

$$G_2(s) = (v_e^2+a_e^2)a_f^2|\chi_Z(s)|^2$$

$$G_3(s) = 2Q_e Q_f a_e a_f \mathrm{Re}\chi_Z(s) + 4v_e v_f a_e a_f |\chi_Z(s)|^2$$

with $\chi_Z(s) \equiv \dfrac{s}{s-M_Z^2+\mathrm{i}M_Z\Gamma_Z}$, $N_c^f = 1$ (3) for f = lepton (quark), β_f = velocity

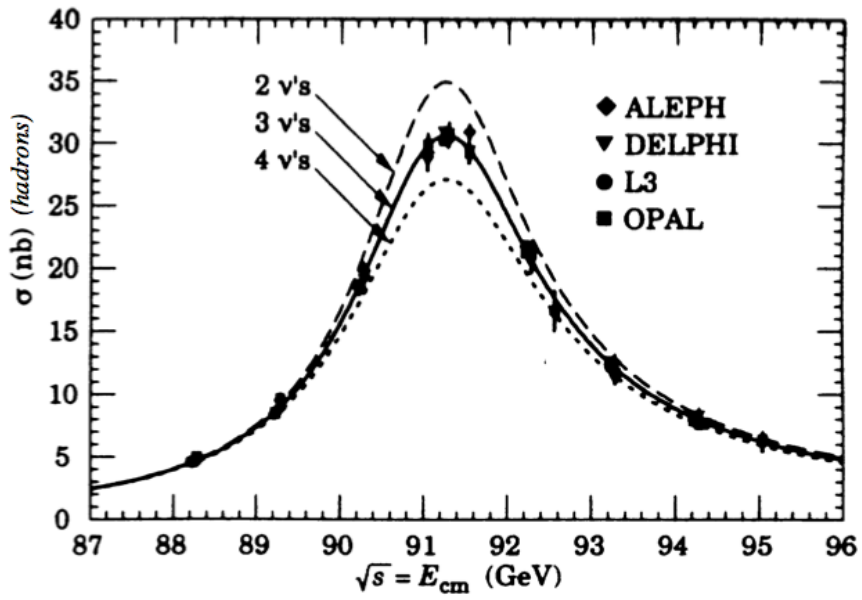
$$\sigma(s) = N_c^f \frac{2\pi\alpha^2}{3s}\beta_f \left[(3-\beta_f^2)G_1(s) - 3(1-\beta_f^2)G_2(s)\right] , \quad \beta_f = \sqrt{1-4m_f^2/s}$$

Observables and experiments

- Z production (LEP1/SLD)

  M_Z, Γ_Z, $\sigma_{\rm had}$, A_{FB}, A_{LR}, R_b, R_c, R_ℓ $\quad\Rightarrow\quad \boxed{M_Z,\, s_W^2}$

  from $e^+e^- \to \bar{f}f$ at the Z pole ($\gamma - Z$ interference vanishes). Neglecting m_f:



$$\sigma_{\rm had} = 12\pi \frac{\Gamma(e^+e^-)\Gamma({\rm had})}{M_Z^2\Gamma_Z^2}$$

$$R_b = \frac{\Gamma(b\bar{b})}{\Gamma({\rm had})} \quad R_c = \frac{\Gamma(c\bar{c})}{\Gamma({\rm had})} \quad R_\ell = \frac{\Gamma({\rm had})}{\Gamma(\ell^+\ell^-)}$$

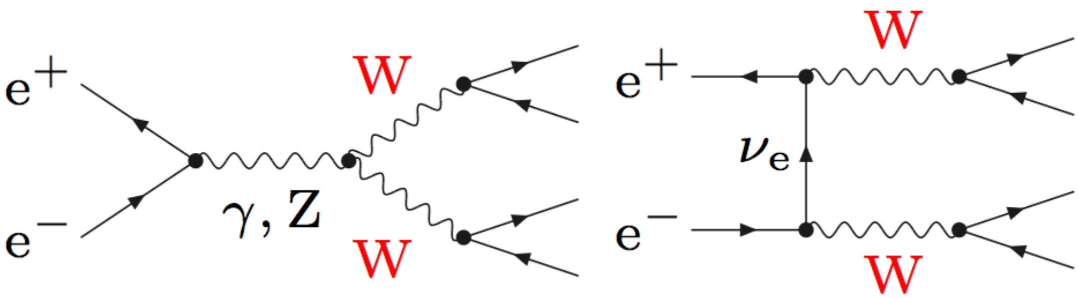
$$\left[\Gamma(Z \to f\bar{f}) \equiv \Gamma(f\bar{f}) = N_c^f \frac{\alpha M_Z}{3}(v_f^2 + a_f^2)\right]$$

Forward-Backward and (if polarized e^-) Left-Right asymmetries due to Z:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_f \frac{A_e + P_e}{1 + P_e A_e} \qquad A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e P_e \quad \text{with } A_f \equiv \frac{2 v_f a_f}{v_f^2 + a_f^2}$$

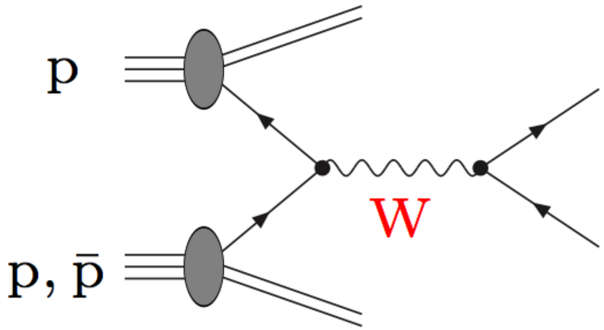
Observables and experiments

- W-pair production (LEP2)
  $e^+e^- \to WW \to 4\,f\ (+\gamma)$



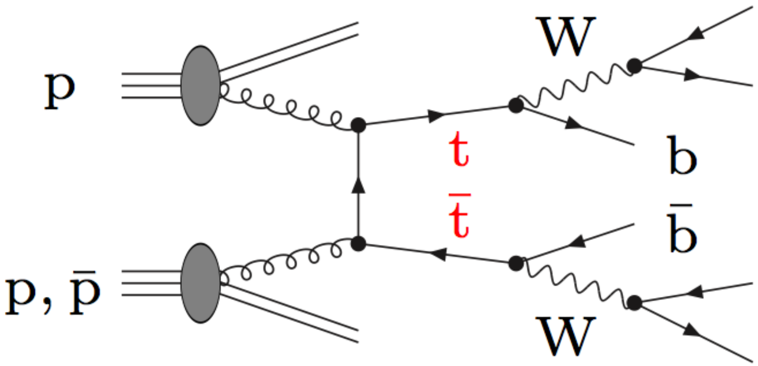
$\Rightarrow \boxed{M_W}$

- W production (Tevatron/LHC)
  $pp/p\bar{p} \to W \to \ell\nu_\ell\ (+\gamma)$



$\Rightarrow \boxed{M_W}$

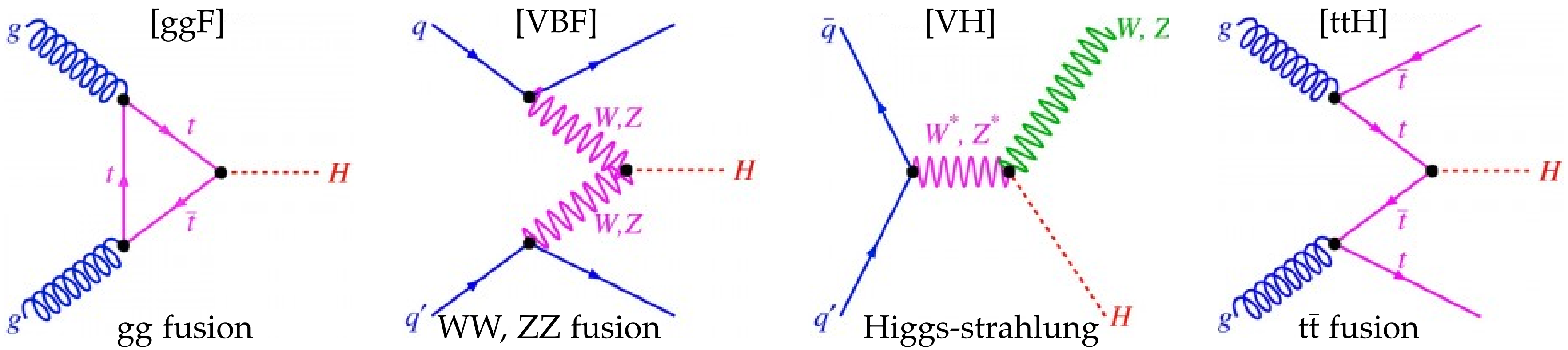
- Top-quark production (Tevatron/LHC)
  $pp/p\bar{p} \to t\bar{t} \to 6\,f$

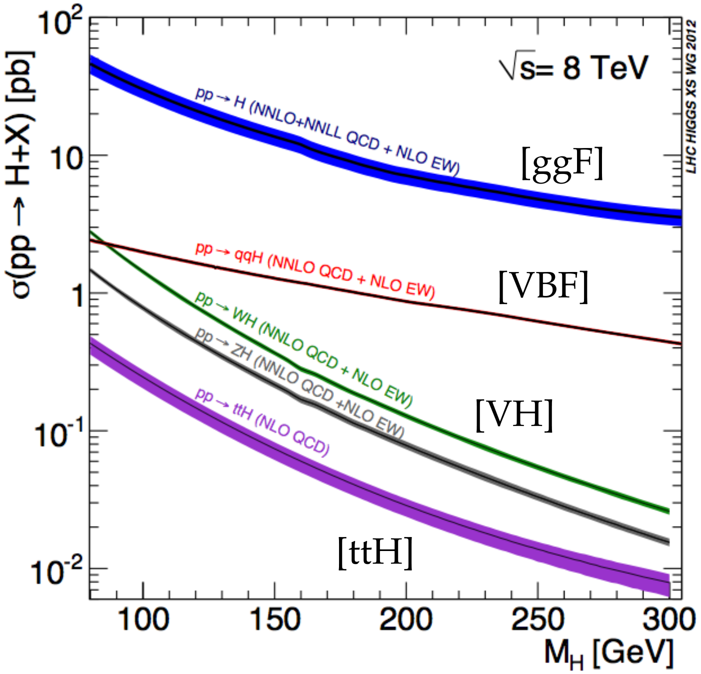


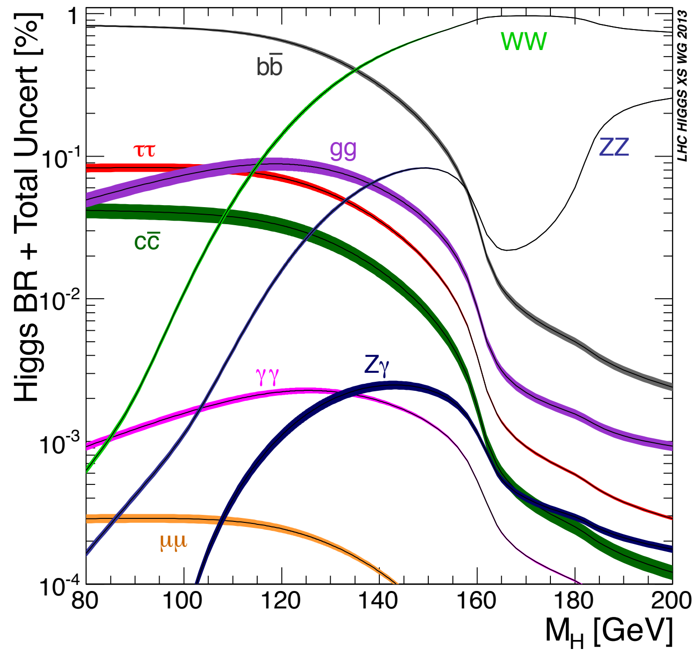
$\Rightarrow \boxed{m_t}$

Observables and experiments

- Higgs production (LHC)
  pp $\to H + X$ and H decays to different channels $\Rightarrow$ $\boxed{M_H}$







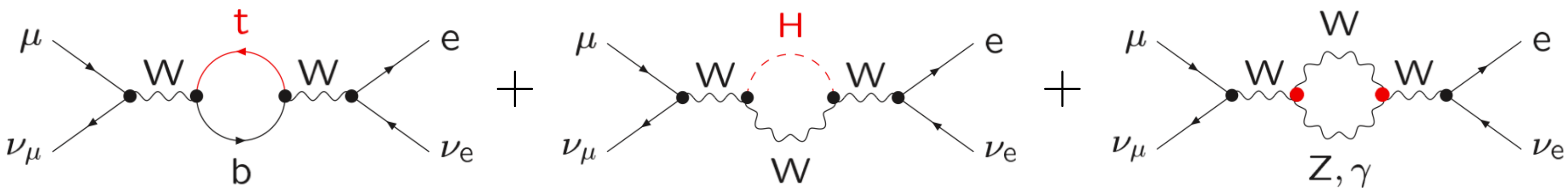
Precise determination of parameters

- Experimental precision requires accurate predictions $\Rightarrow$ quantum corrections (complication: loop calculations involve renormalization)

– Correction to G_F from muon lifetime:

$$\frac{G_F}{\sqrt{2}} \to \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}[1 + \Delta r(m_t, M_H)]$$

when loop corrections are included:



$+ \cdots + 2$ loops

Since muon lifetime is measured more precisely than M_W, it is traded for G_F:

$$\Rightarrow M_W^2(\alpha, G_F, m_t, M_H) = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}[1 + \Delta r(m_t, M_H)]}\right)$$

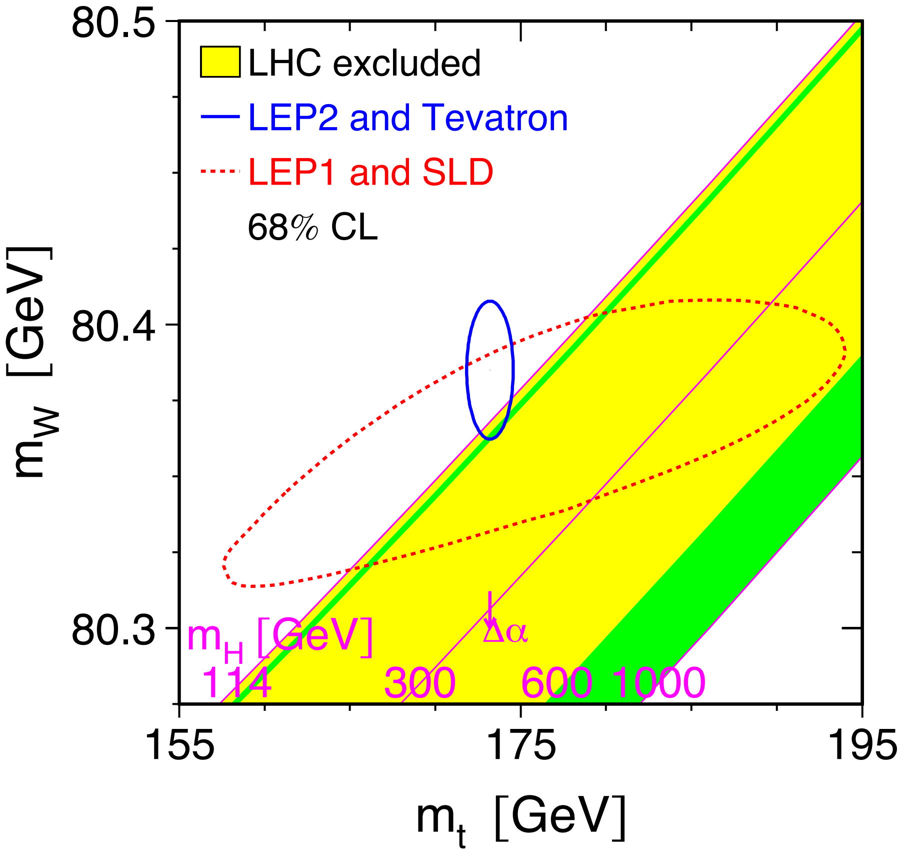
(correlation between M_W, m_t and M_H, given α and G_F)

Precise determination of parameters

Indirect constraints from LEP1/SLD Direct measurements from LEP2/Tevatron

$M_H(M_W, m_t)$ Allowed regions for M_H LHC excluded

[LEPEWWG 2013]



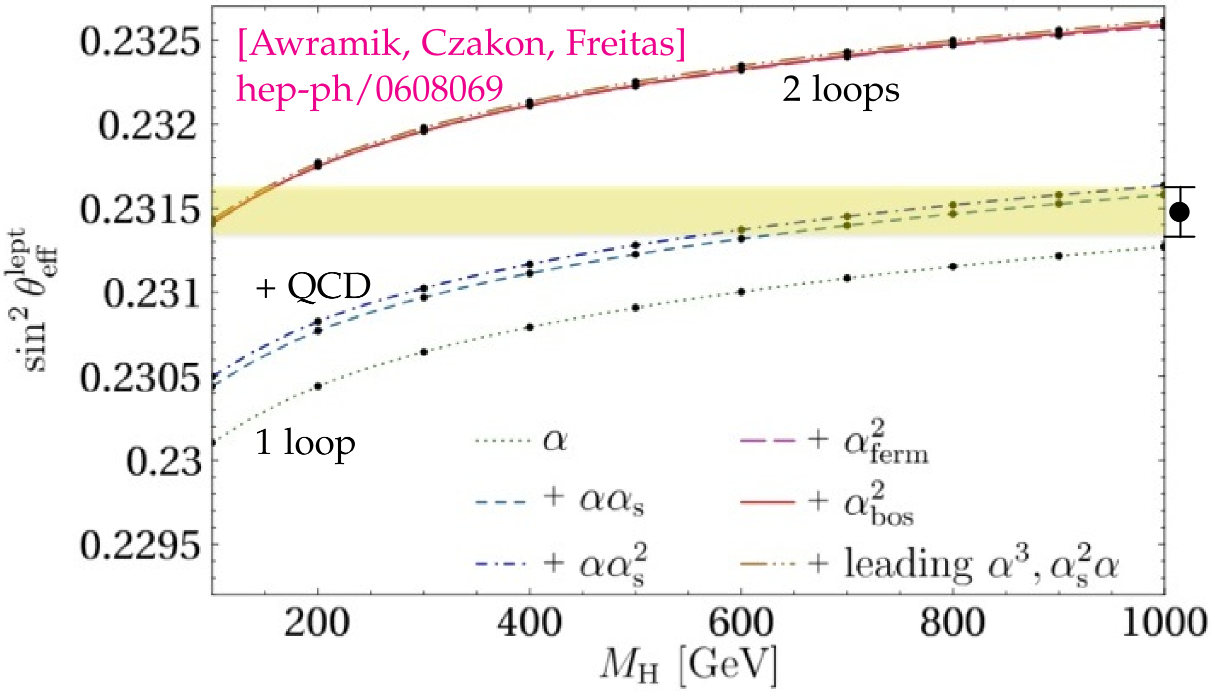
Precise determination of parameters

– Corrections to vector and axial couplings from Z pole observables:

$$v_f \to g_V^f = v_f + \Delta g_V^f \qquad a_f \to g_A^f = a_f + \Delta g_A^f$$

$$\Rightarrow \sin^2\theta_{\rm eff}^f = \frac{1}{4|Q_f|}\left[1 - {\rm Re}(g_V^f/g_A^f)\right] \equiv \overbrace{(1 - M_W^2/M_Z^2)}^{s_W^2}\ \kappa_Z^f$$

(Two) loop calculations are crucial and point to a light Higgs:



$$s_W^2 = 0.22290 \pm 0.00029 \text{ (tree)}$$

$$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23148 \pm 0.000017 \text{ (exp)}$$

Precise determination of parameters

- In addition, experiments and observables testing the flavor structure of the SM: flavor conserving: dipole moments, … flavor changing: $b \to s\gamma$, …

  $\Rightarrow$ very sensitive to new physics through loop corrections

  Extremely precise measurements are:

  – muon anomalous magnetic moment: $a_\mu = (g_\mu - 2)/2$

a_μ^{exp}	$= 116\,592\,089\,(63)$	$\times 10^{-11}$	[Brookhaven '06]
a_μ^{QED}	$= 116\,584\,718$	$\times 10^{-11}$	[QED: 5 loops]
a_μ^{EW}	$= 154$	$\times 10^{-11}$	[W, Z, H: 2 loops]
a_μ^{had}	$= 6\,930\,(48)$	$\times 10^{-11}$	[$e^+e^- \to$ had]
a_μ^{SM}	$= 116\,591\,802\,(49)$	$\times 10^{-11}$	

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287\,(80) \times 10^{-11}$$

3.6σ !

  – electron magnetic moment (new physics suppressed by a factor of m_e^2/m_μ^2):

$$\left.\begin{array}{ll} \text{exp:} & g_e/2 = 1.001\,159\,652\,180\,76\,(27) \\ \text{theo:} & \text{QED (8 loops!)} \end{array}\right\} \Rightarrow \alpha^{-1} = 137.035\,999\,074\,(44)$$

Global fits

- Fit input data from a list of observables (EWPO):

$$M_H,\ M_W,\ \Gamma_W,\ M_Z,\ \Gamma_Z,\ \sigma_{\text{had}},\ A_{FB}^{b,c,\ell},\ A_{b,c,\ell},\ R_{b,c,\ell},\ \sin^2\theta_{\text{eff}}^{\text{lept}},\ \ldots$$

finding the χ^2_{min} for $n_{\text{dof}} = 13\ (14)$ when M_H is included (excluded):

$$\underbrace{\alpha_s(M_Z^2)}_{1\ \text{(QCD)}},\ \underbrace{\Delta\alpha_{\text{had}}(M_Z^2),\ G_F,\ M_Z,\ 9\ \text{fermion masses},\ M_H}_{17-4=13\ \text{(CKM irrelevant)}}$$

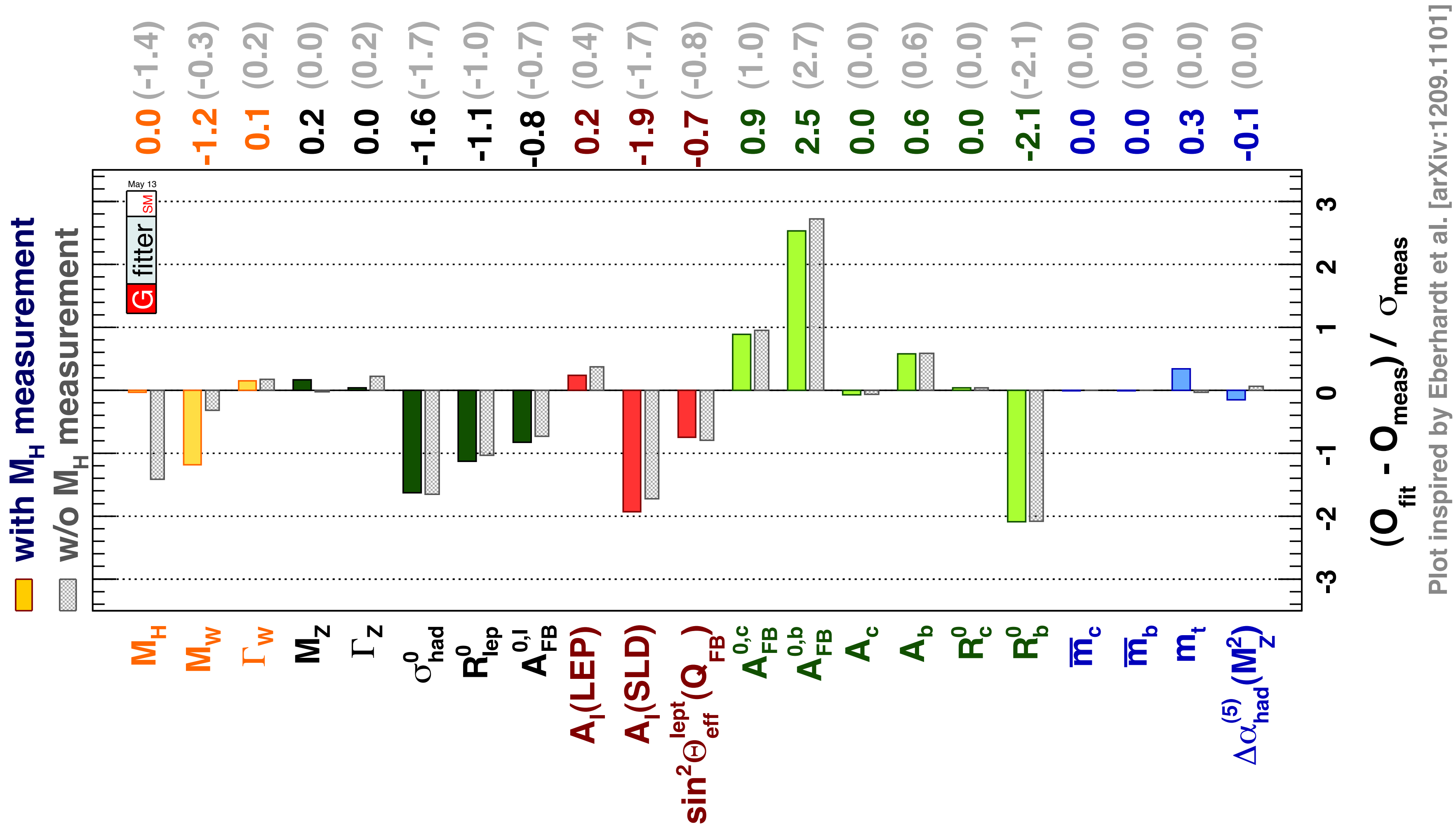
[Gfitter 2013] http://gfitter.desy.de

n_{dof}	χ^2_{min}	p-value
14	20.7	0.11
13	19.3	0.11

$\Rightarrow$ SM describes data to 1.6σ (about 90% CL)

Global fits

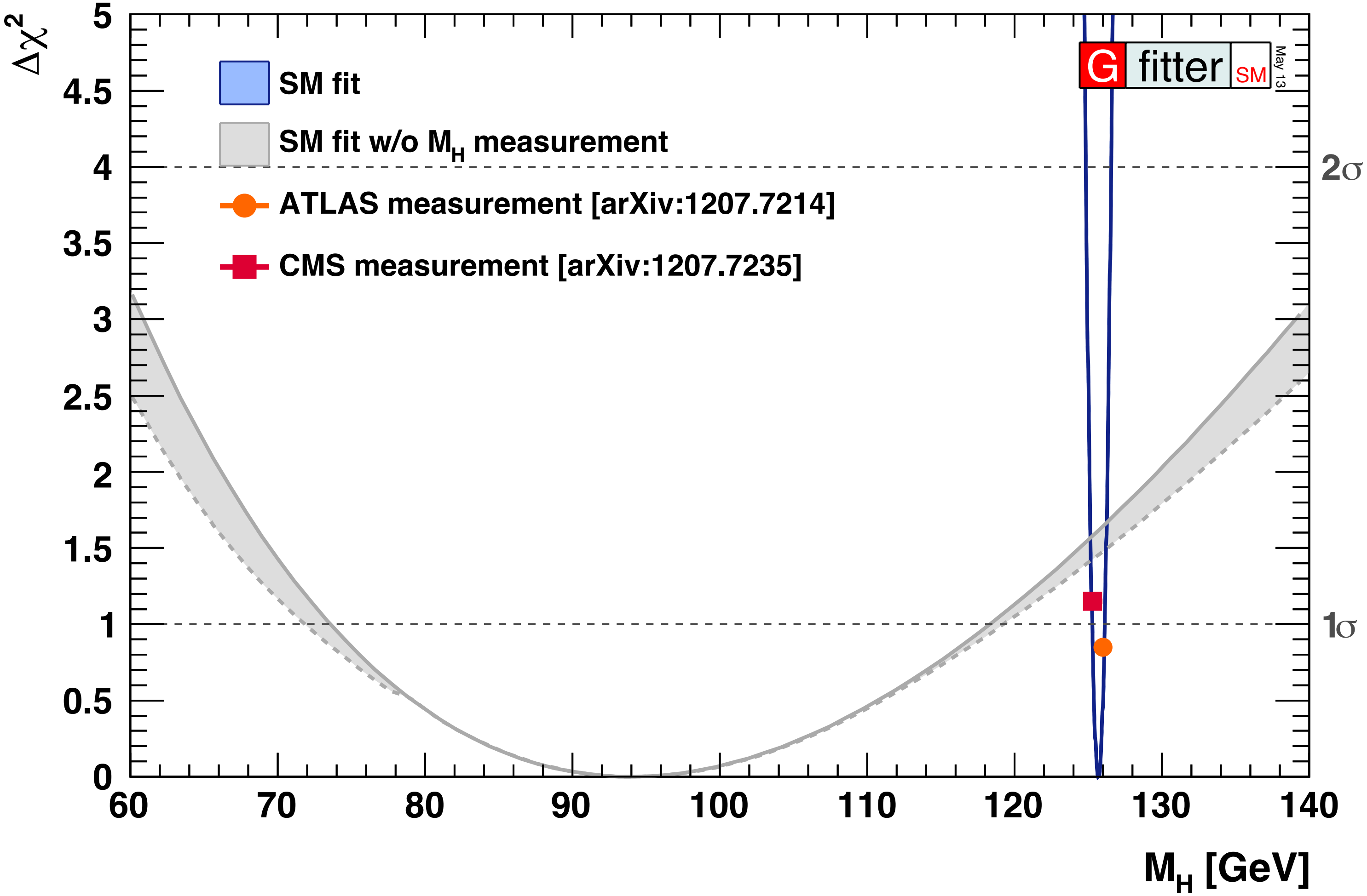
- Compare direct measurements of these observables with fit values:



$\Rightarrow$ some tensions (none above 3σ): A_ℓ(SLD), A^b_{FB}(LEP), R_b, ...

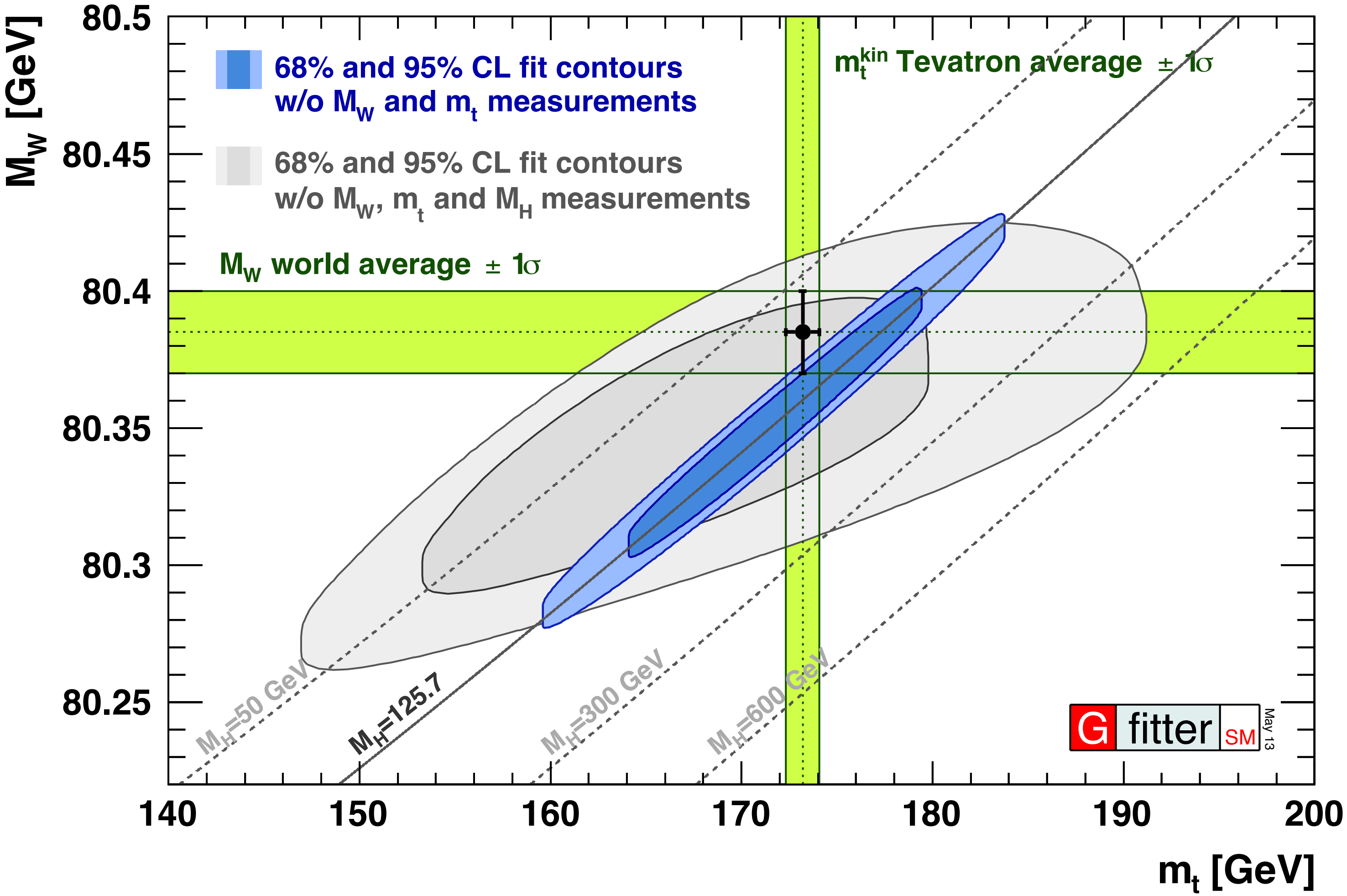
Global fits

⇒ Fits prefer a somewhat lighter Higgs:



Global fits

⇒ In general, impressive consistency of the SM, e.g.:



Summary

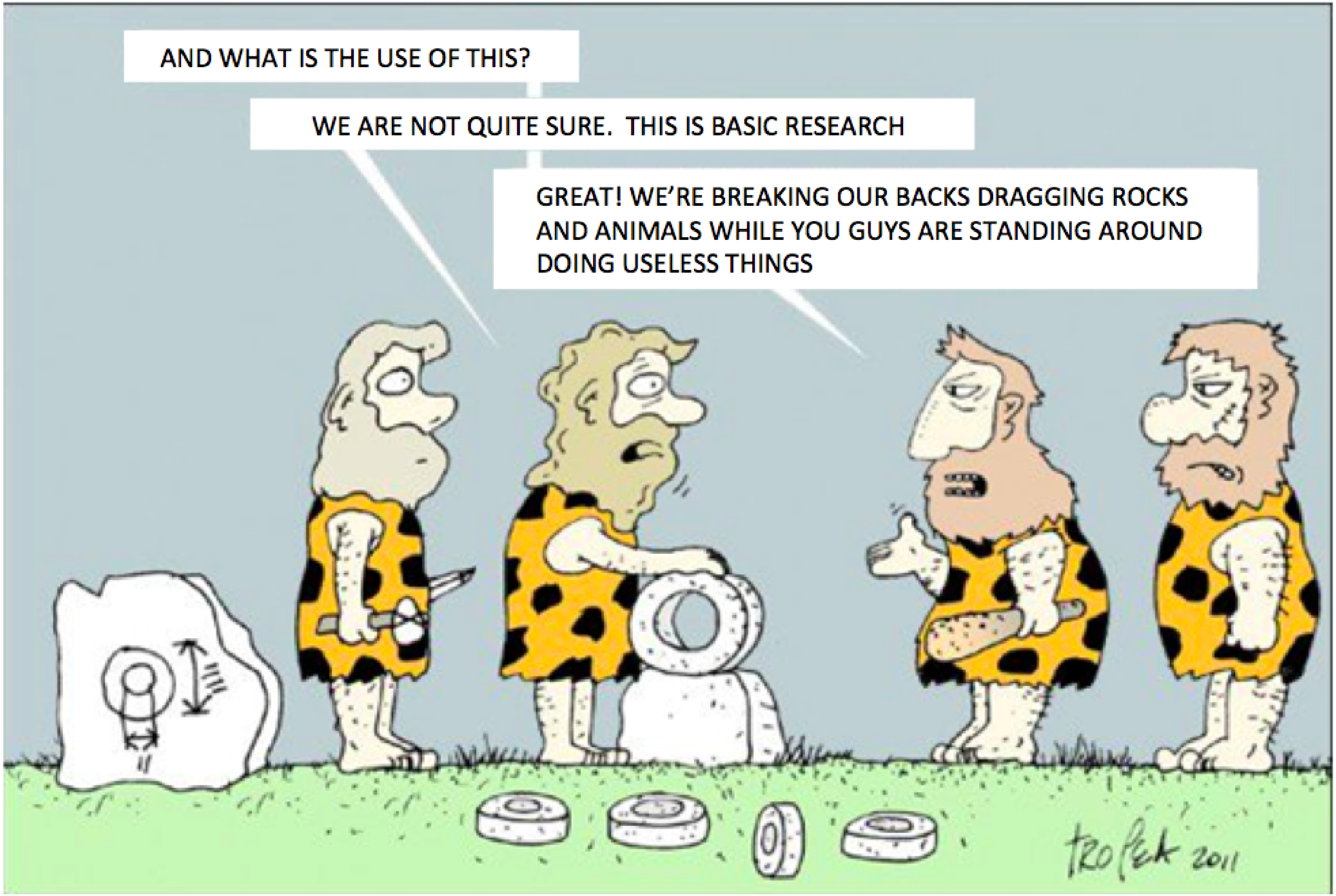
- The SM is a gauge theory with spontaneous symmetry breaking (renormalizable)
- Confirmed by many low and high energy experiments with remarkable accuracy, at the level of quantum corrections, with (almost) no significant deviations
- In spite of its tremendous success, it leaves fundamental questions unanswered:

  why 3 generations? why the observed pattern of fermion masses and mixings?

  and there are several hints for physics beyond:

  - phenomenological:
    - $(g_\mu - 2)$
    - neutrino masses
    - dark matter
    - baryogenesis
    - cosmological constant
  - conceptual:
    - gravity is not included
    - hierarchy problem

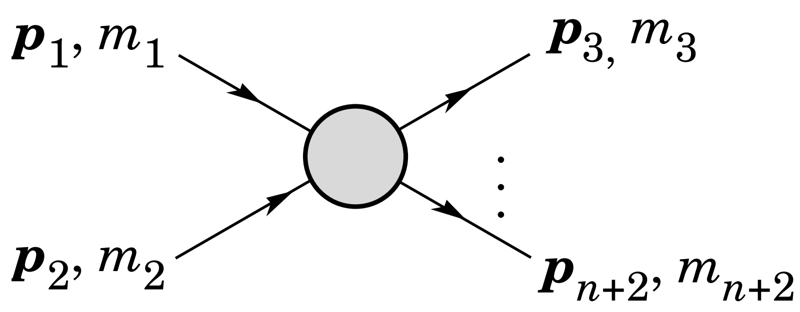
  $\Rightarrow$ The SM is an Effective Theory valid up to electroweak scale?


AND WHAT IS THE USE OF THIS?
WE ARE NOT QUITE SURE. THIS IS BASIC RESEARCH
GREAT! WE'RE BREAKING OUR BACKS DRAGGING ROCKS AND ANIMALS WHILE YOU GUYS ARE STANDING AROUND DOING USELESS THINGS
TROČEK 2011


¿Y PARA QUE SE PUEDE USAR ESTO?
NO SABEMOS, LO QUE HACEMOS ES INVESTIGACION BASICA
QUE BONITO, NOSOTROS NOS MATAMOS EMPUJANDO PIEDRAS Y ARRASTRANDO ANIMALES SALVAJES, MIENTRAS LOS SEÑORES SE ENTRETIENEN HACIENDO COSAS QUE NO SIRVEN PARA NADA
TROPEK 2011

Kinematics

Cross-section

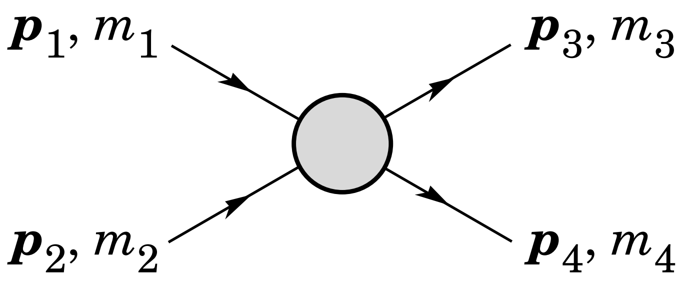


$$\mathrm{d}\sigma(i \to f) = \frac{1}{4\left\{(p_1 p_2)^2 - m_1^2 m_2^2\right\}^{1/2}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_i - p_f) \prod_{j=3}^{n+2} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}$$

- ▷ Sum over initial polarizations and/or average over final polarizations if the initial state is unpolarized and/or the final state polarization is not measured
- ▷ Divide the total cross-section by a symmetry factor $S = \prod_i k_i!$ if there are k_i identical particles of species i in the final state

Cross-section **case $2 \to 2$ in CM frame**

$$\boldsymbol{q} = \boldsymbol{p}_1 = -\boldsymbol{p}_2$$



$$\boldsymbol{p} = \boldsymbol{p}_3 = -\boldsymbol{p}_4$$

$$\Rightarrow \int \mathrm{d}\Phi_2 \equiv (2\pi)^4 \int \delta^4(p_1 + p_2 - p_3 - p_4) \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4} = \int \frac{|\boldsymbol{p}|\mathrm{d}\Omega}{16\pi^2 E_{\mathrm{CM}}}$$

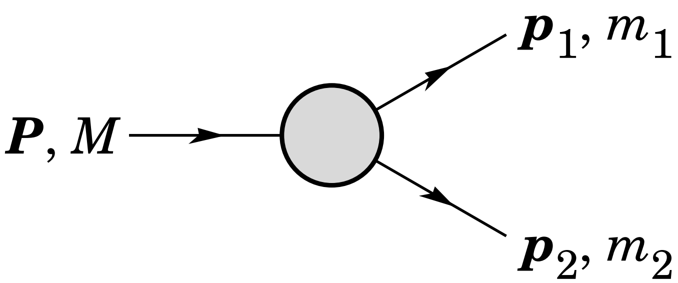
and if $m_1 = m_2 \quad \Rightarrow \quad 4\{(p_1 p_2)^2 - m_1^2 m_2^2\}^{1/2} = 4E_{\mathrm{CM}}|\boldsymbol{q}|$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(1,2 \to 3,4) = \frac{1}{64\pi^2 E_{\mathrm{CM}}^2} \frac{|\mathbf{p}|}{|\mathbf{q}|} |\mathcal{M}|^2}$$

Decay width

$$\mathrm{d}\Gamma(i \to f) = \frac{1}{2M}|\mathcal{M}|^2(2\pi)^4\delta^4(P - p_f)\prod_{j=1}^{n}\frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}$$

case $1 \to 2$



$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega}(i \to 1, 2) = \frac{1}{32\pi^2}\frac{|\boldsymbol{p}|}{M^2}|\mathcal{M}|^2$$

▷ Note that masses M, m_1 and m_2 fix final energies and momenta:

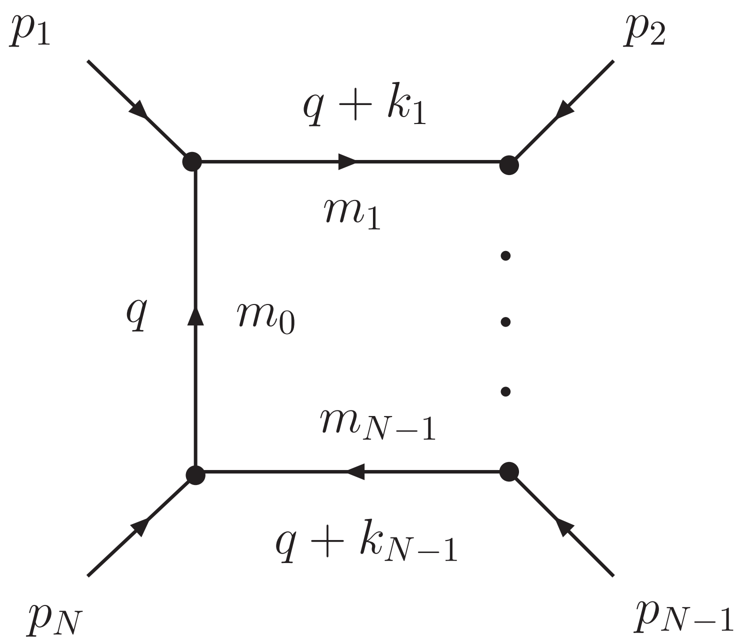
$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \qquad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

$$|\boldsymbol{p}| = |\boldsymbol{p}_1| = |\boldsymbol{p}_2| = \frac{\left\{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]\right\}^{1/2}}{2M}$$

Loop calculations

Structure of one-loop amplitudes

- Consider the following generic one-loop diagram with N external legs:



$$k_1 = p_1, \quad k_2 = p_1 + p_2, \quad \ldots \quad k_{N-1} = \sum_{i=1}^{N-1} p_i$$

- It contains general integrals of the kind:

$$\frac{\mathrm{i}}{16\pi^2} T^N_{\mu_1 \ldots \mu_P} \equiv \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q_{\mu_1} \cdots q_{\mu_P}}{[q^2 - m_0^2][(q + k_1)^2 - m_1^2] \cdots [(q + k_{N-1})^2 - m_{N-1}^2]}$$

Structure of one-loop amplitudes

- D dimensional integration in dimensional regularization
- Integrals are symmetric under permutations of Lorentz indices
- Scale μ introduced to keep the proper mass dimensions
- P is the number of q's in the numerator and determines the tensor structure of the integral (scalar if $P = 0$, vector if $P = 1$, etc.). Note that $P \leq N$
- Notation: A for T^1, B for T^2, etc. For example, the scalar integrals A_0, B_0, etc.
- The tensor integrals can be decomposed as a linear combination of the Lorentz covariant tensors that can be built with $g_{\mu\nu}$ and a set of linearly independent momenta [Passarino, Veltman '79]
- The choice of basis is not unique

  Here we use the basis formed by $g_{\mu\nu}$ and the momenta k_i, where the the tensor coefficients are totally symmetric in their indices [Denner '93]

  This the basis used by the computer package LoopTools [www.feynarts.de/looptools]

Structure of one-loop amplitudes

- We focus here on:

$$
\begin{aligned}
B_\mu &= k_{1\mu} B_1 \\
B_{\mu\nu} &= g_{\mu\nu} B_{00} + k_{1\mu} k_{1\nu} B_{11} \\
C_\mu &= k_{1\mu} C_1 + k_{2\mu} C_2 \\
C_{\mu\nu} &= g_{\mu\nu} C_{00} + \sum_{i,j=1}^{2} k_{i\mu} k_{j\nu} C_{ij} \\
C_{\mu\nu\rho} &= \ldots
\end{aligned}
$$

- We will see that the scalar integrals A_0 and B_0 and the tensor integral coefficients B_1, B_{00}, B_{11} and C_{00} are divergent in $D = 4$ dimensions (ultraviolet divergence, equivalent to take cutoff $\Lambda \to \infty$ in q)

- It is possible to express every tensor coefficient in terms of scalar integrals (scalar reduction) [Denner '93]

Explicit calculation

- Basic ingredients:

– Euler Gamma function:

$$\Gamma(x+1) \quad = \quad x\Gamma(x)$$

Taylor expansion around poles at $x = 0, -1, -2, \ldots$:

$$\begin{aligned} x = 0: \quad \Gamma(x) &= \frac{1}{x} - \gamma + \mathcal{O}(x) \\ x = -n: \quad \Gamma(x) &= \frac{(-1)^n}{n!(x+n)} - \gamma + 1 + \cdots + \frac{1}{n} + \mathcal{O}(x+n) \end{aligned}$$

where $\gamma \approx 0.5772\ldots$ is Euler-Mascheroni constant

– Feynman parameters:

$$\frac{1}{a_1 a_2 \cdots a_n} = \int_0^1 \mathrm{d}x_1 \cdots \mathrm{d}x_n \, \delta\left(\sum_{i=1}^{n} x_i - 1\right) \frac{(n-1)!}{[x_1 a_1 + x_2 a_2 + \cdots x_n a_n]^n}$$

Explicit calculation

– The following integrals, with $\epsilon \to 0^+$, will be needed:

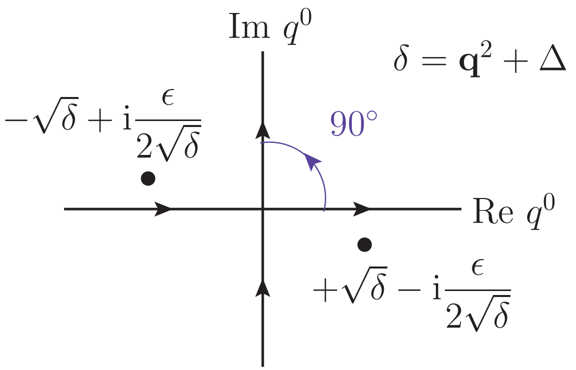
$$\int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - \Delta + i\epsilon)^n} = \frac{(-1)^n i}{(4\pi)^{D/2}} \frac{\Gamma(n - D/2)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-D/2}$$

$$\Rightarrow \int \frac{d^D q}{(2\pi)^D} \frac{q^2}{(q^2 - \Delta + i\epsilon)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{D/2}} \frac{D}{2} \frac{\Gamma(n - D/2 - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-D/2-1}$$

▷ Let's solve the first integral in Euclidean space: $q^0 = i q_E^0$, $\boldsymbol{q} = \boldsymbol{q}_E$, $q^2 = -q_E^2$,

$$\int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - \Delta + i\epsilon)^n} = i(-1)^n \int \frac{d^D q_E}{(2\pi)^D} \frac{1}{(q_E^2 + \Delta)^n}$$

(equivalent to a Wick rotation of 90°). The second integral follows from this



Explicit calculation

In D-dimensional spherical coordinates:

$$\int \frac{\mathrm{d}^D q_E}{(2\pi)^D} \frac{1}{(q_E^2+\Delta)^n} = \int \mathrm{d}\Omega_D \int_0^\infty \mathrm{d}q_E q_E^{D-1} \frac{1}{(q_E^2+\Delta)^n} \equiv \mathcal{I}_A \times \mathcal{I}_B$$

where
$$\mathcal{I}_A = \int \mathrm{d}\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$$

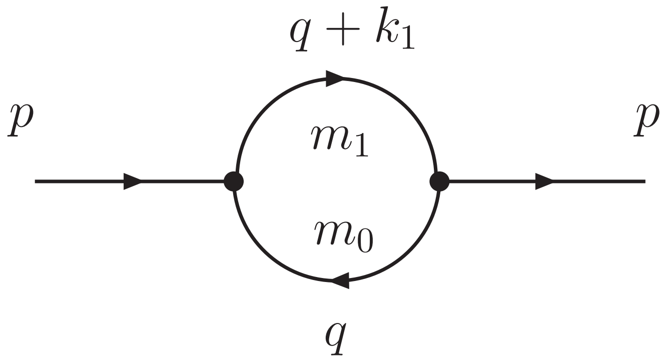
$$\text{since } (\sqrt{\pi})^D = \left(\int_{-\infty}^{\infty} \mathrm{d}x\, \mathrm{e}^{-x^2}\right)^D = \int \mathrm{d}^D x\, \mathrm{e}^{-\sum_{i=1}^D x_i^2} = \int \mathrm{d}\Omega_D \int_0^\infty \mathrm{d}x\, x^{D-1} \mathrm{e}^{-x^2}$$
$$= \left(\int \mathrm{d}\Omega_D\right) \frac{1}{2} \int_0^\infty \mathrm{d}t\, t^{D/2-1} \mathrm{e}^{-t} = \left(\int \mathrm{d}\Omega_D\right) \frac{1}{2}\Gamma(D/2)$$

and, changing variables: $t = q_E^2$, $z = \Delta/(t+\Delta)$, we have

$$\mathcal{I}_B = \frac{1}{2}\left(\frac{1}{\Delta}\right)^{n-D/2} \int_0^1 \partial z\, z^{n-D/2-1}(1-z)^{D/2-1} = \frac{1}{2}\left(\frac{1}{\Delta}\right)^{n-D/2} \frac{\Gamma(n-D/2)\Gamma(D/2)}{\Gamma(n)}$$

where Euler Beta function was used: $B(\alpha,\beta) = \int_0^1 \mathrm{d}z\, z^{\alpha-1}(1-z)^{\beta-1} = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Explicit calculation | Two-point functions



$$\frac{\mathrm{i}}{16\pi^2}\left\{B_0,\ B^\mu,\ B^{\mu\nu}\right\}(\text{args}) = \mu^{4-D}\int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{\left\{1,\ q^\mu,\ q^\mu q^\nu\right\}}{\left(q^2-m_0^2\right)\left[(q+p)^2-m_1^2\right]}$$

▷ $k_1 = p$

▷ The integrals depend on the masses m_0, m_1 and the invariant p^2:

$$(\text{args}) = (p^2; m_0^2, m_1^2)$$

Explicit calculation — Two-point functions

- Using Feynman parameters,

$$\frac{1}{a_1 a_2} = \int_0^1 \mathrm{d}x \frac{1}{[a_1 x + a_2(1-x)]^2}$$

$$\Rightarrow \frac{\mathrm{i}}{16\pi^2}\{B_0,\ B^\mu,\ B^{\mu\nu}\} = \mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1,\ -A^\mu,\ q^\mu q^\nu + A^\mu A^\nu\}}{(q^2 - \Delta_2)^2}$$

with

$$\Delta_2 = x^2 p^2 + x(m_1^2 - m_0^2 - p^2) + m_0^2$$

$$\begin{aligned} a_1 &= (q+p)^2 - m_1^2 \\ a_2 &= q^2 - m_0^2 \end{aligned}$$

and a loop momentum shift to obtain a perfect square in the denominator:

$$q^\mu \to q^\mu - A^\mu, \quad A^\mu = x p^\mu$$

- Then, the scalar function is:

$$\begin{aligned} \frac{\mathrm{i}}{16\pi^2}B_0 &= \mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{1}{(q^2-\Delta_2)^2} \\ \Rightarrow \quad B_0 &= \Delta_\epsilon - \int_0^1 \mathrm{d}x\ \ln\frac{\Delta_2}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D=4-\epsilon] \end{aligned}$$

where $\Delta_\epsilon \equiv \frac{2}{\epsilon} - \gamma + \ln 4\pi$ and the Euler Gamma function was expanded around $x=0$ for $D=4-\epsilon$, using $x^\epsilon = \exp\{\epsilon \ln x\} = 1 + \epsilon \ln x + \mathcal{O}(\epsilon^2)$:

$$\mu^{4-D}\frac{\mathrm{i}\Gamma(2-D/2)}{(4\pi)^{D/2}}\left(\frac{1}{\Delta_2}\right)^{2-D/2} = \frac{\mathrm{i}}{16\pi^2}\left(\Delta_\epsilon - \ln\frac{\Delta_2}{\mu^2}\right) + \mathcal{O}(\epsilon)$$

- Comparing with the definitions of the tensor coefficientes we have:

$$\begin{aligned} \frac{\mathrm{i}}{16\pi^2}B^\mu &= -\mu^{4-D}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{A^\mu}{(q^2-\Delta_2)^2} \\ \Rightarrow \quad B_1 &= -\frac{1}{2}\Delta_\epsilon + \int_0^1 \mathrm{d}x\ x\ln\frac{\Delta_2}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D=4-\epsilon] \end{aligned}$$

Explicit calculation | Two-point functions

and

$$\frac{\mathrm{i}}{16\pi^2} B^{\mu\nu} = \mu^{4-D} \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{(q^2/D) g^{\mu\nu} + A^\mu A^\nu}{(q^2 - \Delta_2)^2}$$

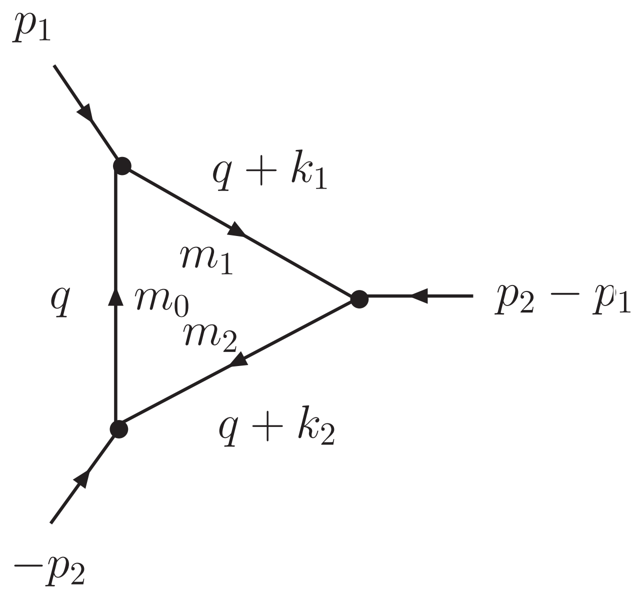
$$\Rightarrow \quad B_{00} = -\frac{1}{12}(p^2 - 3m_0^2 - 3m_1^2)(\Delta_\epsilon + 2\gamma - 1) + \mathcal{O}(\epsilon) \qquad [D = 4 - \epsilon]$$

$$B_{11} = \frac{1}{3}\Delta_\epsilon - \int_0^1 \mathrm{d}x\, x^2 \ln \frac{\Delta_2}{\mu^2} + \mathcal{O}(\epsilon) \qquad [D = 4 - \epsilon]$$

where $q^\mu q^\nu$ have been replaced by $(q^2/D) g^{\mu\nu}$ in the integrand and the Euler Gamma function was expanded around $x = -1$ for $D = 4 - \epsilon$:

$$-\mu^{4-D} \frac{\mathrm{i}\Gamma(1 - D/2)}{(4\pi)^{D/2} 2\Gamma(2)} \left(\frac{1}{\Delta_2}\right)^{1-D/2} = \frac{\mathrm{i}}{16\pi^2} \frac{1}{2} \Delta_2 (\Delta_\epsilon + 2\gamma - 1) + \mathcal{O}(\epsilon)$$

Explicit calculation | Three-point functions



$$\frac{\mathrm{i}}{16\pi^2}\{C_0,\ C^\mu,\ C^{\mu\nu}\}(\text{args}) = \mu^{4-D}\int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{\{1,\ q^\mu,\ q^\mu q^\nu\}}{(q^2 - m_0^2)\left[(q+p_1)^2 - m_1^2\right]\left[(q+p_2)^2 - m_2^2\right]}$$

▷ It is convenient to choose the external momenta so that:

$$k_1 = p_1, \quad k_2 = p_2.$$

▷ The integrals depend on the masses m_0, m_1, m_2 and the invariants:

$$(\text{args}) = (p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2), \quad Q^2 \equiv (p_2 - p_1)^2.$$

Explicit calculation | Three-point functions

- Using Feynman parameters,

$$\frac{1}{a_1 a_2 a_3} = 2 \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \frac{1}{[a_1 x + a_2 y + a_3(1-x-y)]^3}$$

$$\Rightarrow \frac{\mathrm{i}}{16\pi^2} \{C_0,\ C^\mu,\ C^{\mu\nu}\} = 2\mu^{4-D} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1,\ -A^\mu,\ q^\mu q^\nu + A^\mu A^\nu\}}{(q^2 - \Delta_3)^3}$$

with

$$\Delta_3 = x^2 p_1^2 + y^2 p_2^2 + xy(p_1^2 + p_2^2 - Q^2) + x(m_1^2 - m_0^2 - p_1^2) + y(m_2^2 - m_0^2 - p_2^2) + m_0^2$$

$$\begin{aligned} a_1 &= (q + p_1)^2 - m_1^2 \\ a_2 &= (q + p_2)^2 - m_2^2 \\ a_3 &= q^2 - m_0^2 \end{aligned}$$

and a loop momentum shift to obtain a perfect square in the denominator:

$$q^\mu \to q^\mu - A^\mu, \quad A^\mu = x p_1^\mu + y p_2^\mu$$

Explicit calculation — Three-point functions

- Then the scalar function is:

$$\frac{\mathrm{i}}{16\pi^2} C_0 = 2\mu^{4-D} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q^2 - \Delta_3)^3}$$

$$\Rightarrow \quad C_0 = -\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \, \frac{1}{\Delta_3} \qquad [D = 4]$$

- Comparing with the definitions of the tensor coefficientes we have:

$$\frac{\mathrm{i}}{16\pi^2} C^\mu = -2\mu^{4-D} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{A^\mu}{(q^2 - \Delta_3)^3}$$

$$\Rightarrow \quad C_1 = \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \, \frac{x}{\Delta_3} \qquad [D = 4]$$

$$C_2 = \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \, \frac{y}{\Delta_3} \qquad [D = 4]$$

Explicit calculation | **Three-point functions**

$$\begin{aligned}
\frac{\mathrm{i}}{16\pi^2}C^{\mu\nu} &= 2\mu^{4-D}\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\int\frac{\mathrm{d}^Dq}{(2\pi)^D}\frac{(q^2/D)g^{\mu\nu}+A^\mu A^\nu}{(q^2-\Delta_3)^3}\\
\Rightarrow\quad C_{11} &= -\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\,\frac{x^2}{\Delta_3} \qquad [D=4]\\
C_{22} &= -\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\,\frac{y^2}{\Delta_3} \qquad [D=4]\\
C_{12} &= -\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\,\frac{xy}{\Delta_3} \qquad [D=4]\\
C_{00} &= \frac{1}{4}\Delta_\epsilon-\frac{1}{2}\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\,\ln\frac{\Delta_3}{\mu^2}+\mathcal{O}(\epsilon) \qquad [D=4-\epsilon]
\end{aligned}$$

where $\Delta_\epsilon\equiv\frac{2}{\epsilon}-\gamma+\ln 4\pi$ and $q^\mu q^\nu$ was replaced by $(q^2/D)g^{\mu\nu}$ in the integrand

In C_{00} the Euler Gamma function was expanded around $x=0$ for $D=4-\epsilon$:

$$\mu^{4-D}\frac{\mathrm{i}\Gamma(2-D/2)}{(4\pi)^{D/2}\Gamma(3)}\left(\frac{1}{\Delta_3}\right)^{2-D/2}=\frac{\mathrm{i}}{16\pi^2}\frac{1}{2}\left(\Delta_\epsilon-\ln\frac{\Delta_3}{\mu^2}\right)+\mathcal{O}(\epsilon)$$

Note about Diracology in D dimensions

- Attention should be paid to the traces of Dirac matrices when working in D dimensions (dimensional regularization) since

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbf{1}_{4\times4}, \quad g^{\mu\nu}g_{\mu\nu} = \mathrm{Tr}\{g^{\mu\nu}\} = D$$

  Thus, the following identities involving contractions of Lorentz indices can be proven:

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= D \\
\gamma^\mu\gamma^\nu\gamma_\mu &= -(D-2)\gamma^\nu \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} - (4-D)\gamma^\nu\gamma^\rho \\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-D)\gamma^\nu\gamma^\rho\gamma^\sigma
\end{aligned}$$