Determination of Low-Energy Constants of Wilson* Chiral Perturbation Theory

(*) [K. G. Wilson, Phys. Rev. D10, 2445 (1974)]

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SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

with K. Cichy, K. Jansen, C. Michael, K. Ottnad, C. Urbach

ETM Collaboration

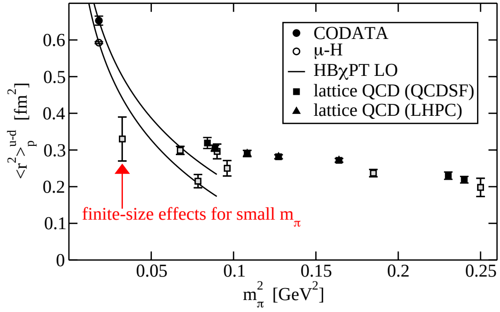
European Twisted Mass Collaboration

IV Workshop on Fermions and Extended Objects on the Lattice

Benasque, June 20, 2013

chiral extrapolation and FSE

Example : charge radius of the nucleon



$$\langle r^2 \rangle_{u-d} = -\frac{1}{6} \frac{d F_1^{u-d}}{d q^2}\Big|_{q^2=0}$$

- benchmark computation in lattice QCD
- further control of FSE, mass dependence, contamination from excited states, . . .

[D. Renner, QNP 2012]

continuum limit scaling

- fix the "physical situation" at a reference point:

  i.e. for every value of g_0, fix $(L\rho)|_{\rm ref}$, $(m_R^{(f)}/\rho)|_{\rm ref}$

- study the dependence of $R = \frac{O}{\rho}$ on the lattice spacing via $a\,\rho$

$$R_{\rm L} = R_{\rm cont} + \widetilde{\Lambda}^2\,(a\,\rho)^2 + \ldots$$

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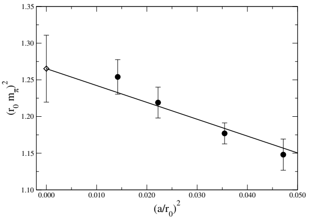
example :

- $N_{\mathrm{f}} = 2$ Wilson twisted mass sea quarks
  $m_\ell = m_u = m_d$
- tree level Symanzik (tlSym) improved gauge action
  $\beta = 3.80,\ 3.90,\ 4.05,\ 4.20$
- scaling variable : $\rho = r_0^{-1}$
- measurements of $aO = am_\pi$ and r_0/a
- reference point : $L\rho = L/r_0 \approx 4.5$
  $m_\ell^{\mathrm{R}}/\rho = m_\ell^{\mathrm{R}} r_0 \approx 0.11$



[ETMC, 1010.3659]

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$$O_{\rm L} = O_{\rm cont} + \Lambda^2\, a^2 + \dots$$

illustration :

- $O = m_\ell^{\rm R}$: $O_{\rm cont}^{\rm R} \approx 4\,$MeV

- What value of a is needed to have O(a^2) effects at 5% level?

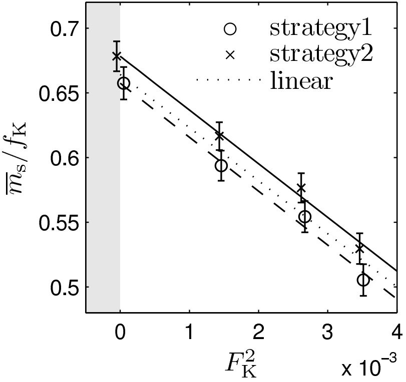
  for $\Lambda \sim 0.3\,$GeV $\rightsquigarrow$ $a \sim 0.02\,$fm ... ...topology freezing

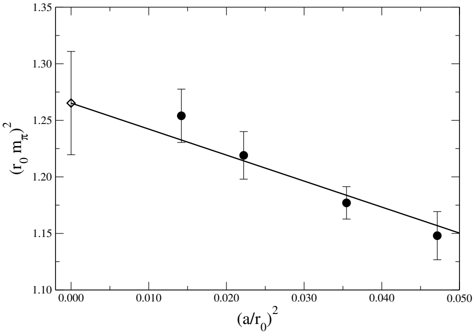
- Then, what level of cutoff effects are expected at $a \approx 0.075\,$fm?

  for $\Lambda \sim 0.3\,$GeV : $a^2\Lambda^3 \sim 4\,$MeV $\rightsquigarrow$ $\sim$ 100% O(a^2) effects

continuum limit scaling : $N_f = 2$

plaq. gauge action + Wilson NP c_{SW} | tlSym gauge action + Wilson twisted mass





M_K / f_K fixed ; $F_K = af_K$ | m_ℓ^R fixed

$a \approx 0.075$ fm : deviation from continuum limit $\sim 28\%$ | $\sim 8\%$

$R_L = R_{cont} + \Lambda^2 a^2$: $\Lambda \sim 1$ GeV | ~ 0.7 GeV

quark mass papers : [ALPHA, 1205.5380] | [ETMC, 1010.3659]

lattice actions

$$S = S_g + S_f$$

gauge action

$$S_{\rm g} = \frac{\beta}{3} \sum_x \left[(1 - 8\, b_1) \sum_{\mu<\nu}^{4} \left(1 - \mathrm{ReTr}\left(U^{1\times 1}_{x,\mu,\nu}\right)\right) + b_1 \sum_{\mu\neq\nu}^{4} \left(1 - \mathrm{ReTr}\left(U^{1\times 2}_{x,\mu,\nu}\right)\right) \right]$$

- Wilson plaquette: $b_1 = 0$
- tlSym: $b_1 = -1/12$ $(N_{\rm f} = 2)$
- Iwasaki: $b_1 = -0.33$ $(N_{\rm f} = 2+1+1)$

Wilson twisted-mass LQCD

- Lattice fermionic action for the light u, d quark doublet $N_\mathrm{f} = 2$

[ALPHA, Frezzotti, Grassi, Sint, Weisz, 1999]

$$S_F^{\mathrm{tmL}} = a^4 \sum_x \bar{\chi}(x) \Big[\gamma_\mu \tilde{\nabla}_\mu - r \frac{a}{2} \nabla^*_\mu \nabla_\mu + m_0 + i\gamma_5 \tau_3 \, \mu_\ell \Big] \chi(x)$$

axial rotation of the quark fields:

$$\psi \to \chi = \exp\Big[- i \frac{\omega}{2} \gamma_5 \tau_3 \Big] \psi \,, \qquad \bar{\psi} \to \bar{\chi}' = \bar{\psi} \exp\Big[- i \frac{\omega}{2} \gamma_5 \tau_3 \Big]$$

$$\text{twist angle} : \quad \tan(\omega) = \mu_\ell / (m_0 - m_{\mathrm{cr}}(r))$$

$$\text{quark mass} : \quad M_{\mathrm{R}} = \sqrt{\mu^2_{\ell,\mathrm{R}} + m^2_R}$$

- maximal twist: $\omega = \pi/2$
  - untwisted quark mass: $m_q = m_0 - m_{\mathrm{cr}} = 0$
  - twisted mass: $\mu_\ell = M_0$

$N_\mathrm{f} = 2 + 1 + 1$

- Wilson twisted-mass action at maximal twist
  - light mass degenerate $\bar{\psi}_\ell = (u, d)$ doublet : $N_\mathrm{f} = 2$

$$S^\ell_\mathrm{tm} = \bar{\psi}_\ell \left[\gamma_\mu \tilde{\nabla}_\mu - i\gamma_5\tau_3 \left(-r\frac{a}{2}\nabla^*_\mu\nabla_\mu + m_0 \right) + \mu_\ell \right] \psi_\ell$$

  - heavy mass non-degenerate $\bar{\psi}_h = (c, s)$ pair : $N_\mathrm{f} = 1 + 1$

$$S^h_\mathrm{tm} = \bar{\psi}_h \left[\gamma_\mu \tilde{\nabla}_\mu - i\gamma_5\tau_1 \left(-r\frac{a}{2}\nabla^*_\mu\nabla_\mu + m_0 \right) + \mu_\sigma + \mu_\delta\tau_3 \right] \psi_h$$

$N_f = 2 + 1 + 1$

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  - light mass degenerate $\bar{\psi}_\ell = (u, d)$ doublet : $N_f = 2$

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properties :

- automatic $O(a)$ improvement of physical observables at maximal twist [Frezzotti & Rossi, 2003]
- in the light-sector, μ_ℓ acts as an infrared cutoff

drawbacks :

- $O(a^2)$ breaking of parity and isospin : $m_{\pi^\pm}$ and m_{π^0}
- $O(a^2)$ contamination from mixing of different parity/flavour states : charm sector

$N_f = 2 + 1 + 1$

- Wilson twisted-mass action at maximal twist
  - light mass degenerate $\bar{\psi}_\ell = (u, d)$ doublet : $N_f = 2$

$$S^\ell_{\rm tm} = \bar{\psi}_\ell \left[\gamma_\mu \tilde{\nabla}_\mu - i\gamma_5 \tau_3 \left(-r \frac{a}{2} \nabla^*_\mu \nabla_\mu + m_0 \right) + \mu_\ell \right] \psi_\ell$$

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- renormalised quark masses :

$$\hat{m}_\ell = 1/Z_{\rm P}\ \mu_\ell$$

$$\hat{m}_s = 1/Z_{\rm P}\ (\ \mu_\sigma - Z_{\rm P}/Z_{\rm S}\ \mu_\delta\ )$$

$$\hat{m}_c = 1/Z_{\rm P}\ (\ \mu_\sigma + Z_{\rm P}/Z_{\rm S}\ \mu_\delta\ )$$

lattice actions

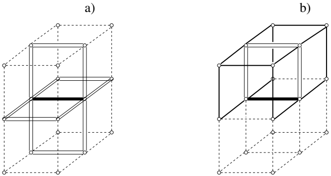
- Sheikholeslami-Wohlert term : c_{SW}
- smearing in the covariant derivative : reduce the short-distance roughness of gauge fields

- stout smearing
  [Morningstar & Peardon, hep-lat/0311018]

$$U'_\mu(x) = e^{iQ_\mu(x,\rho)}\, U_\mu(x)$$

  - $Q_\mu(x,\rho)$ built from staples
    traceless, Hermitian
  - differentiable $\rightsquigarrow$ HMC

- HEX smearing
- iterations : extends the coupling of fermions to gauge links over a larger region



[Hasenfratz & Knechtli, hep-lat/0103029]

mixed actions

- different lattice fermion actions in sea and valence
- eigenvalues and eigenvectors of $D_{\rm sea}$ and $D_{\rm val}$ differ
- unitarity is broken and recovered only in the continuum limit
- study unitarity violations :
  - continuum-limit scaling
  - χPT for mixed actions [Bär, Rupak, Shoresh, 2003]

motivation :

- profit from better properties of valence action (symmetries)
- many examples
  - Ginsparg-Wilson valence quarks
  - variants of same type of action in sea and valence:

    Osterwalder-Seiler valence quarks on twisted-mass sea ...

mixed action: OS valence quarks

- Osterwalder–Seiler (OS) valence quarks are the building blocks of twisted-mass valence quarks at maximal twist (Mtm)
- individual valence flavour χ_f

$$S_{\rm OS} \;=\; \bar{\chi}_f(x)\left[\gamma_\mu \tilde{\nabla}_\mu + \left(-\frac{a}{2}\nabla^*_\mu \nabla_\mu + m_{\rm cr}(r=1)\right) + i\mu_f\,\gamma_5\, r_f\right]\chi_f(x)$$

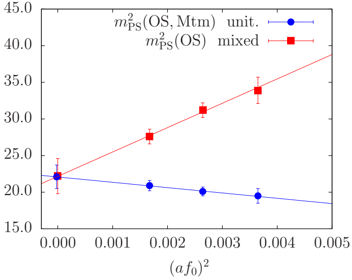
[Osterwalder & Seiler, 1978]

- Mtm corresponds to a pair of OS fermions with $+r_f$ and $-r_f$ [OS, Mtm]
- benefits :
  - $O(a)$ improvement with the same $\kappa_{\rm crit}$ as Mtm [Frezzotti & Rossi, 2004]
  - Mtm and OS fermions share the same renormalisation factors : matching is simplified
  - B_K : $O(a)$ improved and absence of mixing due to breaking of chiral symmetry

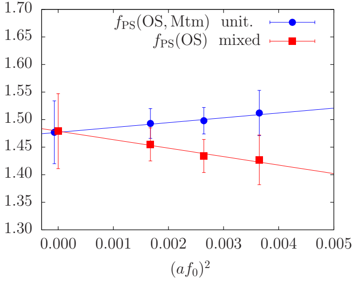
$N_f = 2$: mixed action OS valence quarks

continuum limit scaling

m_K^2



f_K



[ETMC, 2010]

phase structure of Wilson fermions

$$(a, m_q)$$

choice of the gauge action

- Wilson-type fermions have a non-trivial phase structure [Aoki; Sharpe, Singleton]
- The strength of the phase transition depends on details of the action
  - gluonic: b_1
  - fermionic: $c_{\rm sw}$, smearing
- Implications
  - For a given a, simulation is safe if $m_q \gg m_q^{\rm (end-point)} \sim a^2\Lambda^3$
  - Simulations at the physical point require sufficiently small a

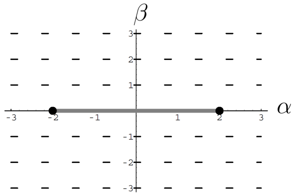
phase structure : Aoki phase $c_2 > 0$

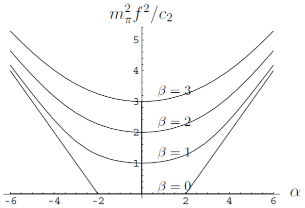
[S. Aoki, 1984]

$$c_2 = -\frac{32 W_0^2}{f^2}(2W'_6 + W'_8)$$

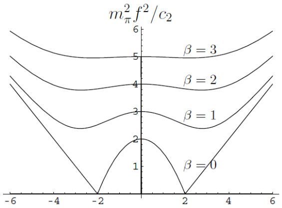
$$\alpha = \frac{(\hat{m}+\hat{a})\, f^2}{16\,|2W'_6 + W'_8|\hat{a}^2} \sim m'/a^2$$

$$\beta = \frac{\hat{\mu}\, f^2}{16\,|2W'_6 + W'_8|\hat{a}^2} \sim \mu/a^2$$





$M_{\pi^\pm}$



M_{π^0}

[Sharpe & Wu, 0407025]

phase structure : first-order scenario $c_2 < 0$

[Sharpe, Singleton, 1998]

$$c_2 = -\frac{32 W_0^2}{f^2}(2W_6' + W_8')$$

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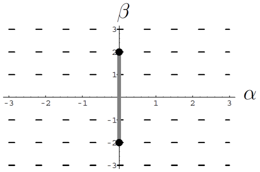
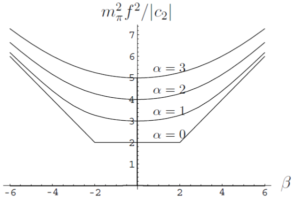
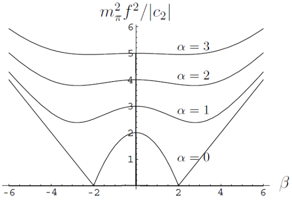




$M_{\pi^\pm}$



M_{π^0}

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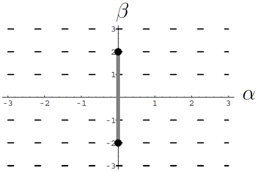
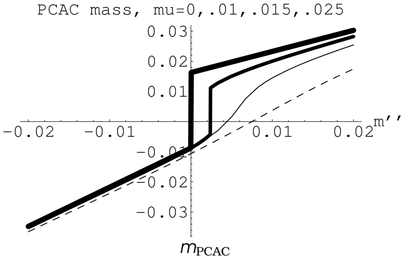
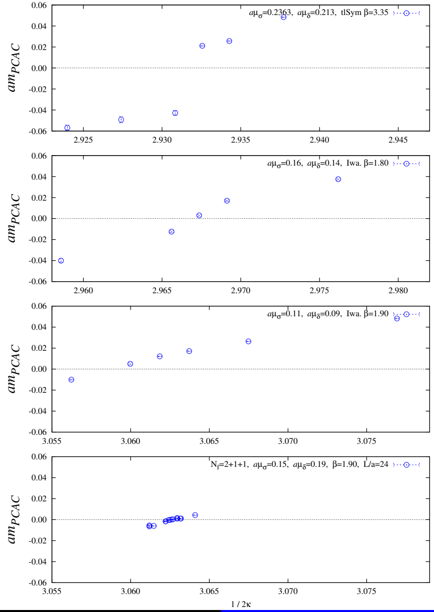




[Sharpe, 0509009]

choice of the gauge action : $am_{\rm PCAC}$ vs. $1/2\kappa$

$N_{\rm f} = 2+1+1$



Wilson χPT

Partially Quenched Wilson χPT (PQWχPT)

power counting : $m_0 \sim \mu_\ell \sim a^2\Lambda^3$ LO : $m_0\,,\mu_\ell\,,p^2\,,a^2$

- chiral Lagrangian

$$\mathcal{L}_\chi = \frac{f^2}{8}\,\mathrm{Str}\left(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right) - \frac{f^2\,B_0}{4}\,\mathrm{Str}\left(M^\dagger\Sigma+\Sigma^\dagger M\right)$$
$$-\,\hat{a}^2\,W'_6\,\left[\mathrm{Str}\left(\Sigma+\Sigma^\dagger\right)\right]^2 - \hat{a}^2\,W'_7\,\left[\mathrm{Str}\left(\Sigma-\Sigma^\dagger\right)\right]^2$$
$$-\,\hat{a}^2\,W'_8\,\mathrm{Str}\left(\Sigma^2+[\Sigma^\dagger]^2\right)$$

[Sharpe, Singleton, 1998; Sharpe & Wu; Münster; Scorzato, 2004]

- $M = m_0^{\rm R} + i\tau_3\mu_\ell^R$ $\hat{a} = 2W_0 a$
- Identify observables which depend on $W'_{6,8}$ ...

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- pseudoscalar meson masses at LO

$$\begin{aligned} M^2_{\pi^\pm} &= 2B_0\mu_\ell\,, \\ M^2_{\pi^0} &= 2B_0\mu_\ell - 8a^2\,(2w'_6 + w'_8)\,, \\ M^2_{\pi^{(0,c)}} &= 2B_0\mu_\ell - 8a^2\,w'_8 \end{aligned}$$

[maximal twist]

[Sharpe & Wu; Münster; Scorzato, 2004; Hansen & Sharpe, 2011]

$\hat{a} = 2W_0 a$

- c_2

$$M^2_{\pi^0} - M^2_{\pi^\pm} = -a^2\,\frac{128\,W_0^2}{f^2}\,(2W'_6 + W'_8) \;=\; 4c_2a^2$$

$$w'_k = \frac{16W_0^2\,W'_k}{f^2} \qquad (k=6,8)$$

contraints on Wilson LECs

- For any flavour non-singlet meson X : $|C_X^{(2)}| \leq |C_\pi^{(2)}| \rightsquigarrow M_X \geq M_\pi$

  [Weingarten, 1983]

  therefore $M_{\pi^{(0,c)}} \geq M_{\pi^\pm} \rightsquigarrow W'_8 < 0$

  [Hansen & Sharpe, 1111.2404]

- Consistent with γ_5-Hermiticity argument in ϵ-regime

  [P. Damgaard, K. Splittorff and J. Verbaarschot, 1001.2937]
  [G. Akemann, P. Damgaard, K. Splittorff and J. Verbaarschot, 1012.0752]

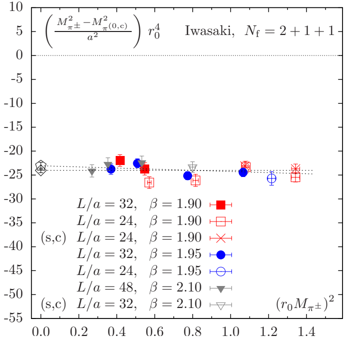
- $N_{\rm f} = 0 \rightsquigarrow$ Aoki phase

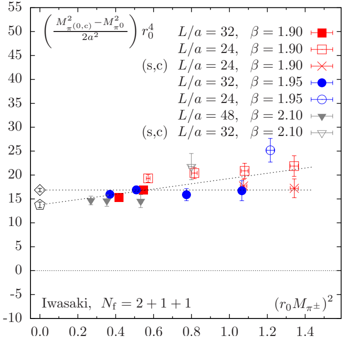
Wilson LECs

$$W'_{6,8} \;\; \& \;\; c_2$$

$M_{\rm PS}$: $W'_{6,8}$

- lattice action : Wtm $N_f = 2+1+1$ + Iwasaki gauge action





[G.H., K. Jansen, C. Michael, K. Ottnad, C. Urbach, 1303.3516]

$$M^2_{\pi^\pm} - M^2_{\pi^{(0,c)}} = 8a^2\, w'_8 \,; \qquad \frac{1}{2}\left(M^2_{\pi^{(0,c)}} - M^2_{\pi^0}\right) = 8a^2\, w'_6$$

signs of $W'_{6,8}$: consistent with

[P. Damgaard, K. Splittorff and J. Verbaarschot, 1001.2937]
[G. Akemann, P. Damgaard, K. Splittorff and J. Verbaarschot, 1012.0752]
[M. Hansen and S. Sharpe, 1111.2404]
[M. Kieburg, K. Splittorff and J. Verbaarschot, 1202.0620]

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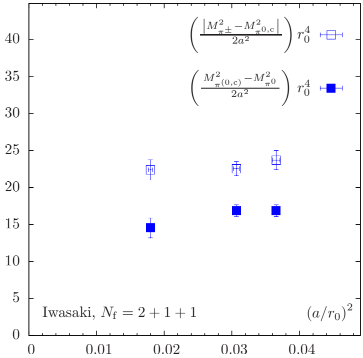

$(M_{\pi^\pm} r_0)^2 \approx 0.55$

[G.H., K. Jansen, C. Michael, K. Ottnad, C. Urbach, 1303.3516]

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WχPT LECs : c_2

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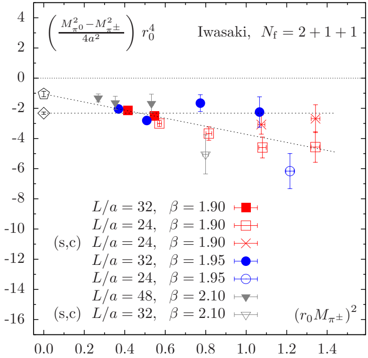

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$$M^2_{\pi^0} - M^2_{\pi^\pm} = -a^2 \frac{128\, W_0^2}{f^2} (2W_6' + W_8') = 4 c_2 a^2$$

data is consistent with $c_2 < 0$

[Sharpe, Singleton, 1998]

WχPT LECs : $W'_{6,8}$, c_2

lattice action : Wtm $N_f = 2+1+1$ + Iwasaki gauge action

	$w'_8\, r_0^4$	w'_8	$W'_8\, (r_0^6 W_0^2)$
syst.	-2.9(4)	$-[571(32)\ \mathrm{MeV}]^4$	-0.0138(22)
	$w'_6\, r_0^4$	w'_6	$W'_6\, (r_0^6 W_0^2)$
syst.	+1.7(7)	$+[502(58)\ \mathrm{MeV}]^4$	+0.0082(34)
	$c_2\, r_0^4$	c_2	$-2\,(2W'_6 + W'_8)\,(r_0^6 W_0^2)$
lin.	-1.1(2)	$-[444(28)\ \mathrm{MeV}]^4$	-0.0050(10)
cst.	-2.3(1)	$-[541(24)\ \mathrm{MeV}]^4$	-0.0111(10)

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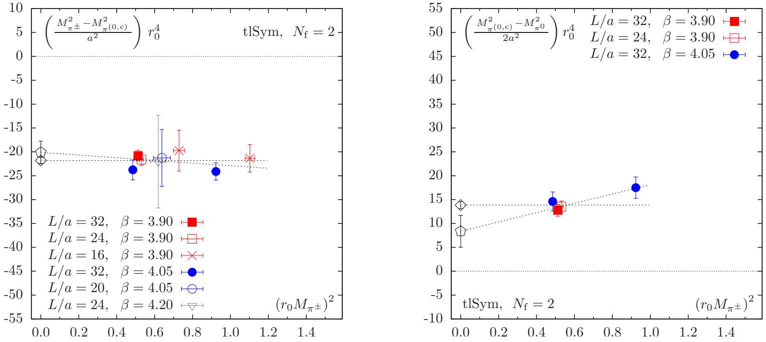


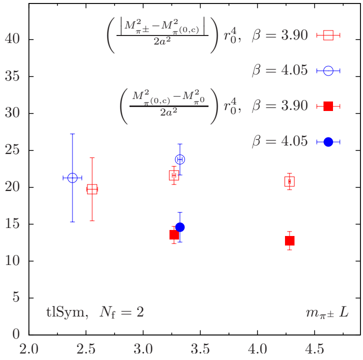
[G.H., K. Jansen, C. Michael, K. Ottnad, C. Urbach, 1303.3516]

$$M^2_{\pi^\pm} - M^2_{\pi(0,c)} = 8a^2\, w'_8\,;$$

$$\frac{1}{2}\left(M^2_{\pi(0,c)} - M^2_{\pi^0}\right) = 8a^2\, w'_6$$

$M_{\rm PS}$: $W'_{6,8}$

- lattice action : Wtm $N_f = 2$ + tlSym gauge action



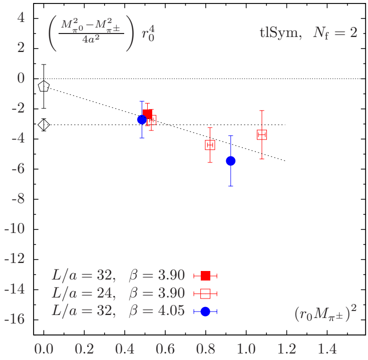
$(M_{\pi^\pm} r_0)^2 \approx 0.55$

[G.H., K. Jansen, C. Michael, K. Ottnad, C. Urbach, 1303.3516]

$$M^2_{\pi^\pm} - M^2_{\pi(0,c)} = 8a^2\, w'_8 \; ; \qquad \frac{1}{2}\left(M^2_{\pi(0,c)} - M^2_{\pi^0}\right) = 8a^2\, w'_6$$

WχPT LECs : c_2

- lattice action : Wtm $N_f = 2$ + tlSym gauge action



[G.H., K. Jansen, C. Michael, K. Ottnad, C. Urbach, 1303.3516]

$$M^2_{\pi^0} - M^2_{\pi^\pm} = -a^2 \, \frac{128 \, W_0^2}{f^2} \, (2W'_6 + W'_8) \; = \; 4c_2 a^2$$

data is consistent with $c_2 < 0$ ... but mass dependence is difficult to address

WχPT LECs : $W'_{6,8}$, c_2

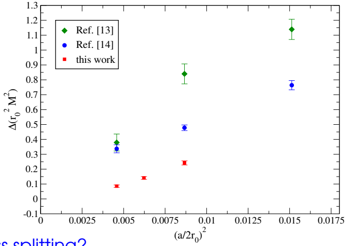
lattice action : Wtm $N_{\rm f} = 2$ + tlSym gauge action

$w'_8\ r_0^4$	w'_8	$W'_8\ (r_0^6 W_0^2)$
-2.5(4)	$-[552(025)\ \mathrm{MeV}]^4$	-0.0119(17)
$w'_6\ r_0^4$	w'_6	$W'_6\ (r_0^6 W_0^2)$
+1.0(8)	$+[443(138)\ \mathrm{MeV}]^4$	+0.0049(38)

Wtm with clover term : $N_f = 0$

- [ALPHA, P. Dimopoulos, H. Simma, A. Vladikas, 0902.1074]

  observed for $N_f = 0$



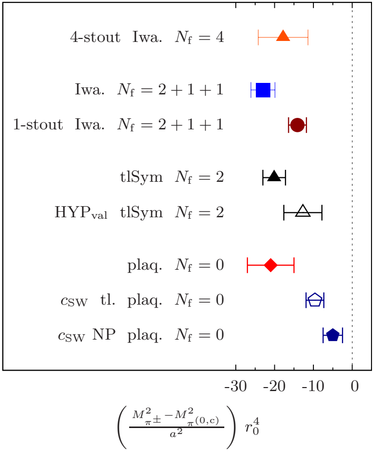
- tune c_{SW} to minimize the (connected) mass splitting?

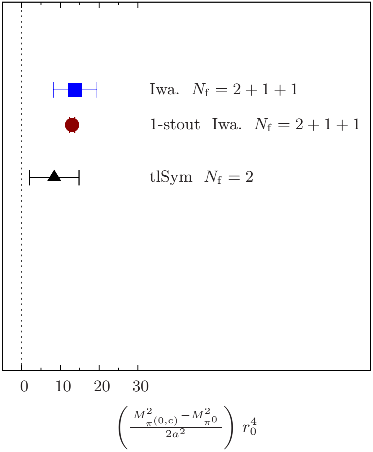
- $N_f = 0 \quad \beta = 6.0\,,\ r_0/a \approx 5.4\,,\ a\mu_q = 0.0135$

$$(M^2_{\pi^\pm} - M^2_{\pi^{(0,c)}})\, r_0^2 \;=\; \Lambda_c\, (a/r_0)^2$$

c_{SW}	κ	Λ_c	ref.
0	0.157409	−24	[χ_LF , 2005]
1	0.145550	−11	[P. Dimopoulos, G.H.]
1.769	0.135196	−7	[ALPHA, 2009]

WχPT LECs : $W'_{8,6}$





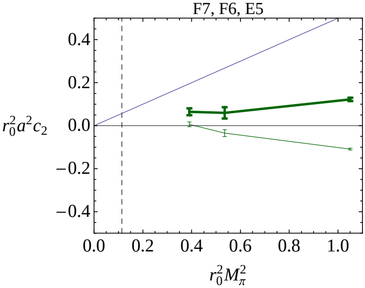
mass-splittings related to W'_8 (left) and W'_6 (right)

[G.H., K. Jansen, C. Michael, K. Ottnad, C. Urbach, 1303.3516]

WχPT LECs : c_2

- lattice action : $N_f = 2$ Wilson NP O(a) improved + Wilson plaquette gauge action
- S-wave π-π scattering length, $I = 2$

$$M_\pi a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 + \frac{3M_\pi^2 + 12c_2a^2}{32\pi^2F_\pi^2}\ln\frac{M_\pi^2}{\tilde{\mu}_2^2} + O(a^2, m)\right]$$
$$-\frac{2c_2a^2}{16\pi F_\pi^2}\left[1 + \frac{11c_2a^2 - 2M_\pi^2}{16\pi^2F_\pi^2}\ln\frac{M_\pi^2}{\tilde{\mu}_3^2} + O(a^2, m)\right]$$



[ALPHA, Bernardoni, Bulava, Sommer, 1111.4351]

$a = 0.065\,\text{fm}$: $r_0^2\, a^2\, c_2 \approx 0.05$ $\rightsquigarrow$ $c_2 \approx [520\,\text{MeV}]^4$

extension to PQ case [Hansen & Sharpe, 1112.3998] overlap valence quarks on

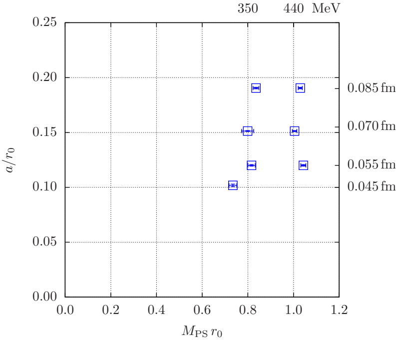
$N_\mathrm{f} = 2$ Wilson twisted mass

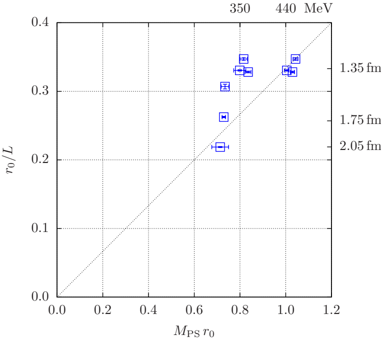
$N_f = 2$ ensembles

ETMC ensembles

- tree-level Symanzik improved gauge action
- $a = \{0.045,\ 0.055,\ 0.070,\ 0.085\}$ fm
  $\beta = \{4.35,\ 4.20,\ 4.05,\ 3.90\}$
- $M_{\rm PS} = \{350,\ 440\}$ MeV

- Wilson twisted-mass at maximal twist
  [ALPHA, Frezzotti et al.,2001; Frezzotti & Rossi,2003]
- $L = \{1.35,\ 1.75,\ 2.05\}$ fm





Neuberger overlap valence fermions

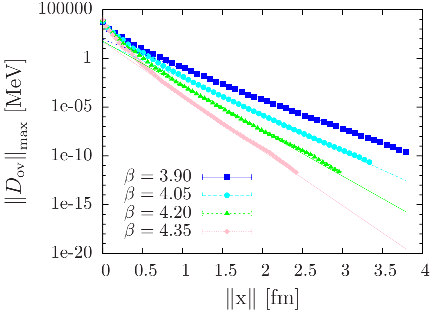
- Massive Neuberger-Dirac Operator

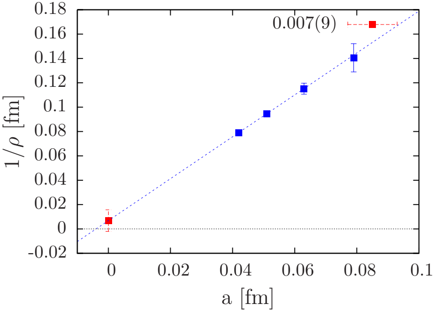
$$D_{\rm ov} = \frac{1}{a}\left(1 - \frac{am_q}{2}\right)\left(1 - A(A^\dagger A)^{-1/2}\right) + m_q$$

$$A = (1 + s) - aD_{\rm W}, \qquad |s| < 1$$

- Ginsparg-Wilson relation
- $\mathcal{O}(a)$ improved
- exact chiral zero-modes

- HYP-smearing in A
- locality : $||D_{\rm ov}|| \propto e^{-\rho||x||}$





$a = \{0.045, 0.055, 0.070, 0.085\}$ fm

[K. Cichy, V. Drach, E. García Ramos, G.H., K. Jansen, 1211.1605]

determination of LECs of MAχPT

W_M $W'_{6,8}$

mixed action PQ χPT (MAPQχPT)

power counting : $m_q \sim \mu_q \sim a^2$ LO : $m_q\,,\mu_q\,,p^2\,,a^2$

- mixed action : overlap/Wtm
- $\mathcal{O}(a^2)$ contribution to the chiral Lagrangian

$$\mathcal{L}\left[a^2\right] = -\hat{a}^2\,W'_6\langle P_S\Sigma^\dagger + \Sigma P_S\rangle^2 - \hat{a}^2\,W'_7\langle P_S\Sigma^\dagger - \Sigma P_S\rangle^2 - \hat{a}^2\,W'_8\langle P_S\Sigma^\dagger P_S\Sigma^\dagger + \Sigma P_S\Sigma P_S\rangle - \hat{a}^2\,W_M\left\langle P_S\Sigma P_s\Sigma^\dagger\right\rangle$$

[Sharpe, Singleton, 1998; Bär, Rupak, Shoresh, 2003; Sharpe & Wu; Münster; Scorzato, 2004]

- W_M is the extra LEC at $\mathcal{O}(a^2)$ for Ginsparg-Wilson valence quarks
- $\chi_{\rm sea} = 2B_0(m_{0,\rm sea} + i\tau_3\mu_q)$

$$\hat{a} = 2W_0 a$$
$$m_s \equiv \mu_q$$
$$m_v \equiv m_q$$

- Identify observables which depend on $W'_{6,8}$, W_M ...

mixed action PQ χPT (MAPQχPT)

power counting : $m_s \sim m_v \sim a^2$

LO : m_s , m_v , p^2 , a^2

- pseudoscalar meson masses at LO

$$M_{\pm}^2 = 2B_0 m_s \qquad \text{[maximal twist]}$$

$$M_0^2 = 2B_0 m_s - \hat{a}^2 \frac{32}{f^2} (2W'_6 + W'_8)$$

$$M_{\rm VV}^2 = 2B_0 m_v$$

$$M_{\rm VS}^2 = B_0(m_v + m_s) - \hat{a}^2 \frac{4}{f^2} (W_M - 2W'_8) = B_0(m_v + m_s) + a^2 \Delta\text{mix}$$

[Sharpe & Wu; Münster; Scorzato, 2004; Bär & Furchner, 2010; Ueda & Aoki, 2011]

$$\hat{a} = 2W_0 a$$

- $\Delta_{\rm mix}$

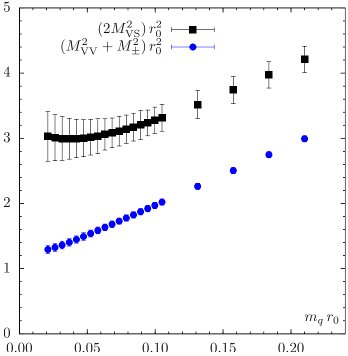
$$M_{\rm VS}^2 - \frac{1}{2}(M_{\rm VV}^2 + M_{\pm}^2) = \Delta_{\rm mix} a^2 = a^2 \frac{16\, W_0^2}{f^2} (W_M - 2W'_8)$$

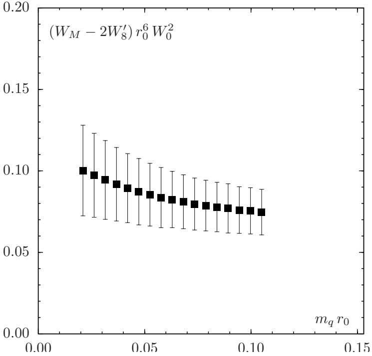
- c_2

$$M_0^2 - M_{\pm}^2 = 4c_2 a^2 = -a^2 \frac{128\, W_0^2}{f^2} (2W'_6 + W'_8)$$

$M_{\rm PS}$: $W_M - 2W'_8$ and $W'_8 + 2W'_6$

- $W_M - 2W'_8$ from $M_\pm$, $M_{\rm VV}$, $M_{\rm VS}$

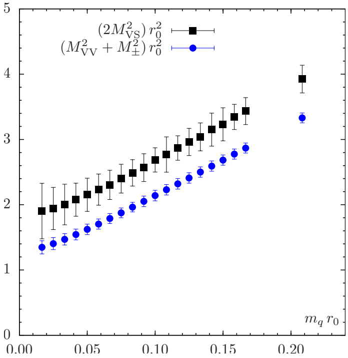


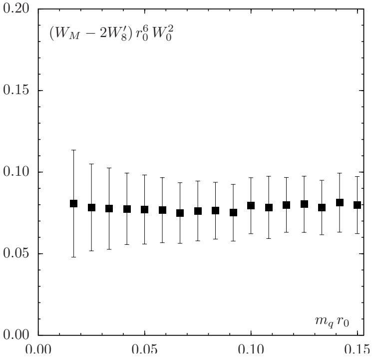


$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.085\,\rm fm$

$M_{\rm PS}$: $W_M - 2W'_8$ and $W'_8 + 2W'_6$

- $W_M - 2W'_8$ from $M_\pm$, $M_{\rm VV}$, $M_{\rm VS}$





$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055\,\text{fm}$

determination of $W_M - 2W'_8$

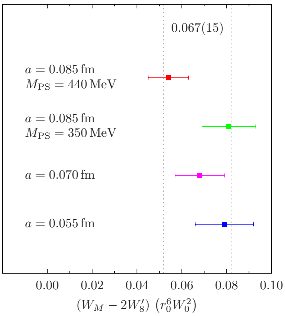
- W'_8 for $N_f = 2$ Wilson, tlSym
- $2W_M - W'_8 > 0$

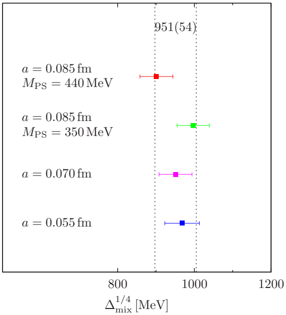
  [Bär, Golterman, Shamir, 2011]
- comparison $\Delta_{mix}^{1/4}$

  ov. on smeared-clover : 861(90) MeV

  domain wall on stagg. : 678(13) MeV

  ov. on domain wall : 416(27) MeV





[K. Cichy, V. Drach, E. García Ramos, G.H., K. Jansen, 1211.1605]

mixed action PQ χPT (MAPQχPT)

power counting : $m_s \sim m_v \sim a^2$

LO : m_s , m_v , p^2 , a^2

- non-singlet scalar correlator (mixed action) at large euclidean time

$$C^{\rm VV}_{\rm sca}(t) \;\rightarrow\; \frac{B_0^2}{2L^3}\left[\frac{e^{-2M_{\rm VS}t}}{{M_{\rm VS}}^2} - \frac{e^{-2M_{\rm VV}t}}{M^4_{\rm VV}}\left(M^2_{\rm VV} + \hat{a}^2\,\frac{16}{f^2}\,W'_8(1+M_{\rm VV}\,t)\right)\right] + Ae^{-m_{a_0}t}$$

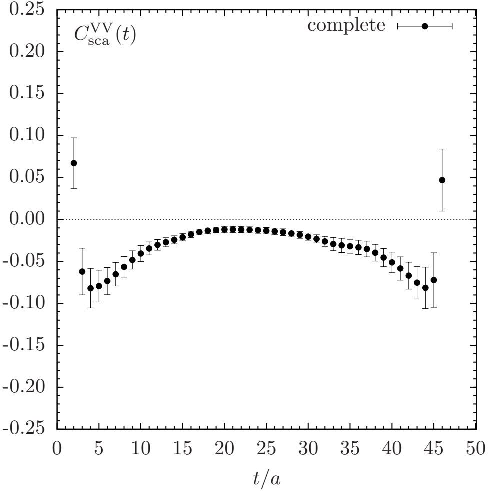
[Golterman, Izubuchi, Shamir, 2005; Bär & Furchner, 2010]

for maximal twist

at the matching mass $M_{\pm} = M_{\rm VV}$

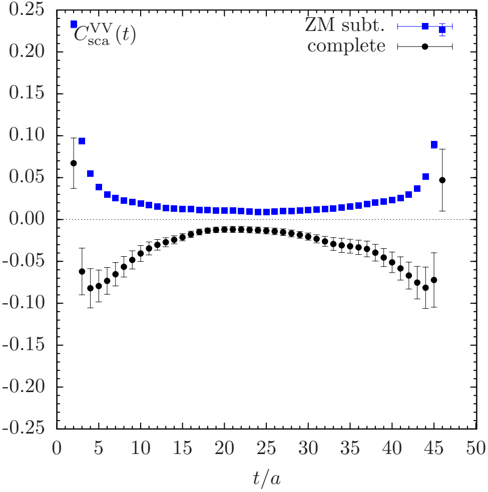
- combining measurements of pseudoscalar masses and scalar correlator $\rightsquigarrow$ W'_8 , W_M

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



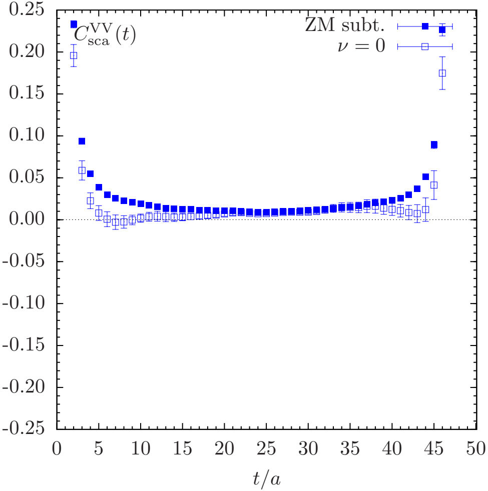
$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055\,\text{fm}$

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



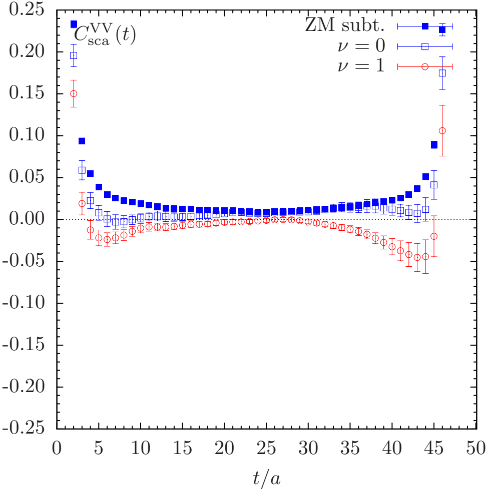
$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055$ fm

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



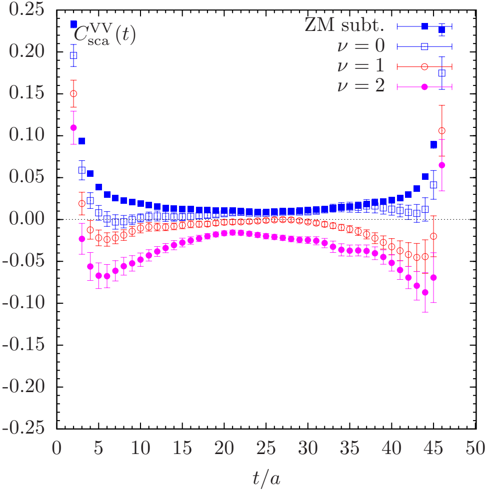
$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055$ fm

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



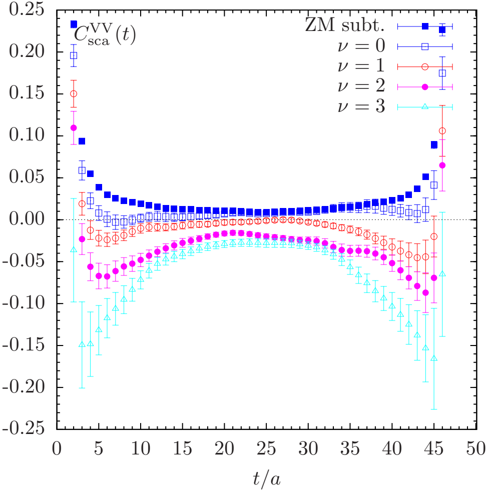
$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055$ fm

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



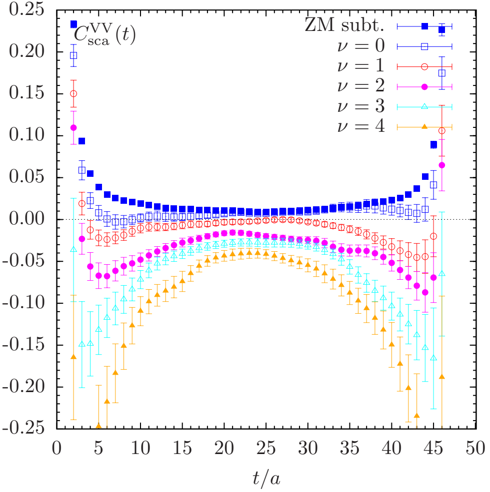
$M_{\rm PS} r_0 = 0.8\,;\quad L/r_0 = 3\,;\quad a = 0.055\,\text{fm}$

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



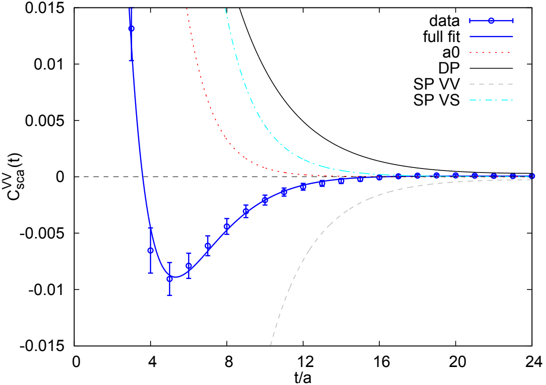
$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055$ fm

$C^{VV}_{\rm sca}(t)$: $W_M + 2W'_8$



$M_{\rm PS} r_0 = 0.8$; $L/r_0 = 3$; $a = 0.055$ fm

$C^{VV}_{sca}(t)$: $W_M + 2W'_8$



$M_{PS} r_0 = 1.0$; $L/r_0 = 4.6$; $a = 0.08$ fm

[K. Cichy, V. Drach, E. García Ramos, G.H., K. Jansen, 1211.1605]

determination of W_M and $W'_{6,8}$

Mixed action : overlap on $N_f = 2$ Wtm with tlSym

- $\Delta_{\rm mix}^{1/4} = 951(54)$ MeV
- $w_M = 901(65)$ MeV
- $w'_8 = -528(51)$ MeV
- $r_0^6\, W_0^2\, W'_8 = -0.0064(24)$
- $r_0^6\, W_0^2\, W'_8 = -0.0127(08)$ [subtracting zero-modes]

$$w_M = \frac{16 W_0^2\, W_M}{f^2}$$

$$w'_k = \frac{16 W_0^2\, W'_k}{f^2} \qquad (k = 6, 8)$$

Unitary action : $N_f = 2$ Wtm with tlSym

- $w'_8 = -552(25)$ MeV
- $r_0^6\, W_0^2\, W'_8 = -0.0119(17)$
- $r_0^6\, W_0^2\, W'_6 = 0.0049(38)$
- $W'_6 / W'_8 = -0.4(3)$ [w.r.t. $1/N_c$]

- systematic effects :
  larger volume
  higher orders in MAχPT and WχPT
  zero-mode subtraction in $C_{\rm sca}^{VV}(t)$
  other observables

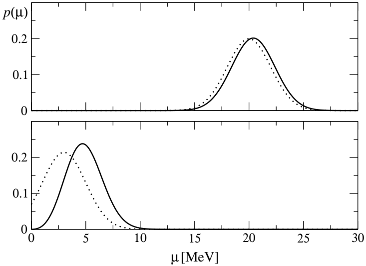
spectrum of Wilson Dirac operator

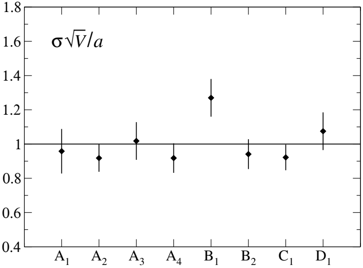
stability of simulations with Wilson fermions

(a) phase structure of Wilson fermions : c_2 is a LEC of WχPT
$\rightsquigarrow$ Aoki or Singleton-Sharpe scenarios in LCE region

(b) distribution of $\lambda_{\rm min}$ of $\gamma_5 D_W$

- similar conclusions from (a) and (b)



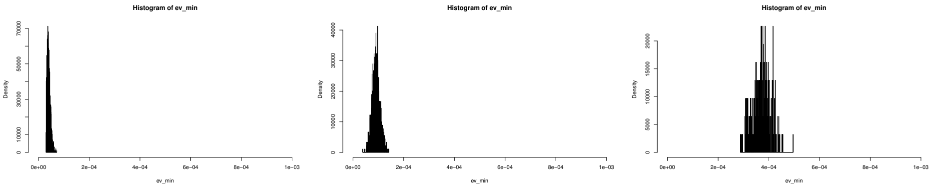


[CERN-ToV, 2005]

- CP-PACS & JLQCD with $N_f = 2+1$ Clover + Iwasaki : $0.5 < \sigma\sqrt{V}/a < 0.75$

Wtm at maximal twist

- $\lambda_{\min}$ of $D_W^\dagger D_W + \mu_\ell^2$
- $N_f = 4 \quad \beta = 1.95 \quad a\mu = 0.0085 \quad L/a = 24$

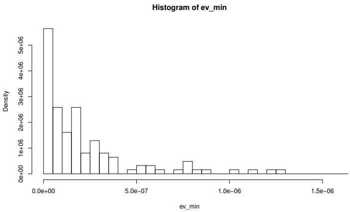


- close to maximal twist, the distribution of $\lambda_{\min}$ of $D_W^\dagger D_W$ is not Gaussian
- is the width of $\lambda_{\min}$ useful to monitor the stability?

Wtm at maximal twist

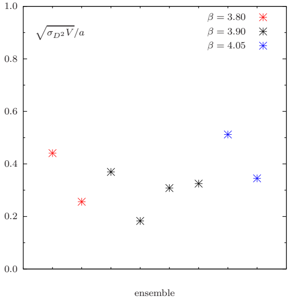
- close to maximal twist, the distribution of $\lambda_{\rm min}$ of $D_W^\dagger D_W$ is not Gaussian

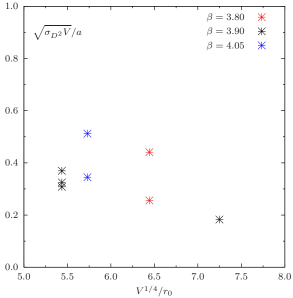
  example from $N_{\rm f} = 2,\ \beta = 3.9,\ L/a = 24$



  examples

- $N_{\rm f} = 2$

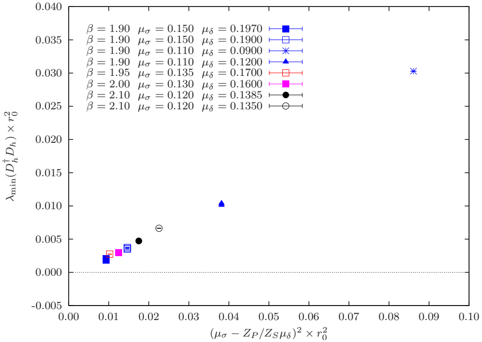




(c-s)-doublet : $\det(D^h_{\rm tm})$

$$D^h_{\rm tm} = D_W[U] + m_{0h} + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3$$
$$m_s = 1/Z_{\rm P}\,(\mu_\sigma - Z_{\rm P}/Z_{\rm S}\,\mu_\delta)$$

- $\mu_\sigma^2 > \mu_\delta^2 \;\Rightarrow\; \det(D^h_{\rm tm}) > 0$ [Frezzotti, Rossi, 2003]
- $\mu_\sigma^2 \le \mu_\delta^2$ sign of $\det(D^h_{\rm tm})$ is controlled by m_s ... it is therefore, in practise, positive
- $\lambda_{\rm min}$ of $Q_h^\dagger Q_h$ where $Q_h = \gamma_5\tau_3 D^h_{\rm tm,eo}$



[PRELIMINARY]

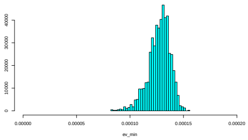
$(c$-$s)$-doublet : $\det(D^h_{\rm tm})$

$$D^h_{\rm tm} = D_W[U] + m_{0h} + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3$$
$$m_s = 1/Z_{\rm P}\,(\mu_\sigma - Z_{\rm P}/Z_{\rm S}\,\mu_\delta)$$

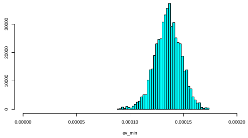
- $\lambda_{\rm min}$ of $Q_h^\dagger Q_h$
- spectral gap
- width $\propto 1/L$

[Del Debbio et al., 2005]

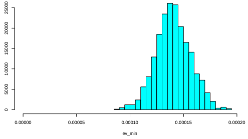
$$\beta = 1.90\,;\ a \approx 0.086\,\mathrm{fm}$$
$$\kappa = 0.163270$$
$$a\mu_l = 0.004$$
$$a\mu_\sigma = 0.15;\ a\mu_\delta = 0.19$$



$L/a = 32$



$L/a = 24$



$L/a = 20$

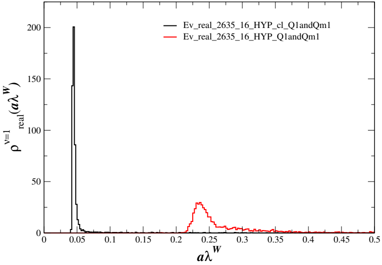
Spectrum

- $N_f = 0$, fixed topology (e.g. $\nu = 1$) , power-counting : $m \sim a^2$

  distribution of the single real eigenvalues of D_W has a width

$$\sigma = \frac{\sqrt{8a^2 W_8}}{\Sigma\sqrt{V}}$$

[G. Akemann, P. Damgaard, K. Splittorff and J. Verbaarschot, 1012.0752]



[P. Damgaard, U. Heller, K. Splittorff, 1301.3099]

- mode number

[L. Giusti, M. Lüscher, 2009]
[S. Necco, A. Shindler, 2011]
[ETMC, K. Cichy, E. García Ramos, K. Jansen, 2013]

conclusions

- $O(a^2)$ cutoff effects in the light-quark mass regime can be large for Wilson fermions
- determination of Wilson χPT LECs can be useful :
  - estimate expected size of cutoff effects
  - identify a lattice action with reduced $O(a^2)$ lattice artifacts
  - combined fits of mass, volume and lattice spacing dependence