QUANTUM TIME TRAVELS: FROM FICTION TO OPTIMAL INFORMATION PROCESSING

Giulio Chiribella
Institute for Interdisciplinary Information Sciences
Tsinghua University
Beijing

Causal Structure in Quantum Theory
$|6 \text{ June } 2013\rangle$, Benasque

AIMS OF THIS TALK (I)

- To counteract some common misconceptions:

  - The SCI-FI effect: time-travels are not a topic for serious scientific investigation

  - The free-will issue: closed timelike curves are incompatible with free will (cf. grandfather paradox)

  - The "no-operational consequences" idea:
    even if closed timelike curves are possible at all
    in some weird GR solution,
    the dynamics of the the universe should include a
    mechanism that prevents us from observing them

AIMS OF THIS TALK (II)

On the positive side, this talk aims at putting forward an approach to CTCs based on notions from quantum information/foundations and from theoretical computer science.

Key point: higher-order computation as (the) model for arbitrary causal structures

(will explain later what I mean)

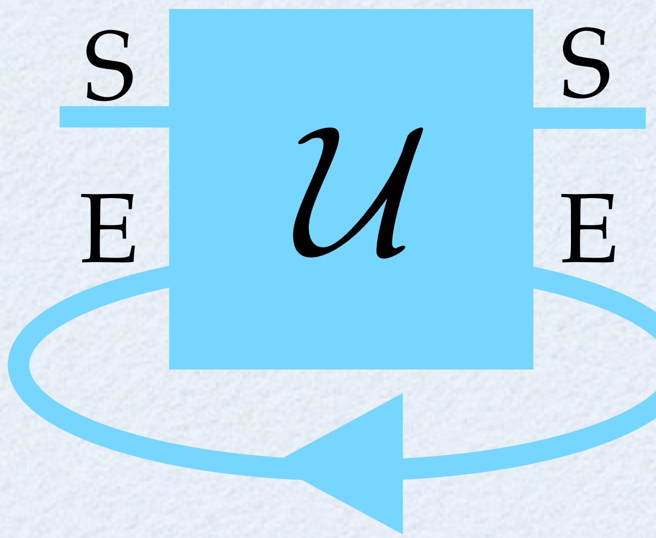
WHAT I WILL
NOT
TALK ABOUT
(AND WHY)

SCI-FI


Wait a minute, Doc. Ah... Are you telling me that you built a time machine... out of a DeLorean?
The way I see it, if you're gonna build a time machine into a car, why not do it with some style?

DEUTSCH'S MODEL OF CLOSED TIMELIKE CURVES (CTCS)

Unitary dynamics U, coupling a quantum system outside the CTC with quantum system inside the CTC



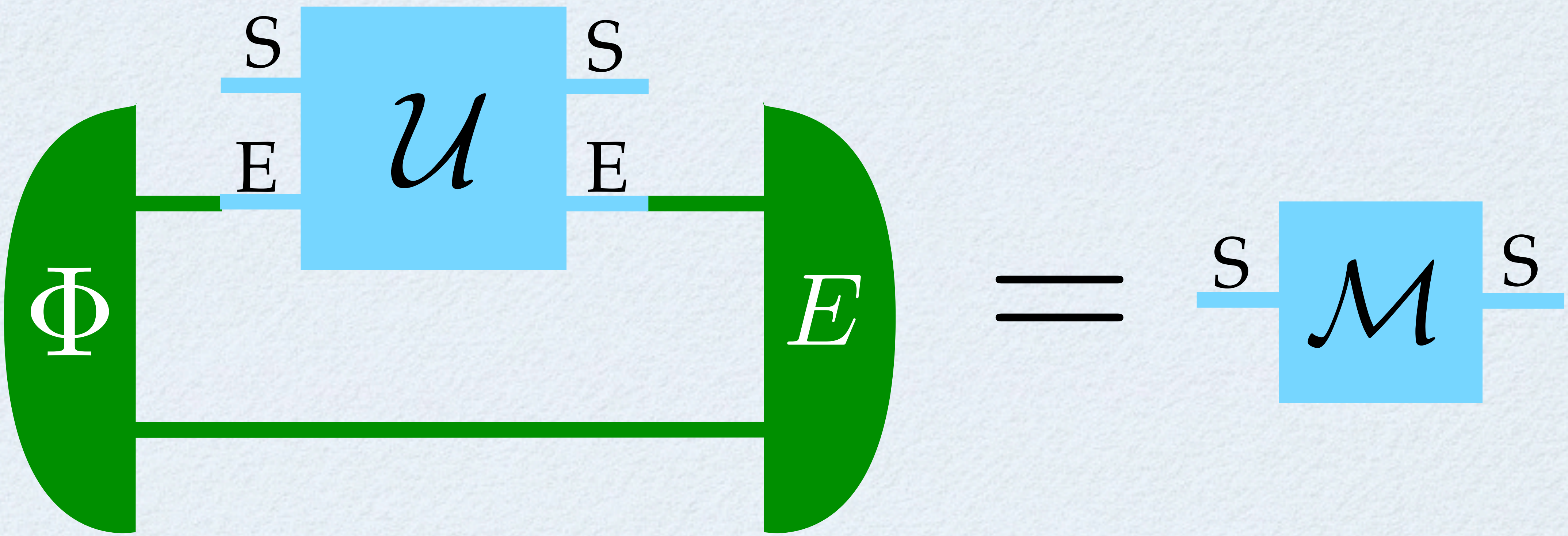
State of the system inside the CTC depends on the state of S:

$$\rho_E = \mathrm{Tr}_S[U(\rho_S \otimes \rho_E)U^\dagger]$$

Non-linear evolution of S

NON-LINEAR POSTSELECTED TELEPORTATION

Recipe: represent time-travel as probabilistic teleportation, and then forget about the probability (cf. Lloyd et al, 2010).



non-linear evolution of S: $$\rho'_S = \frac{\mathcal{M}(\rho_S)}{\mathrm{Tr}[\mathcal{M}(\rho_S)]}$$

WHAT'S WRONG WITH NON-LINEARITY?

Theorem (GC, D'Ariano, Perinotti, PRA 2010): in any probabilistic theory, the evolution of the state is linear.
(see also Hardy 2001, different proof requiring convexity of state space)

Evolution non-linear in the density matrix implies that

- the state space no longer the set of density matrices.
- mixed quantum states can become pure in the new state space
- quantum theory would be falsified for all systems that interacted with a CTC (virtually, all systems)

Not surprisingly, non-linear models can be used to violate most of the usual quantum laws
(again, recall that they imply that QT does not hold):

- non-orthogonal states can be perfectly cloned
- “ “ “ “ “ “ distinguished
- one can efficiently solve PSPACE problems
- BB84 is not secure
- an adversary can steer the state in your lab to any desired state
- ...

for a comprehensive critical discussion,
see forthcoming work by Debbie Leung

WHAT I WILL TALK ABOUT

PLAN OF THE TALK

- Probabilistic simulation of time-travels within ordinary QT:

  -non-classical features,

  -fundamental role of CTCs in quantum information

- The consistency requirement for CTCs

- Higher-order quantum computation as the generator of causal and non-causal structures

  -quantum supermaps

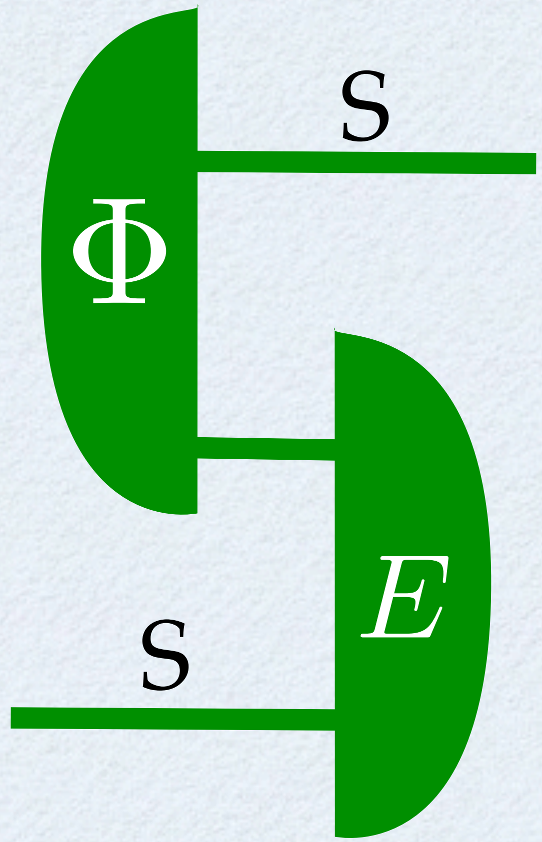
  -the Quantum Switch

PROBABILISTIC SIMULATION OF TIME TRAVELS WITHIN A GIVEN CAUSAL STRUCTURE

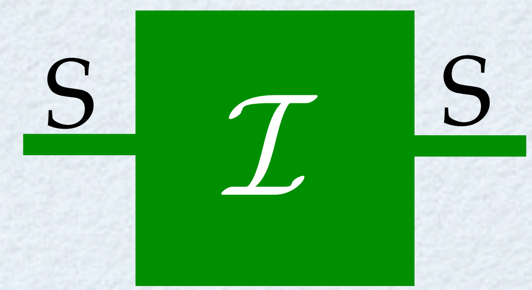
(with Dina Genkina and Lucien Hardy, PRA 2011)

PROBABILISTIC TELEPORTATION

- Probabilistic teleportation (BBCJPW)



$$= \frac{1}{d_S^2}$$



$$\Phi := d_S^{-1} \sum_{m,n} |m\rangle|m\rangle\langle n|\langle n| \qquad \text{(Bell state)}$$

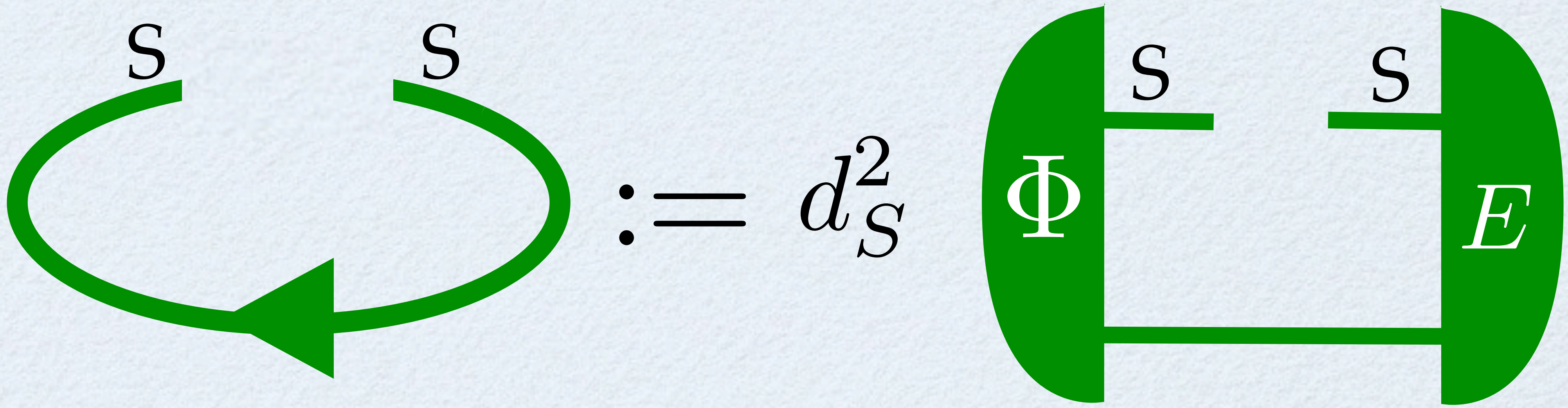
$$E(\cdot) := \mathrm{Tr}\left[d_S^{-1} \sum_{m,n} |m\rangle|m\rangle\langle n|\langle n| \ \cdot \right] \qquad \text{(Bell effect)}$$

DEFINITION: TIME TRAVEL

- Idea (Bennett-Schumacher, Coecke, Svetlichny)

stretch the wires in the teleportation diagram, so to obtain an identity channel from the future to the past.

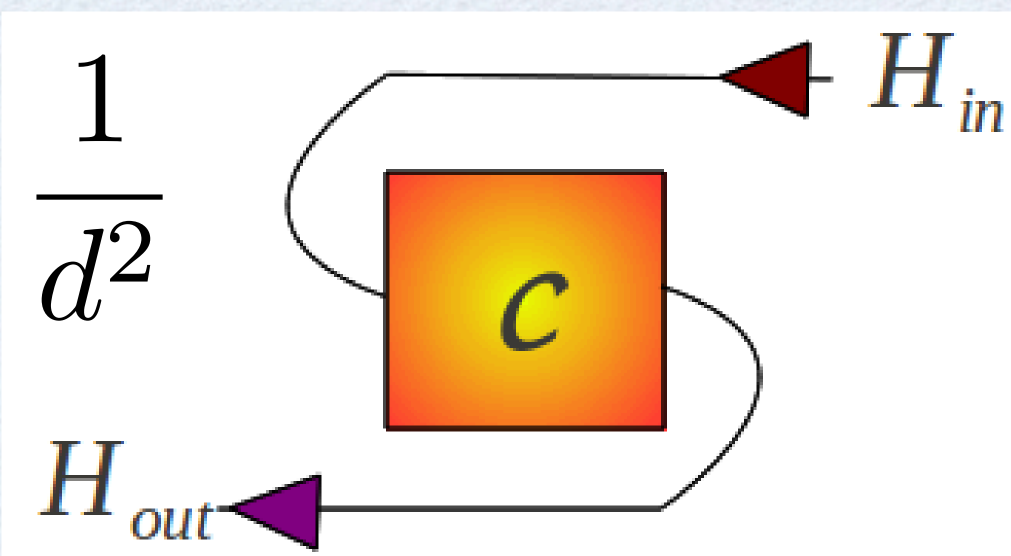
- Definition:



Linear, deterministic time travel.

PROBABILISTIC SIMULATIONS

Quantum operations with the input in the future and the output in the past can be simulated probabilistic using probabilistic teleportation



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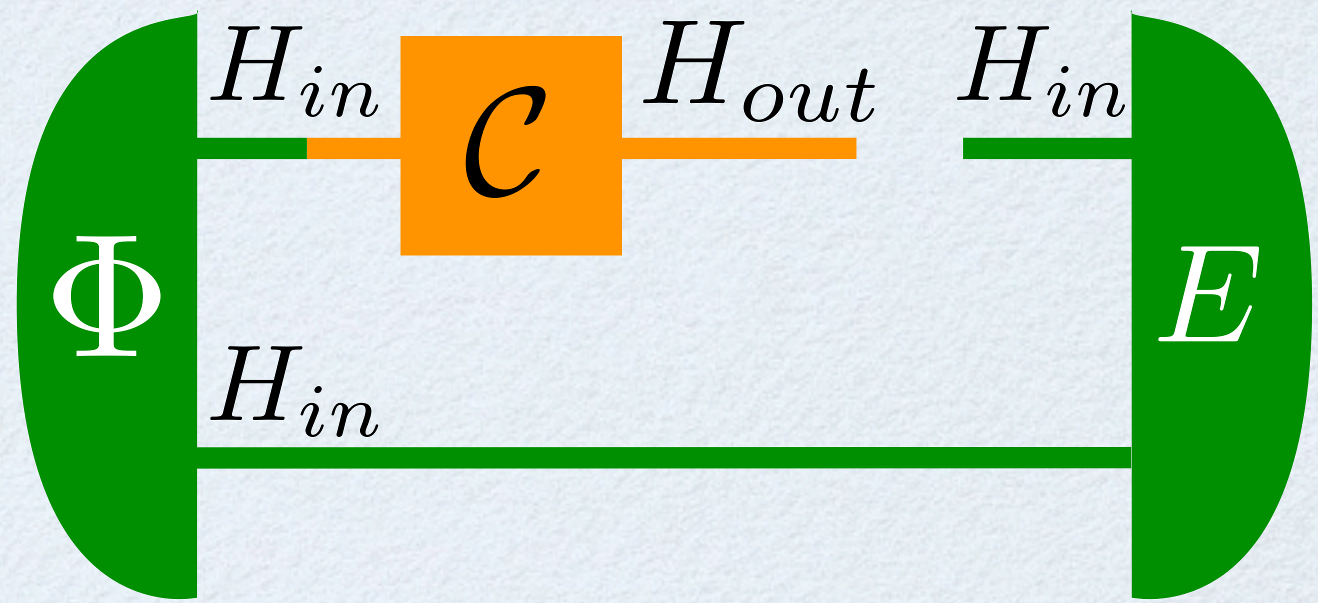
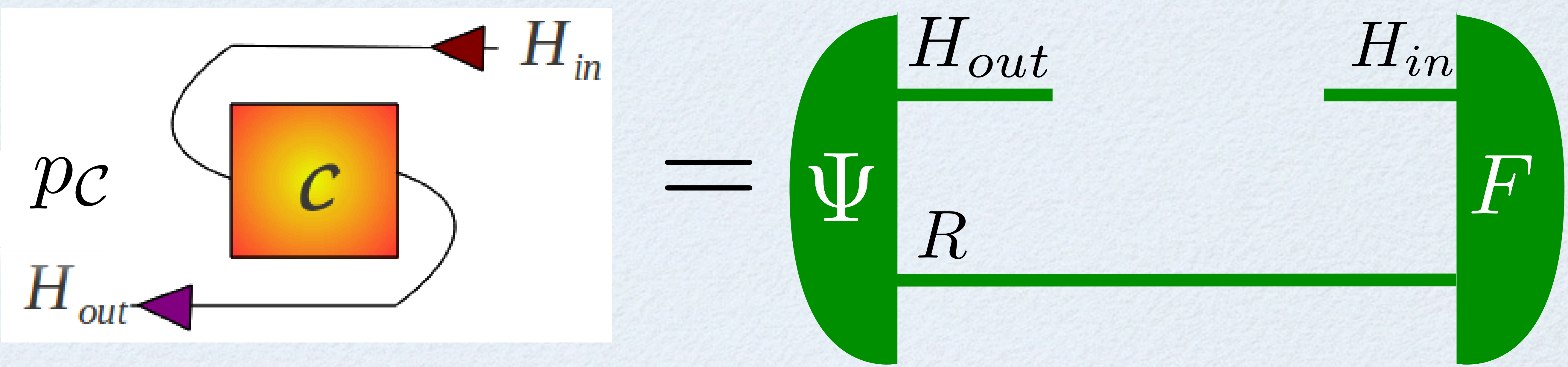


OPTIMAL PROBABILISTIC SIMULATIONS

Teleportation allows for a probabilistic simulation of time-travel channels with probability $p_{tele} = \frac{1}{d^2}$

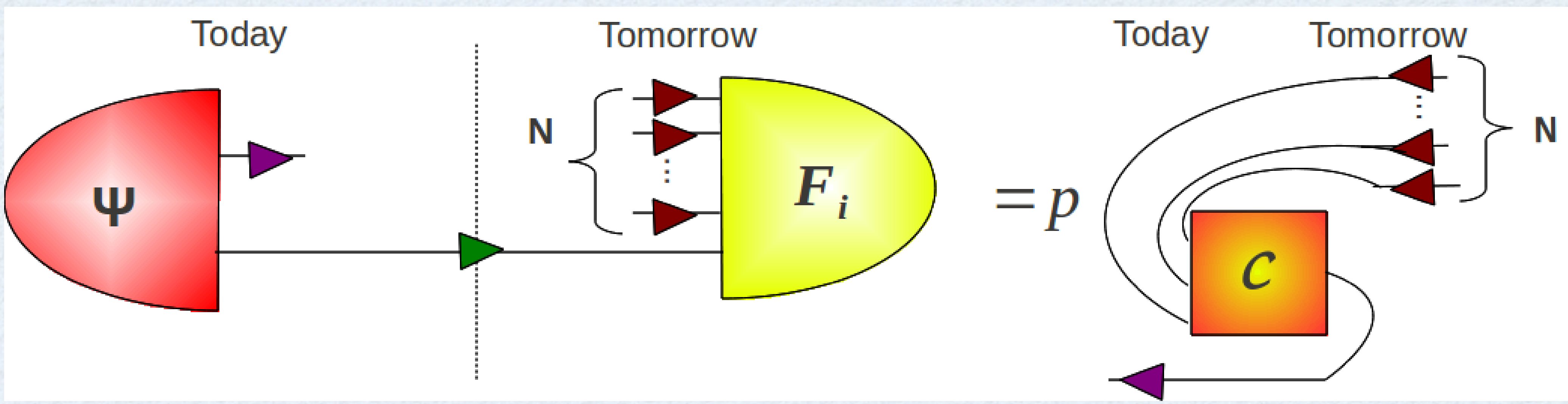
Question: what is the maximum probability $p_{\mathcal{C}}$ such that



for some state Ψ and some effect F ?

EXAMPLE: TELEPORTING PURE STATES WITH MANY COPIES

N identical copies of a pure state in the future, we want to transfer one copy to the past



Equivalent to the probabilistic simulation of the channel

$$Trace_{N\to 1}(\rho) = \mathrm{Tr}_{2,\dots,N}\left[P_+^{(N)}\rho P_+^{(N)}\right]$$

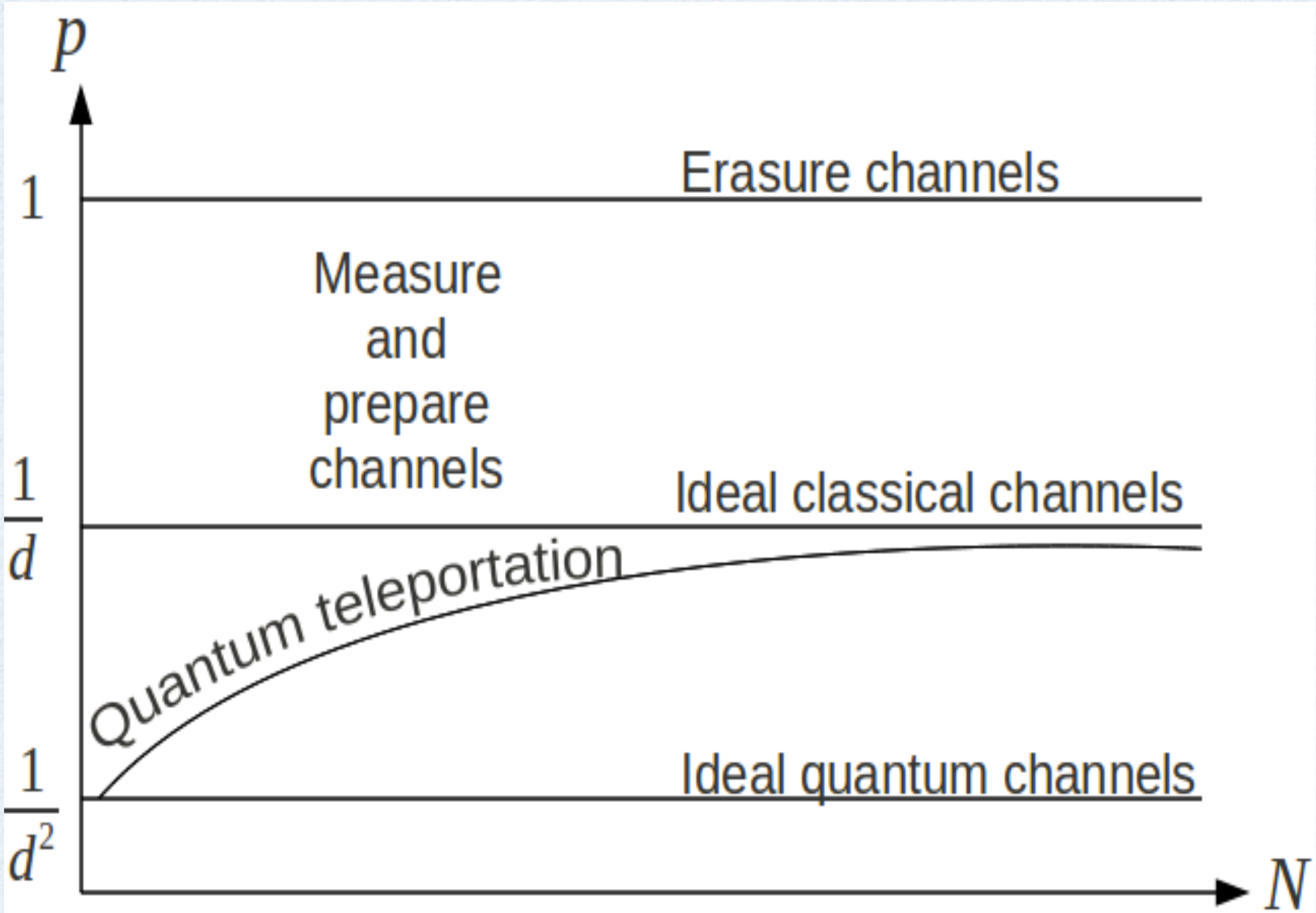
CAN THE PROBABILITY INCREASE WITH N?

Question: Does the probability depend on the number of input copies N?

In the classical world, no dependence:

since pure states can be perfectly copied,
the probability to simulate a classical transfer of data from the future to the past
does not depend on how many copies of these data are available.

ANSWER TO THE QUESTION: $p_{N \to 1} = \dfrac{N}{d(d+N-1)}$



Non-classical feature in the interplay between information flow and causal structure!

IN OTHER WORDS

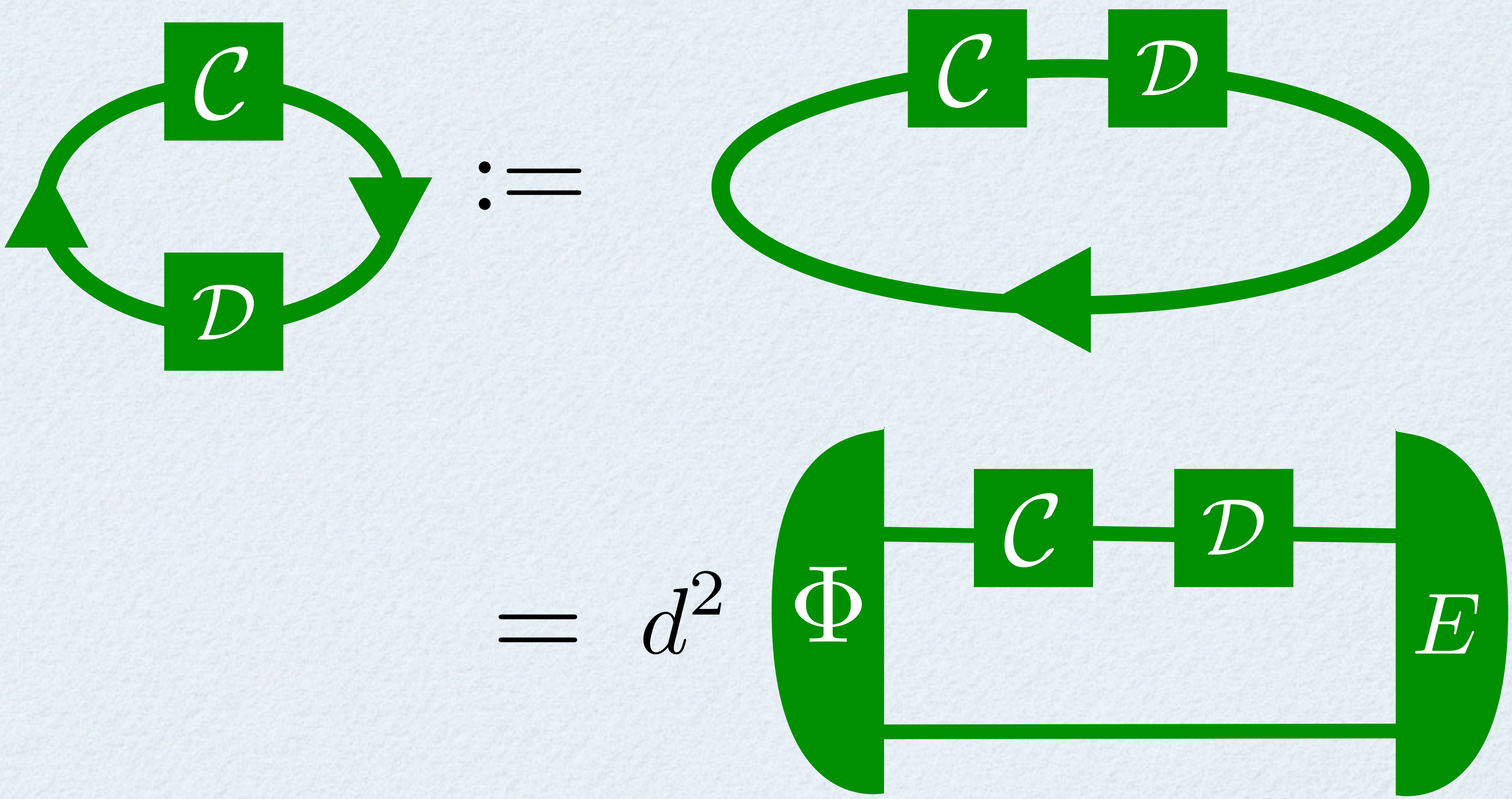
Travelling back in time would be more likely
if you had at many identical twins.

(well, the probability would be still less than $2^{-N_{avogadro}}$ ...)

DUALITY

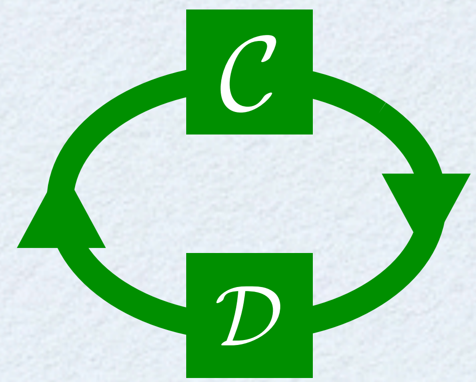
TIME LOOPS

Definition of time loop:



DUAL EXPRESSION FOR THE OPTIMAL PROBABILITY

$$p_{\mathcal{C}} = \frac{1}{\max_{\mathcal{D}}}$$

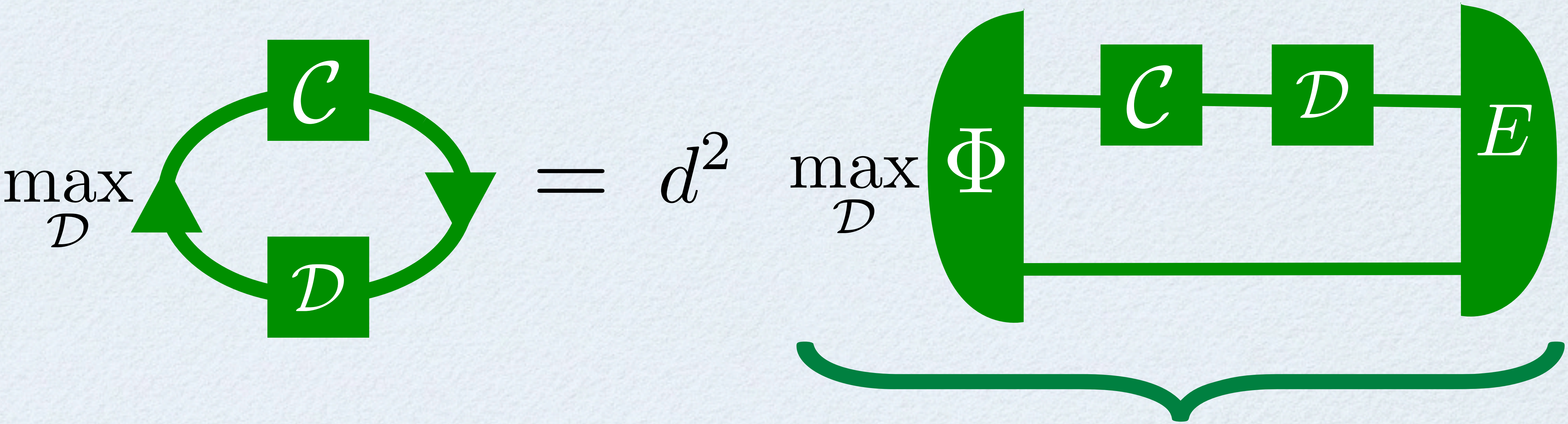


$\mathcal{D} = \text{quantum channel}$

Interpretation: the probability of successful simulation is the maximum one compatible with the fact that probabilities cannot be larger than 1.

QI LINK #1: ENTANGLEMENT FIDELITY IN QUANTUM ERROR CORRECTION

By definition



entanglement fidelity:
quantifies how well we can restore entanglement by correcting the action of $\mathcal{C}$

QI LINK #2: FIDELITY OF OPTIMAL CHANNELS

Consider a set of input states $(\rho_x)_{x \in \mathsf{X}}$

given with prior probabilities $(p_x)_{x \in \mathsf{X}}$

Suppose that we want to realize the transformation

$$\rho_x \mapsto \phi_x$$

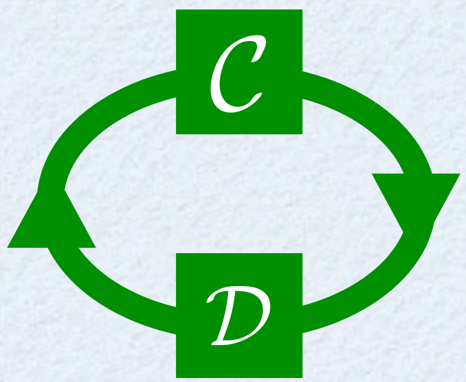
where $(\phi_x)_{x \in \mathsf{X}}$ is a set of pure target states

Examples: cloning, state estimation, transpose, purification, broadcasting, ...

Maximum fidelity over all quantum channels:

$$F_{\max} = \max_{\mathcal{D}} \sum_{x \in \mathsf{X}} p_x \langle \phi_x | \, \mathcal{D}(\rho_x) \, | \phi_x \rangle$$

$$= \gamma$$



with $\quad \gamma = \left\| \sum_{x \in \mathsf{X}} p_x \rho_x \right\|_\infty$

$$\mathcal{C}(\sigma) := \gamma^{-1} \sum_{x \in \mathsf{X}} p_x \langle \phi_x | \sigma | \phi_x \rangle \; \rho_x^T$$

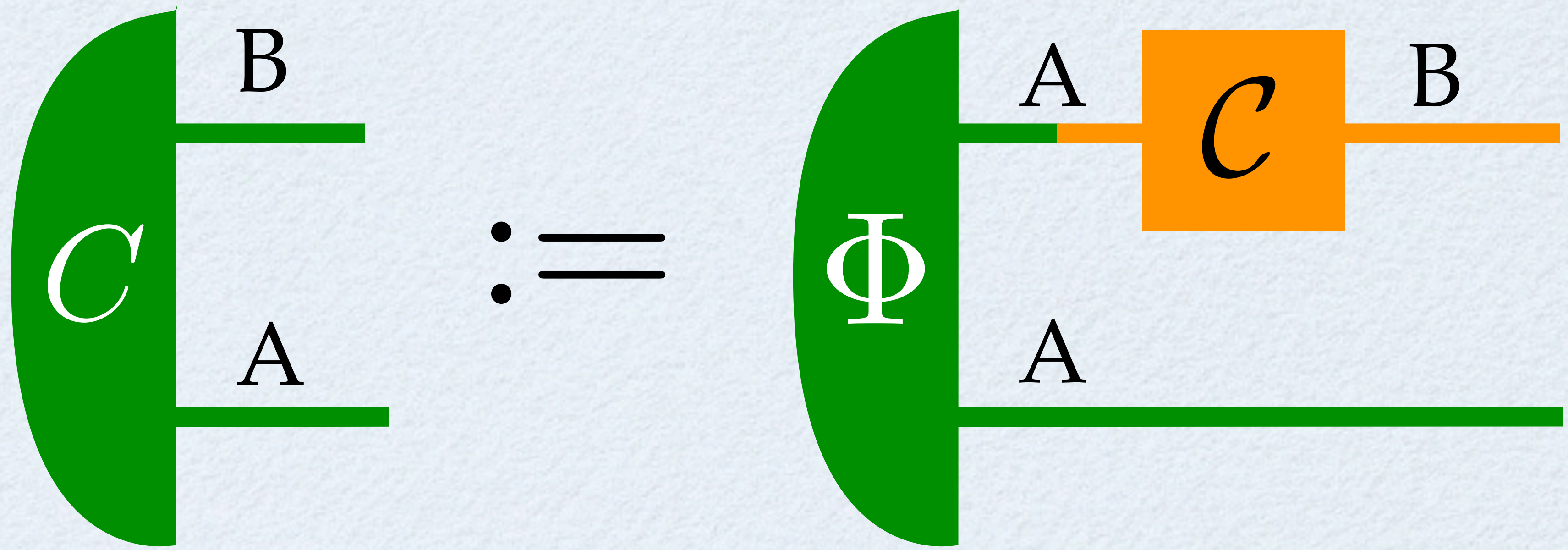
Hence: $$F_{\max} = \frac{\gamma}{p_C}$$

The optimal quantum fidelities inversely proportional to the optimal probabilities of simulated time-travels.

cf. optimal cloning, ...

QI LINK #3: MIN ENTROPY

Choi state:

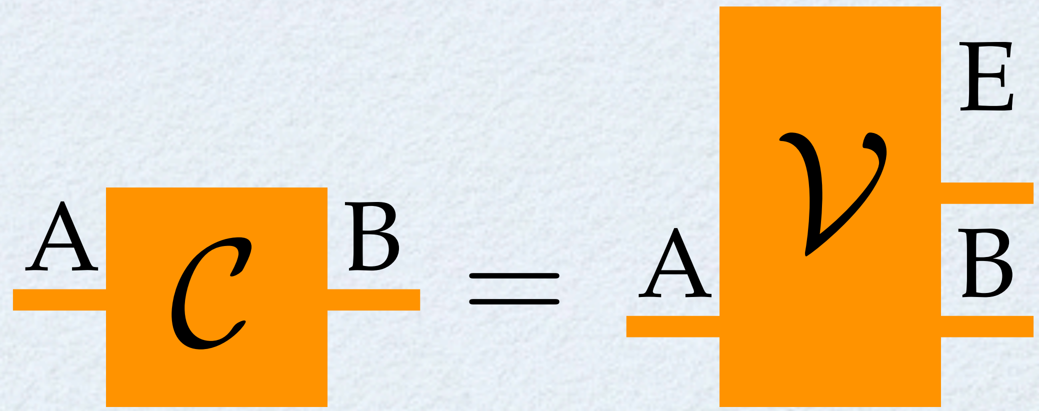


Min-entropy (cf. Renner, Koenig-Renner-Schaffner) important quantity in quantum communication and cryptography:

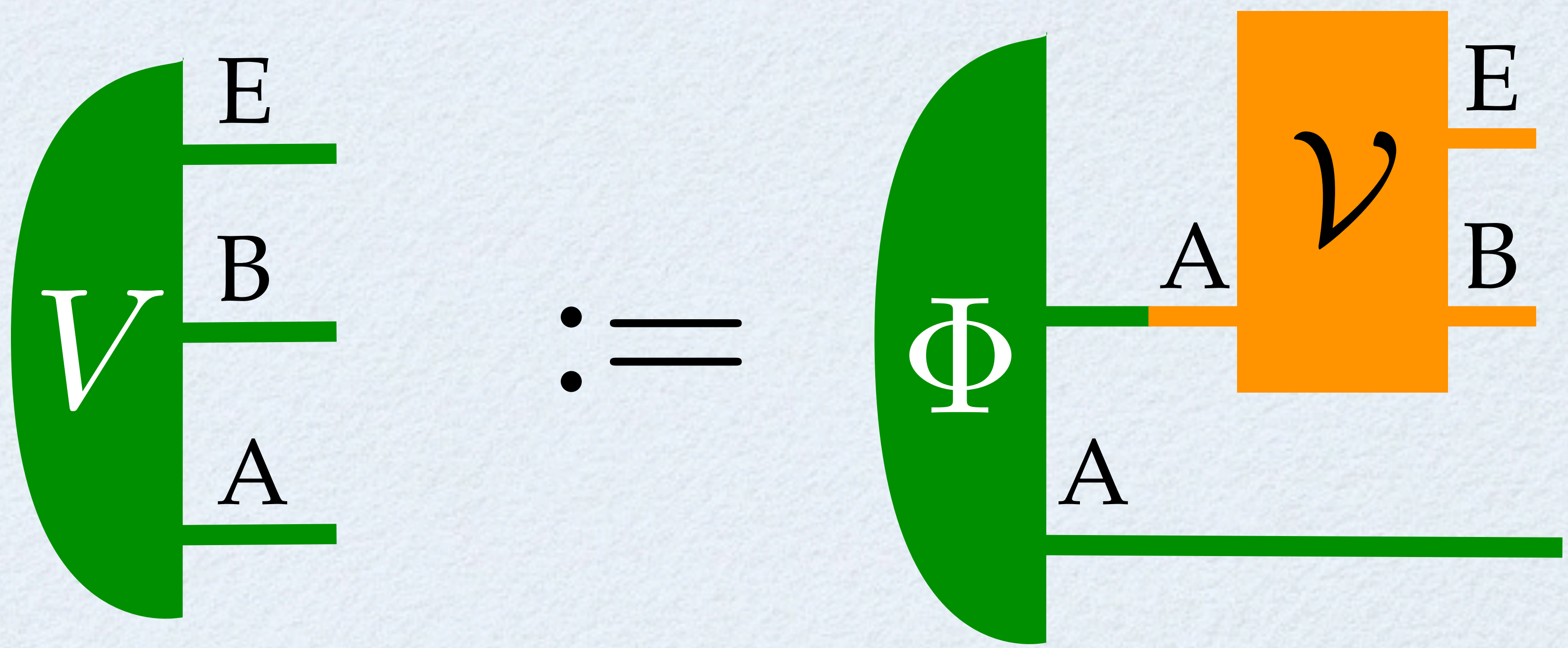
$$p_{\mathcal{C}} = \frac{2^{H_{\min}(A|B)}}{d_A}$$

QI LINK # 3.5: STATE MERGING

Stinespring dilation:



Purified Choi state:



State merging problem: transfer A's state to E, preserving the correlations with B.

Amount of quantum communication needed: $H_{\min}(A|B)$

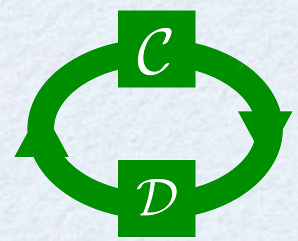
(Mario Berta, diploma thesis, ETH)

Hence: the log of the maximum probability of simulated time-travel is proportional to the amount of quantum communication needed to throw information from the input into the environment.

SUMMARY OF PROBABILISTIC SIMULATION

- non-classical effects: increase of the probability of simulation with the number of copies

- duality $p_{\mathcal{C}} = \dfrac{1}{\max_{\mathcal{D}}}$



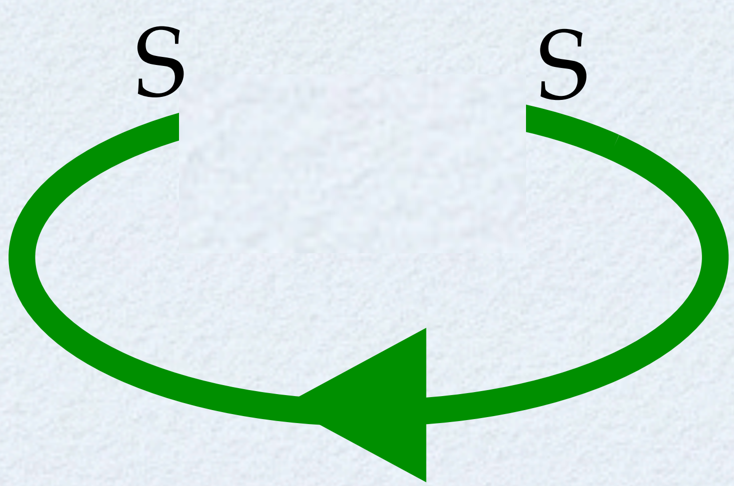
- the simulation of time travels has deep roots in quantum info: error correction, optimal quantum fidelities, min-entropy, state merging...

QUANTUM CIRCUITS WITH TIME LOOPS

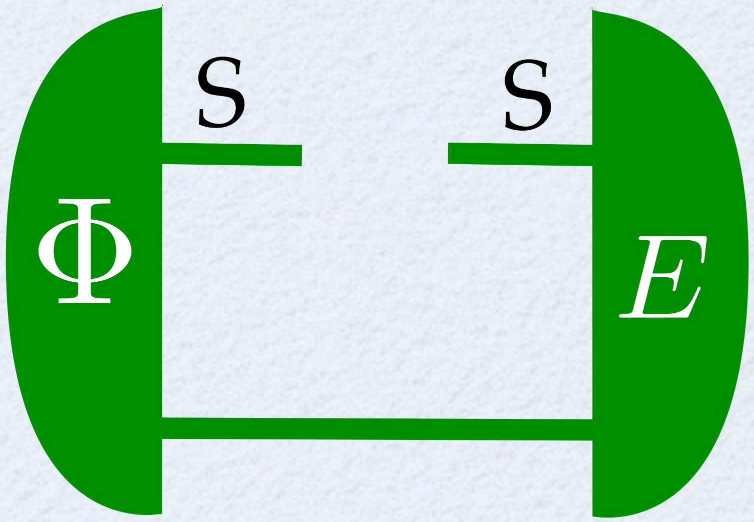
INCLUDING LOOPS IN THE CIRCUIT

Until now we simulated time travels within a given causal structure (= DAG = computational circuit)

Now we want to add loops in the picture, i.e. to introduce CTCs



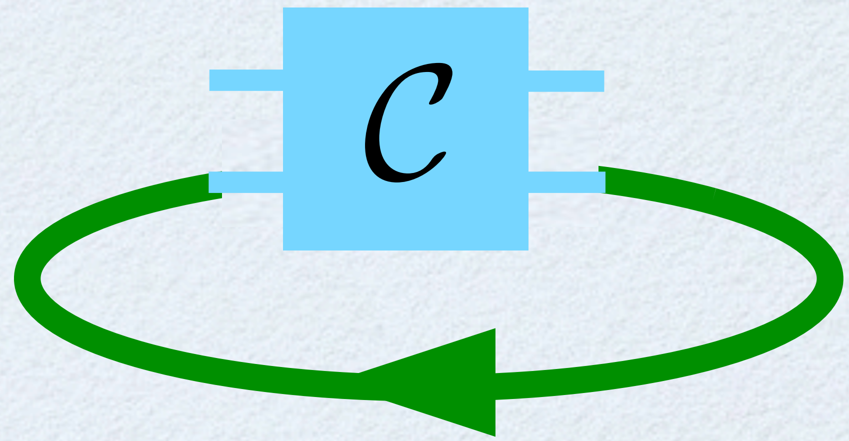
$$= d_S^2$$



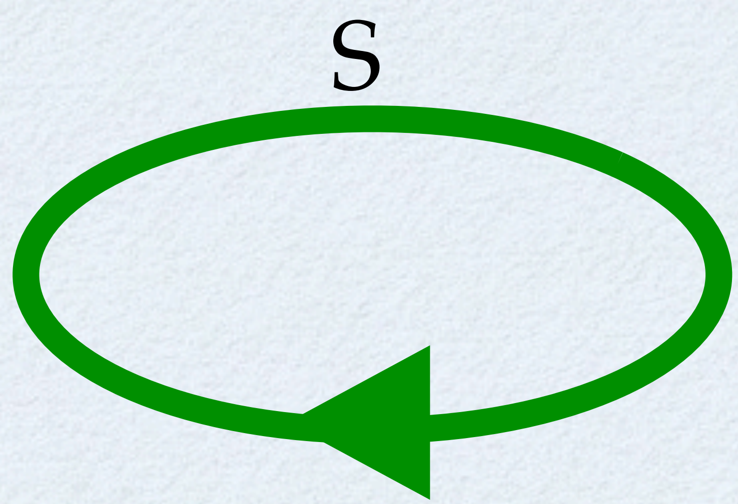
The problem is how to do it consistently.

INTERACTING WITH THE CTC

Connecting quantum devices to the CTC:



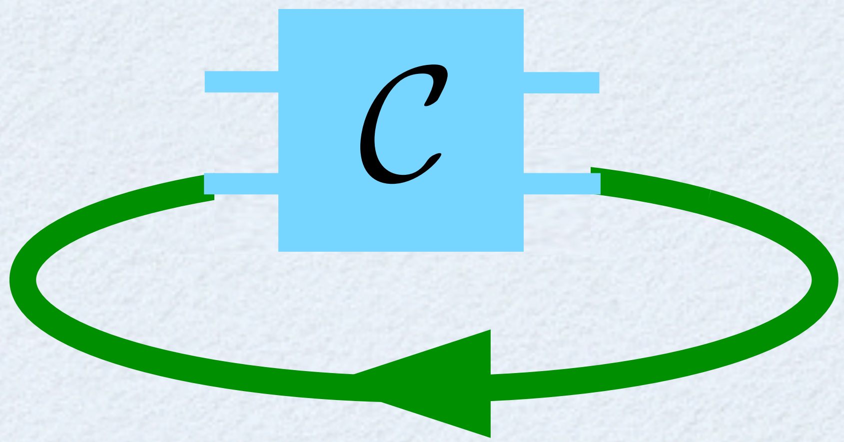
However, we cannot plug arbitrary channels in the loop: e.g. plugging the identity gives probability larger than 1



$$= d_S^2$$

cf.grandfather paradox, where the probability would be 0

CONSISTENCY REQUIREMENT



When we plug part of a channel in a CTC,
the remaining part outside the CTC must be a valid quantum channel (CPTP map)

- the requirement imposes that ordinary quantum theory holds outside the CTC

- limits the set of boxes that can be connected with the CTC

LIMITATION TO FREE WILL..?

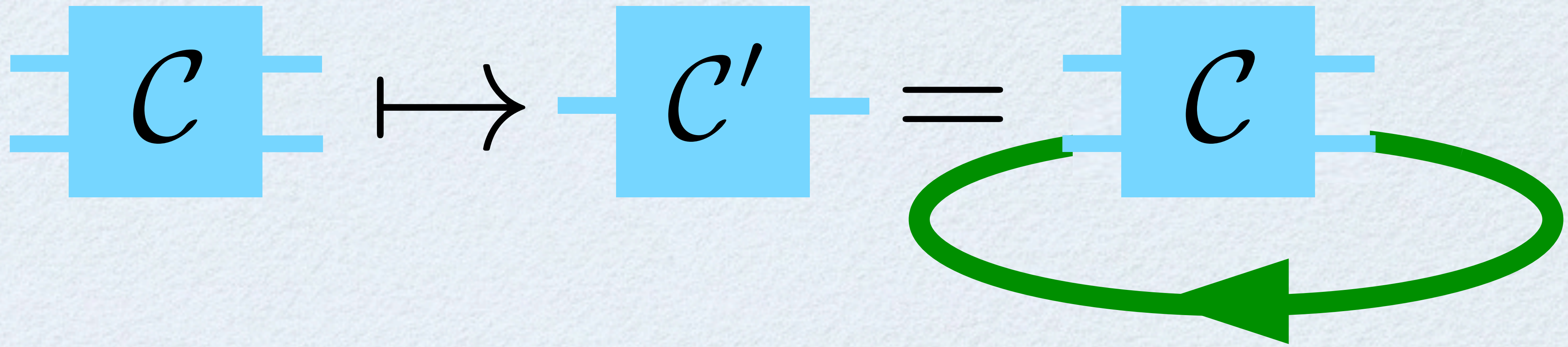
Well, no: in this operational theory,
we are free to choose within the set of allowed channels.

There is no physical reason to expect that arbitrary quantum channels should be implementable.

[cf. in ordinary quantum theory we cannot implement PR-boxes, but this does not mean that we have no free-will...]

HIGHER ORDER COMPUTATIONS

The mapping



is an example of higher-order transformation: transforms an input channel into an output channel.

Idea: the structure of higher-order transformations identifies all possible quantum circuits containing CTCs.

HIGHER-ORDER QUANTUM COMPUTATION

LET'S PLAY A GAME

Forget everything you know about quantum theory,
forget causal structure,
forget CTCs

just remember the fact that quantum states are density matrices.

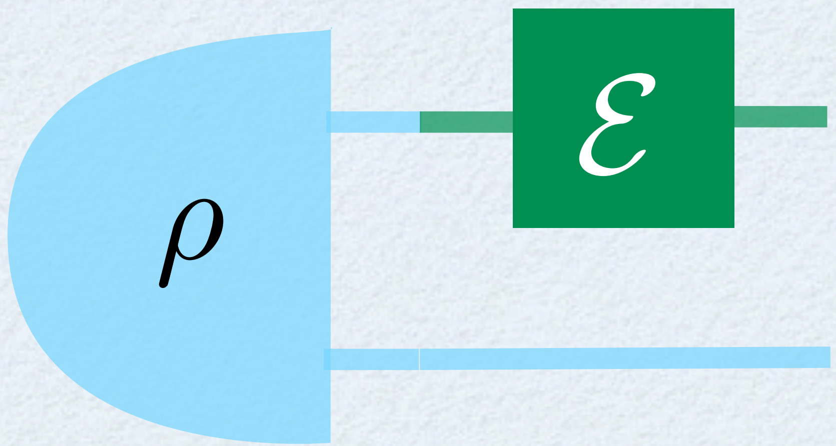
THE MOST GENERAL EVOLUTIONS

Question: What are the most general deterministic transformations of quantum states?

Answer: Quantum channel (linear, completely positive, trace-preserving maps)

Linear: mixture of input states is mapped into mixture of output states

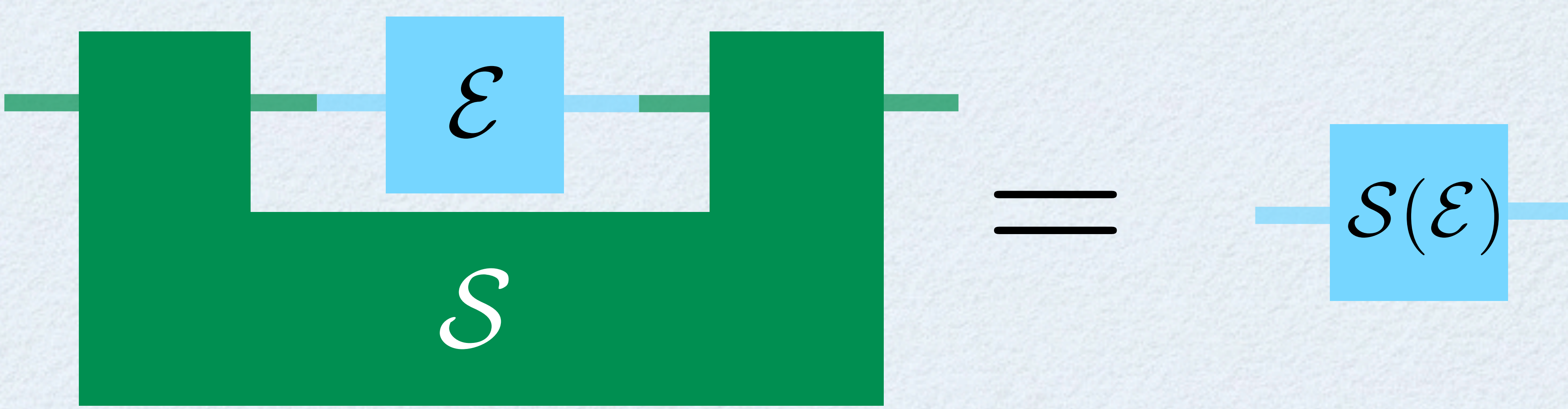
Completely positive and trace-preserving: output states should be valid density matrices for all possible inputs



THE MOST GENERAL TRANSFORMATIONS OF TRANSFORMATIONS

Question: What are the most general deterministic transformations of quantum states?

Let us call these transformations supermaps and represent them as follows:

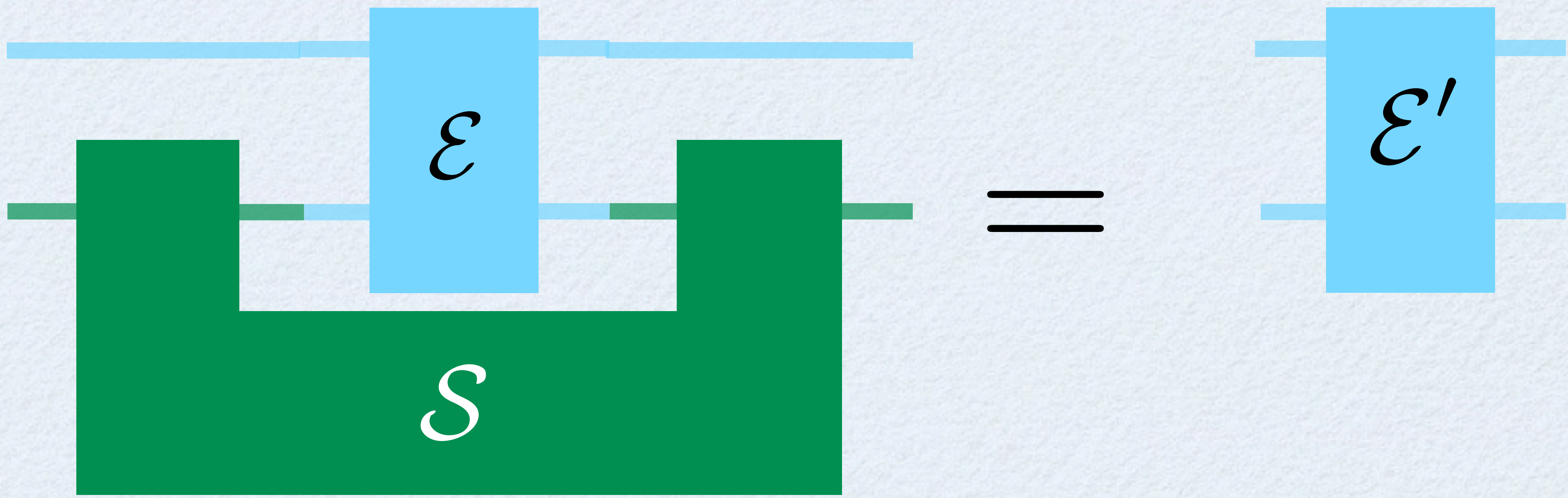


ADMISSIBLE SUPERMAPS

Requirements:

- supermaps must be linear in their input (same reason as before)

- must map quantum channels into quantum channels, even when acting on parts of larger quantum devices

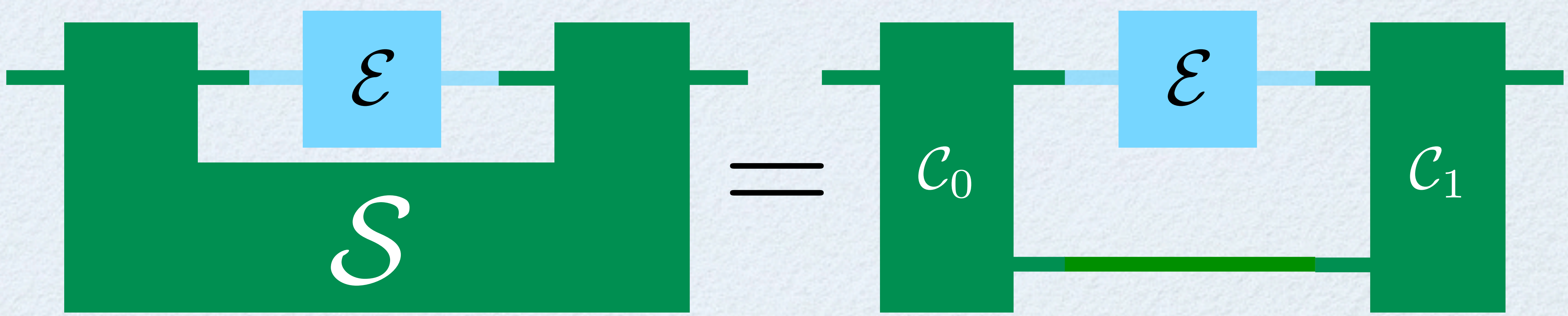


REALIZATION OF ADMISSIBLE SUPERMAPS: SEQUENTIAL QUANTUM NETWORKS

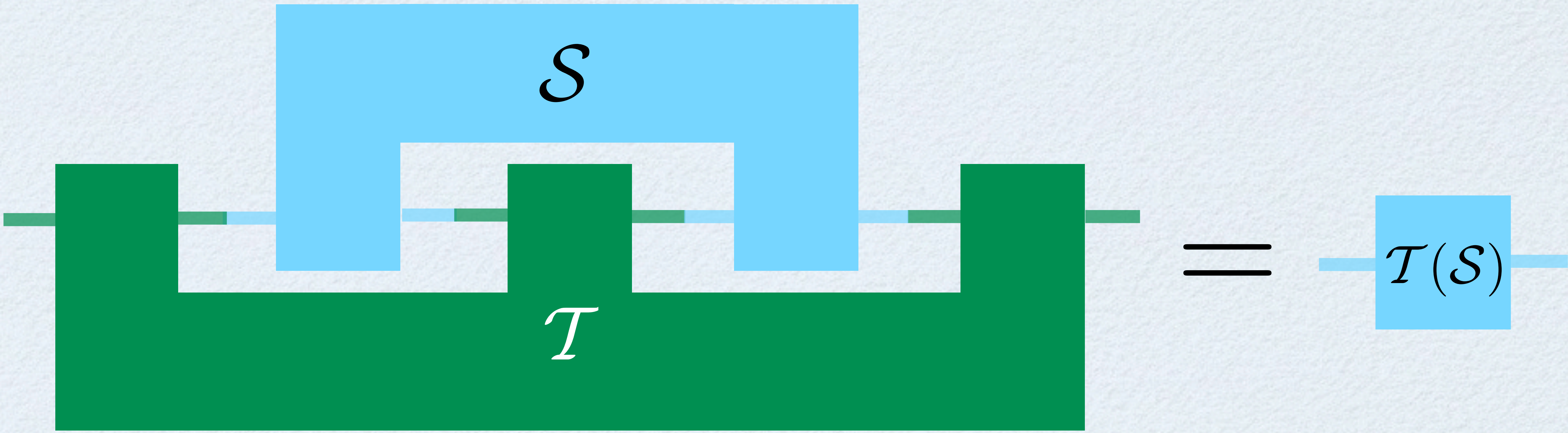
Theorem (GC, G M D'Ariano, and P Perinotti, EPL 2008)

any admissible supermap can be realized by a quantum network consisting in

- a pre-processing channel
- a post-processing channel



THE HIERARCHY OF ADMISSIBLE SUPERMAPS

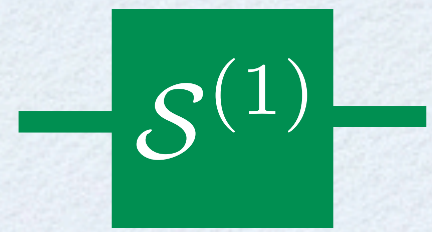


Recursive definition of admissible (deterministic) transformations:
a deterministic 1-map is a quantum channel,
for N>1 a deterministic N-map transforms deterministic (N-1)-maps into quantum channels, and must be

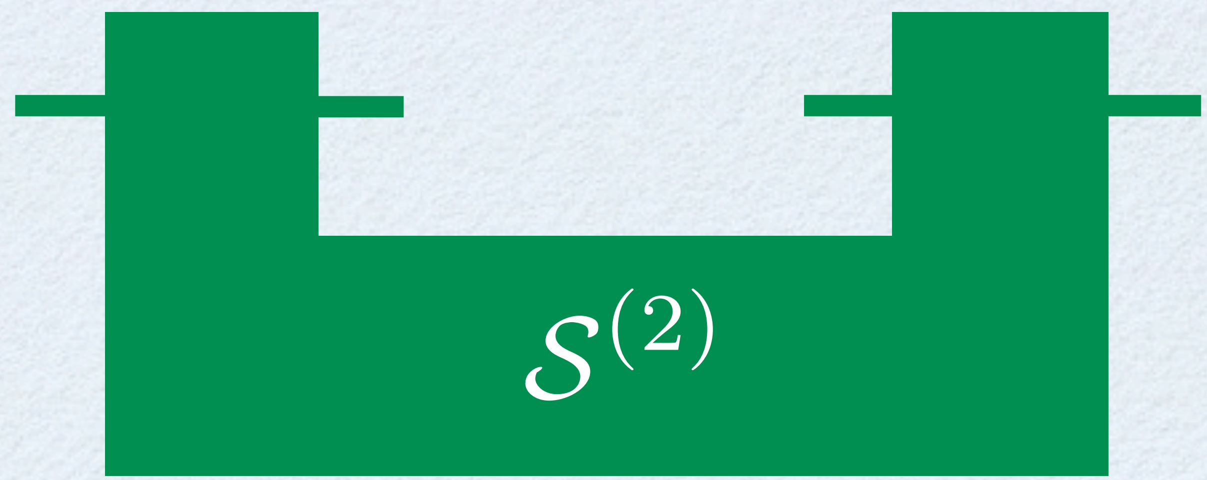
- linear
- sending (N-1) maps into quantum channels also when applied locally on one side of a bipartite input

HIERARCHY OF ADMISSIBLE SUPERMAPS

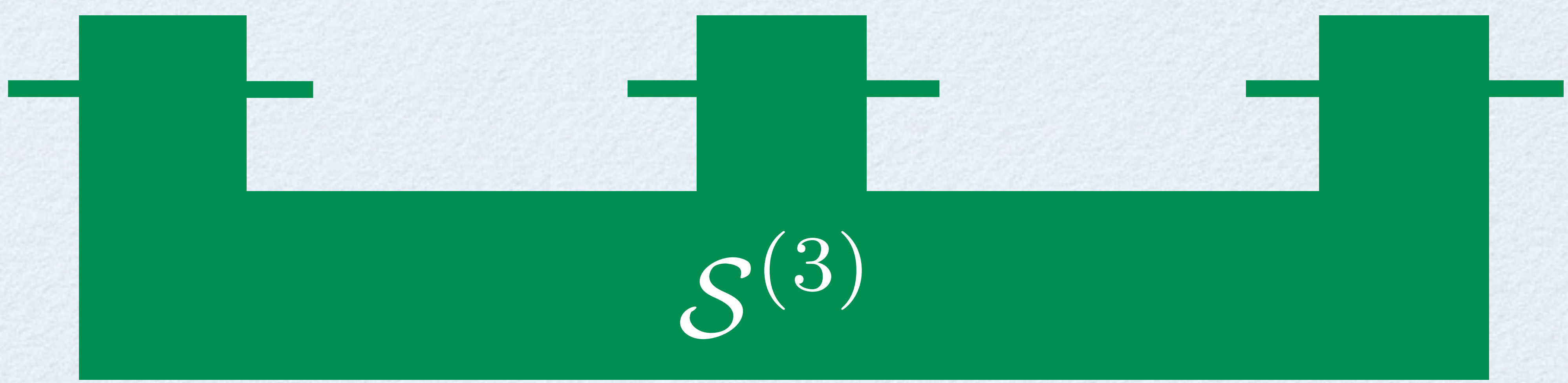
N=1 quantum channel



N=2



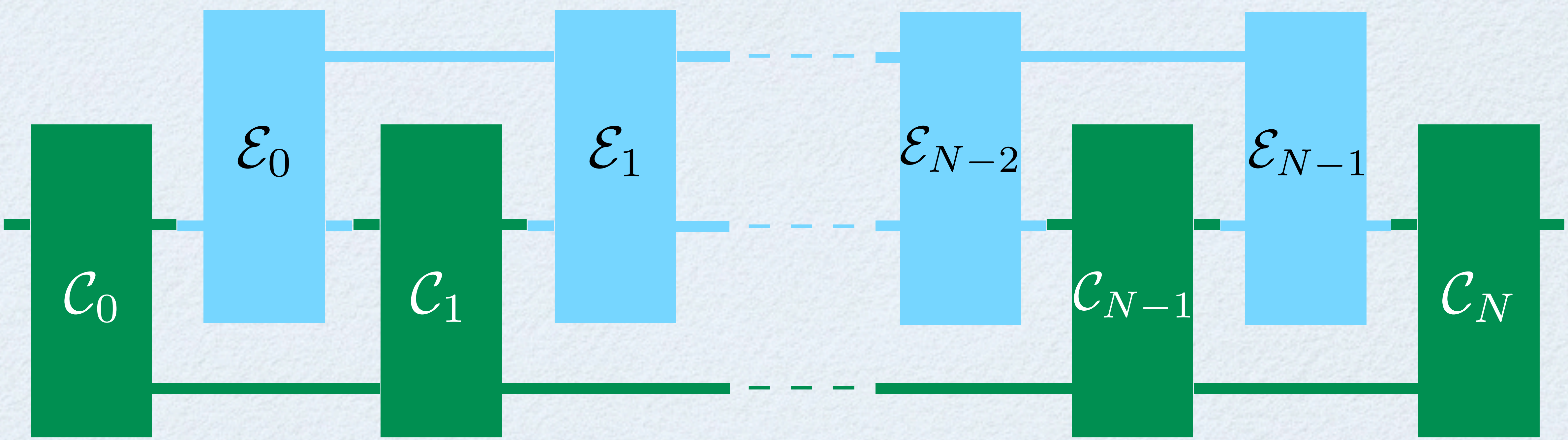
N=3



REALIZATION OF ADMISSIBLE N-MAPS

Theorem (GC, M D'Ariano, and P Perinotti, PRA 80, 2009):
any admissible N-map can be realized by a sequential network of quantum channels with memory.

The outcome of the application of an N-map to an (N-1)-map is the channel resulting from the interlinking of the corresponding networks.



RECONSTRUCTING CAUSAL SEQUENCES

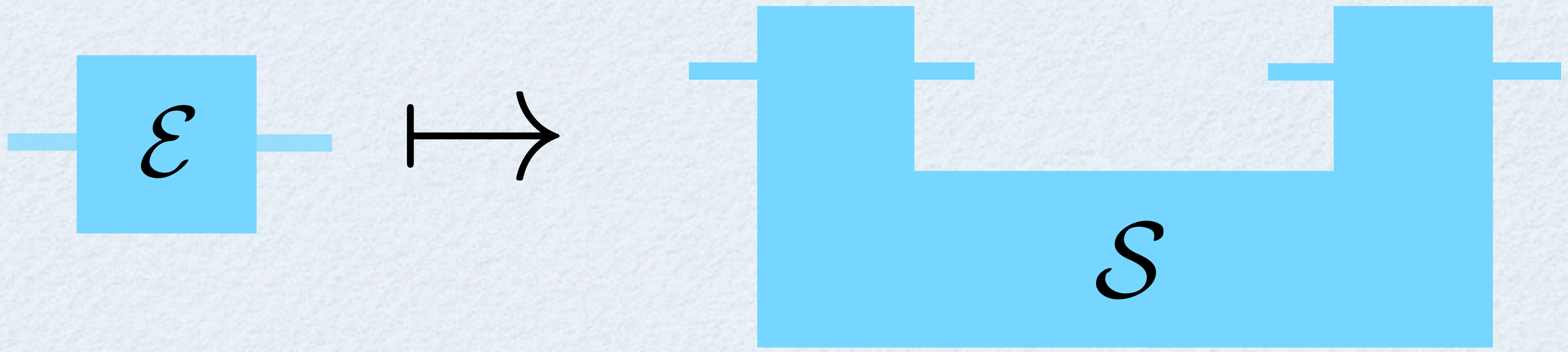
Remember the game we were playing:
forget everything except the fact that quantum states are density matrices.

Now, just by basic compositional reasoning,
we reconstructed causal sequences
of quantum channels!

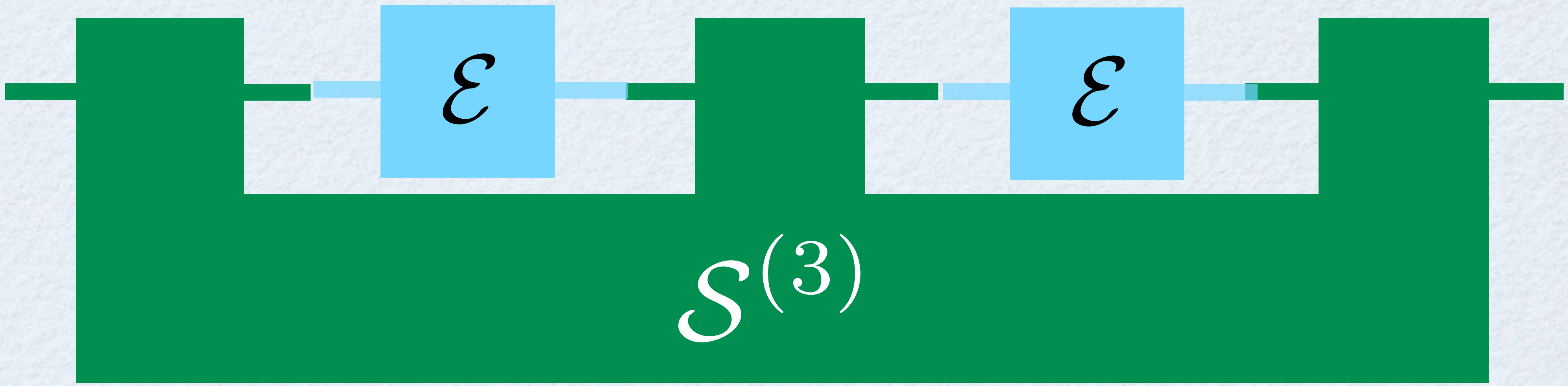
However, as anticipated, the hierarchy of higher-order transformations includes also a lot of new, non-causal stuff,
some of which are equivalent to CTCs.

THE EASIEST NON-CAUSAL EXAMPLE

Question: what is the most general transformation that maps a quantum channel into a quantum supermap?



In this case, there are at least two possible realizations:



MIXTURE AND SUPERPOSITION OF CAUSAL STRUCTURES

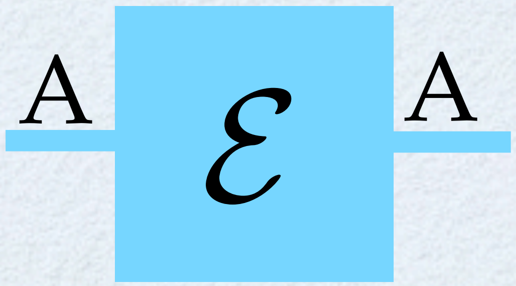
Since there are two possible choices of circuits,
surely we can choose randomly between them.

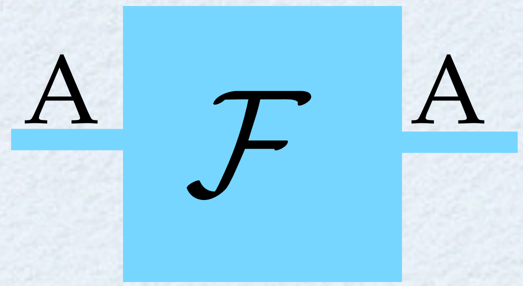
What's more, since quantum mechanics satisfies the
purification principle,
we can also generate a coherent superposition of these two scenarios...

THE QUANTUM SWITCH

THE TASK "SWITCH"

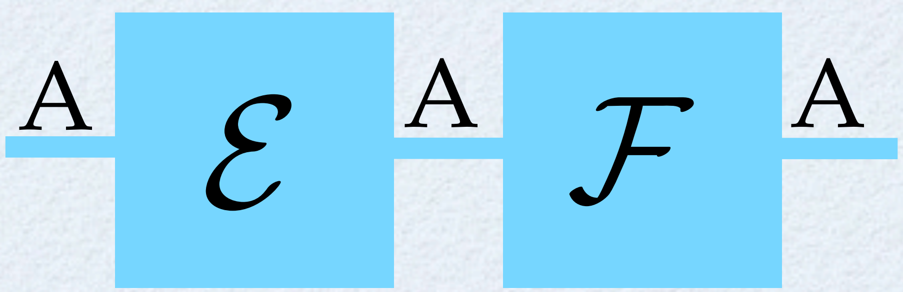
Suppose we are given two black boxes, implementing two generic channels $\mathcal{E}$ and $\mathcal{F}$:



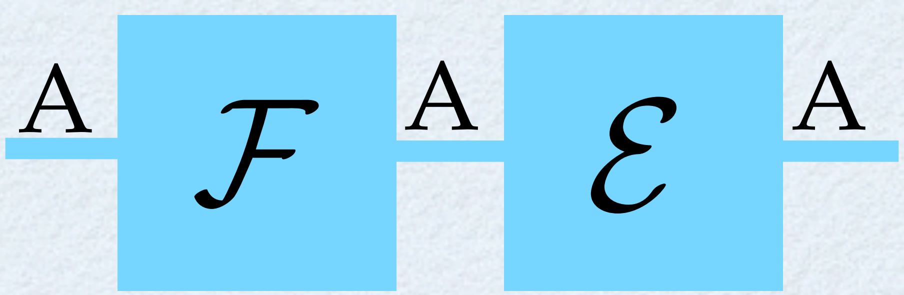


Suppose that we are given a qubit system Q

You want to connect the boxes as



if the state of the qubit is $\varphi_0 = |0\rangle\langle 0|$

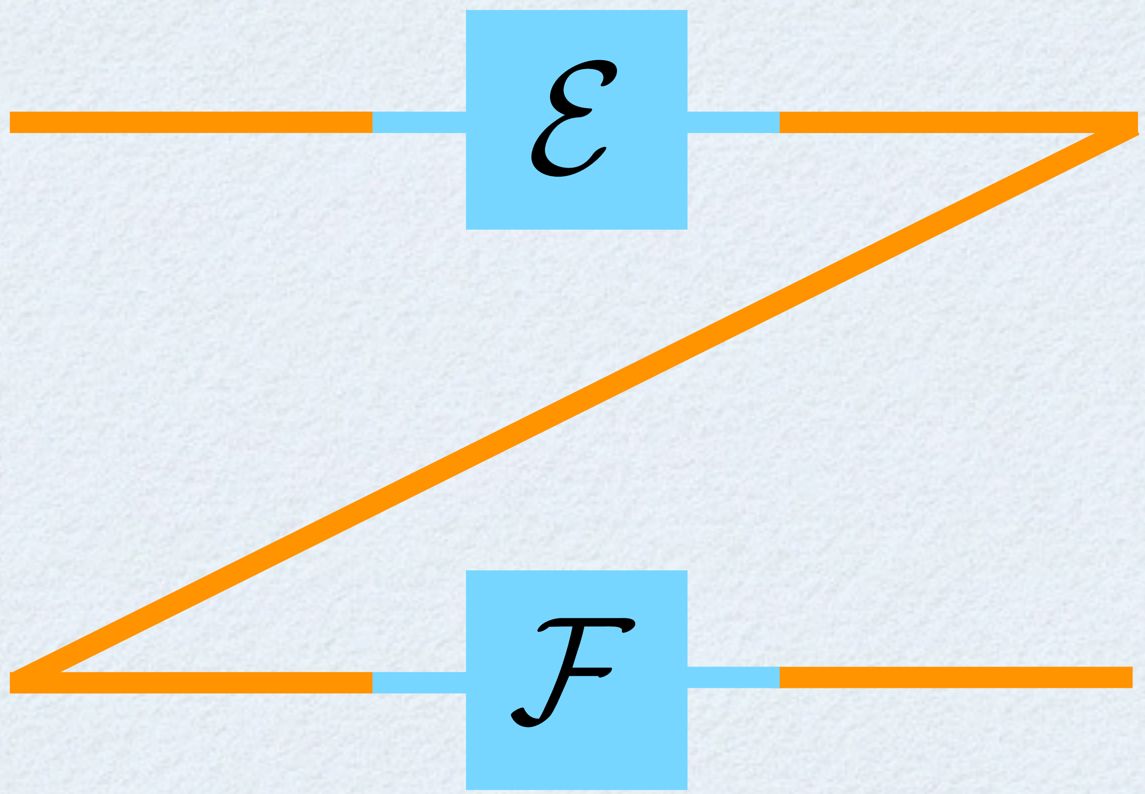


if the state of the qubit is $\varphi_1 = |1\rangle\langle 1|$

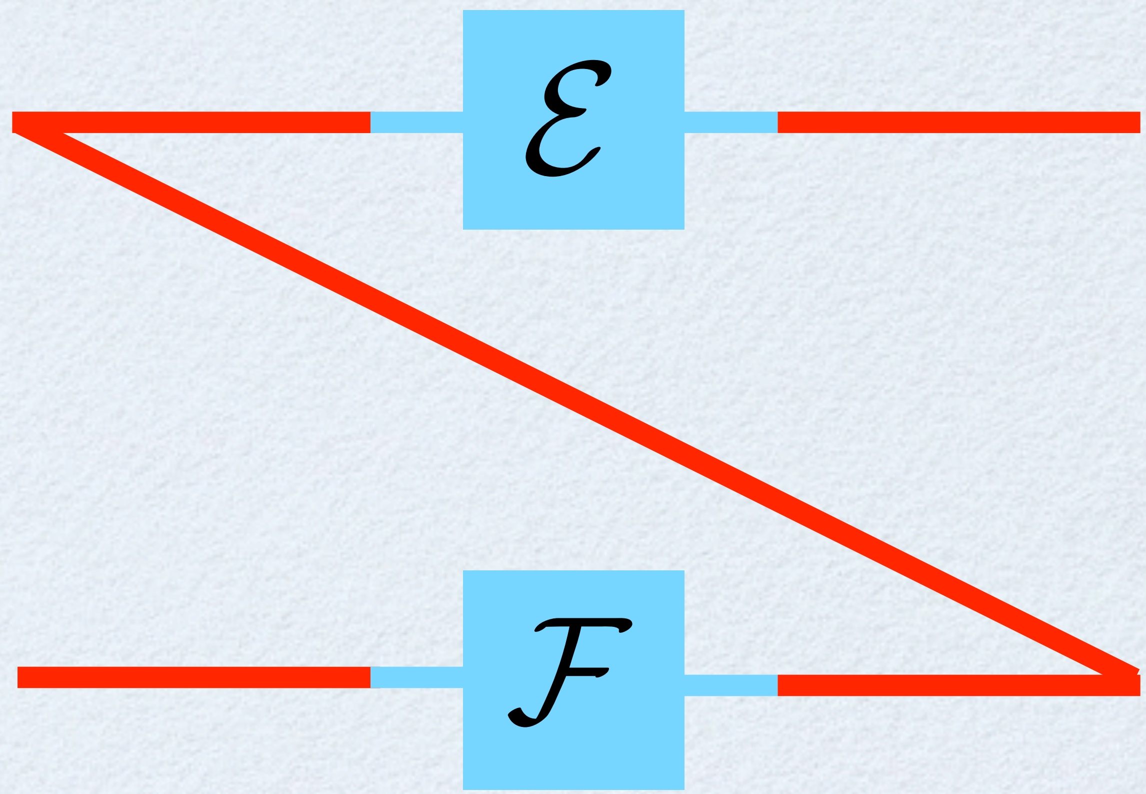
PHYSICAL INTUITION (I)

Imagine that the control qubit Q is able to control the path of system A:

State $|0\rangle$,
causal structure 0:
the system is routed first to $\mathcal{E}$ and then to $\mathcal{F}$



State $|1\rangle$
causal structure 1:
the system is routed first to $\mathcal{F}$ and then to $\mathcal{E}$

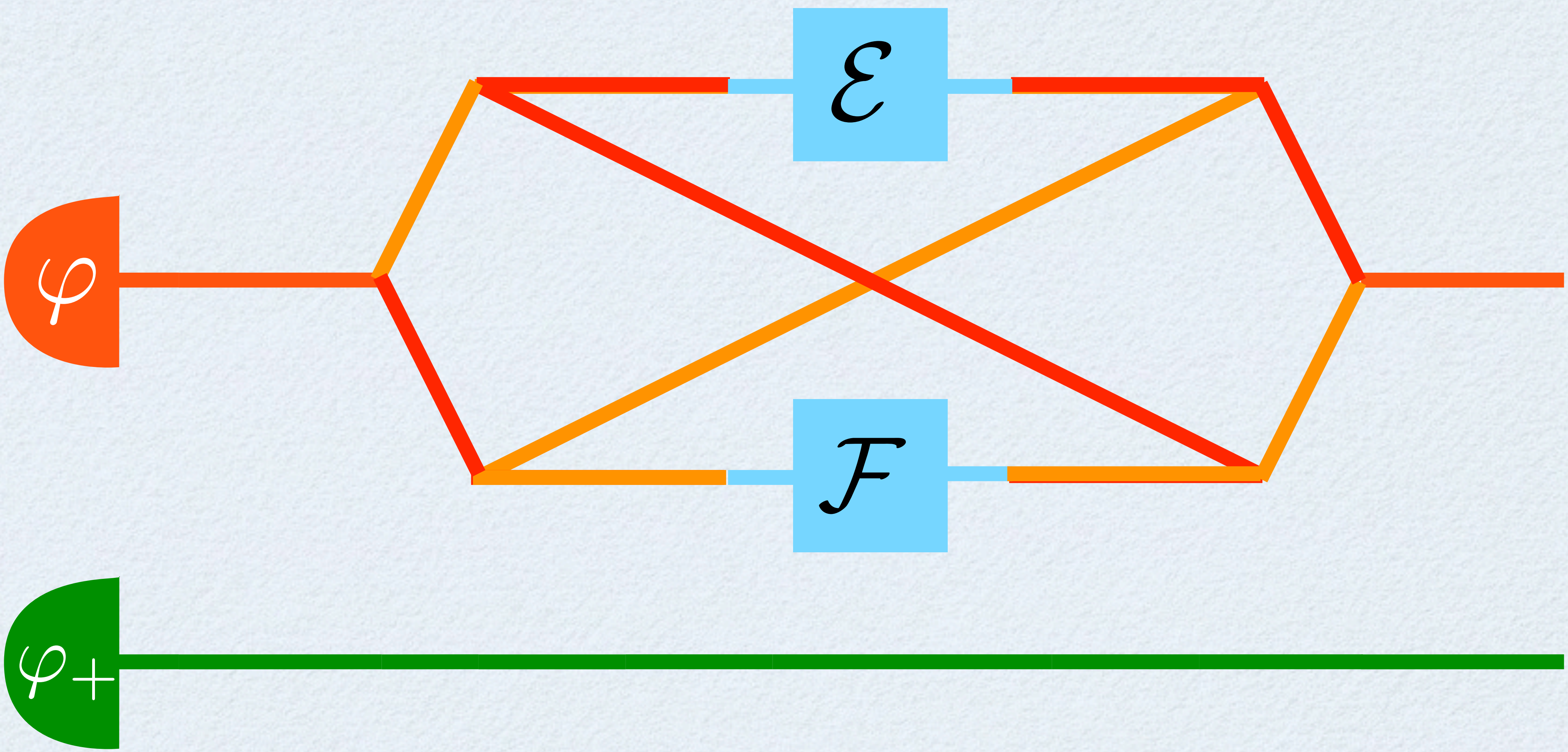


PHYSICAL INTUITION (II)

If Q is prepared in the superposition state

$$\varphi_+ = |+\rangle\langle+|, \quad |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

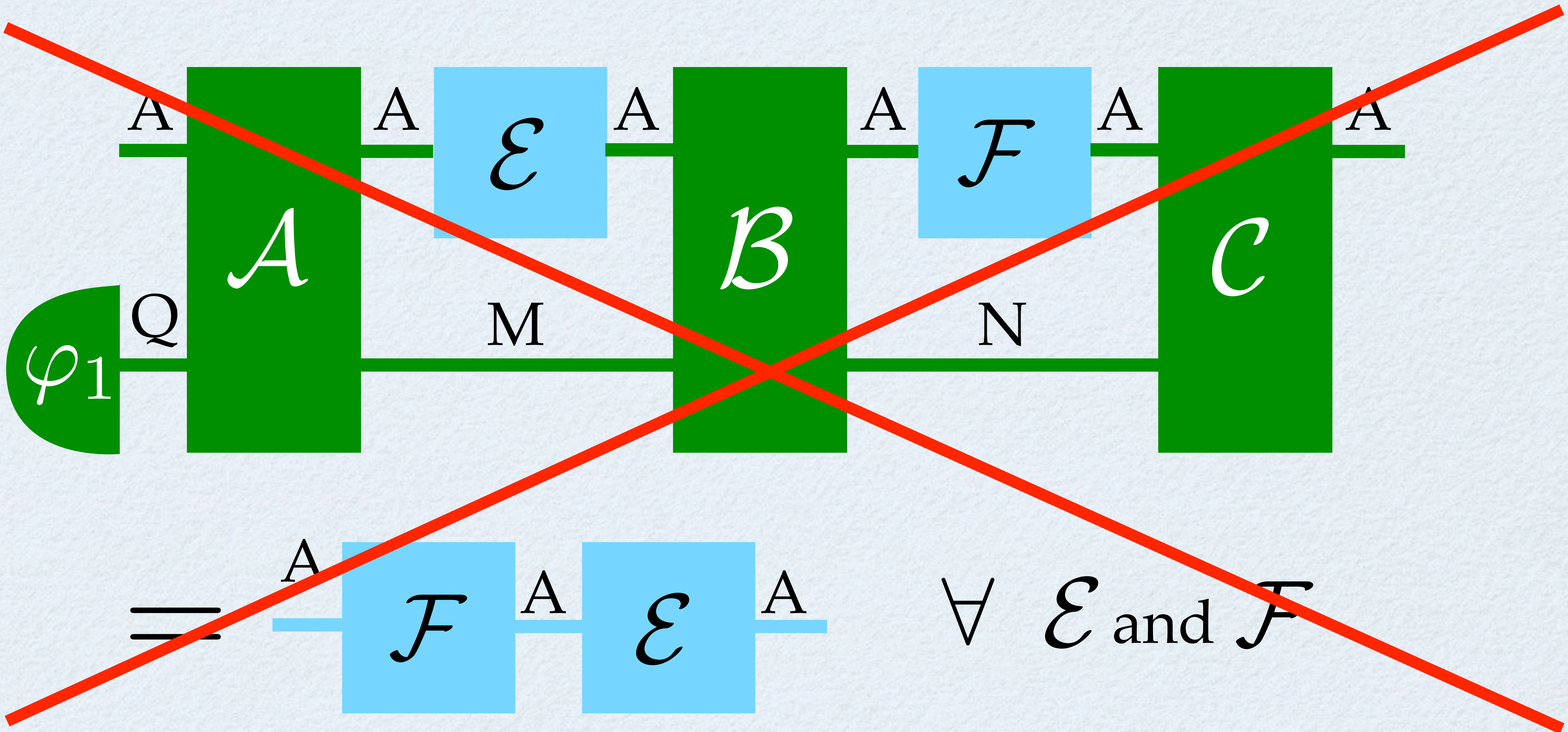
we will have the joint evolution:



NO-SWITCH OF BLACK BOXES IN A CIRCUIT

Theorem (GC, D'Ariano, Perinotti, Valiron, 2009)

It is impossible to find quantum systems M and N and quantum channels $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ such that



IN OTHER WORDS

Once two black boxes have been inserted in a circuit in a given order,
there is no way to invert their causal relation.

Theorem: The task SWITCH cannot be implemented by a quantum circuit that is both deterministic and causal
(i.e. a circuit where the two input channels are composed with quantum channels in a given causal sequence)

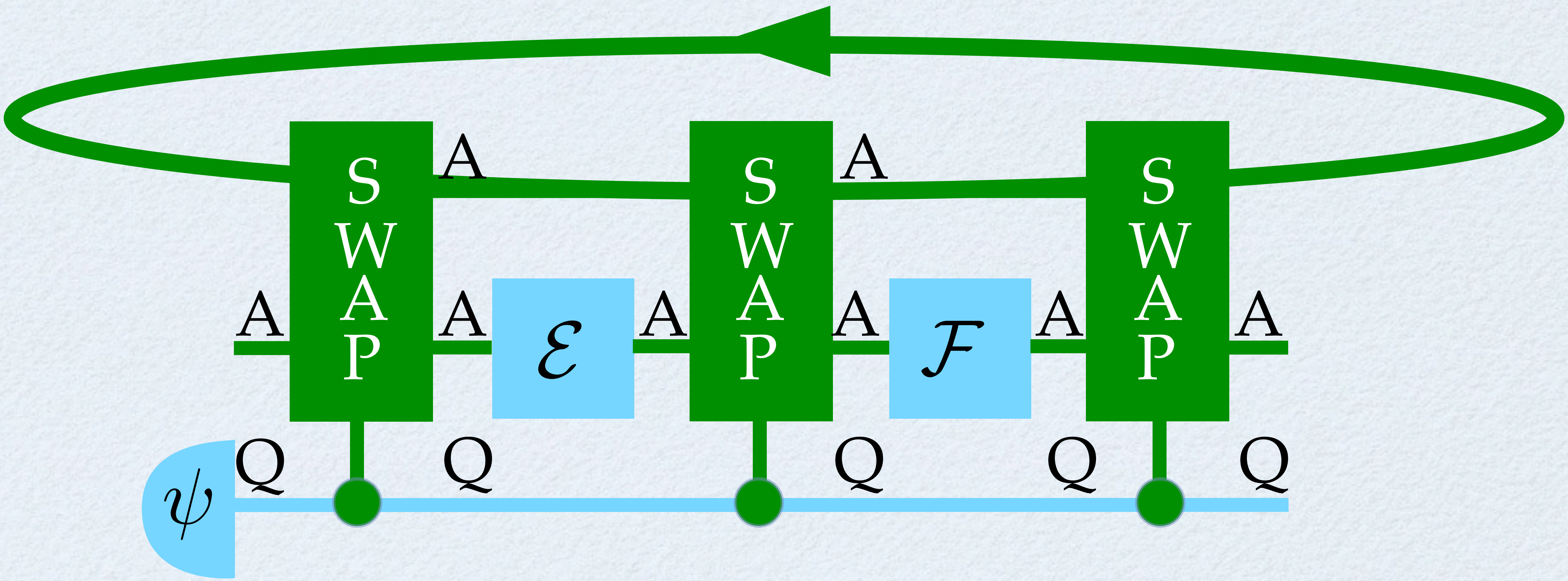
RELATION WITH TIME TRAVELS

Theorem: If a circuit
implements the task SWITCH,
then it must contain a CTC.

The converse can also be proved:

If we have access to a circuit of
that implements a deterministic time travel,
then we use it to construct a circuit that implements the task SWITCH.

REALIZATION OF THE SWITCH IN A CIRCUIT WITH CTC



INFORMATION-THEORETIC ADVANTAGE OF THE QUANTUM SWITCH:

DISCRIMINATION OF BLACK BOXES

A CLASSIFICATION PROBLEM

Problem: You are given two black boxes

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger \qquad \mathcal{F}(\rho) = \sum_i F_i \rho F_i^\dagger$$

with the following promise:

either (+) $\quad E_i F_j = F_j E_i \quad \forall i, j$

or (-) $\quad E_i F_j = -F_j E_i \quad \forall i, j$

Task: Find out whether the two black boxes are of type (+) or type (-)

PERFECT CLASSIFICATION PROTOCOL USING QUANTUM SWITCH

- Prepare the control qubit in the state $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
- Apply the Quantum Switch
- For every i,j in the (+) case you will obtain

$$|\varphi\rangle|+\rangle \mapsto \frac{F_j E_i|\varphi\rangle \otimes |0\rangle + E_i F_j|\varphi\rangle \otimes |1\rangle}{\sqrt{2}} = F_j E_i|\varphi\rangle \otimes |+\rangle$$

while in the (-) case you will obtain

$$|\varphi\rangle|+\rangle \mapsto \frac{F_j E_i|\varphi\rangle \otimes |0\rangle + E_i F_j|\varphi\rangle \otimes |1\rangle}{\sqrt{2}} = F_j E_i|\varphi\rangle \otimes |-\rangle$$

IMPOSSIBILITY OF PERFECT DISCRIMINATION WITHIN THE QUANTUM CIRCUIT MODEL

Theorem (GC, PRA 2012): No causal deterministic circuit can perfectly discriminate between the two classes of black boxes (+) and (-) using a single query.

For this classification problem
there is always a non-zero error in the framework of quantum circuits!

REFUTING THE "NO OPERATIONAL CONSEQUENCES" IDEA

It was longly believed that the only way CTCs can exist and be consistent with the resto of physics is that they do not lead to any observable consequence.

Here, instead, we have a consistent CTC that leads to an observable advantage in an operational setting.

Similar comment for the non-causal game invented by Oreshkov, Costa, and Brukner
(which is also an example of higher order map realizable in a circuit with CTC)

CONCLUSIONS

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- Probabilistic simulation of time travels within ordinary quantum theory:
  non-classical features, and roots in quantum info

- Introducing CTCs in quantum circuits:
  consistence requirement

- The hierarchy of higher-order quantum maps:
  abstract way to reconstruct causal (and non-causal) structures

- The quantum SWITCH: example of higher-order map that is not compatible with a pre-defined causal structure.

- Switching boxes is equivalent to introducing a CTC in the circuit

- Observable consequences of the CTC, without inconsistencies and without violations of standard quantum theory outside the CTC:
  classification of pair of boxes

RELATED WORKS:

- GC, D'Ariano, Perinotti,

Transforming quantum operations: quantum supermaps, EPL 2008
Quantum circuits architecture, PRL 2008
Theoretical framework for quantum networks, PRA 2009
Probabilistic theories with purification, PRA 2010
Beyond causally ordered quantum computers, arXiv:0912.0195

- Oreshkov, Costa, Brukner, Quantum correlations without causal order, Nature Commun, 2012

- Genkina, Hardy, GC, Optimal probabilistic simulation of quantum channels from the future to the past, PRA 2011

- GC, Perfect discrimination of no-signalling channels via quantum superposition of causal structures, PRA 2012