Local orthogonality: a multipartite principle for (quantum) correlations

Antonio Acín
ICREA Professor at ICFO-Institut de Ciencies Fotoniques, Barcelona

Causal Structure in Quantum Theory, Benasque, Spain, June 2013

ICFO Collaborators

Remigiusz Augusiak

Jonatan Bohr Brask

Rafael Chaves

Tobias Fritz

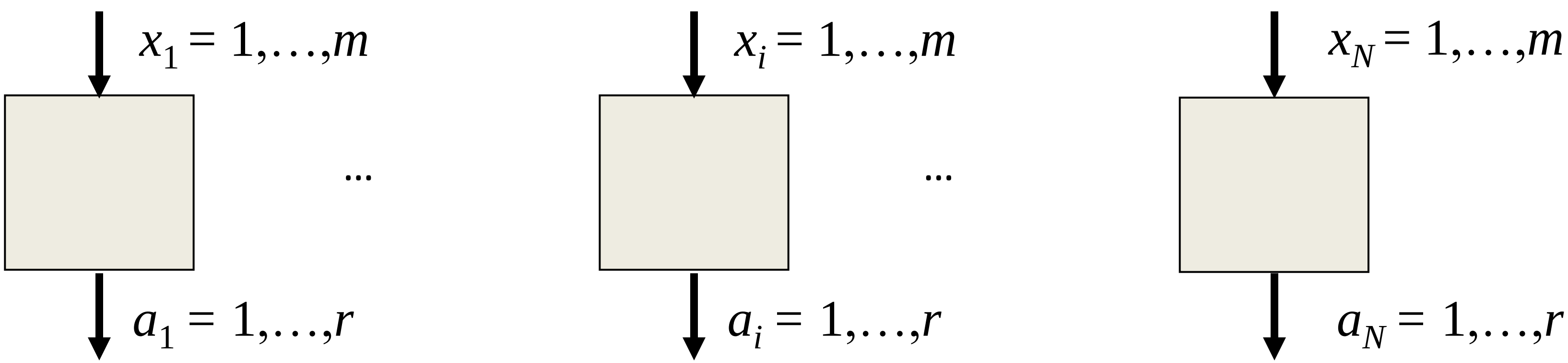
Anthony Leverrier

Belén Saínz

arXiv:1210.3018

Box-World Scenario

N distant parties performing m different measurements of r outcomes.



$$p(a_1,\ldots,a_N | x_1,\ldots,x_N)$$

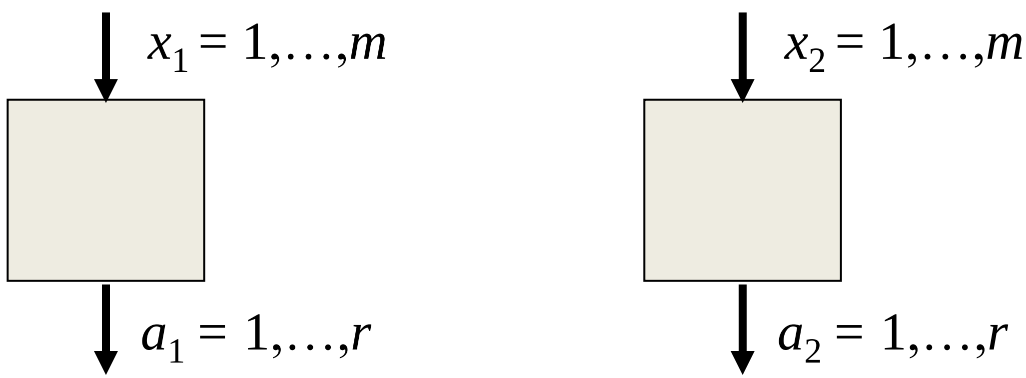
Physical Correlations

Physical principles translate into limits on correlations.

No-signalling correlations: correlations compatible with the no-signalling principle, i.e. the impossibility of instantaneous communication.

$$\sum_{a_{k+1},\ldots,a_N} p(a_1,\ldots,a_N|x_1,\ldots,x_N) = p(a_1,\ldots,a_k|x_1,\ldots,x_k)$$

$p(a_1|x_1)$



Physical Correlations

Classical correlations: correlations established by classical means.

$$p(a_1,\ldots,a_N|x_1,\ldots,x_N) = \sum_\lambda p(\lambda) D(a_1|x_1,\lambda)\ldots D(a_N|x_N,\lambda)$$

These are the standard "EPR" correlations. Independently of fundamental issues, these are the correlations achievable by classical resources. Bell inequalities define the limits on these correlations.

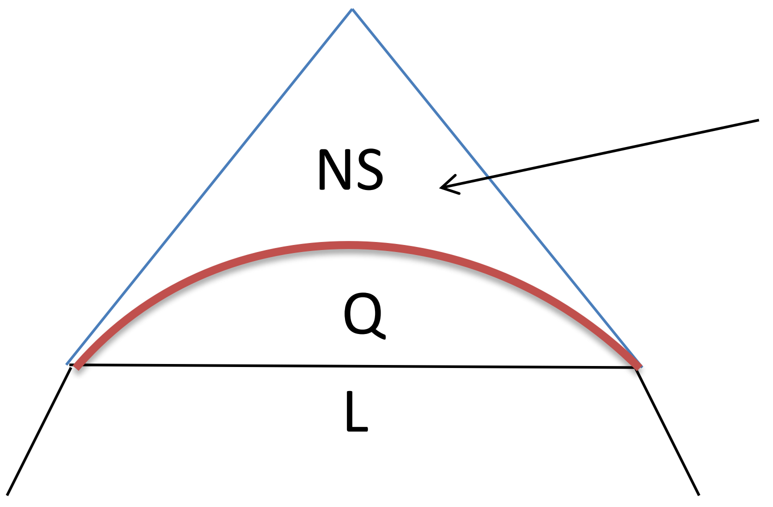
Physical Correlations

Quantum correlations: correlations established by quantum means.

$$p(a_1,\ldots,a_N|x_1,\ldots,x_N)=\mathrm{tr}\left(\rho\, M_{a_1}^{x_1}\otimes\cdots\otimes M_{a_N}^{x_N}\right)$$

$$\sum_{a_i} M_{a_i}^{x_i}=1 \qquad M_{a'_i}^{x_i}M_{a_i}^{x_i}=\delta_{a_i a'_i}M_{a_i}^{x_i}$$

Why quantum correlations?



Q: Why are these correlations not possible in Nature?

A: They are incompatible with quantum laws. That is, there is no quantum state and measurements able to reproduce them.

What would their existence imply operationally?

Information principles have been proposed as the mechanism to bound quantum correlations. Examples: non-trivial communication complexity, information causality, macroscopic locality.

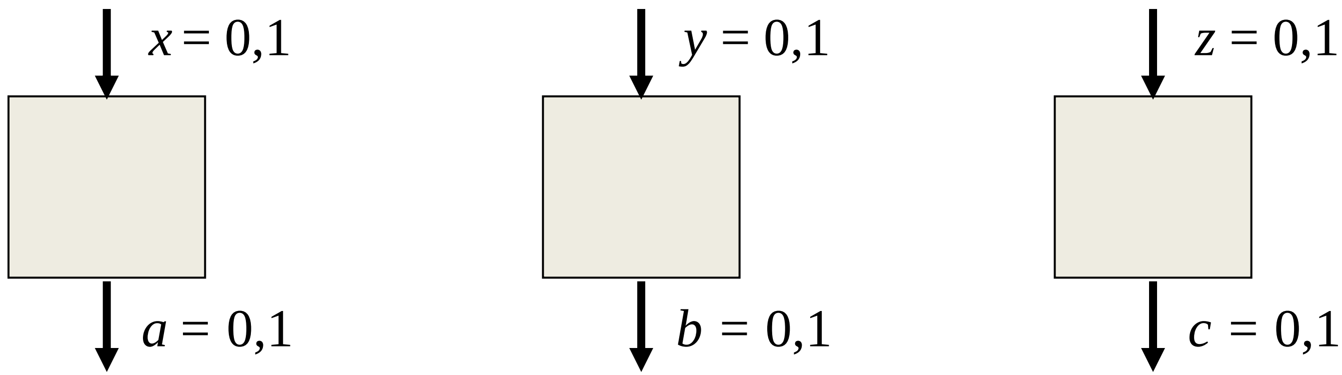
Guess Your Neighbour's Input (GYNI)

Alice and Bob receive two random bits, x and y. Their goal is to compute the bit the other party received. Clearly, winning too often would imply signalling.

$$P_{ok} = \frac{1}{4}\big(p(00|00) + p(01|10) + p(10|01) + p(11|11)\big)$$

Optimal classical strategy: the parties give their input as output → $P_{ok} = 1/2$.
This value is "universal", as violating it would imply signalling between the parties.
That is, quantum and supra-quantum non-signalling correlations do not improve it.

Guess Your Neighbour's Input (GYNI)



Alice has to guess the bit received by Bob, who has to guess the one received by Charlie, who has to guess Alice's bit.

$$P_{ok} = \frac{1}{8}\big(p(000|000) + p(010|001) + p(100|010) + p(110|011) + p(001|100) + p(011|101) + p(101|110) + p(111|111)\big)$$

Optimal classical strategy: the parties give their input as output → $P_{ok} = 1/4$.
This value is "universal", as violating it would imply signalling between the parties.
That is, quantum and supra-quantum non-signalling correlations do not improve it.

Guess Your Neighbour's Input (GYNI)

$$P_{ok} = \frac{1}{8}(p(000|000) + p(010|001) + p(100|010) + p(110|011) + p(001|100) + p(011|101) + p(101|110) + p(111|111))$$

Promise: the sum of the inputs is zero, ie $x \oplus y \oplus z = 0$.

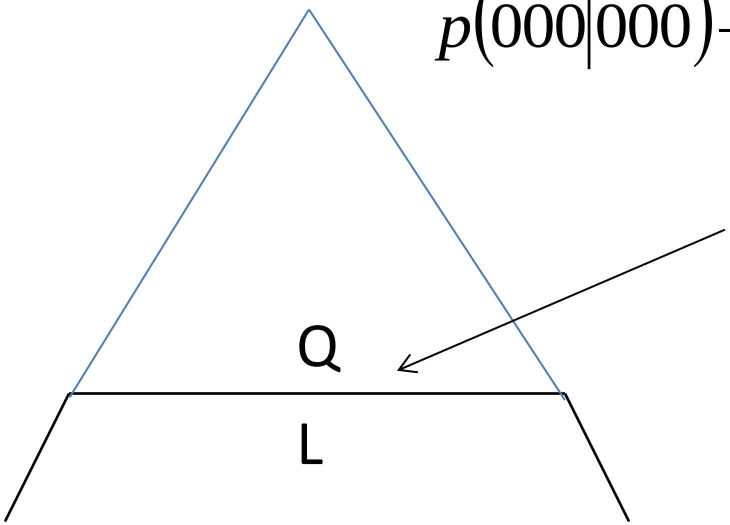
$$P_{ok} = \frac{1}{4}(p(000|000) + p(110|011) + p(011|101) + p(101|110))$$

Intuition: it should be the same as Alice's bit does not provide any information about Bob's, and the same applies for all the parties.

Optimal classical strategy: the parties give their input as output → $P_{ok} = 1/4$.
This limit is again valid for parties having access to correlated quantum particles.
Yet, it is possible to get a larger probability without violating the no-signalling principle! Why?!

Guess Your Neighbour's Input (GYNI)

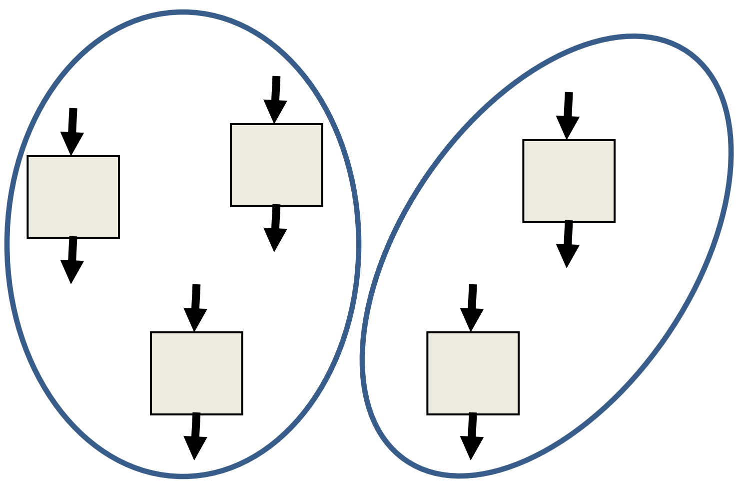
$$p(000|000) + p(110|011) + p(011|101) + p(101|110) \leq 1$$



First tight task with no quantum violation.

Almeida et al, PRL'10

The no-signalling principle is intrinsically bipartite.



Local orthogonality:
a multipartite principle

Local orthogonality

Local orthogonality: different outcomes of the same measurement by one of the observers define orthogonal events, independently of the rest of measurements.

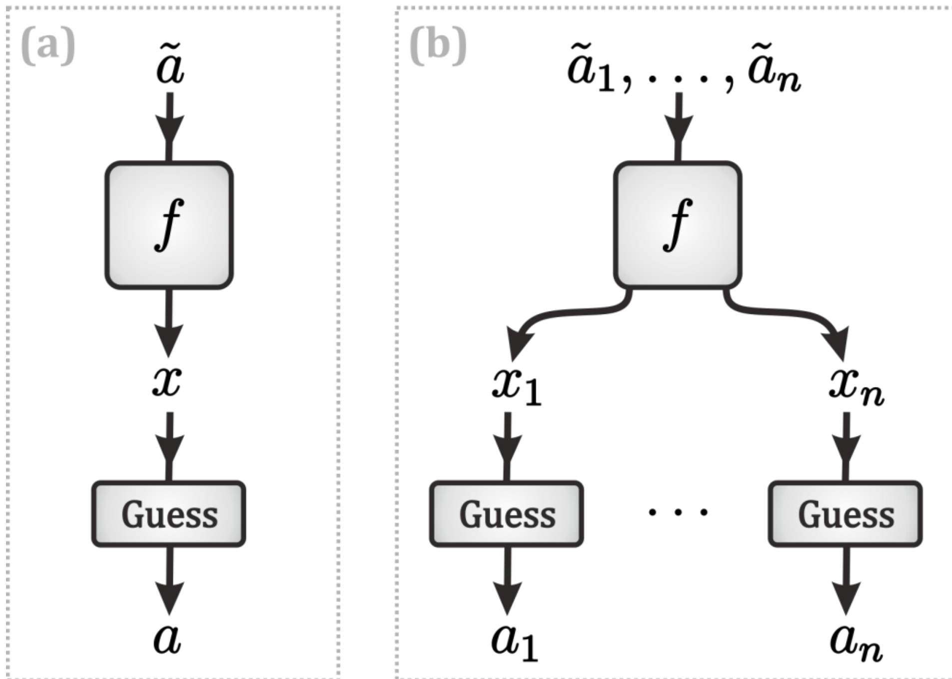
Event	Input	Output
e_1	$x_1 \ldots \boldsymbol{x_i} \ldots x_N$	$a_1 \ldots \boldsymbol{a_i} \ldots a_N$
e_2	$x'_1 \ldots \boldsymbol{x_i} \ldots x'_N$	$a'_1 \ldots \boldsymbol{\bar{a}_i} \ldots a'_N$

N events are orthogonal if they are pairwise orthogonal.

Operationally: the sum of probabilities of pairwise orthogonal events is bounded by 1.

$$\sum_{e_i} p(e_i) \leq 1$$

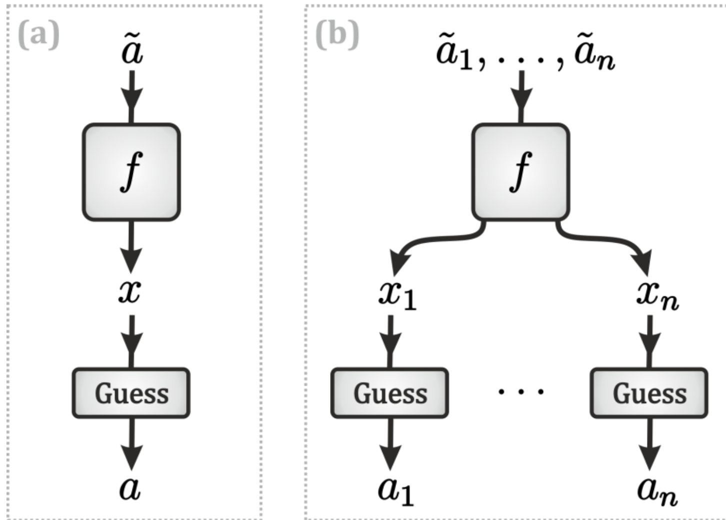
LO as a distributed guessing problem



(a) In a standard guessing problem, a value $\tilde{a}$ to be guessed is encoded by a function f and the goal is to make a guess a about the encoded value.

(b) In a Distributed Guessing Problem (DGP) a string of bit is encoded on a string of N bits that are distributed among distant parties, who have to make a guess.

LO as a distributed guessing problem



- The figure of merit is the probability of making a right guess.
- If the initial bit string can take S values, this probability is lower bounded by $1/S$.
- There exist functions for which the optimal guessing probability for classically correlated players is equal to $1/S$. We call these functions maximally difficult.
- In non-distributed problems, the only maximally difficult function is the trivial one in which the function maps all the values into one, it erases all the information.
- In distributed versions, there exist other non-trivial maximally difficult functions.
- Correlations violating LO turn maximally difficult functions for classical players into non-maximally difficult.

LO and quantum correlations

Quantum correlations satisfy LO.

Proof:

Event	Input	Output
e_1	$x_1 \ldots \boldsymbol{x_i} \ldots x_N$	$a_1 \ldots \boldsymbol{a_i} \ldots a_N$
e_2	$x'_1 \ldots \boldsymbol{x_i} \ldots x'_N$	$a'_1 \ldots \boldsymbol{\bar{a}_i} \ldots a'_N$

$$\max p(e_1) + p(e_2) = \max\langle\psi|\Pi^{x_1,a_1} \otimes \cdots \otimes \Pi^{x_i,a_i} \otimes \cdots \otimes \Pi^{x_N,a_N} + \Pi^{x'_1,a'_1} \otimes \cdots \otimes \Pi^{x_i,\bar{a}_i} \otimes \cdots \otimes \Pi^{x'_N,a'_N} |\psi\rangle \leq \langle\psi|I|\psi\rangle=1$$

Local orthogonality is satisfied both by classical and quantum theory.
Indeed, while quantum physics breaks the orthogonality of preparations, it keeps the orthogonality of measurement outcomes .
Intuition: measurement outcomes are always of classical nature.

LO and the no-signalling principle

For two parties: compatibility with LO $\leftrightarrow$ non-signalling correlations.

Cabello, Severini and Winter

For more parties: LO is strictly more restrictive than no-signalling.

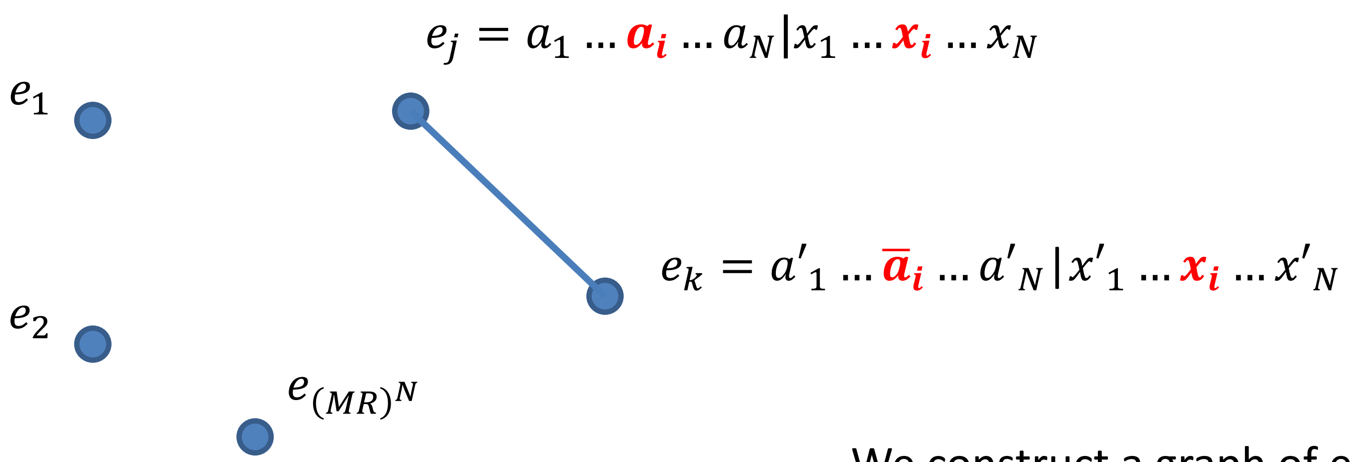
Example: GYNI.

$$p(000|000)+p(110|011)+p(011|101)+p(101|110)\leq 1$$

All events in GYNI are pairwise orthogonal.

LO and graph theory

How to get LO inequalities in a general scenario consisting of *N* parties making *M* measurements of *R* possible outcomes?
There are M^N possible combination of inputs. For each of them, there are R^N possible results. This makes $(MR)^N$ different events.

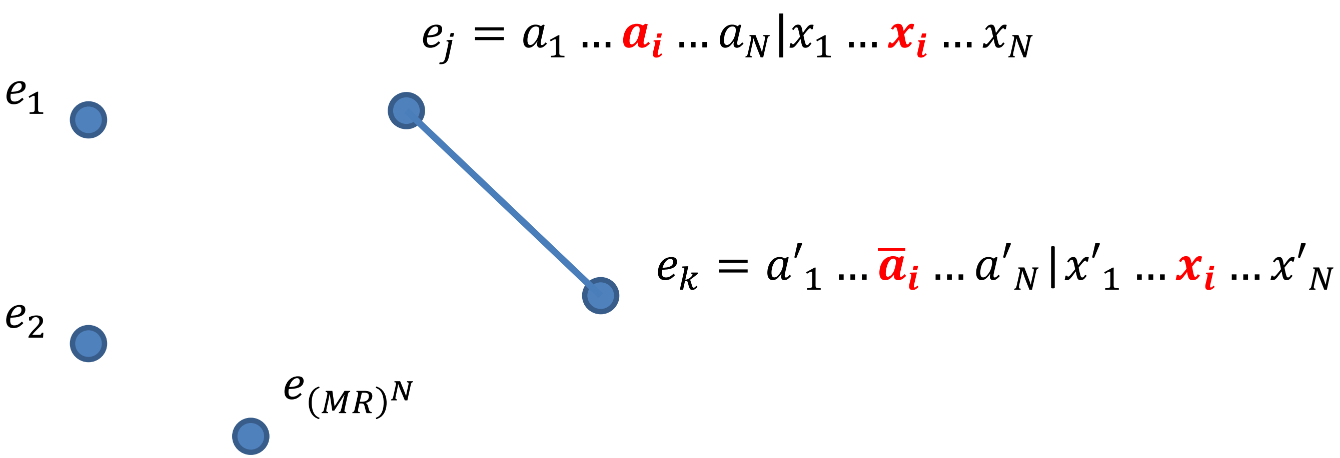


Cabello, Severini and Winter

We construct a graph of events:

- Nodes: events.
- Edges: orthogonality condition.

LO and graph theory



Clique: fully connected subgraph → set of pairwise orthogonal events.
Maximum clique → optimal LO inequality.
There exist algorithm to find cliques of a graph. Recall that finding the maximum clique of an arbitrary graph is an NP-hard problem. These graphs are not arbitrary.

LO and extremal tripartite correlations

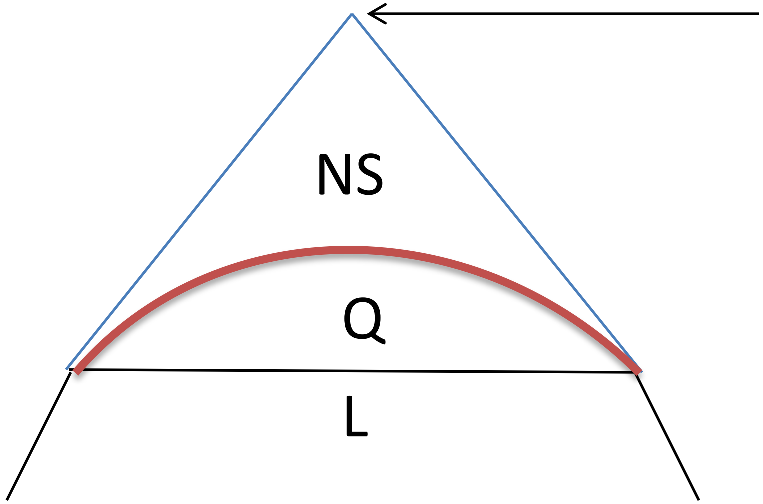
- All extremal non-signalling correlations for 3 observers performing 2 measurements of 2 outcomes were listed in **S. Pironio et al, JPA'11**. They can be classified into 46 classes (one of them corresponding to local points).

- All but one of the 45 classes of non-local correlations can be ruled out by information causality (**Tzyh Haur et al, NJP'12**).

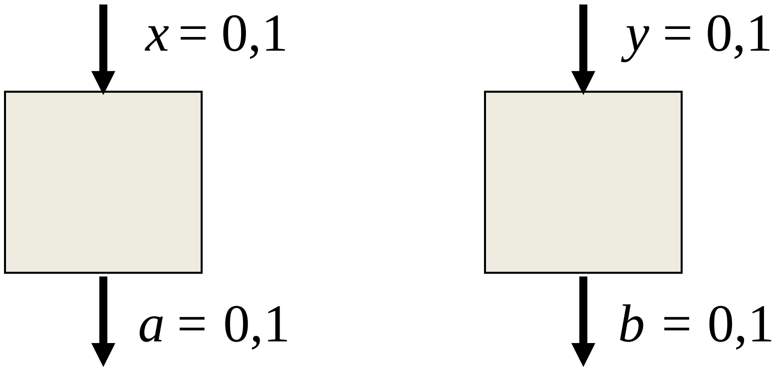
- The remaining point, box 4, is an example of a point that cannot be falsified by bipartite principles.

- All the tripartite boxes contradict LO and, thus, do not have a quantum realization. In particular, it rules out box 4 because of its intrinsically multipartite formulation.

LO and bipartite correlations



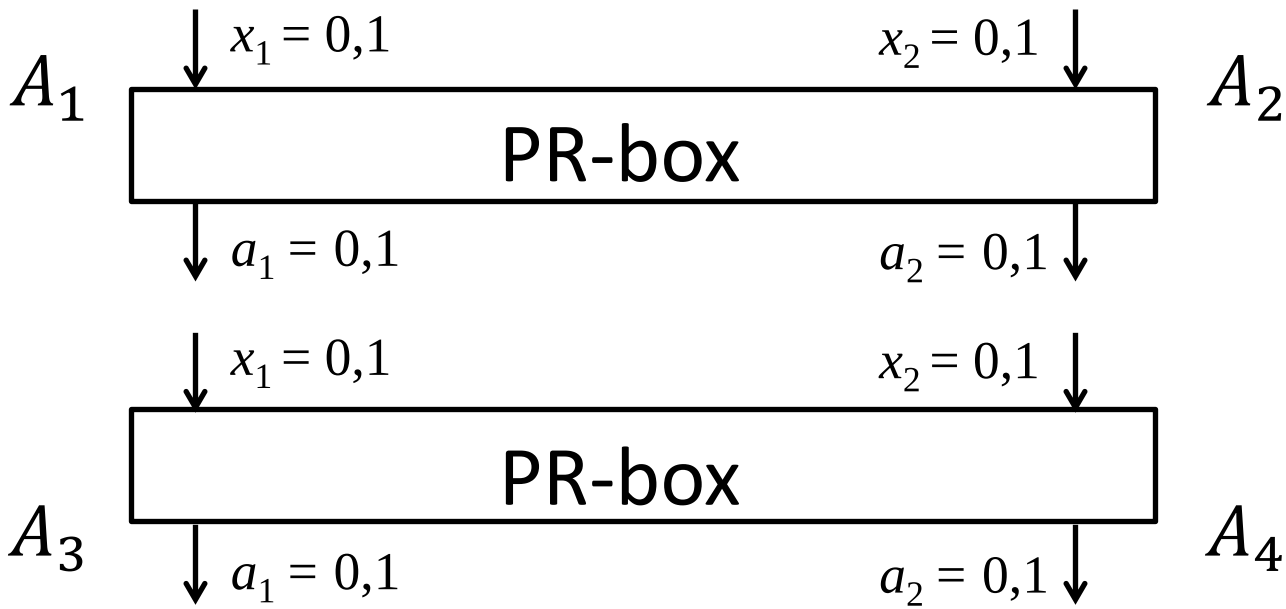
Popescu-Rohrlich (PR)-box



$$p(ab|xy) = \left(\frac{1}{2}, 0,0, \frac{1}{2}; \frac{1}{2}, 0,0, \frac{1}{2}; \frac{1}{2}, 0,0, \frac{1}{2}; 0, \frac{1}{2}, \frac{1}{2}, 0\right)$$

LO and bipartite correlations

Despite the equivalence with NS for two parties, LO can be used to rule out supra-quantum bipartite correlations. How? Use **networks**.

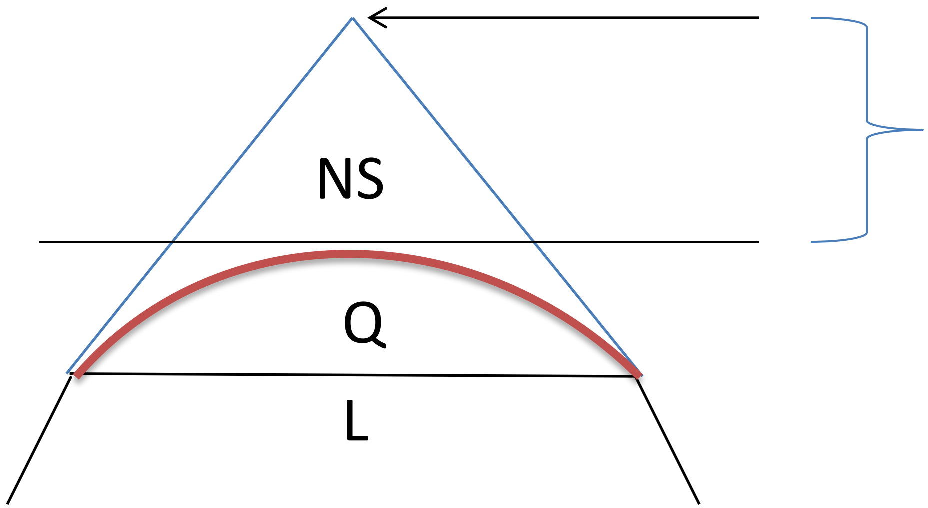


Check now for violation of LO inequalities for 4 parties.

LO and bipartite correlations

Two PR-boxes distributed among 4 observers violate the LO inequality:

$$p(0000|0000) + p(1110|0011) + p(0011|0110) + p(1101|1011) + p(0111|1101) \leq 1$$



All supra-quantum correlations in this region violate LO.

Conjecture

Conjecture: Local orthogonality defines the quantum set.

Principle: there is always someone smarter than you!

Navascués: there are supra-quantum correlations compatible with LO!

In fact, the set of LO correlations is not even convex!

LO and contextuality

Our approach easily extends to non-contextuality scenario. This has been studied for instance in:

T. Frizt, A. Leverrier and A.B. Sainz, arXiv:1212.4084

A. Cabello, Phys. Rev. Lett. 110 (2013) 060402

B. Yan, arXiv:1303.4357

Conclusions

- Multipartite principle are needed for our understanding of quantum correlations.
- Local orthogonality is an intrinsically multipartite principle.
- It captures the classical nature of measurement outcomes: outcomes of the same measurement define incompatible events.
- It is a powerful method when combined with graph-theory concepts and network geometries.
- It rules out supra-quantum correlations, both in the bipartite and multipartite case.
- The principle alone does not give quantum correlations.
- What else is needed to define quantum correlations?