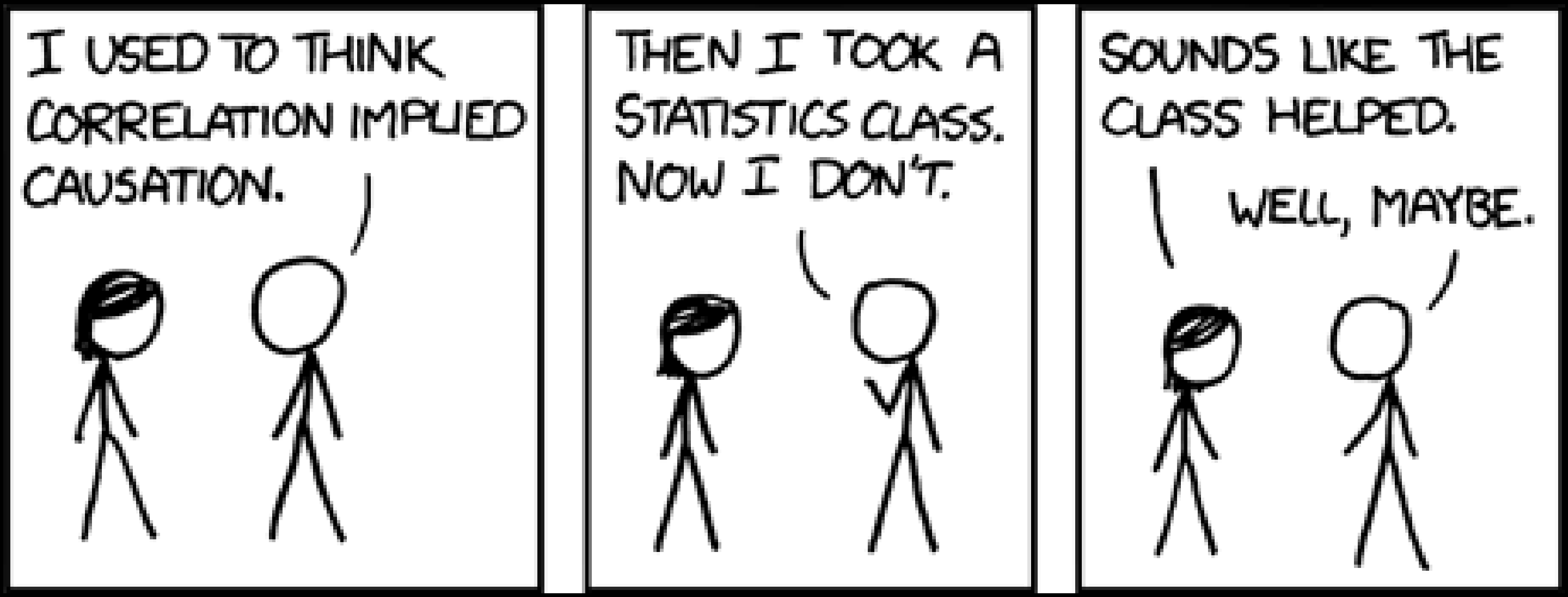
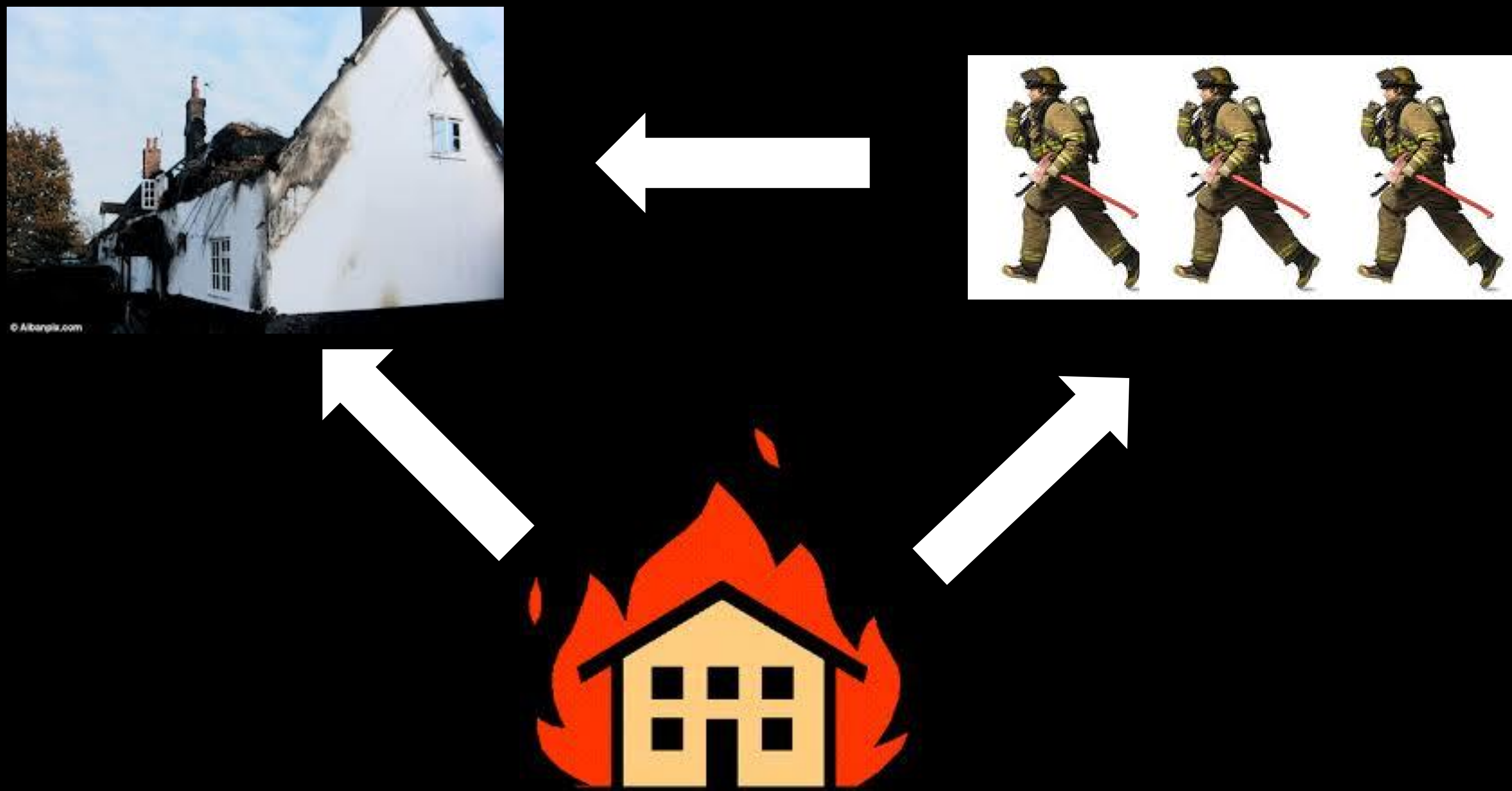
If Correlation Doesn't Imply Causation, What Does?

Rob Spekkens



From XKCD comics

Causal Structure in Quantum Theory, Benasque, June 3, 2013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Simpson's Paradox

P(recovery | drug) > P(recovery | no drug)

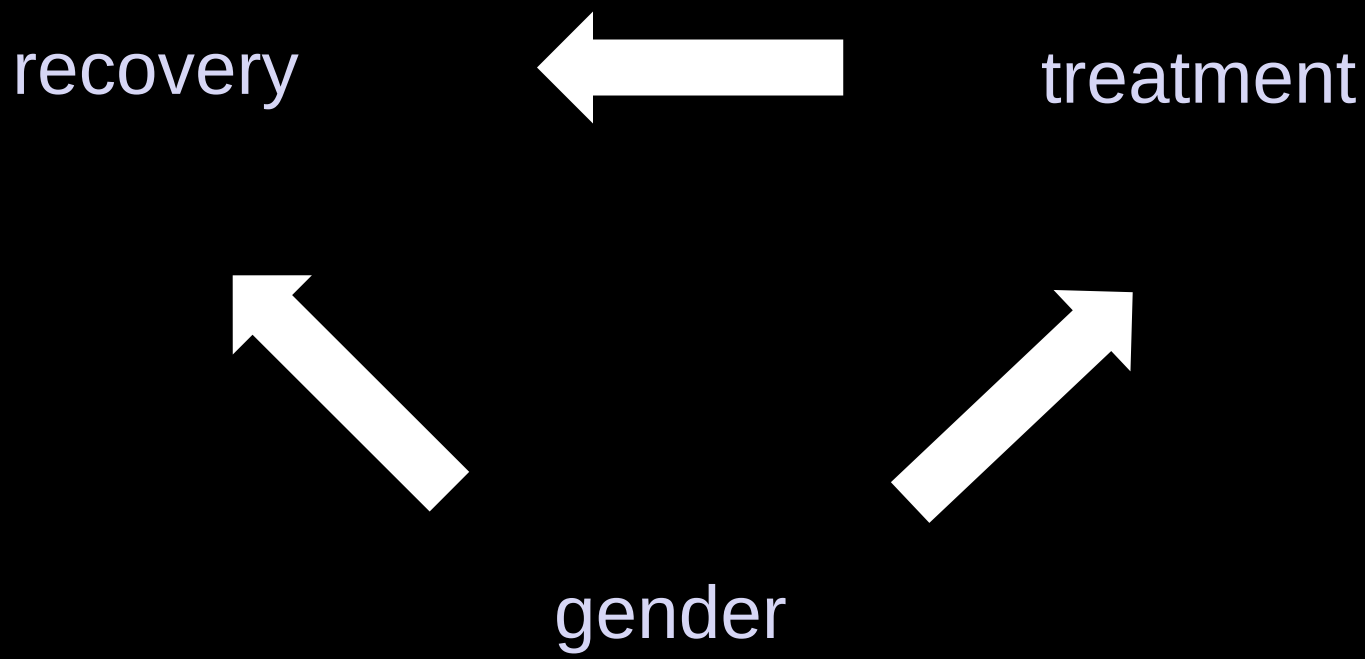
P(recovery | drug, male) < P(recovery | no drug, male)

P(recovery | drug, female) < P(recovery | no drug, female)

Recovery probability

	drug	**no drug**
male	180/300 = 60%	70/100 = 70%
female	20/100 = 20%	90/300 = 30%
combined	200/400 = 50%	160/400 = 40%

Simpson’s Paradox



Simpson's Paradox

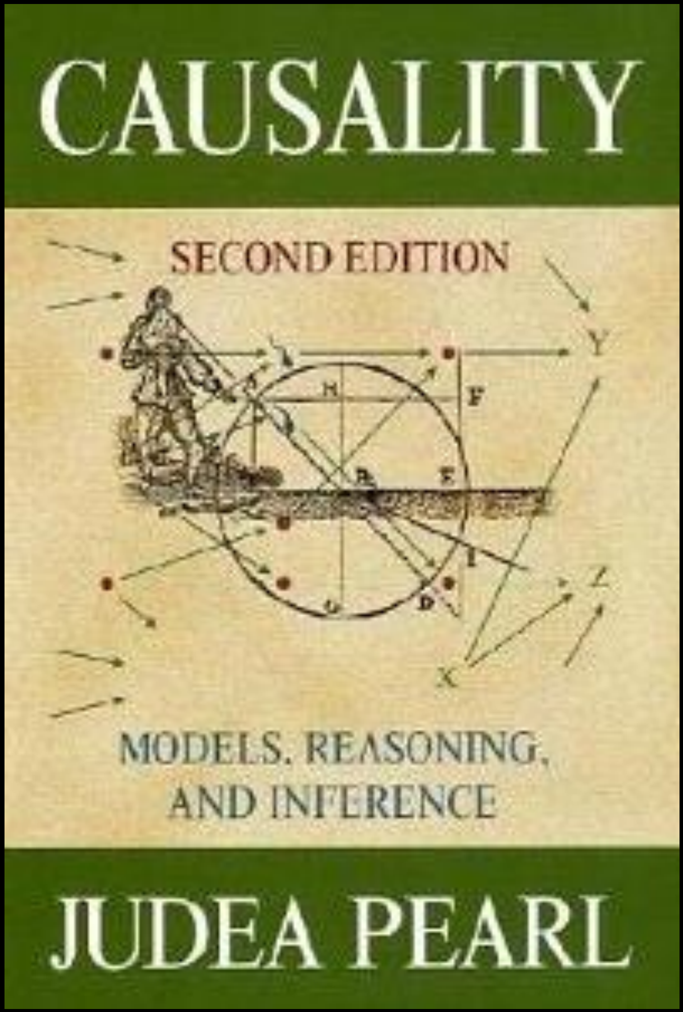
P(recovery | do (drug)) $\neq$ P(recovery | observe (drug))

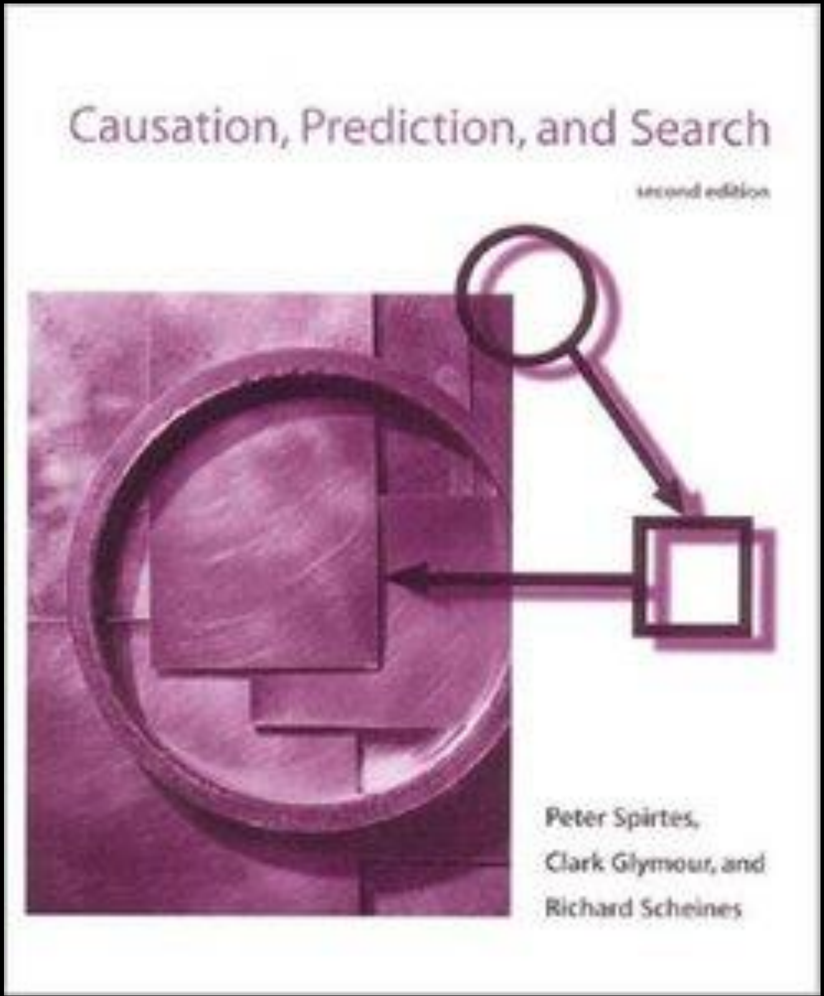
causation correlation

What formalism can we use to describe causal relations?

How do we come to have knowledge of causal relations?
(“we” = children, scientists, machine learning systems)

How do we come to have knowledge of causal relations in uncontrolled experiments?

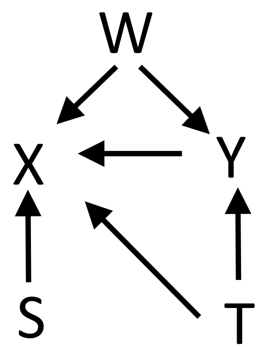

CAUSALITY
SECOND EDITION
MODELS, REASONING,
AND INFERENCE
JUDEA PEARL


Causation, Prediction, and Search
second edition
Peter Spirtes,
Clark Glymour, and
Richard Scheines

What is a Causal Model?

Causal Model

Causal Structure



Causal-Statistical Parameters

$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

Reichenbach’s principle

No correlation without causation!

If X and Y are correlated, then either

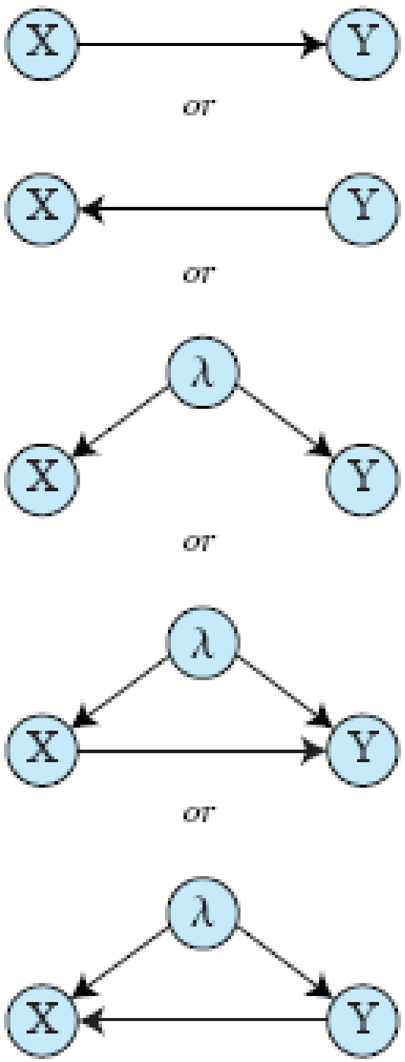
(i) X causes Y

(ii) Y causes X

(iii) X and Y have a common cause

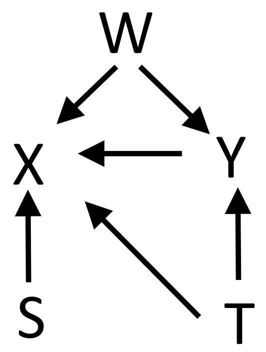
(iv) both (i) and (iii)

(v) both (ii) and (iii)



Causal Model

Causal Structure



Causal-Statistical Parameters

$P(W)$

$P(S)$

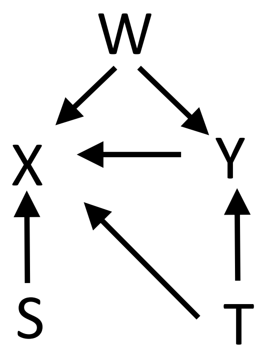
$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

- Parentless variables are independently distributed
- Conditionals arise from *autonomous* mechanisms

Given a causal model, what sorts of correlations can arise?



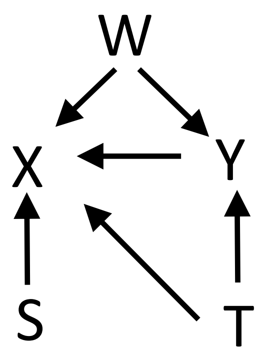
$$P(W)$$
$$P(S)$$
$$P(T)$$
$$P(X|S,T,W,Y)$$
$$P(Y|T,W)$$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal inference algorithms seek to solve the inverse problem

Inferring facts about the causal structure from statistical independences

Given a causal model, what sorts of correlations can arise?



$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Def'n: A and B are marginally independent

$$P(A|B) = P(A)$$

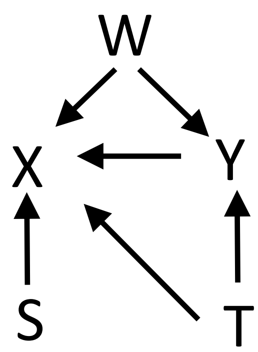
$$P(B|A) = P(B)$$

$$P(A,B) = P(A)P(B)$$

Denote this

$(A \perp B)$

Given a causal model, what sorts of correlations can arise?



$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

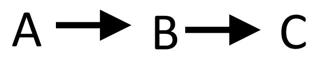
$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Def'n: A and B are conditionally independent given C

$$P(A|B,C) = P(A|C)$$
$$P(B|A,C) = P(B|C)$$
$$P(A,B|C) = P(A|C)P(B|C)$$

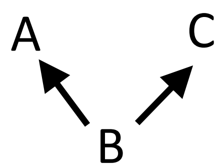
Denote this $(A \perp B|C)$

chain



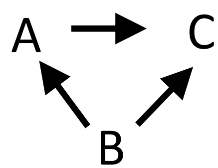
$$A \not\perp C$$
$$(A \perp C | B)$$

fork



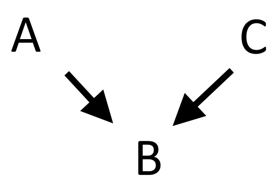
$$A \not\perp C$$
$$(A \perp C | B)$$

confounded cause



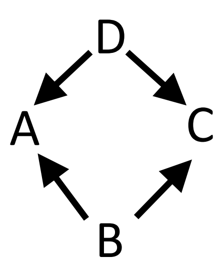
$$A \not\perp C$$
$$(A \not\perp C | B)$$

collider



$$A \perp C$$
$$(A \not\perp C | B)$$

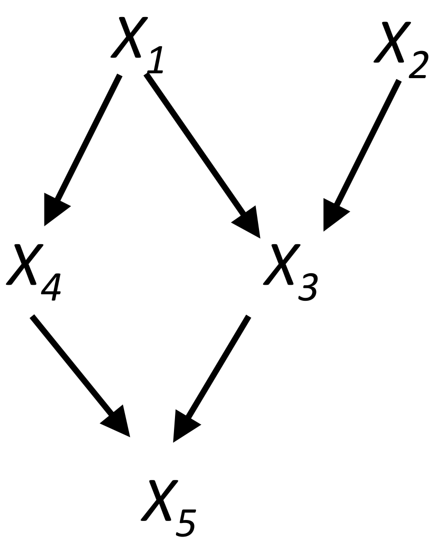
Pair of forks



$$A \not\perp C$$
$$(A \not\perp C | B)$$
$$(A \perp C | B, D)$$

Markov condition: The joint distribution induced by a causal model is such that every variable X is conditionally independent of its nondescendants given its parents,

$$(X \perp \text{Nondescendants}(X) \mid \text{Parents}(X))$$



$$(X_1 \perp X_2)$$
$$(X_2 \perp \{X_1, X_4\})$$
$$(X_3 \perp X_4 \mid \{X_1, X_2\})$$
$$(X_4 \perp \{X_2, X_3\} \mid X_1)$$
$$(X_5 \perp \{X_1, X_2\} \mid \{X_3, X_4\})$$

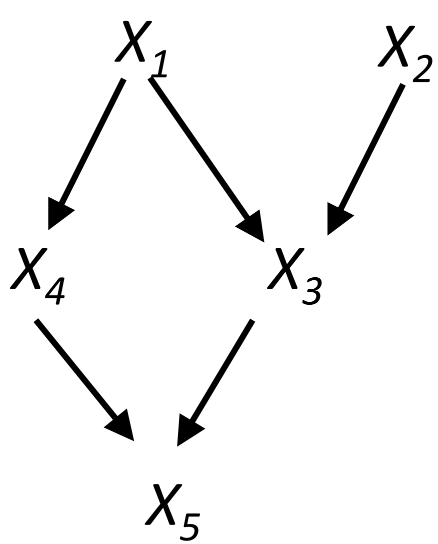
Semi-graphoid axioms

Symmetry: $(X \perp Y \mid Z) \Leftrightarrow (Y \perp X \mid Z)$

Decomposition: $(X \perp YW \mid Z) \Rightarrow (X \perp Y \mid Z)$

Weak Union: $(X \perp YW \mid Z) \Rightarrow (X \perp Y \mid ZW)$

Contraction: $(X \perp Y \mid Z)$ and $(X \perp W \mid ZY)$ $\Rightarrow (X \perp YW \mid Z)$

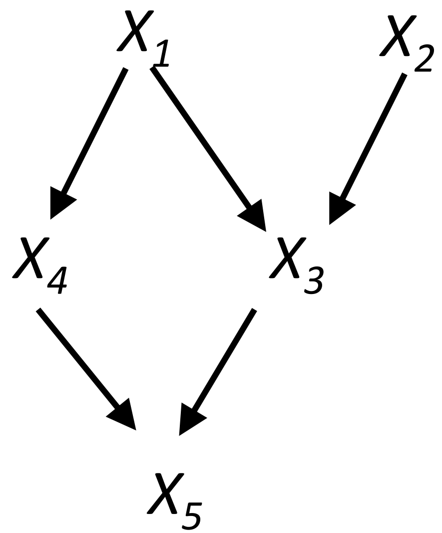


$(X_1 \perp X_2)$
$(X_2 \perp \{X_1, X_4\})$
$(X_3 \perp X_4 \mid \{X_1, X_2\})$
$(X_4 \perp \{X_2, X_3\} \mid X_1)$
$(X_5 \perp \{X_1, X_2\} \mid \{X_3, X_4\})$

The semi-graphoid axioms then imply

$(X_4 \perp X_2 \mid X_1)$
$(\{X_4, X_5\} \perp X_2 \mid \{X_1, X_3\})$
$\ldots$

The values of the causal-statistical parameters can imply further CI relations



Suppose:

$$X_3 = (X_1 + X_2)\mathrm{mod}2$$

$$P(X_2 = 0) = P(X_2 = 1) = \frac{1}{2}$$

Then:

$$X_3 \perp X_1$$

$$A \perp B$$

and no other independence relations

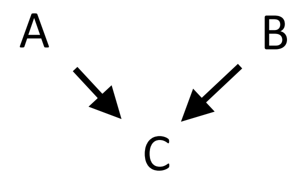
A ? B

C

$A \perp B$

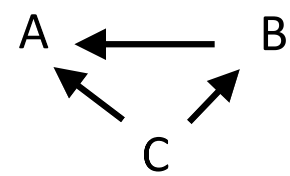
and no other independence relations

$\Rightarrow$



$P(A)$
$P(B)$
$P(C|A, B)$

✓



$P(C)$
$P(B|C)$
$P(A|B, C)$

✗

No Fine-tuning!

A key assumption of causal discovery algorithms

No fine-tuning (a.k.a. stability, a.k.a. faithfulness):
A causal model M is not fine-tuned relative to a probability distribution P if the conditional independences that hold in P continue to hold for any variation of the parameters in M

$(A \perp B | C)$

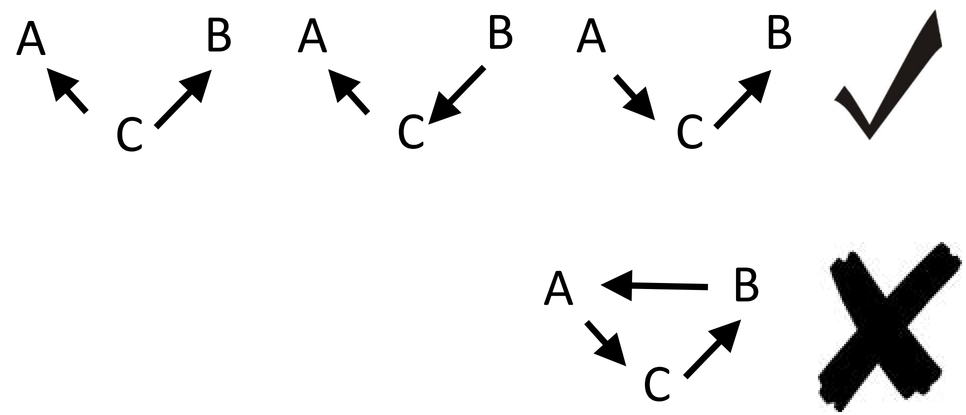
and no other independence relations

A ? B

C

$$(A \perp B | C)$$

and no other independence relations

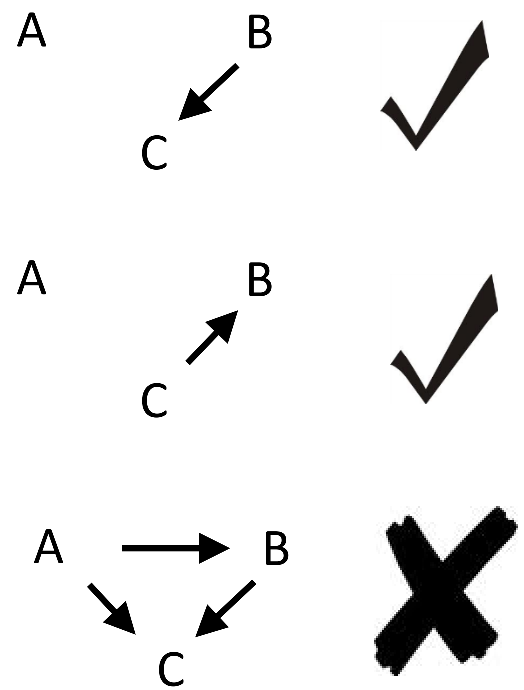


$$A \perp B$$
$$A \perp B \mid C$$
$$A \perp C$$
$$A \perp C \mid B$$

and no other independence relations

$$A \perp B$$
$$A \perp B \mid C$$
$$A \perp C$$
$$A \perp C \mid B$$

and no other independence relations



Allowing latent variables in the causal structure

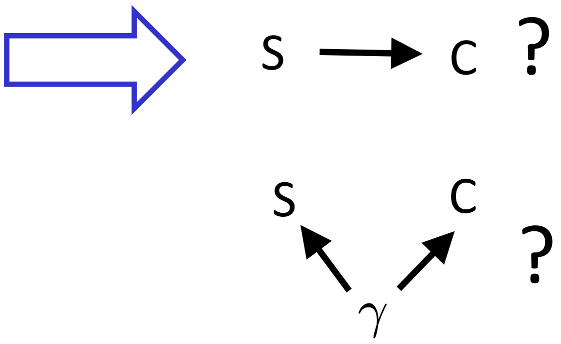
What's given: probability distribution over observed variables

What we must infer: a causal structure over a set of variables that includes the observed variables and may include one or more latent variables

Notational Convention
Observed variables: A, B, C,…
Latent variables: λ, μ, ν, …

Does smoking cause lung cancer?

$S \not\perp C$



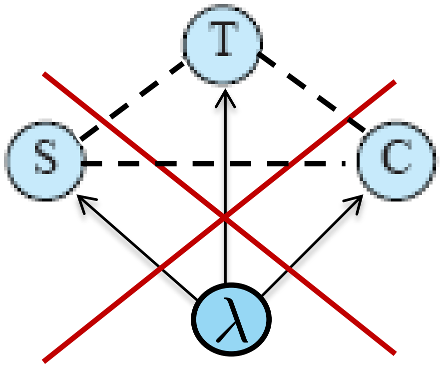
Suppose you also observe

$$S \perp C \mid T$$

and no other independences

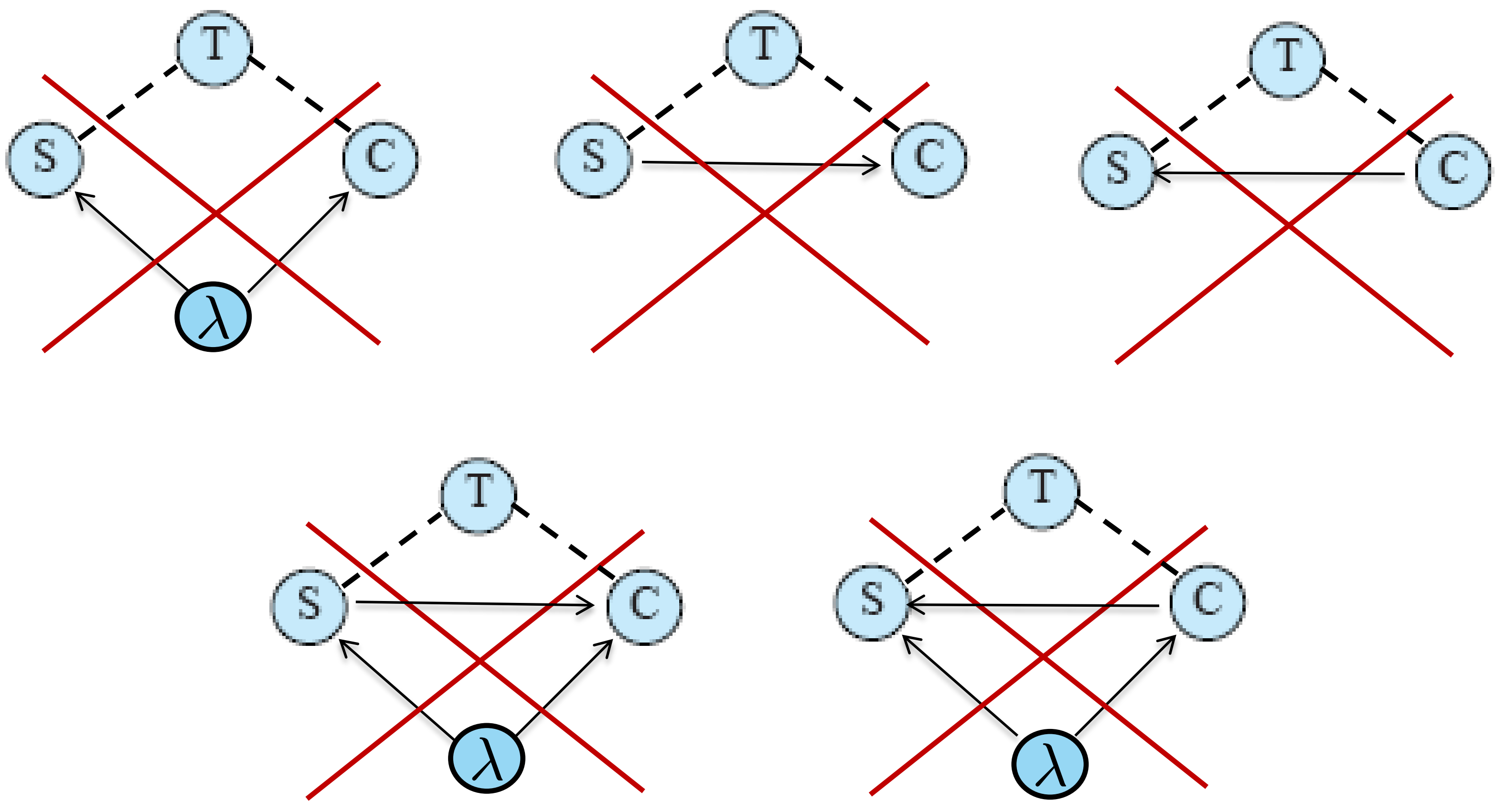
$(S \perp C \mid T)$

Latent common cause for S, C and T?



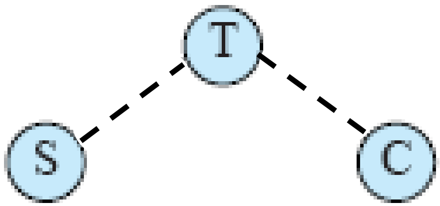
$(S \perp C \mid T)$

Latent common cause or direct causal relation (or both) between S and C?



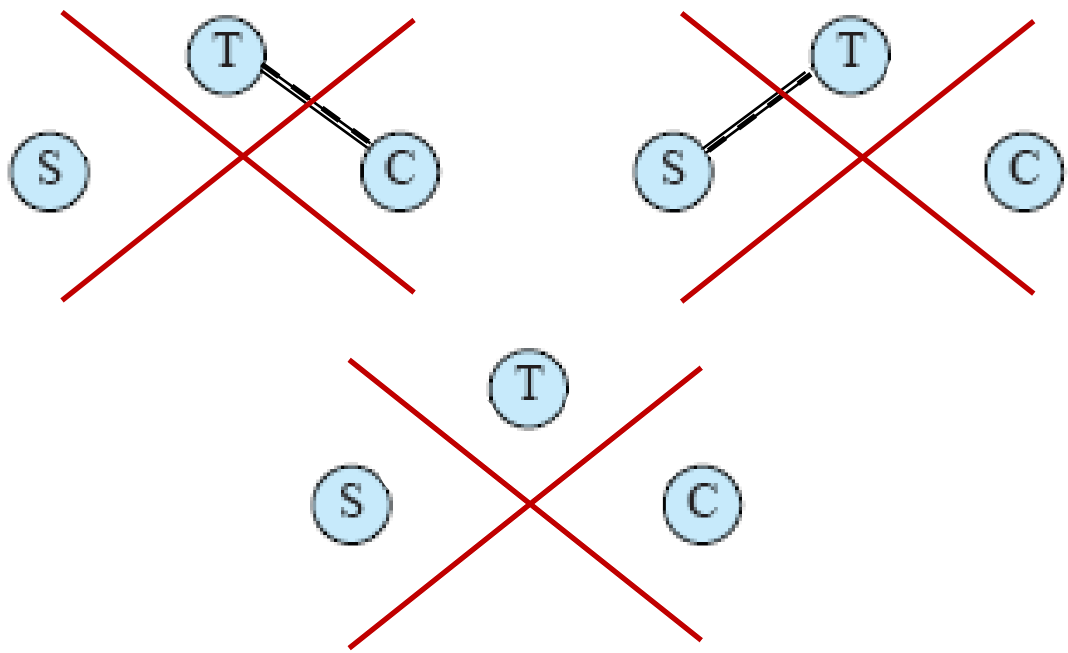
$(S \perp C \mid T)$

So the causal structure must be of the form



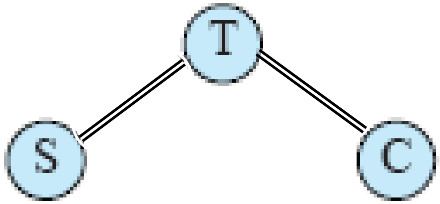
$(S \perp C \mid T)$

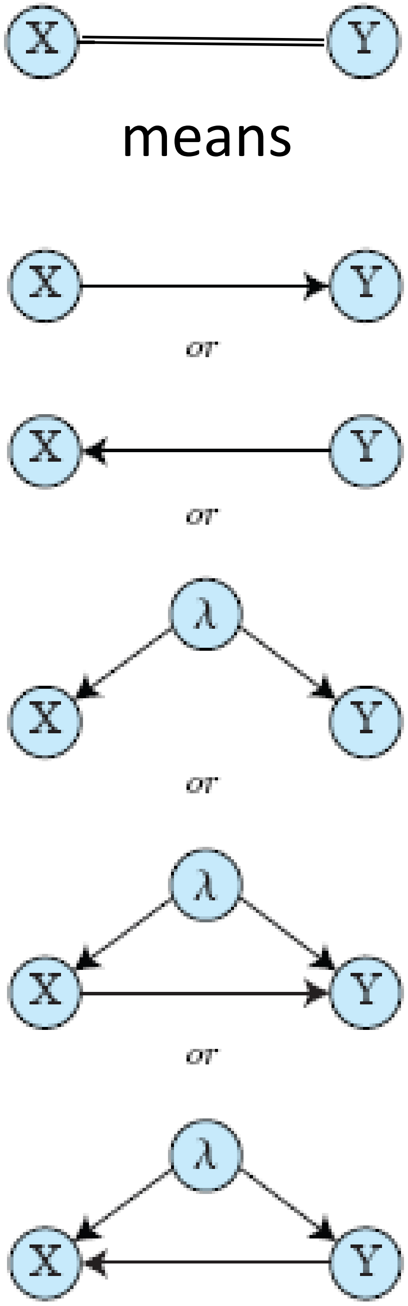
Marginal independence between remaining pairs?



$(S \perp C \mid T)$

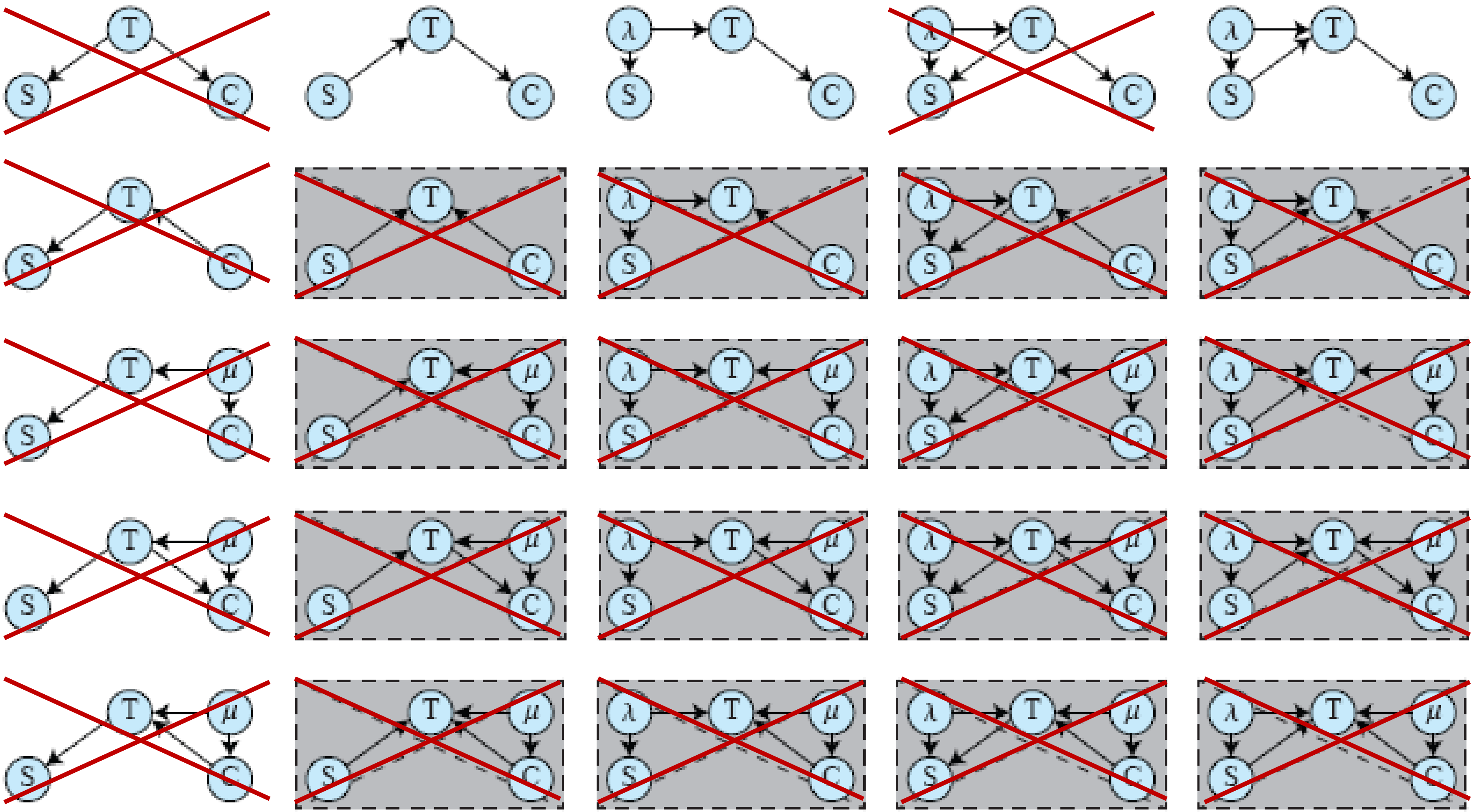
So the causal structure must be of the form





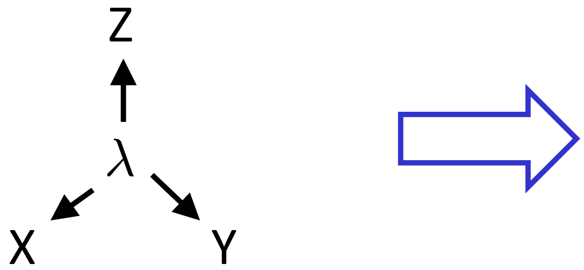
$(S \perp C \mid T)$

Assume one extra piece of data: *S* always precedes *T*

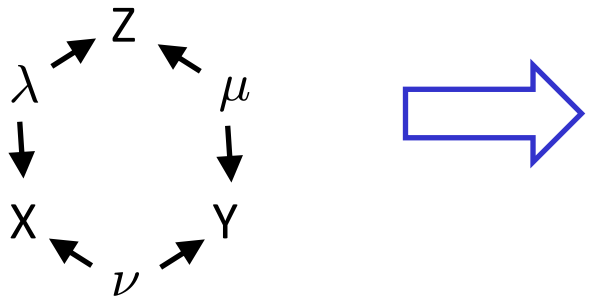


Inferring facts about the causal structure from the strength of correlations

Strength of Correlations



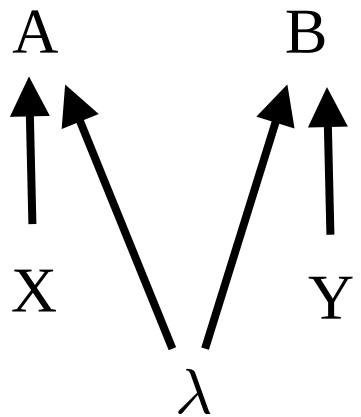
P(X,Y,Z) can have perfect three-way correlation



P(X,Y,Z) is bounded away from perfect three-way correlation

Janzing and Beth, arXiv:quant-ph/0208006
Steudel and Ay, arXiv:1010:5720
Fritz, New J. Phys. 14, 103001 (2012)
Branciard, Rosset, Gisin, Pironio, arXiv:1112.4502

Strength of Correlations



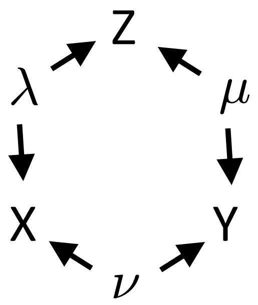
Inequalities on *P(A,B|X,Y)*

$$P(A=B|0,0)+P(A=B|0,1)$$
$$+P(A=B|1,0)+P(A\neq B|1,1) \cdot \;\; 3$$

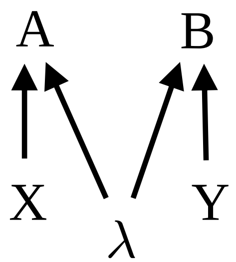
where

$$P(A=B|X,Y) := \sum_{a=b} P(A=a,B=b|X,Y)$$
$$P(A\neq B|X,Y) := \sum_{a\neq b} P(A=a,B=b|X,Y)$$

Testing candidate causal structures



Inequalities on P(X,Y,Z)



Inequalities on P(A,B,X,Y)

Assumptions about causal structure

"Instrumental inequalities" (Chap. 8 of Pearl)

The lesson of causal inference
for Bell-inequality-violating correlations

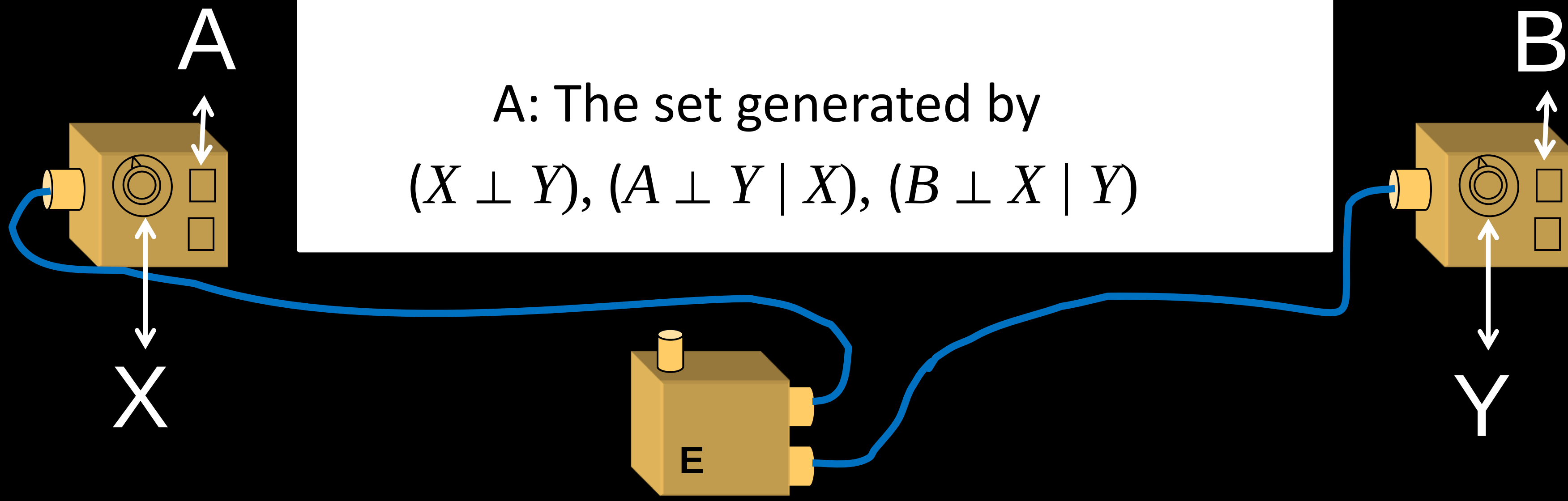
Joint work with Christopher Wood

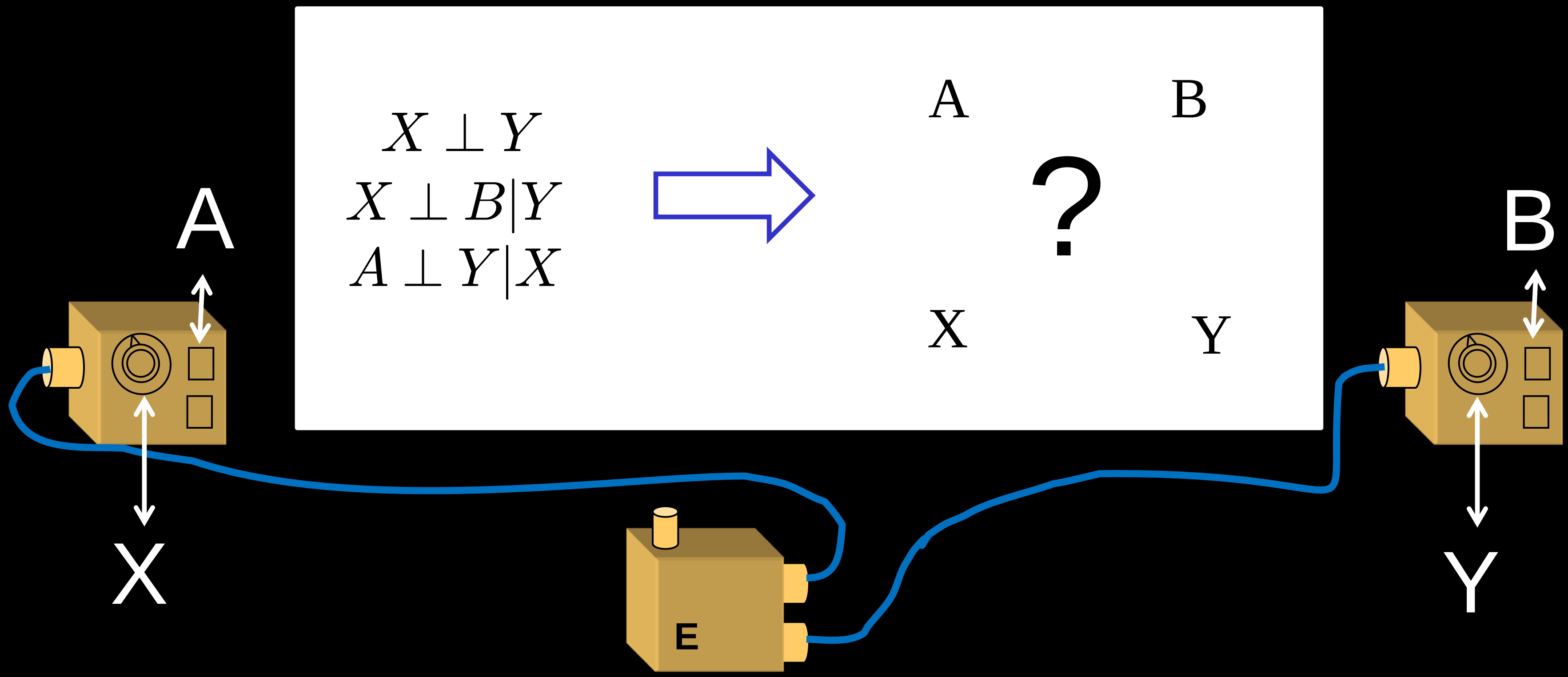
See: arXiv:1208.4119

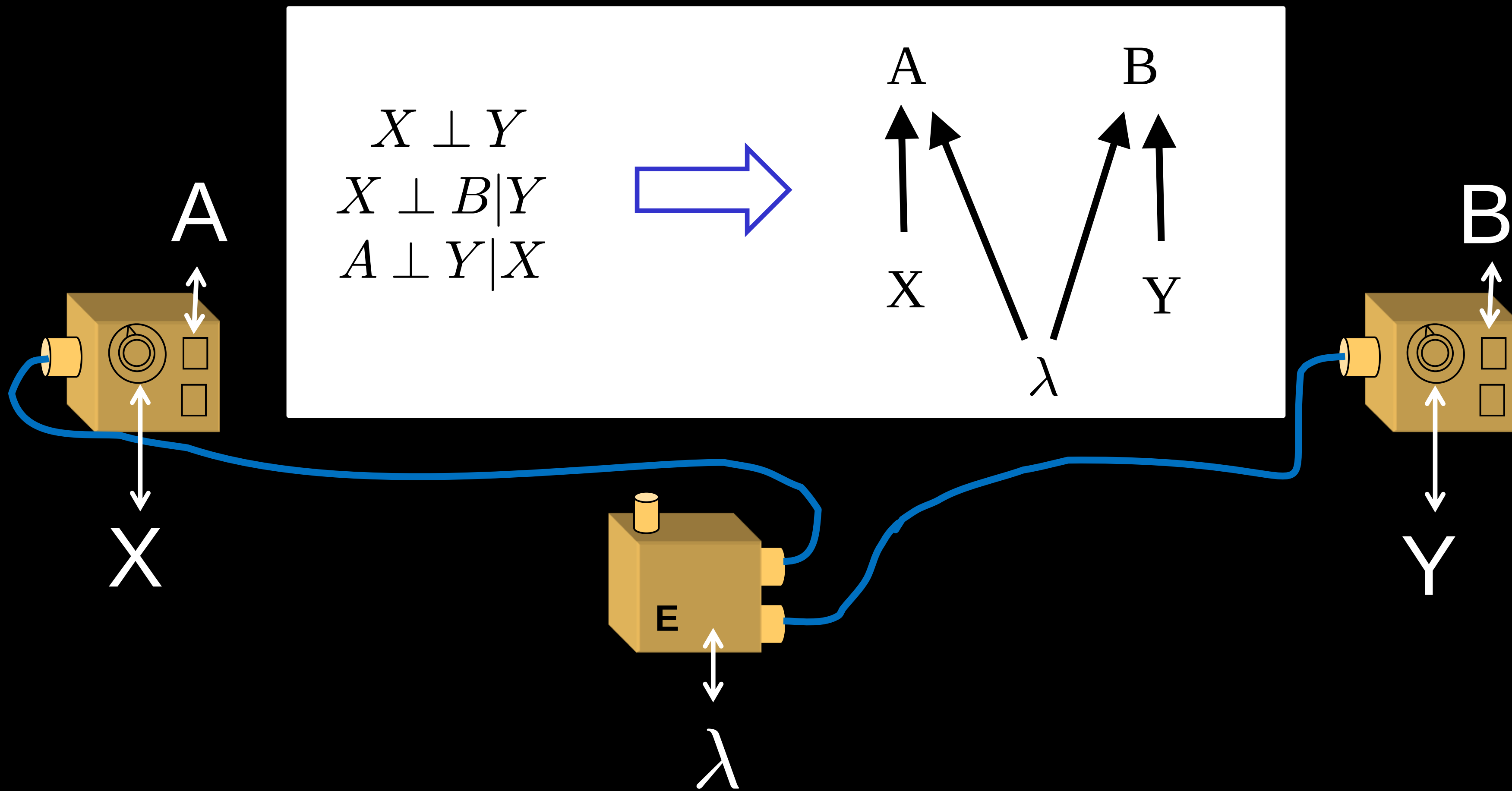
Q: For the observed correlations *P(A,B,X,Y)* what are the independences?

A: The set generated by

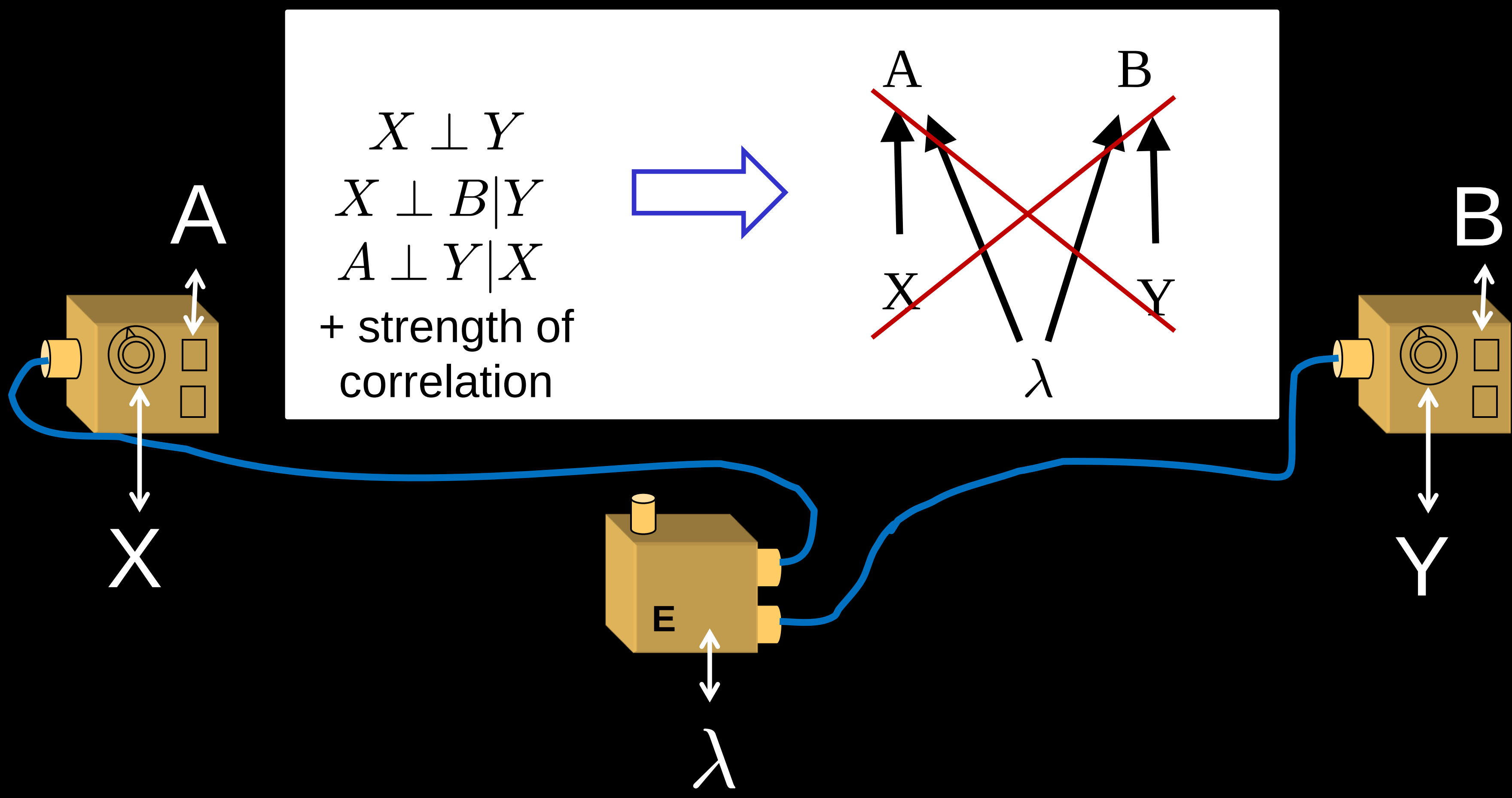
$(X \perp Y), (A \perp Y \mid X), (B \perp X \mid Y)$




$X \perp Y$
$X \perp B|Y$
$A \perp Y|X$
A
B
?
X
Y
A
X
E
B
Y



$X \perp Y$
$X \perp B|Y$
$A \perp Y|X$
A
B
X
Y
λ
A
X
B
Y
E
λ


$X \perp Y$
$X \perp B|Y$
$A \perp Y|X$
+ strength of correlation
A
B
X
Y
λ
A
X
B
Y
E
λ

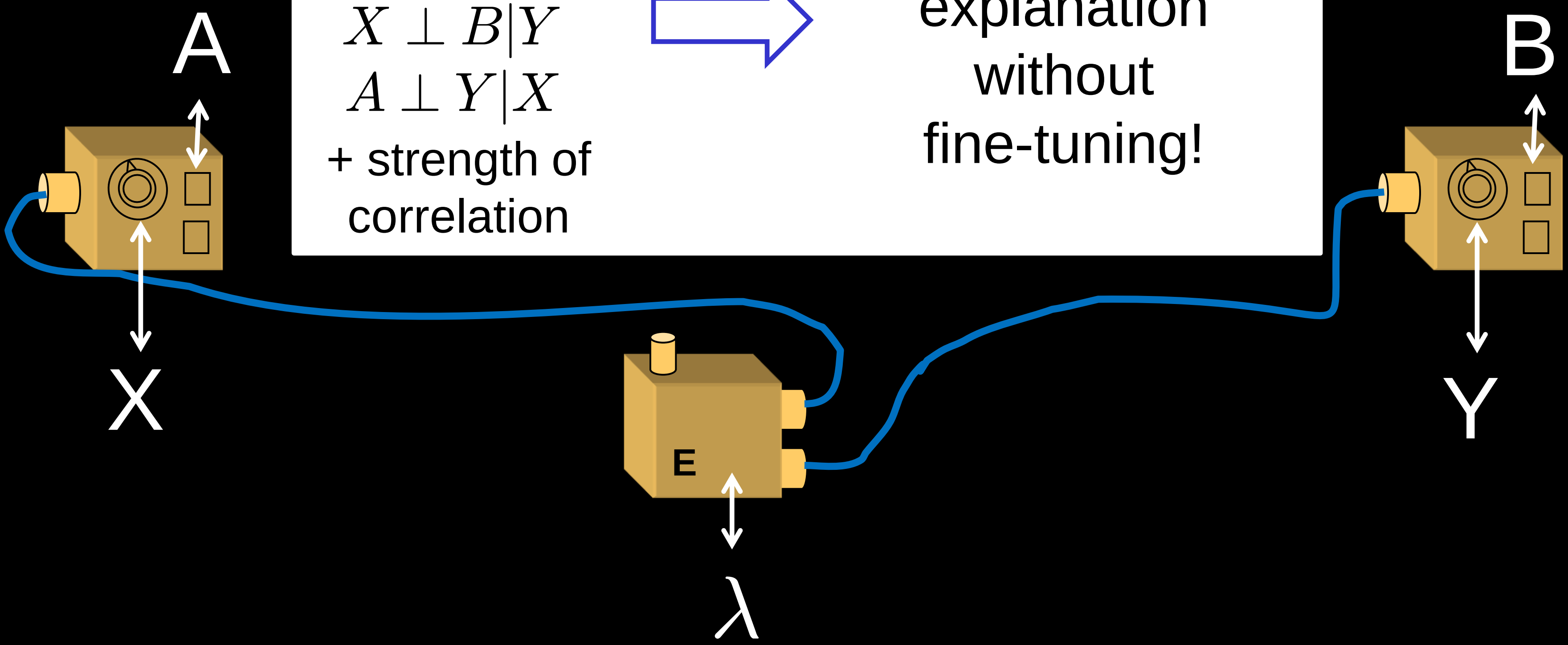
$X \perp Y$

$X \perp B|Y$

$A \perp Y|X$

+ strength of correlation

No causal explanation without fine-tuning!



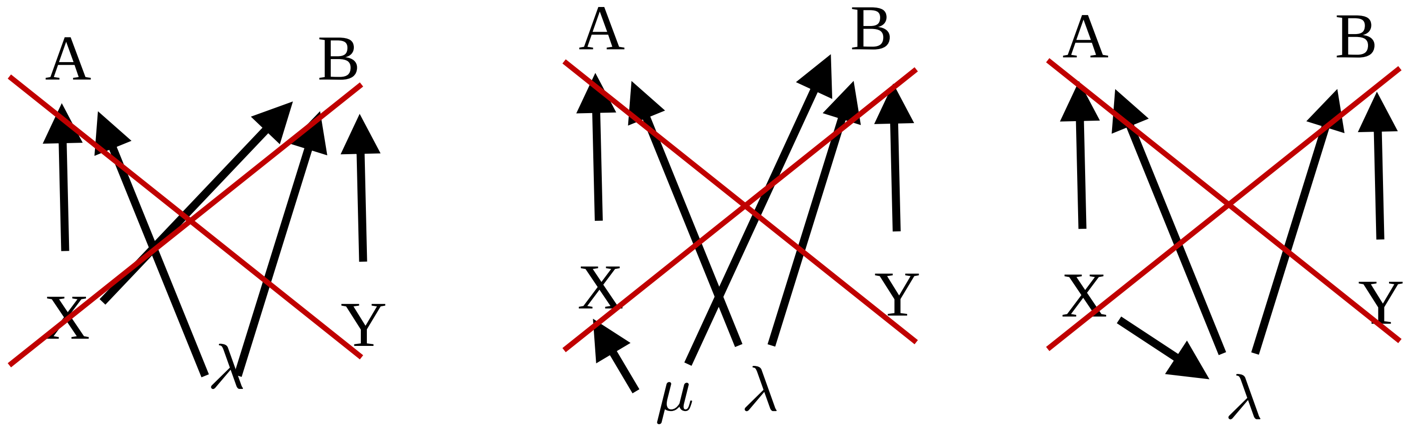
What are the key assumptions of Bell's theorem?

<u>A "standard" response:</u>

- Realism
- Local causality
- No superdeterminism
- No retrocausation

<u>What is proposed here:</u>

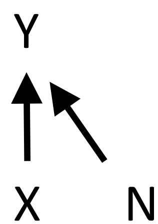
- Reichenbach's principle
- No fine-tuning
- A causal model is a directed acyclic graph supplemented with conditional probabilities



Distinguishing X → Y from Y → X under assumption of additive noise

Linear functional model with additive noise

distinguish

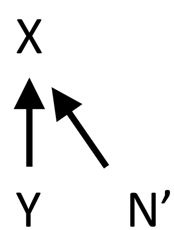


$$Y = \alpha X + N$$

$$P(X)$$

$$P(N)$$

from

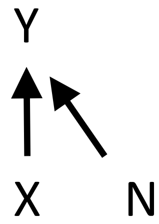


$$X = \alpha' Y + N'$$

$$P(Y)$$

$$P(N')$$

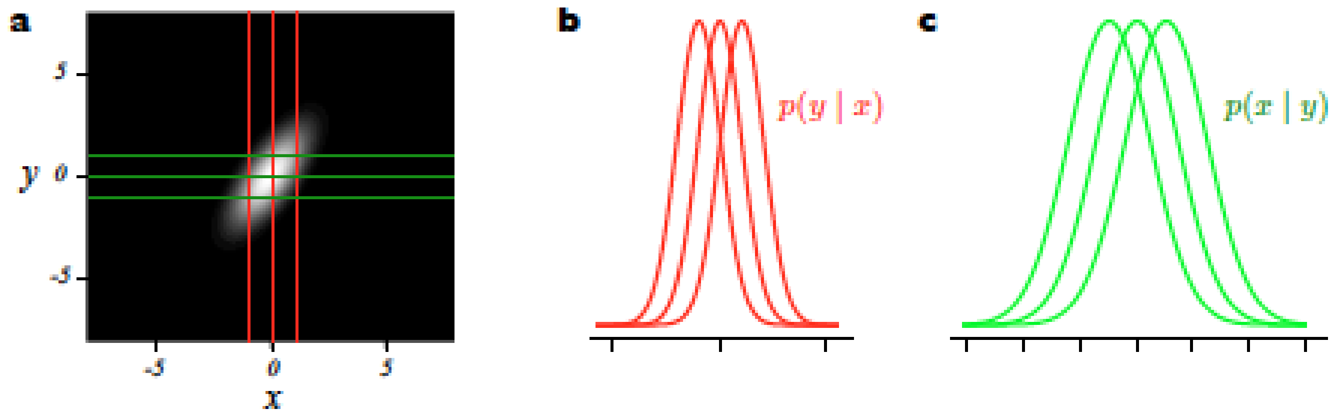
Linear functional model with additive noise



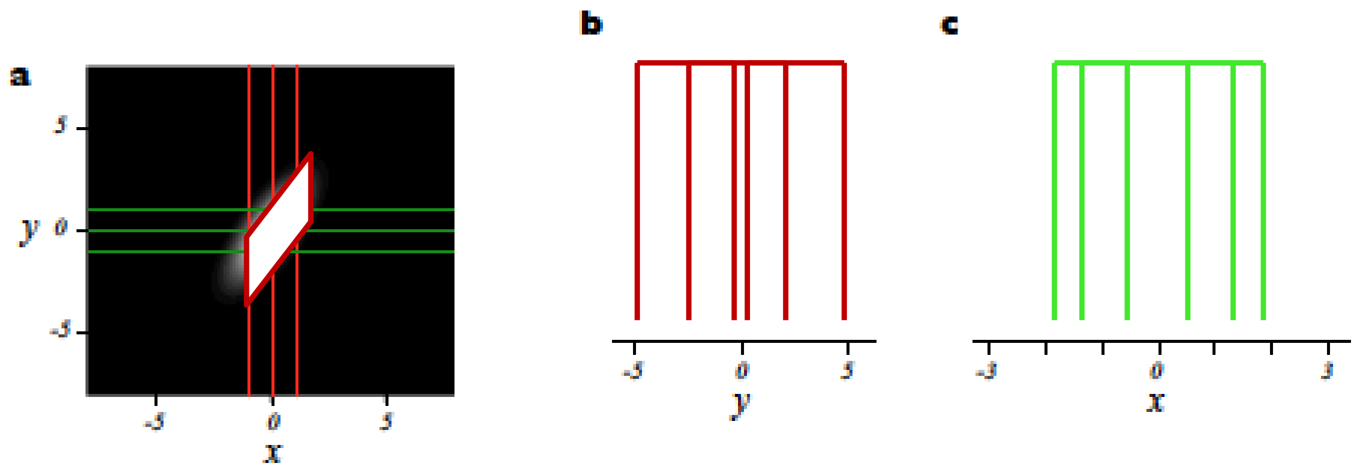
$$Y = \alpha X + N$$
$$P(X)$$
$$P(N)$$

Hoyer et al. *NIPS 21*, Vancouver (2009)

Gaussian
X and N

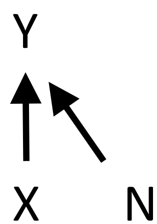


NonGaussian
X and N



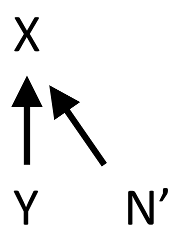
Nonlinear functional model with additive noise

distinguish



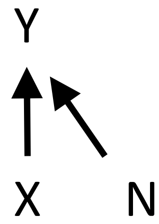
$$Y = f(X) + N$$
$$P(X)$$
$$P(N)$$

from



$$X = f'(Y) + N'$$
$$P(Y)$$
$$P(N)$$

Nonlinear functional model with additive noise

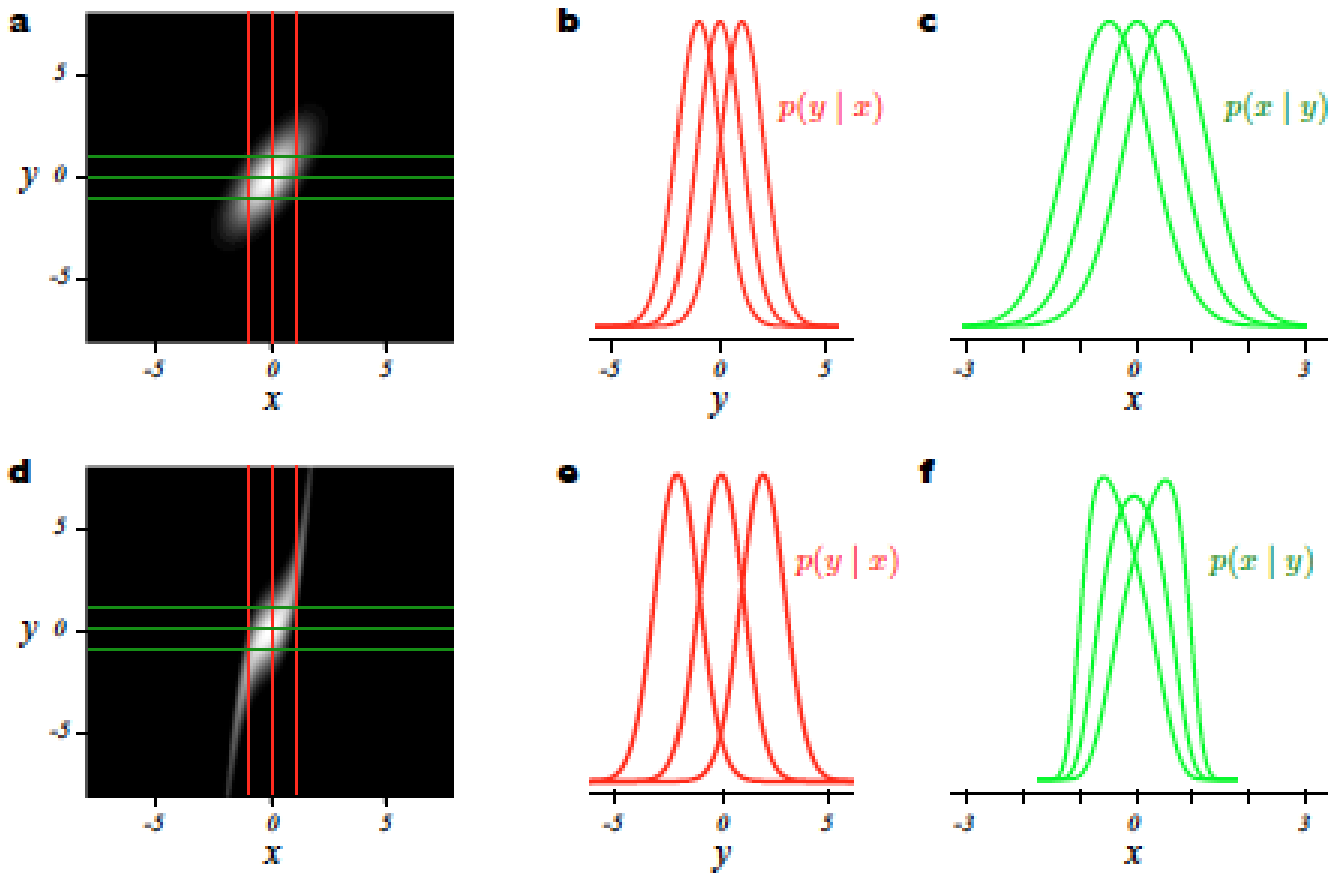


$$Y = f(X) + N$$
$$P(X)$$
$$P(N)$$

Hoyer et al. *NIPS 21*, Vancouver (2009)

Linear fn’
+ Gaussian
X and N

Nonlinear fn’
+ Gaussian
X and N

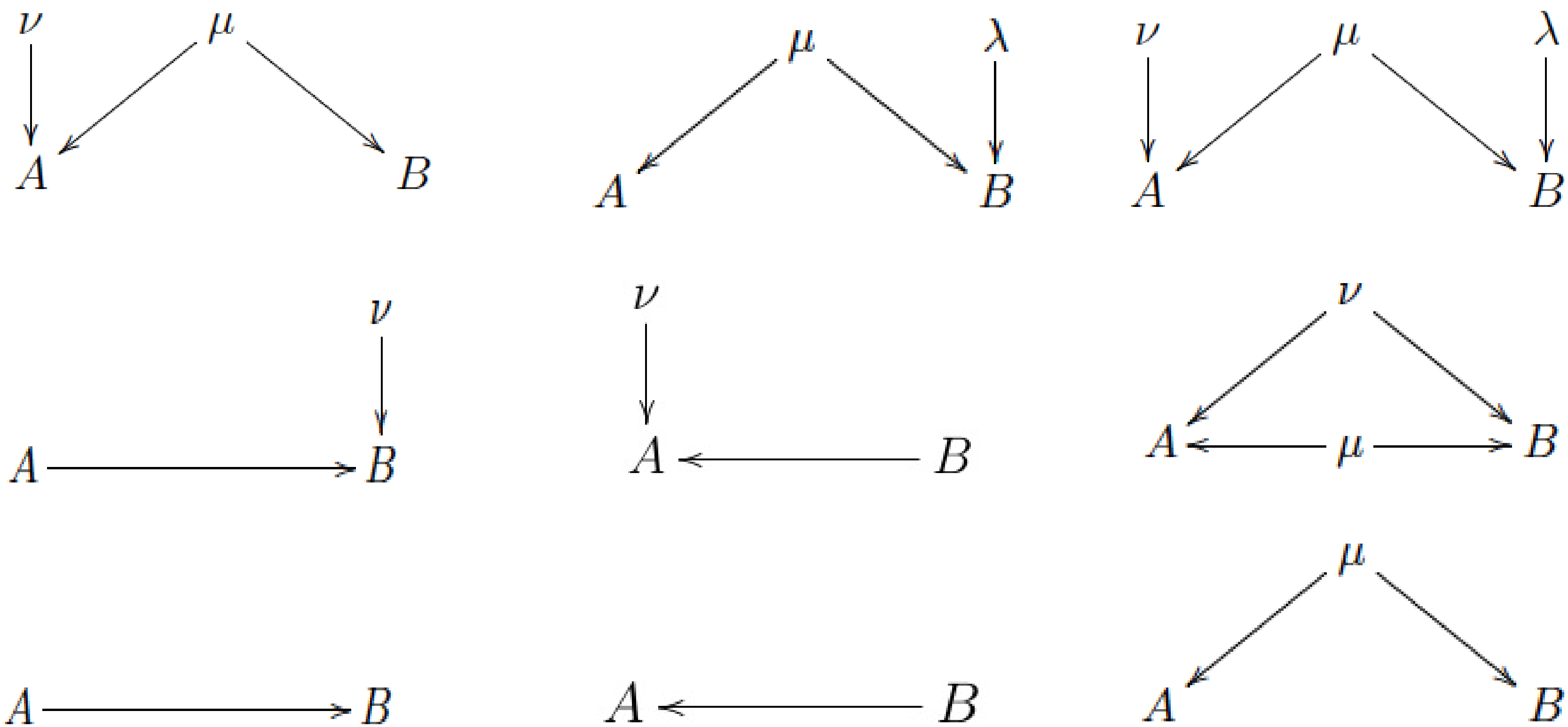


Causal inference
from correlations on a pair of binary variables

Joint work with Ciaran Lee

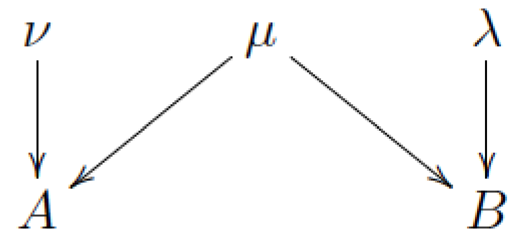
Functional Causal Models where A and B have at most two binary variables as parents

Possible Causal structures



Note: all noise is assumed to come from the root nodes

Possible functional dependences of A and B on their parents



$$A = \nu \oplus \mu \qquad B = \lambda \oplus \mu$$

$$A = \nu \oplus \mu \oplus 1 \qquad B = \lambda \oplus \mu \oplus 1$$

$$A = \nu\mu \qquad B = \lambda\mu$$

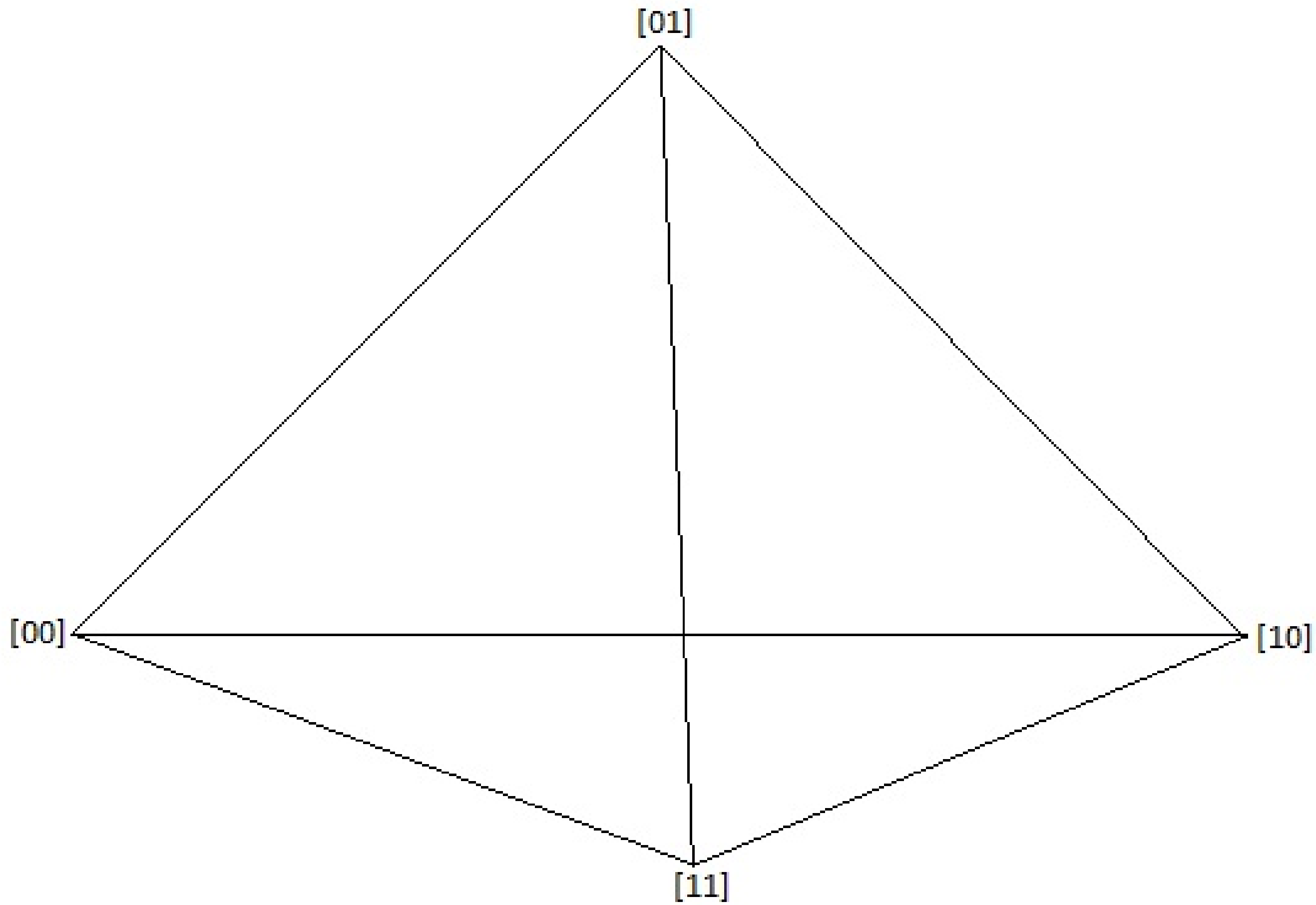
$$A = \nu\mu \oplus 1 \qquad B = \lambda\mu \oplus 1$$

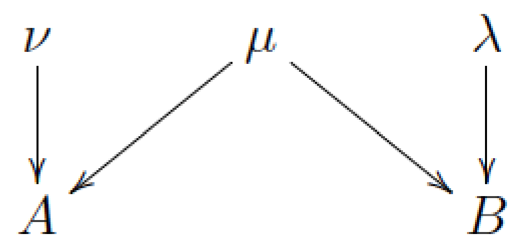
Note:

μ	ν	$f = \mu \oplus \nu \oplus \mu\nu$
0	0	0
0	1	1
1	0	1
1	1	1

μ	ν	$g = (\mu \oplus 1)(\nu \oplus 1) \oplus 1$
0	0	0
0	1	1
1	0	1
1	1	1

$$P(A,B) = p_{00}[00] + p_{01}[01] + p_{10}[10] + p_{11}[11]$$

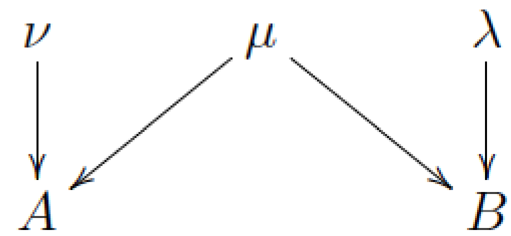




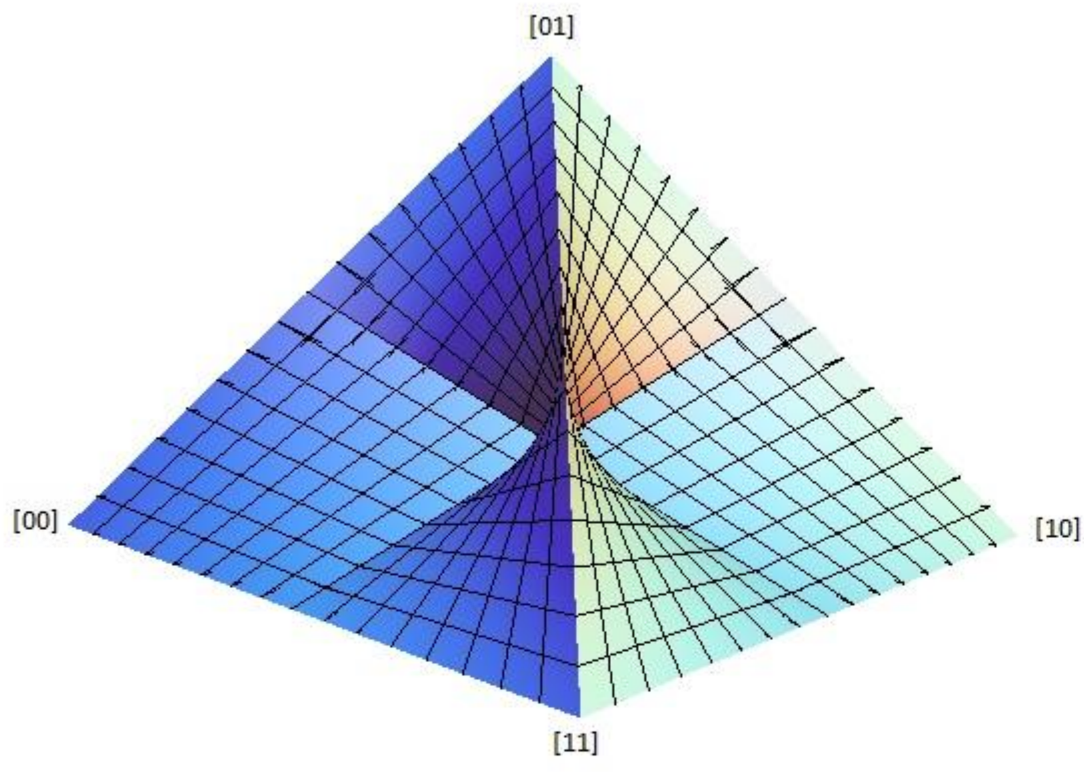
$A = \nu \oplus \mu$ $\qquad\qquad$ $B = \lambda \oplus \mu$

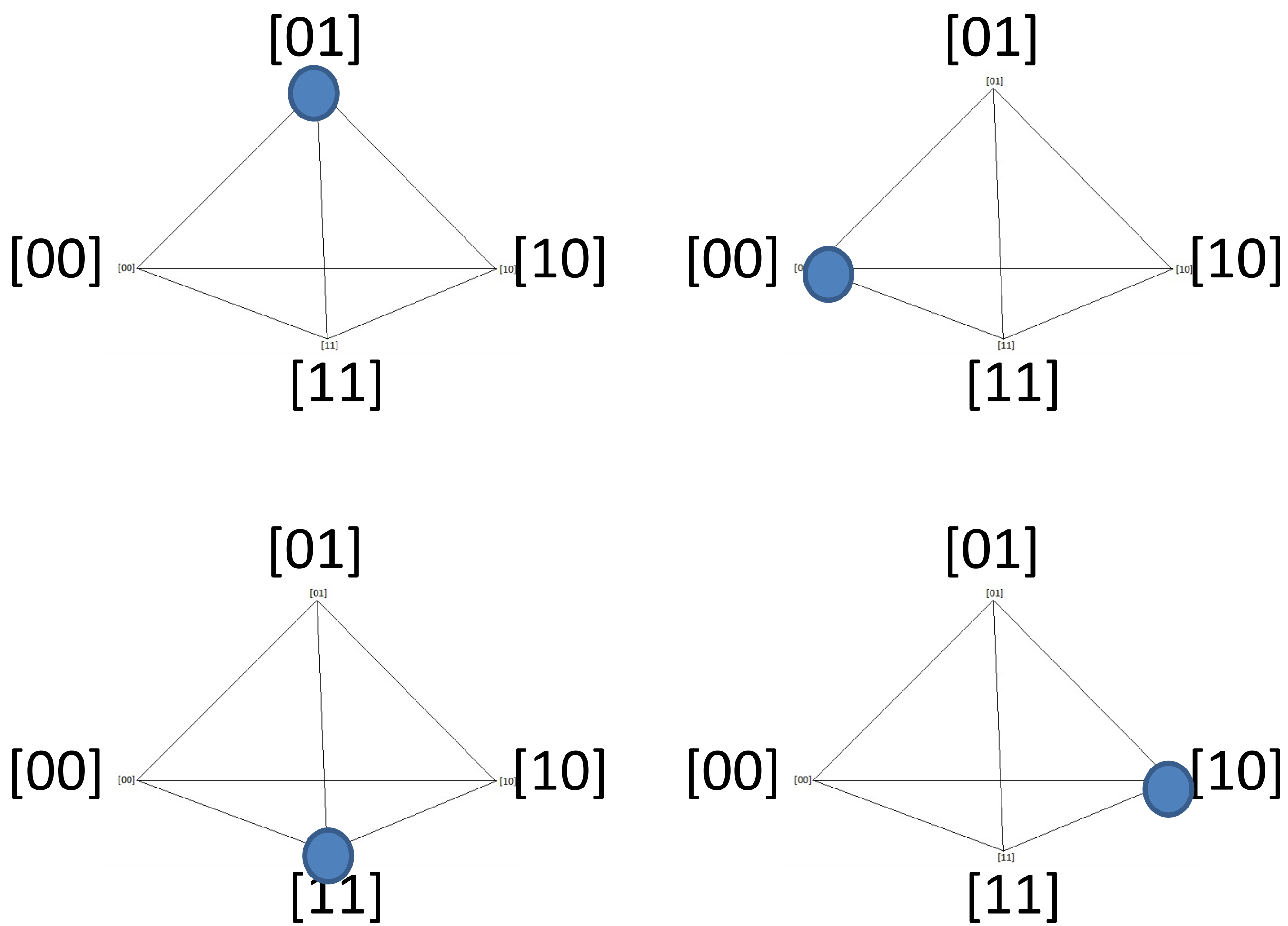
μ	ν	λ	$A = \mu \oplus \nu$	$B = \mu \oplus \lambda$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
1	0	0	1	1
1	1	0	0	1
0	1	1	1	1
1	0	1	1	0
1	1	1	0	0

$$P(A,B) = (q_1q_2q_3 + \bar{q}_1\bar{q}_2\bar{q}_3)[00] + (q_1q_2\bar{q}_3 + \bar{q}_1\bar{q}_2q_3)[01]$$
$$+(q_1\bar{q}_2q_3 + \bar{q}_1q_2\bar{q}_3)[10] + (\bar{q}_1q_2q_3 + q_1\bar{q}_2\bar{q}_3)[11],$$

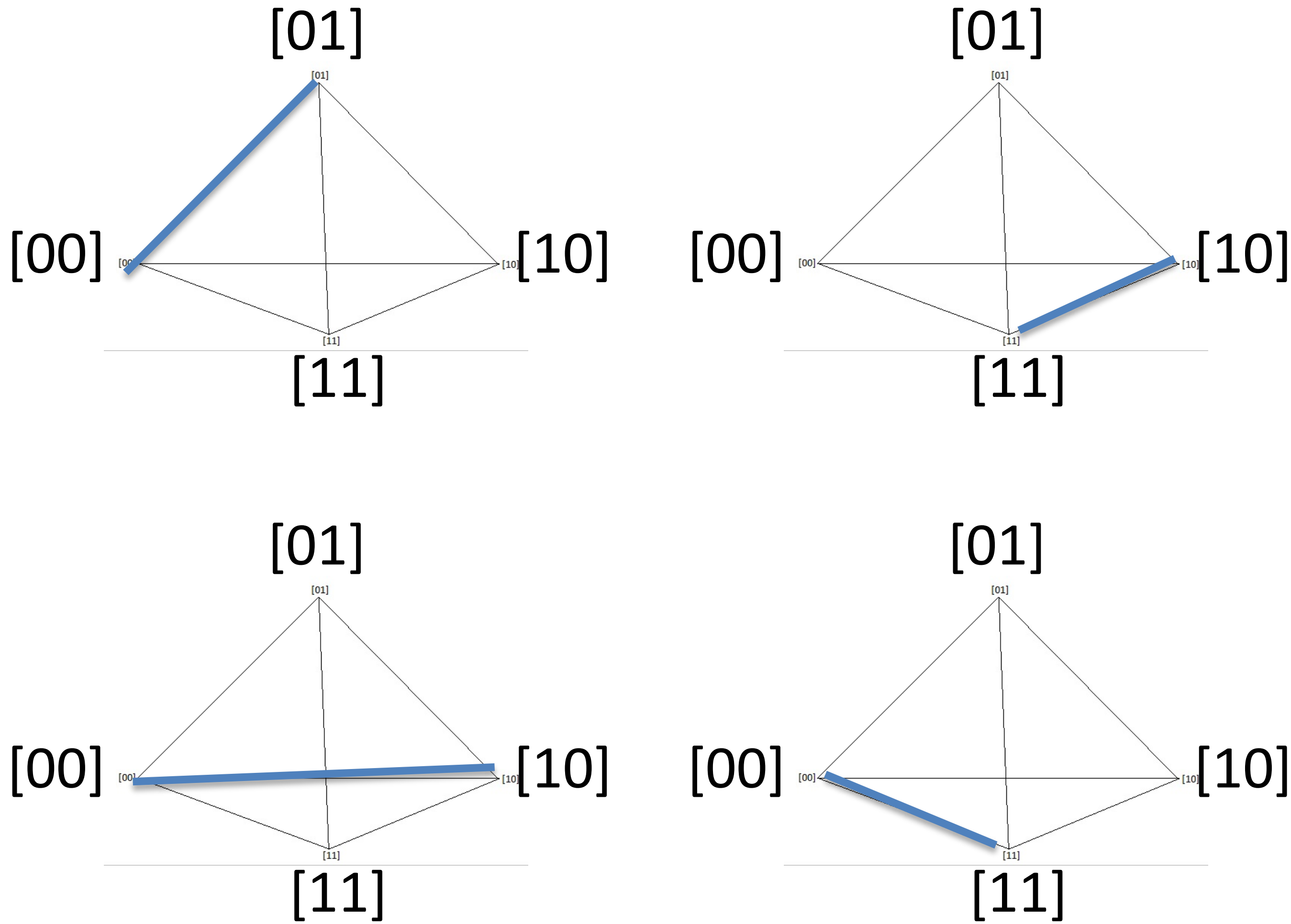


$$A = \nu \oplus \mu \qquad\qquad B = \lambda \oplus \mu$$





A B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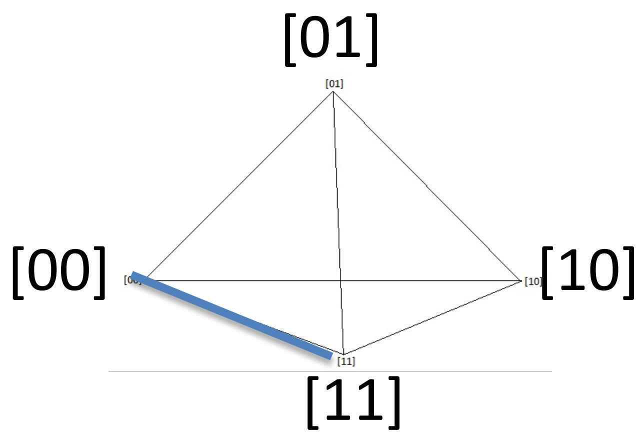
A
B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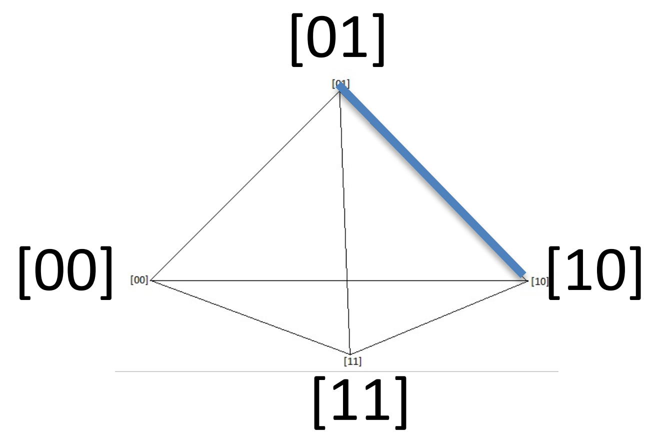
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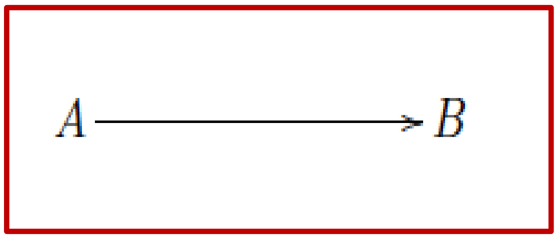
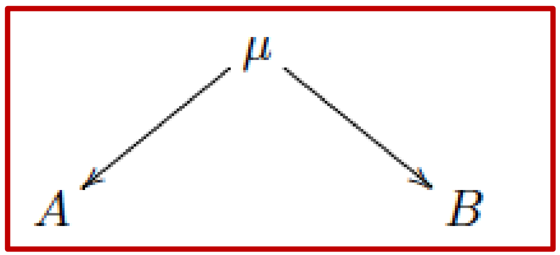
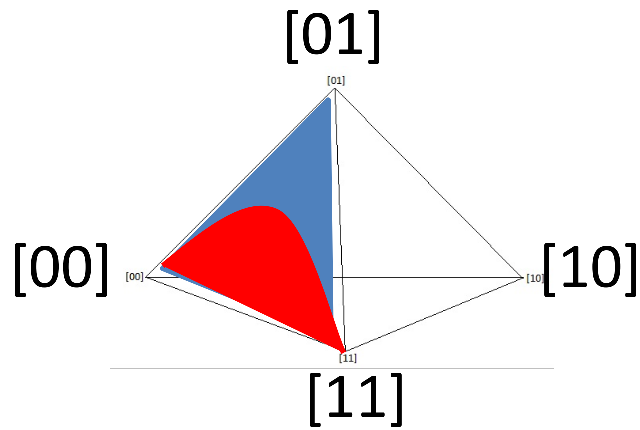
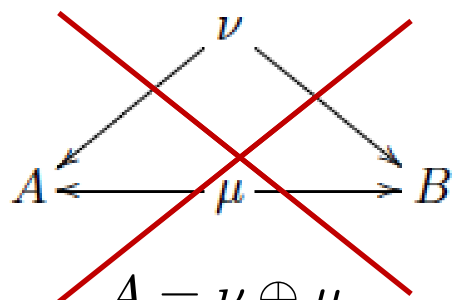

A
B


A
B


μ
A
B

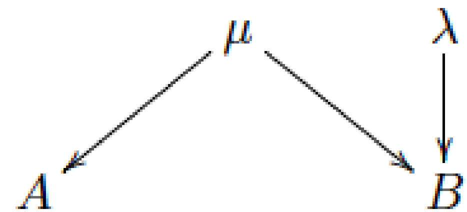


$$B = \nu(A \oplus 1) \oplus 1$$



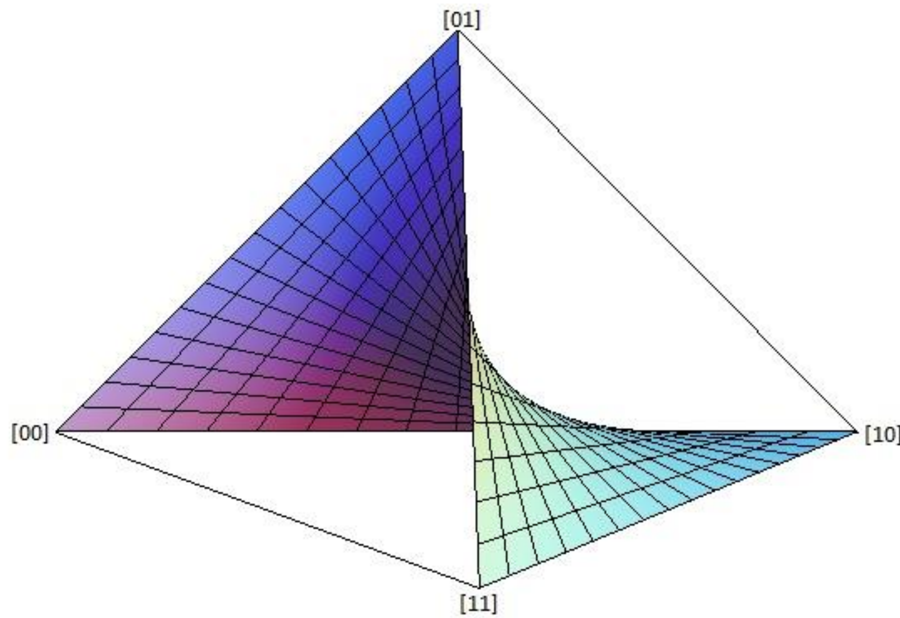
$$A = \nu \oplus \mu$$

$$B = \nu\mu$$



$$A = \mu \oplus 1$$

$$B = \lambda\mu \oplus 1$$



[01]
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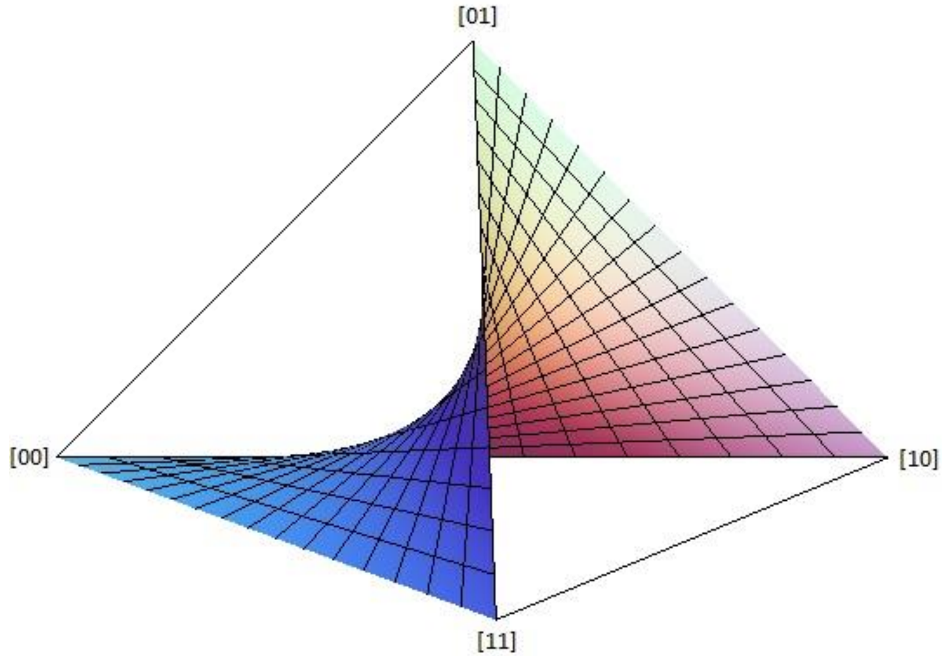
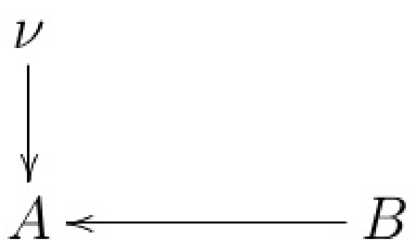
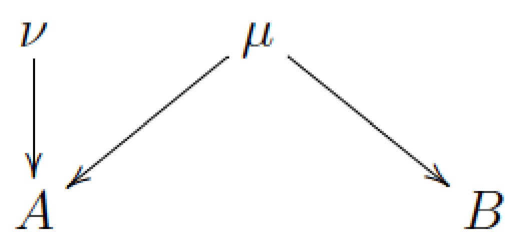
A
B



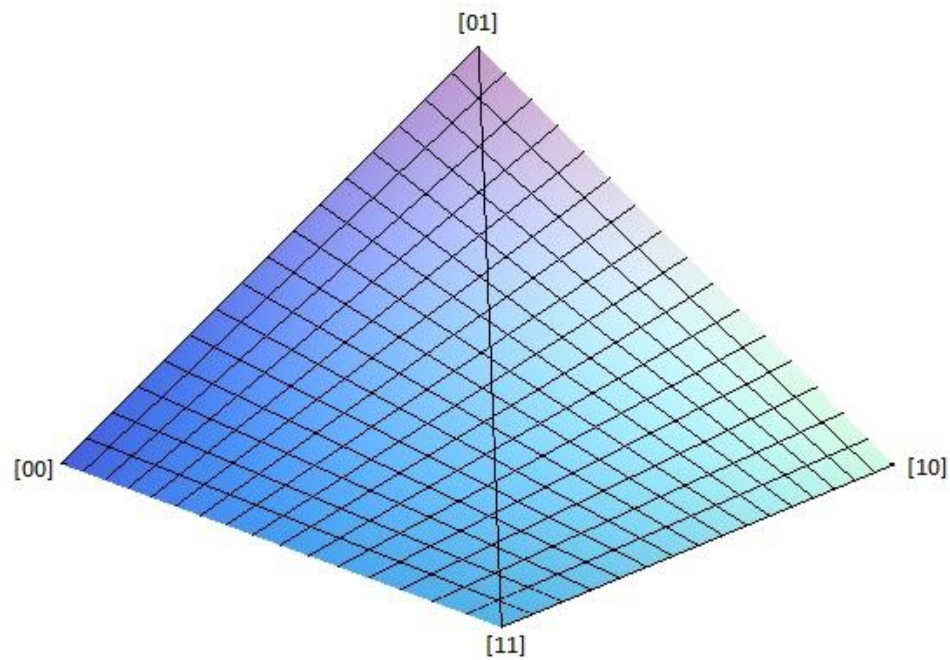


$$A = B \oplus \nu$$

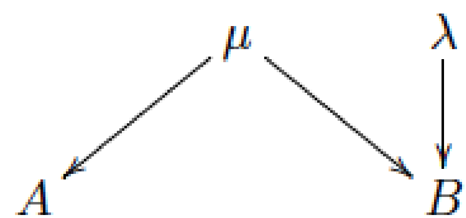


$$A = \nu \oplus \mu$$

$$B = \mu$$

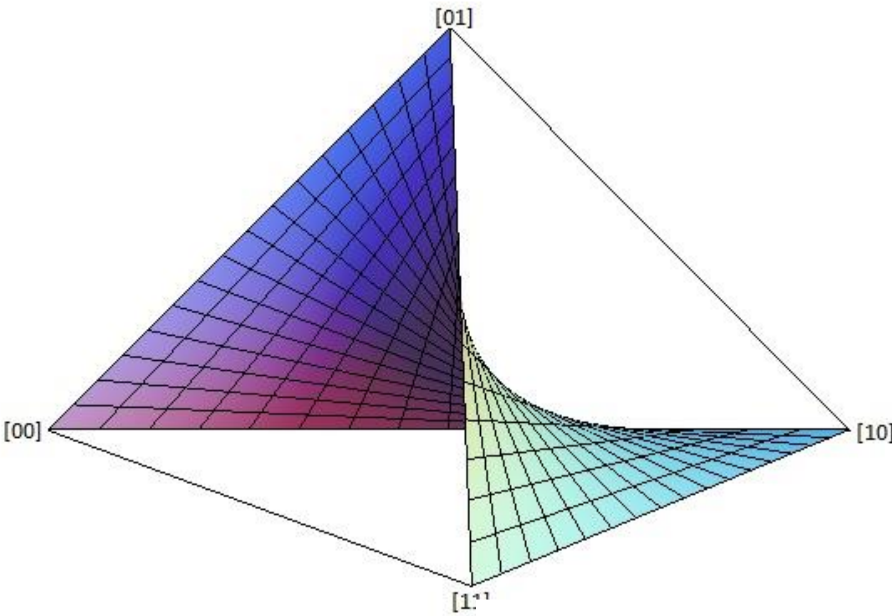


$$B = A \oplus \nu$$



$$A = \mu$$

$$B = \mu \oplus \lambda$$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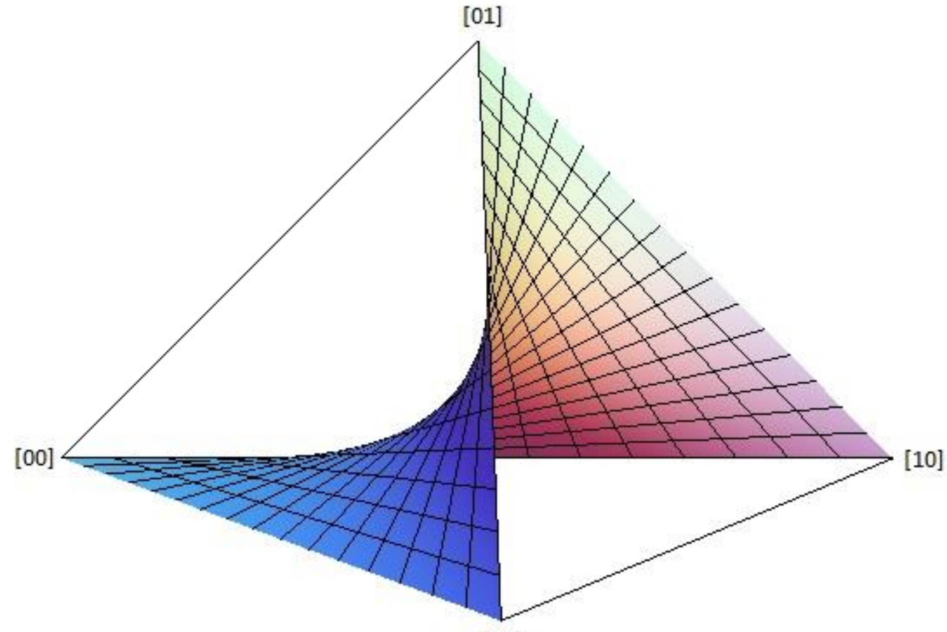
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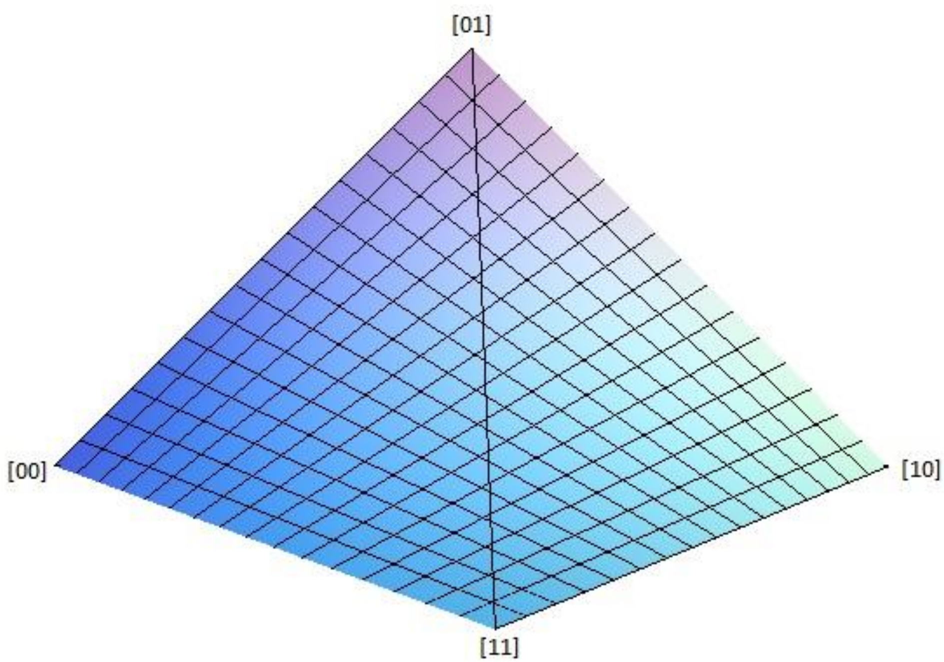
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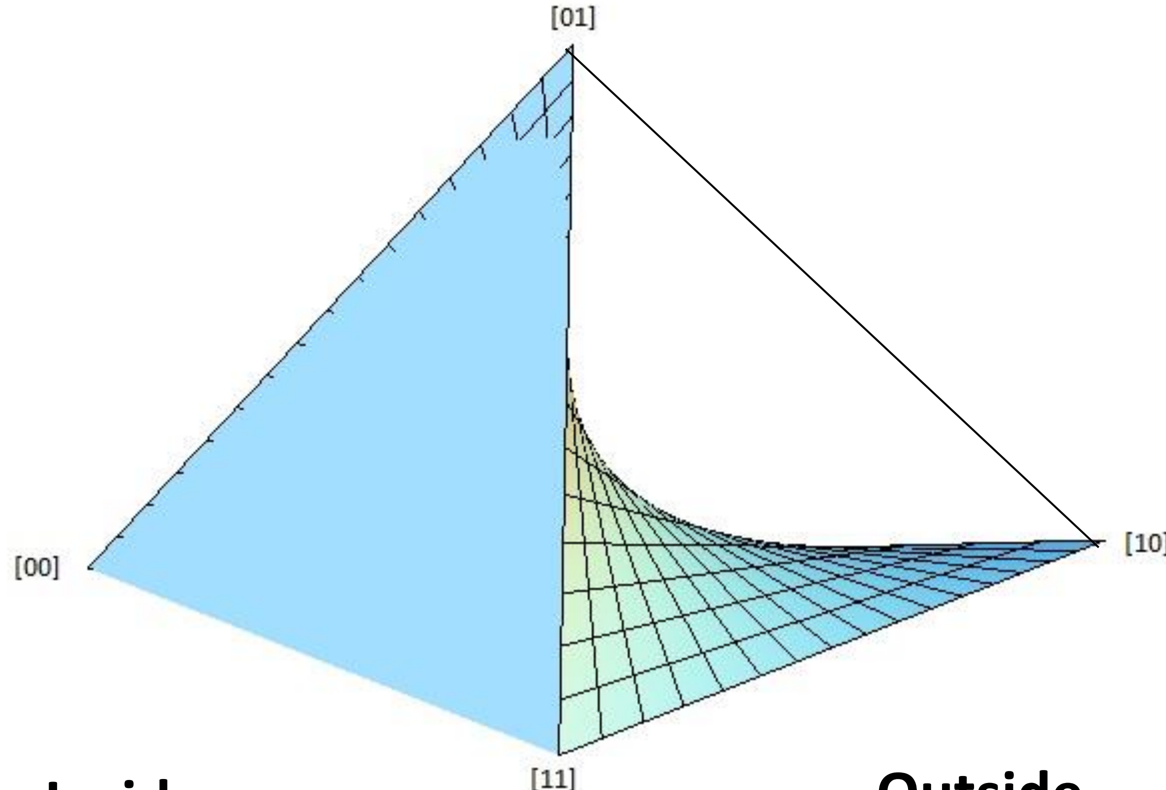
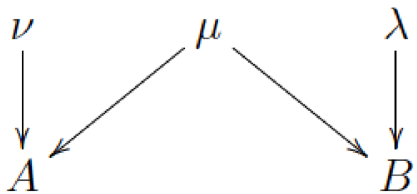


Inside

Consistent with

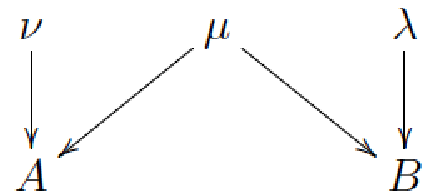


$$A = \mu\nu$$

$$B = \mu\lambda$$

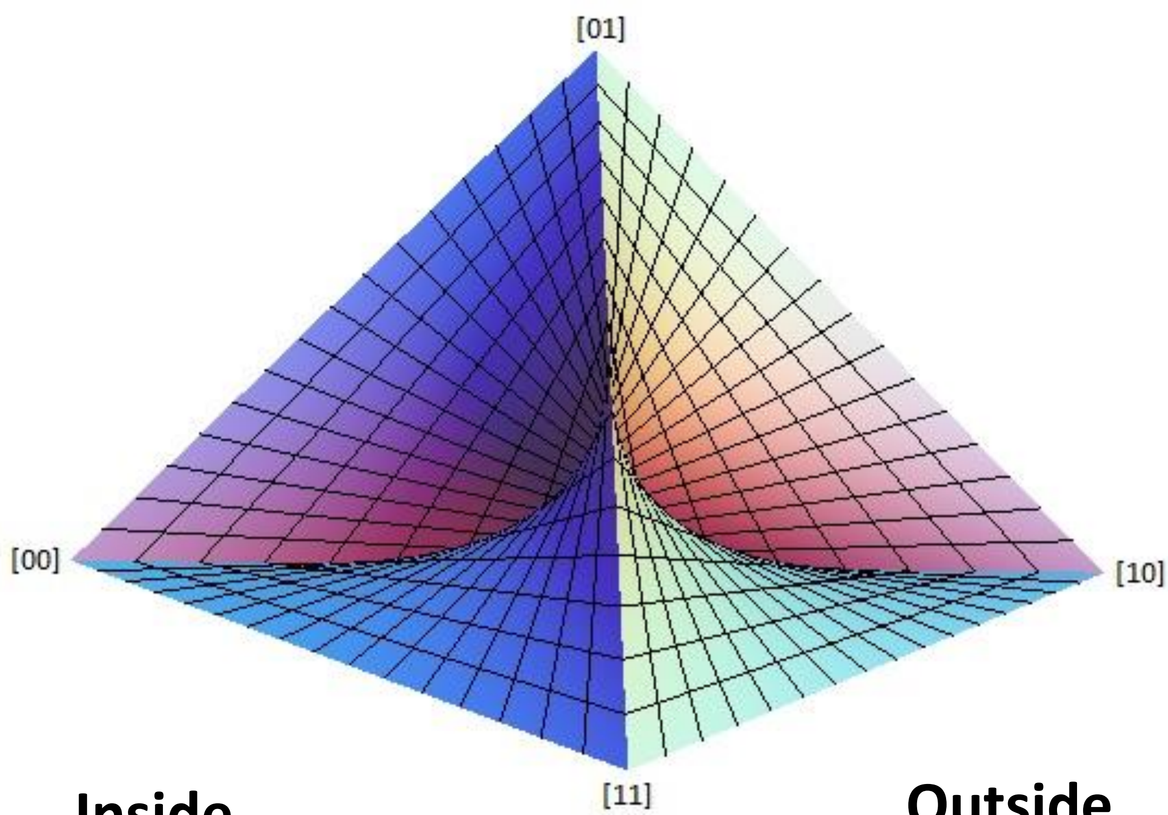
Outside

Consistent with



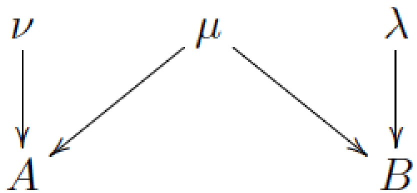
$$A = \mu\nu$$

$$B = \mu\lambda \oplus 1$$



Inside

Consistent with

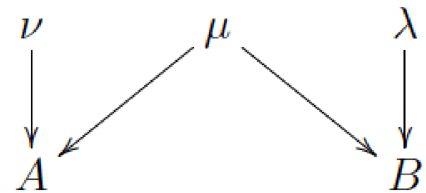


$$A = \mu \oplus \nu$$

$$B = \mu\lambda$$

Outside

Consistent with



$$A = \mu \oplus \nu$$

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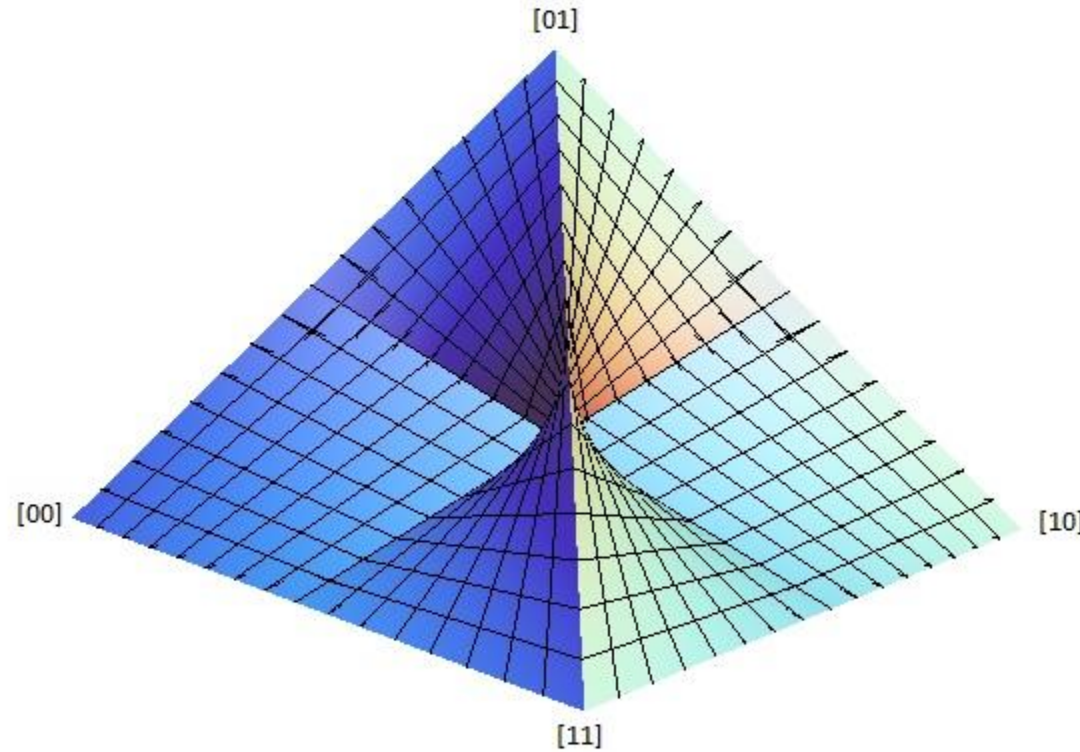
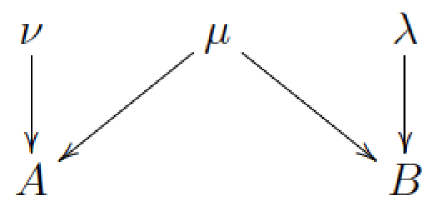
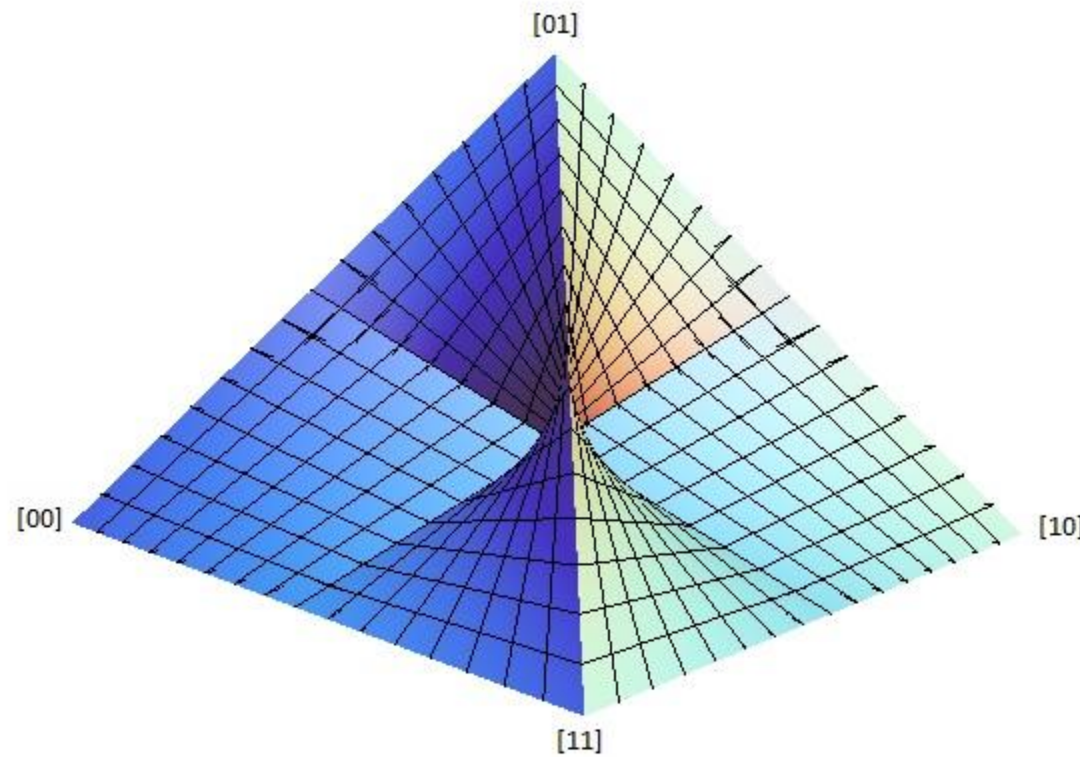


Inside

Consistent with

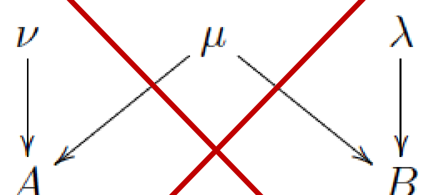


$$A = \nu \oplus \mu$$
$$B = \lambda \oplus \mu$$



Outside

Consistent with



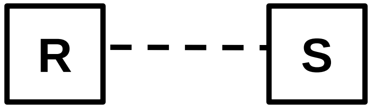
$$A = \nu \oplus \mu$$
$$B = \lambda \oplus \mu$$

Quantum Bayesian Inference and Quantum Causal Models

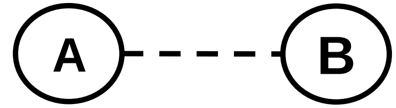
joint work with Matt Leifer

See: arXiv:1107.5849, arXiv:1110.1085

	Classical	Quantum
Joint state	$P(R,S)$	ρ_{AB}
Marginalization	$P(S) = \sum_R P(R,S)$	$\rho_B = \mathrm{Tr}_A \rho_{AB}$
Conditional state	$P(S\|R)$	$\rho_{B\|A}$
	$\sum_S P(S\|R) = 1$	$\mathrm{Tr}_B(\rho_{B\|A}) = I_A$
Belief propagation	$P(S) = \sum_R P(S\|R)P(R)$	$\rho_B = \mathrm{Tr}_A(\rho_{B\|A}\rho_A)$



$P(R, S)$



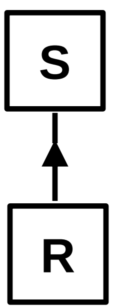
ρ_{AB}

$P(S|R) = P(R, S)/P(R)$????

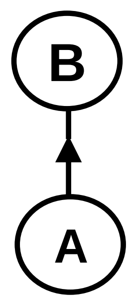
$P(S) = \sum_R P(S|R)P(R)$????

$P(S) = \Gamma_{R \to S}[P(R)]$????

$P(R, S)$????



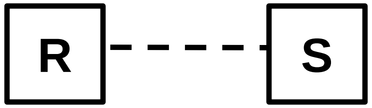
$P(S|R) = P(R, S)/P(R)$????



$P(S) = \sum_R P(S|R)P(R)$????

$P(S) = \Gamma_{R \to S}[P(R)]$

$\rho_B = \mathcal{E}_{A \to B}(\rho_A)$



$$P(R,S)$$

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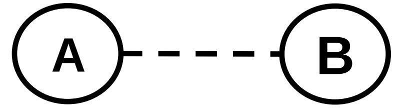
$$P(S) = \sum_R P(S|R)P(R)$$

$$P(S) = \Gamma_{R\to S}[P(R)]$$

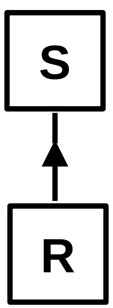


$$\rho_{AB}$$

$$\rho_{B|A} = \rho_A^{-1/2}\rho_{AB}\rho_A^{-1/2}$$

$$\rho_B = \mathrm{Tr}_A(\rho_{B|A}\rho_A)$$

$$\rho_B = \mathfrak{E}_{A\to B}(\rho_A)$$



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$$P(S) = \sum_R P(S|R)P(R)$$

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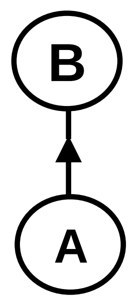

????

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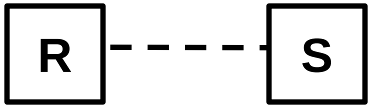
????

$$\rho_B = \mathcal{E}_{A\to B}(\rho_A)$$



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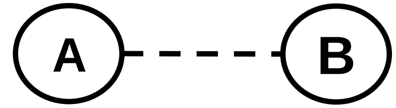

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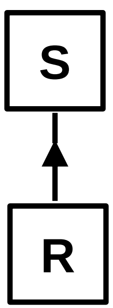
$$\rho_B = \mathfrak{E}_{A\to B}(\rho_A)$$



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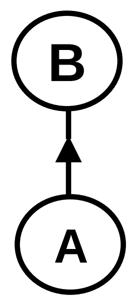

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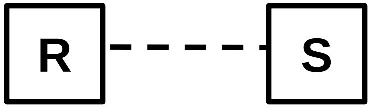

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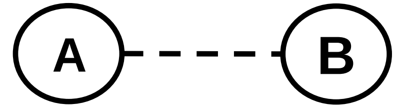

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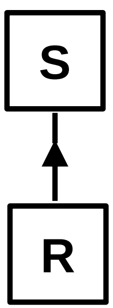
$$\rho_B = \mathrm{Tr}_A(\rho_{B|A}\rho_A)$$

$$\rho_B = \mathfrak{E}_{A\to B}(\rho_A)$$

$$\rho_{B|A} \geq 0$$

$$\mathfrak{E}_{A\to B}\circ T_A \ \text{ is CP}$$



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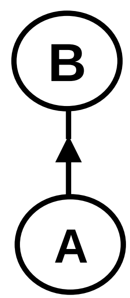
$$P(S) = \sum_R P(S|R)P(R)$$

$$P(S) = \Gamma_{R\to S}[P(R)]$$



$$\varrho_{AB}$$

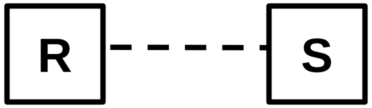
$$\varrho_{B|A} = \varrho_A^{-1/2}\varrho_{AB}\rho_A^{-1/2}$$

$$\rho_B = \mathrm{Tr}_A(\varrho_{B|A}\rho_A)$$

$$\rho_B = \mathcal{E}_{A\to B}(\rho_A)$$

$$\varrho_{B|A}^{T_A} \geq 0$$

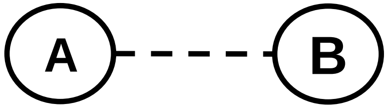
$$\mathcal{E}_{A\to B} \ \text{ is CP}$$



$$P(R,S)$$

$$P(S|R) = P(R,S)/P(R)$$

$$P(S) = \sum_R P(S|R)P(R)$$



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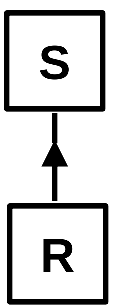

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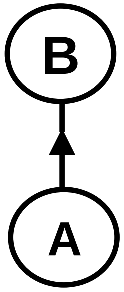

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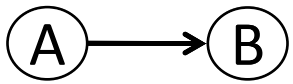
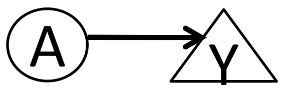
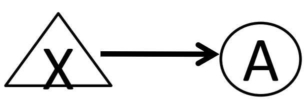
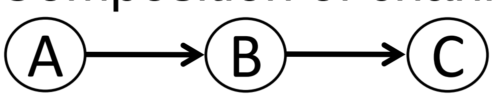
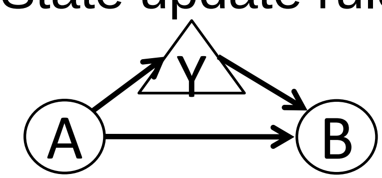
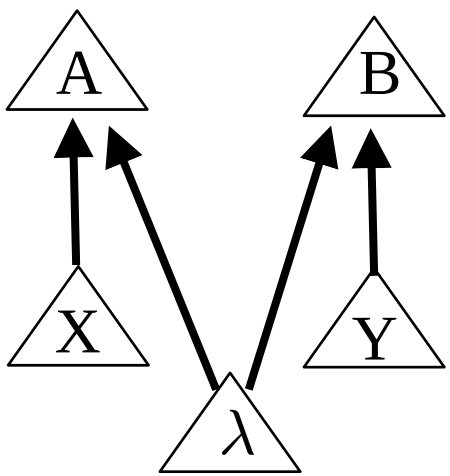
$$\varrho_{B|A}^{T_A} \geq 0$$

	Conventional expression	In terms of conditional states
Probability distribution for X	$P(X)$	ρ_X
Set of states on A	$\{\rho_x^A\}$	$\rho_{A\|X}$
POVM on A	$\{E_x^A\}$	$\rho_{X\|A}$
Channel from A to B	$\mathcal{E}^{A\to B}$	$\rho_{B\|A}$
Instrument	$\{\mathcal{E}_x^{A\to B}\}$	$\rho_{XB\|A}$

	Conventional expression	In terms of conditional states
Action of quantum channel 	$\rho_B = \mathcal{E}^{A\to B}(\rho_A)$	$\rho_B = \mathrm{Tr}_A(\rho_{B\|A}\rho_A)$
Born's rule 	$P(Y=y) = \mathrm{Tr}_A(E_y^A \rho_A)$	$\rho_Y = \mathrm{Tr}_A(\rho_{Y\|A}\rho_A)$
Ensemble averaging 	$\rho_A = \sum_x P(X=x)\rho_x^A$	$\rho_A = \mathrm{Tr}_X(\rho_{A\|X}\rho_X)$
Composition of channels 	$\mathcal{E}^{A\to C} = \mathcal{E}^{B\to C} \circ \mathcal{E}^{A\to B}$	$\rho_{C\|A} = \mathrm{Tr}_B(\rho_{C\|B}\rho_{B\|A})$
State update rule 	$P(Y=y)\rho_y^B = \mathcal{E}_y^{A\to B}(\rho_A)$	$\rho_{YB} = \mathrm{Tr}_A(\rho_{YB\|A}\rho_A)$

Quantum Causal Models



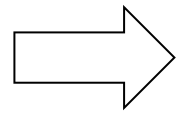
$P(X)$

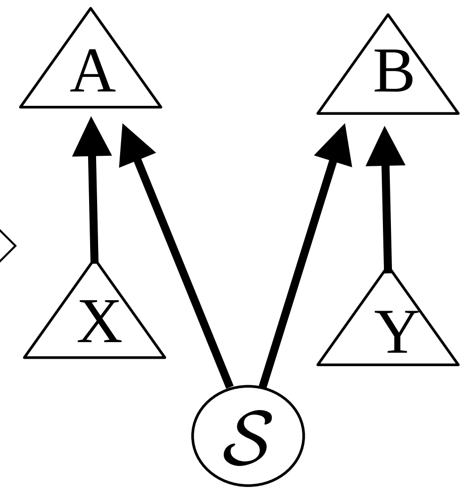
$P(Y)$

$P(\lambda)$

$P(A|\lambda, X)$

$P(B|\lambda, Y)$





ρ_X

ρ_Y

$\rho_{\mathcal{S}}$

$\rho_{A|X\mathcal{S}}$

$\rho_{B|Y\mathcal{S}}$

$$P(A, B|X, Y) = \sum_\lambda P(A|\lambda, X)P(B|\lambda, Y)P(\lambda)$$

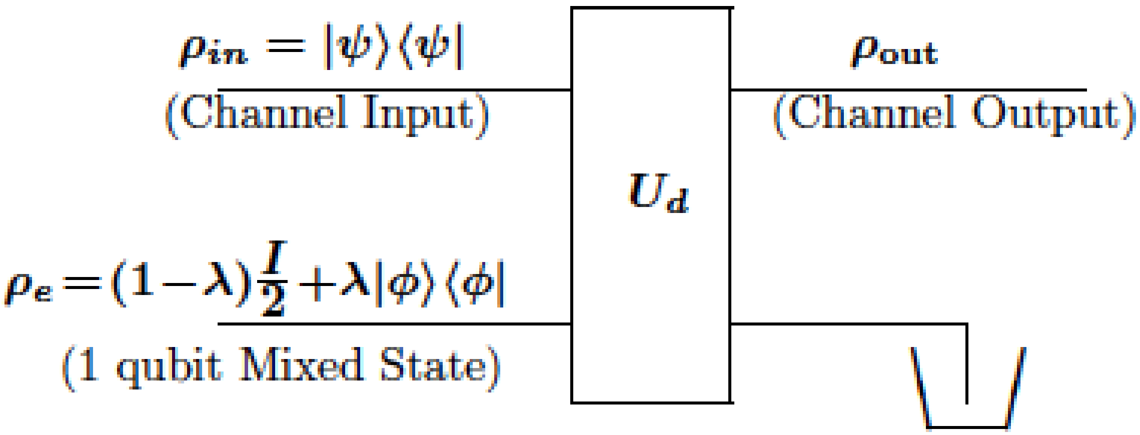
$$\rho_{AB|XY} = \mathrm{Tr}_{\mathcal{S}}(\rho_{A|X\mathcal{S}}\rho_{B|Y\mathcal{S}}\rho_{\mathcal{S}})$$

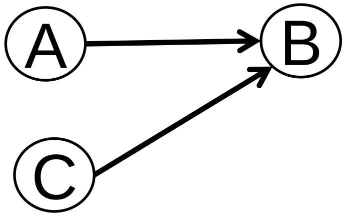
Deriving quantum correlations

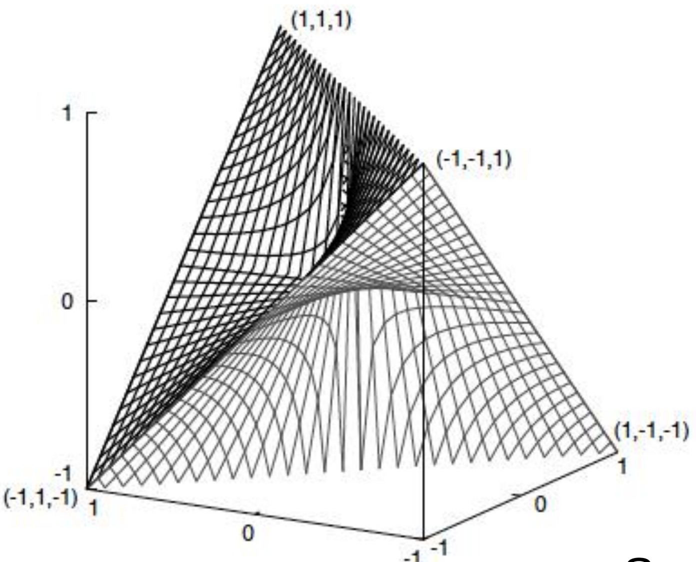
A possible line of attack:
Principles about inference → Quantum Bayesian inference
+ Assumptions about causal structure

See: Coecke and RWS, Synthese 186, 651 (2012)

Understanding the subset of qubit channels induced by a single qubit ancilla

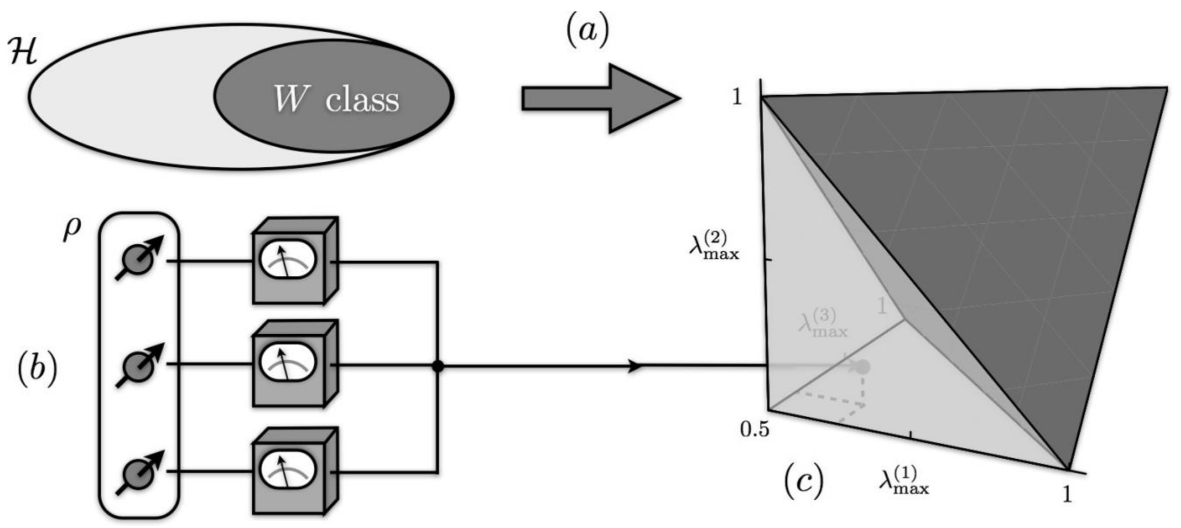






See: Narang and Arvind, arXiv:quant-ph/0611058

Understanding multipartite entanglement SLOCC classes



See: Walter et al. arXiv:1208.0365

Fin