

STRONG-FIELD QED AND HIGH-POWER LASERS

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QFEXT2011, BENASQUE, SPAIN

23 SEPTEMBER 2011

**DISCOVER
WITH
PLYMOUTH
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Outline



1. Introduction
2. Strong Laser Fields: Theory
3. Historical Interlude
4. Strong Laser Fields: Examples
5. Conclusion and Outlook

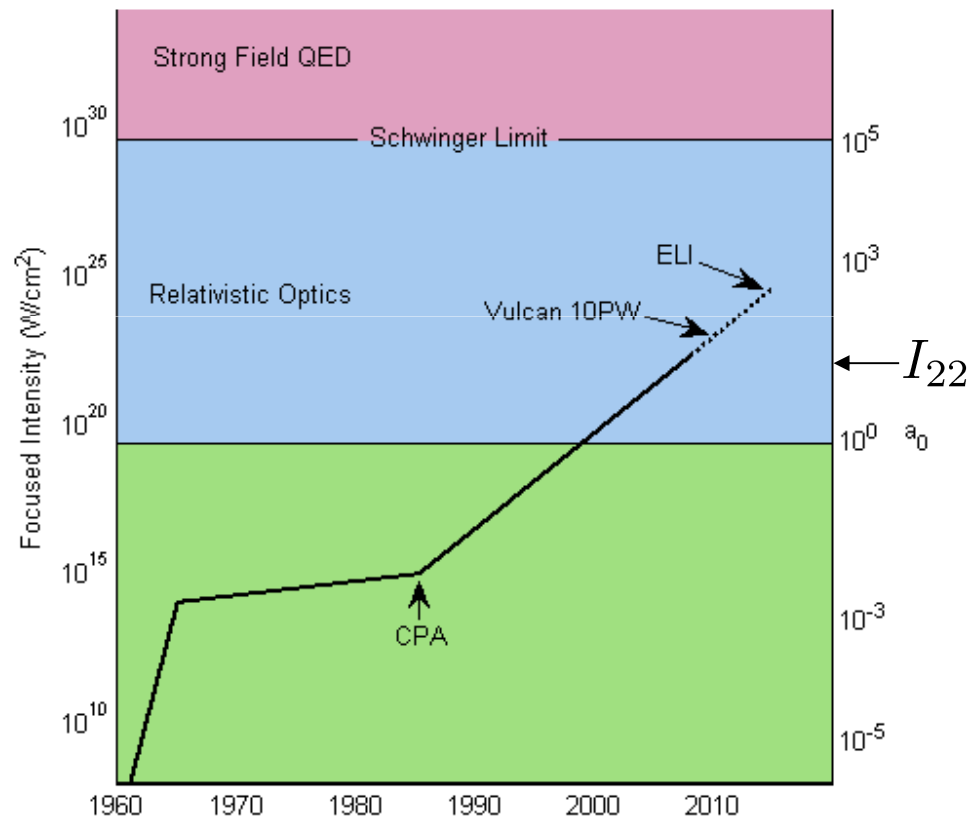


Introduction

Context: new physics

- Increase sensitivity/versatility of experiments:
- Explore new regimes for **known** parameters
 - ▣ Resolution/energy
 - ▣ Temperature/density/pressure
 - ▣ External conditions: boundaries (Casimir effect)
 - ▣ Magnitude of external field
 - Gravitational: e.g. ultracold neutrons (peV!)
 - Electromagnetic: E, B and combinations thereof, e.g. **Laser**
 - ▣ **NB:** may need to go beyond concept of “external”

Laser intensity: Time evolution



- Important parameter: dim.less amplitude

$$a_0 \equiv \frac{eE\lambda_L}{mc^2} \sim I^{1/2}$$

- Energy gain of e^- per λ_L
- $a_0 \gtrsim 1$: e^- relativistic
- magnitude:

$$a_0 = 60\sqrt{I/I_{22}} \lambda/\mu\text{m}$$

(adapted from Mourou, Tajima, Bulanov, RMP **78**, 2006)

Regime of Extremes

- Current magnitudes:

Power	$P \gtrsim 10^{15} \text{ W} \equiv 1 \text{ PW}$
Intensity	$I \gtrsim 10^{22} \text{ W/cm}^2$
Electric field	$E \gtrsim 10^{14} \text{ V/m}$
Magnetic field	$B \gtrsim 10^{10} \text{ G} \equiv 10^6 \text{ T}$

- Fields **huge** but...
- ... **pulsed** and **alternating**

2 Laser Projects (of many)



Building (projected)

- CLF Vulcan 10 PW

- 10^{23} Wcm^{-2}

- Completion by 2016

- Budget: 20 M£

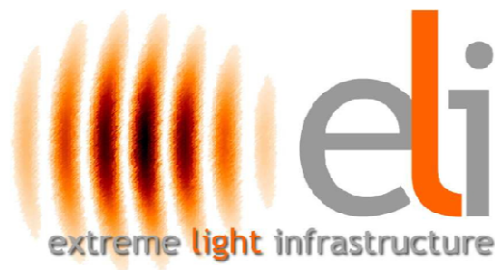
- ELI ('4th pillar')

- $>100 \text{ PW}$ (Exawatt ?)

- $>10^{25} \text{ Wcm}^{-2}$

- Budget: several 100 M€

- Decision by 2012 (?)





2. Strong Laser Fields: Theory

Modelling a laser

□ In order of increasing complexity:

□ Plane wave



■ Infinite (IPW)

■ Pulsed (PPW)

■ Finite T-duration

■ Infinite transverse extension

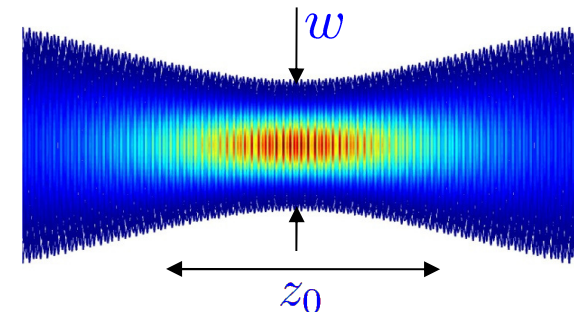
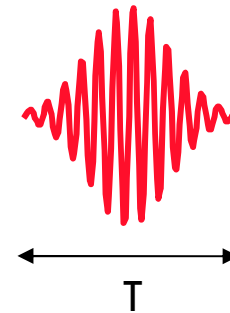
□ Gaussian beam:

■ Finite transverse waist w

■ Finite longitudinal extension z_0

■ parameter: $\kappa \equiv w/z_0 \lesssim 1/2\pi$

■ measures deviation from PW



Modelling a laser: Plane wave

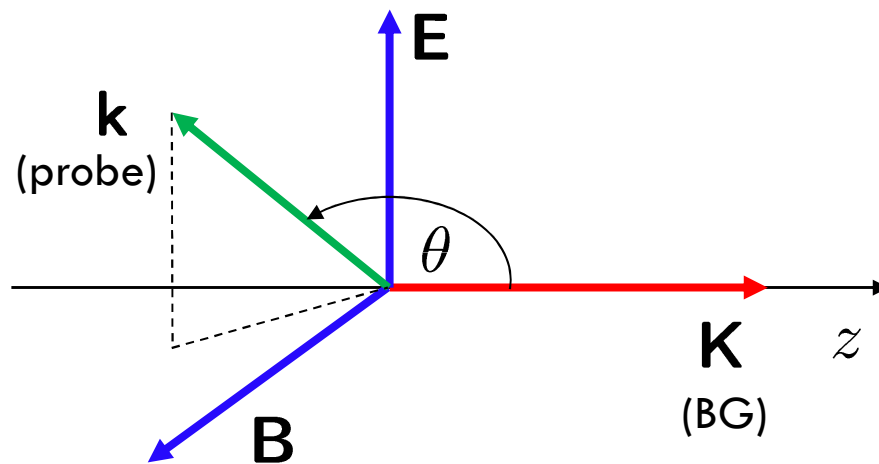
- **Null** wave vector K , $K^2 = 0$
- E.M. field: $F^{\mu\nu}$
 - univariate: $F^{\mu\nu} = F^{\mu\nu}(K \cdot x)$, $K \cdot x = \Omega x^-$
 - transverse: $F^{\mu\nu} K_\nu = 0$
 - **Null:**

$$\mathcal{S} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = 0, \quad \mathcal{P} = -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = 0, \quad F^3 = 0$$

- **NB:** for Gaussian beams $\mathcal{S}, \mathcal{P} = O(\kappa^2)$
- No intrinsic invariant scale – but note: $T \equiv F^2 \neq 0$
- Need (probe) momentum to build invariants

Kinematic invariants

□ Sketch:



□ Lab momenta
(head-on collⁿ)

$$K^\mu = \Omega(1, \mathbf{z})$$

$$k = \omega(1, -\mathbf{z})$$

□ Two basic invariants

$$I_1 \equiv K \cdot k \stackrel{\text{lab}}{=} 2\Omega\omega$$

$$I_2^2 \equiv k_\mu T^{\mu\nu} k_\nu \stackrel{\text{lab}}{=} (2\omega E)^2, \quad T^{\mu\nu} = (E^2/\Omega^2)K^\mu K^\nu$$

Dimensionless parameters

- Measure E in units of QED electric field

$$E_S \equiv m^2/e = 1.3 \times 10^{18} \text{ V/m} \quad (\text{Sauter 1931})$$

- Define dimensionless field and probe frequency

$$\epsilon \equiv E/E_S \quad \nu \equiv \omega/m$$

- In terms of invariants

$$I_1/m^2 \stackrel{\text{lab}}{=} 2\nu \Omega/m$$

$$eI_2/m^3 \stackrel{\text{lab}}{=} 2\epsilon\nu$$

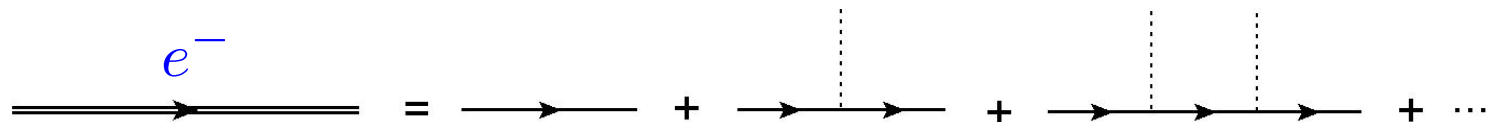
$$a_0 \equiv eI_2/mI_1 \stackrel{\text{lab}}{=} \epsilon m/\Omega$$

Strong-field QED

- Split $A \rightarrow A + a$: BG (laser) and fluctuation (probe)

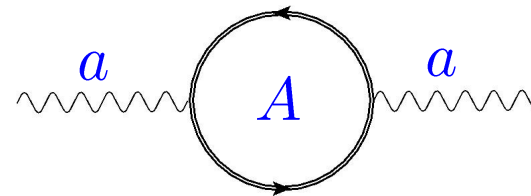
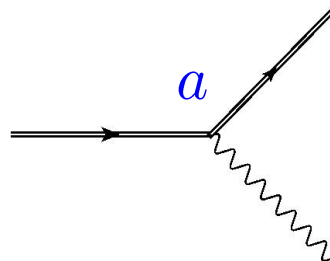
- Probe photons: γ  k

- Electrons 'dressed' by laser photons:  K



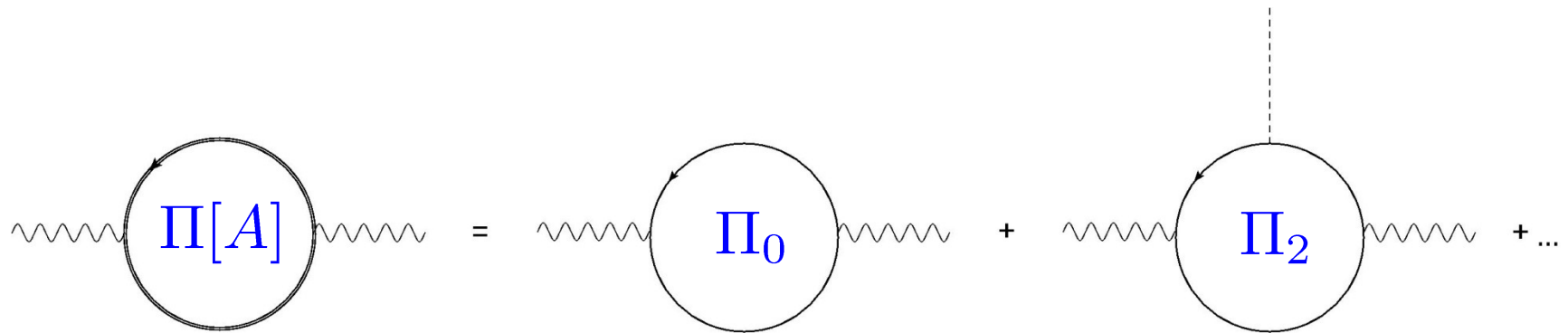
- “Furry Picture” Diagrams:

Scattering
(\rightarrow A. Ilderton)



Vacuum polarisation

SF Vacuum polarisation expanded



The diagram shows the expansion of the vacuum polarization tensor $\Pi[A]$ into a series of terms. On the left, a wavy line enters a circle labeled $\Pi[A]$ with a clockwise arrow, and another wavy line exits. This is equal to the sum of three terms: 1) a wavy line entering a circle labeled Π_0 with a clockwise arrow, and another wavy line exiting; 2) a wavy line entering a circle labeled Π_2 with a clockwise arrow, and another wavy line exiting; and 3) an ellipsis $+ \dots$. A vertical dashed line extends upwards from the Π_2 circle.

$$\text{wavy line} \rightarrow \text{circle } \Pi[A] \leftarrow \text{wavy line} = \text{wavy line} \rightarrow \text{circle } \Pi_0 \leftarrow \text{wavy line} + \text{wavy line} \rightarrow \text{circle } \Pi_2 \leftarrow \text{wavy line} + \dots$$

$$\mathcal{L}_{\text{eff}} = \frac{2}{45} \frac{\alpha^2}{m^4} (c_1 \mathcal{S}^2 + c_2 \mathcal{P}^2)$$

$$c_1 = 4, \quad c_2 = 7$$

$$\rightarrow \mathcal{L}_{\text{eff}} = \frac{1}{2} a_\mu \Pi_2^{\mu\nu} [A] a_\nu$$

LO light-by-light scattering

Euler, Kockel, Naturwiss. **15**, 246 (1935)

Students of Heisenberg's

3. Historical Interlude

Heisenberg & Euler (\rightarrow G. Dunne)

Heisenberg & Euler



Werner Heisenberg (1901-76)
Foto from 1933 (Wikipedia)



Hans Euler (1909-41)
Foto ca. 1935 (H. Wergeland)

Hans Euler: a very short biography

6 Oct. 1909	Born in Meran, South Tyrol
1929-1933	Graduate student (Munich, Bonn, Göttingen, Frankfurt)
1933-35	PhD student in Leipzig with Heisenberg Thesis on light-by-light scattering
25 June 1936	PhD award (“very good”)
2 June 1938	Submits habilitation on cosmic rays
Oct. 1938	Assistant (Leipzig)
1940	Though leftist volunteers for army after phase of depression
23 June 1941	Lost in action, Sea of Asow, Crimea

Light-by-light scattering: Timeline

- O. Halpern, Phys. Rev. **44**, 855 (1933):
 - ▣ *Scattering Processes Produced by Electrons in Negative Energy States*
- Discussion Heisenberg-Debye, Leipzig 1934
- H. Euler, B. Kockel, Naturwiss. **23**, 246 (1935):
 - ▣ *On the scattering of light by light according to Dirac's theory*
 - ▣ Short note, 2p, LO correction to Maxwell
- H. Euler, Thesis 1935, Ann. Phys. **26**, 398 (1936), same title
 - ▣ PT calculation of LO correction coefficients ("4 : 7")
- Akhiezer, Landau, Pomeranchuk, Nature **138**, 206 (1936)
 - ▣ *Scattering of Light by Light*
- A. Akhiezer, Physik. Z. Sowjetunion **11**, 263 (1937)
 - ▣ *On the scattering of light by light (high-energy limit)*

Heisenberg & Euler: Z. Phys. **98**, 714 (1936)

- W. Heisenberg, H. Euler: low-energy correction to all orders

Folgerungen aus der Diracschen Theorie des Positrons.

Von W. Heisenberg und H. Euler in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur **Paarerzeugung** neigt, eine Abänderung der Maxwell'schen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur **wenig ändert**. Es ergibt sich für das Feld eine Lagrange-Funktion:

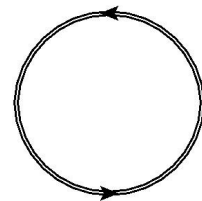
$$\Omega = \frac{1}{2} (\mathcal{E}^2 - \mathcal{B}^2) + \frac{e^2}{\hbar c} \int_0^\infty e^{-\eta} \frac{d\eta}{\eta^3} \left\{ i \eta^2 (\mathcal{E} \mathcal{B}) \cdot \frac{\cos\left(\frac{\eta}{|\mathcal{E}_k|} \sqrt{\mathcal{E}^2 - \mathcal{B}^2 + 2i(\mathcal{E} \mathcal{B})}\right) + \text{konj}}{\cos\left(\frac{\eta}{|\mathcal{E}_k|} \sqrt{\mathcal{E}^2 - \mathcal{B}^2 + 2i(\mathcal{E} \mathcal{B})}\right) - \text{konj}} + |\mathcal{E}_k|^2 + \frac{\eta^2}{3} (\mathcal{B}^2 - \mathcal{E}^2) \right\}.$$

$$\left(\begin{array}{l} \mathcal{E}, \mathcal{B} \text{ Kraft auf das Elektron.} \\ |\mathcal{E}_k| = \frac{m^2 c^3}{e \hbar} = \frac{1}{„137“} \frac{e}{(e^2/m c^2)^2} = \text{„Kritische Feldstärke“} \end{array} \right)$$

Ihre Entwicklungsglieder für (gegen $|\mathcal{E}_k|$) kleine Felder beschreiben Prozesse der **Streuung von Licht an Licht**, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwell'schen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

Heisenberg & Euler: achievements

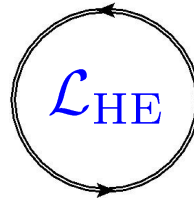
- Eff. Lagrangian to all orders in const. external field


$$= \mathcal{L}_{\text{HE}}(\mathcal{S}, \mathcal{P})$$

- Critical field strength (Sauter)
- Renormalisation: no infinities (cf. Pauli: 'subtraction physics')
- Anticipation of
 - evaluation of BG determinants
 - Low-energy effective field theory
- Vacuum polarisation

Heisenberg & Euler: Vacuum polarisⁿ

andererseits wird selbst dort, wo die Energie zur Materieerzeugung nicht ausreicht, aus ihrer virtuellen Möglichkeit eine Art „Polarisation des Vakuums“ und damit eine Änderung der Maxwell'schen Gleichungen resultieren.

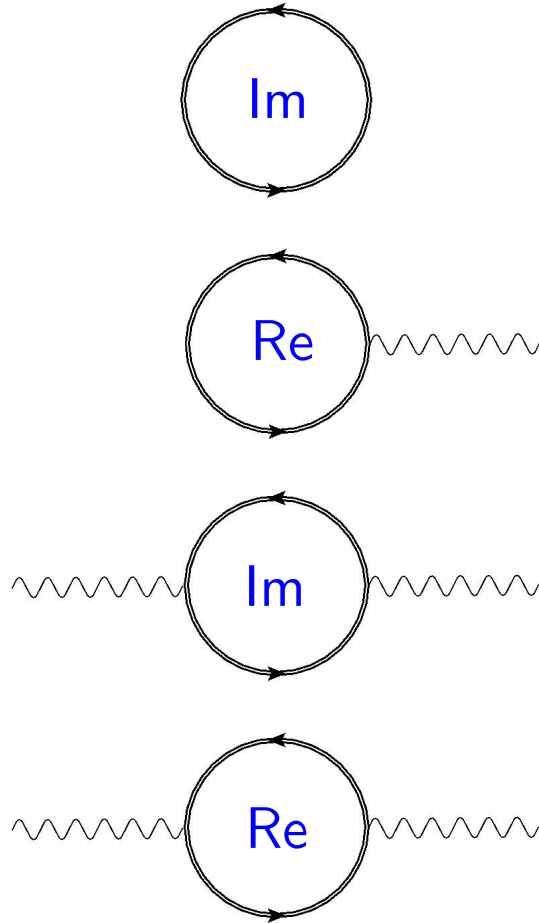


“...even in situations where the [photon] energy is not sufficient for matter production, its virtual possibility will result in a ‘**polarization of the vacuum**’ and hence in an alteration of Maxwell’s equations.”

4. Strong Laser Fields: Examples

(Vacuum Polarisation)

Overview



Exotic loop particles?
→ PVLAS (G. Zavattini)

- Vacuum PP: ‘Schwinger effect’
 - ▣ zero for null fields
 - ▣ exponential suppression
- Vacuum emission
 - ▣ Power suppression
- ‘Stimulated’ PP
 - ▣ threshold ‘suppression’ (→ A. Ilderton)
- Vacuum Birefringence
 - ▣ Power suppression

4. Strong Laser Fields: Examples

4.1 Vacuum Pair Production

Spontaneous (vacuum) PP

- Feynman diagram (optical theorem)

$$\text{Im} \left[\text{Diagram} \right] \sim \left| \text{Diagram} \right|^2 \quad \text{'vacuum breakdown'}$$

- Identically **zero** for PWs as $\mathcal{S} = \mathcal{P} = 0$
- Substantial when

$$E_0 \equiv \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} + \mathcal{S} \right)^{1/2} \gtrsim E_S$$

- Rate exponentially suppressed (Schwinger 1951)

$$\mathfrak{R} \sim \exp(-\pi E_S / E_0)$$

Vacuum PP cont^d

- With lasers: **very difficult!**
- Need to fight both
 - ▣ Exponential suppression
 - ▣ Null field (plane wave) character ($E_0 = 0$)
- Expect rate for e.g. Gaussian beams

$$\mathfrak{R} \sim \kappa^2 \coth(\pi B/E) \exp(-\pi E_S/\kappa E)$$

- Alternative: counter-propagating lasers (standing wave)?

$$\mathcal{S} \neq 0 \quad \text{and/or} \quad \mathcal{P} \neq 0$$

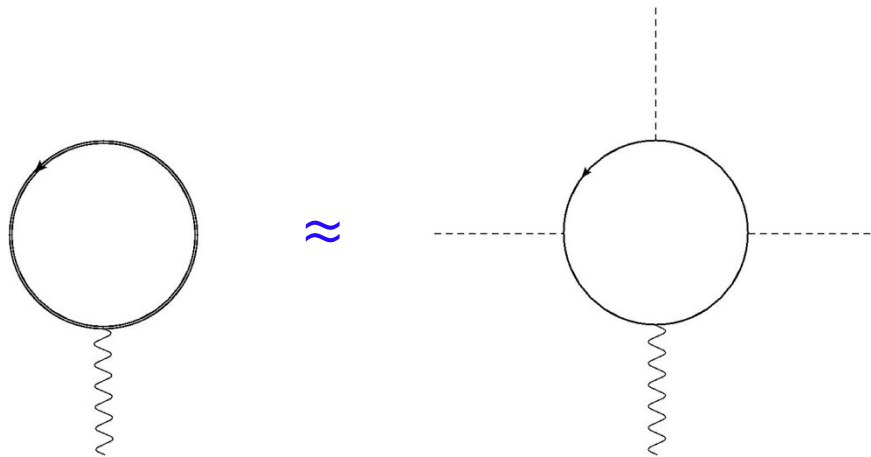
4. Strong Laser Fields: Examples

4.2 Light-by-Light Scattering

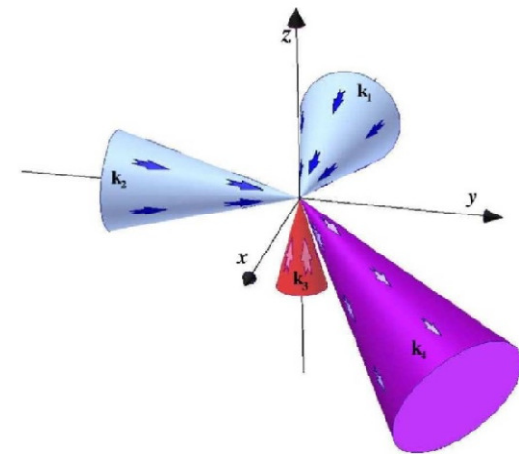
γ - γ scattering

- Recall prediction: Halpern 1934, Euler/Kockel 1935, Euler; Heisenberg/Euler 1936
- But (for real γ 's) never observed in lab!
- Idea: $3\gamma_L \rightarrow \gamma$ ('3-wave mixing') (Lundström et al., 2005)
(Monden, Kodama, 2011)

Feynman diagrams:



Artistic view:



γ - γ scattering cont^d

- Low-energy X-section ('Euler-Kockel' approxⁿ):

$$\sigma_{\gamma\gamma} = \frac{973}{10125\pi^2} \alpha^2 r_e^2 \nu_L^6 \simeq 10^{-67} \text{ cm}^2$$

- Laser photon density:

$$n_L \simeq 10^{14} a_0^2 / \mu\text{m}^3$$

- Photon number in focus volume $(10 \mu\text{m})^3$

$$N_\gamma \simeq 10^{17} a_0^2$$

- Number of scattered γ' s @ $a_0 \simeq 10^2$

$$N_{\gamma'} \simeq \frac{\sigma_{\gamma\gamma}}{(10 \mu\text{m})^2} N_\gamma^3 \simeq 10^{-4}$$

4. Strong Laser Fields: Examples

4.3 Vacuum Birefringence (VB)

HE vacuum dispersion

- Recall: for *single* plane wave BG $\mathcal{L}_{\text{HE}} = 0$
- But: for *two* plane waves, $A \rightarrow A + a$ (BG & probe) find to 2nd order in fluctuation

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{\text{HE}} = \frac{1}{2} a_\mu (\square g^{\mu\nu} + \Pi^{\mu\nu}[A]) a_\nu$$

- vac.pol. tensor $\Pi^{\mu\nu}(A; k) = C^{\mu\nu}_{\alpha\beta}(A) k^\alpha k^\beta$
- **Two** nontrivial eigenvals $\Pi_\pm = c_\pm k_\mu T^{\mu\nu} k_\nu$
- **Two** dispersion relations ('deformed LC')

$$k^2 - \Pi_\pm = (g^{\mu\nu} - c_\pm T^{\mu\nu}) k_\mu k_\nu = 0$$

Vacuum birefringence



Calcite crystal

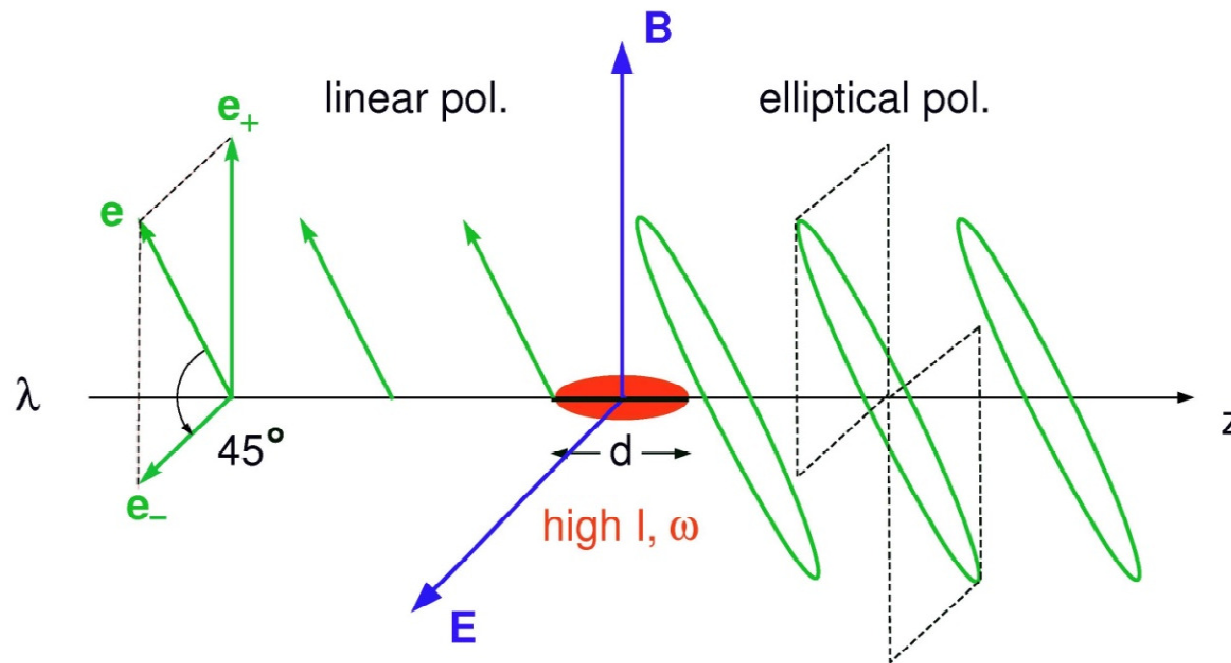
- **Two** indices of refraction (Toll 1952; Narozhny 1968; Brezin, Itzykson 1970)

$$n_{\pm} = 1 + \frac{\alpha}{45\pi} (11 \pm 3) \epsilon^2 \quad \epsilon \equiv E/E_S$$

- Experimental signature: ellipticity (squared)

$$\delta^2 \sim (n_+ - n_-)^2$$

Experiment: measure ellipticity



Phase retardation of \mathbf{e}_+

Analysis (TH et al., Opt. Commun., 2006)

- ellipticity (squared)

$$\delta^2 = 3.2 \times 10^5 \epsilon^2 \left(\frac{d}{\mu\text{m}} \epsilon \nu \right)^2, \quad \epsilon \nu \ll 1$$

- Optimal scenario: XFEL ($\nu \simeq 10^{-2}$) & HP laser

- PW laser ($\epsilon \simeq 10^{-4}$): $\delta^2 \simeq 10^{-11}$

- ELI ($\epsilon \simeq 10^{-2}$): $\delta^2 \simeq 10^{-7} \dots 10^{-4}$

- X-ray polarimetry:

- New record in polarisation purity: 1.5×10^{-9} @ 6 keV

(Marx et al., Opt. Comm., 2010)

- Recently (I. Uschmann)

$$2.4 \times 10^{-10}$$

Beyond Heisenberg-Euler I

- For large probe frequency ($\nu > 1$) HE breaks down
- need full polarisation tensor (→ F. Karbstein)
 - Crossed fields (Toll 1952, Narozhny 1968, Batalin/Shabad 1968)
 - Plane waves (Becker/Mitter 1975, Baier et al. 1975)
- for simpler case of crossed fields

$$n = 1 + \frac{2\alpha}{\pi} \epsilon^2 f(\epsilon\nu)$$

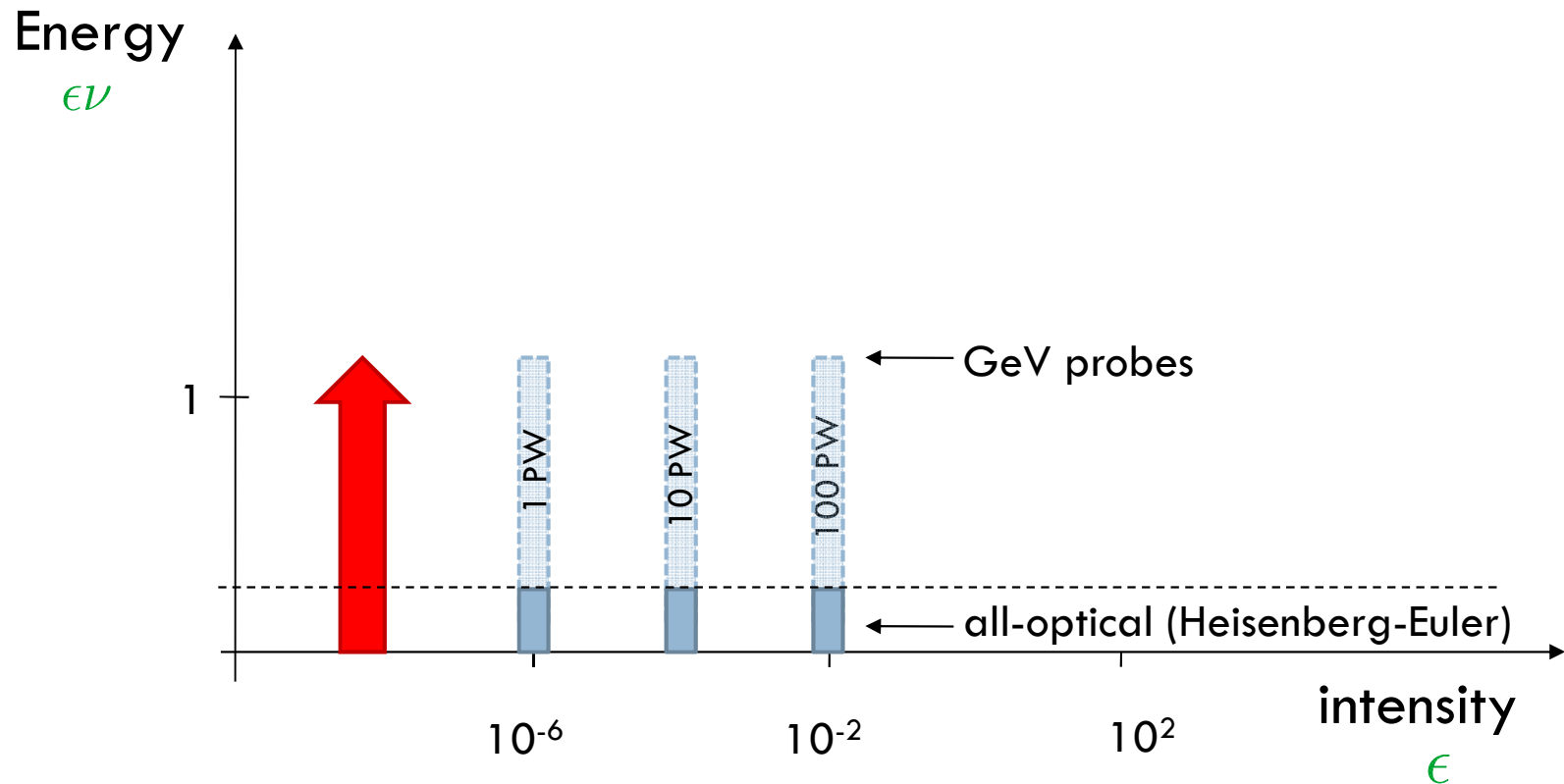
Toll 1952
TH, O. Schröder 2006
Shore 2007

- With *dispersive and absorptive* parts

$$f = f_R + if_I$$

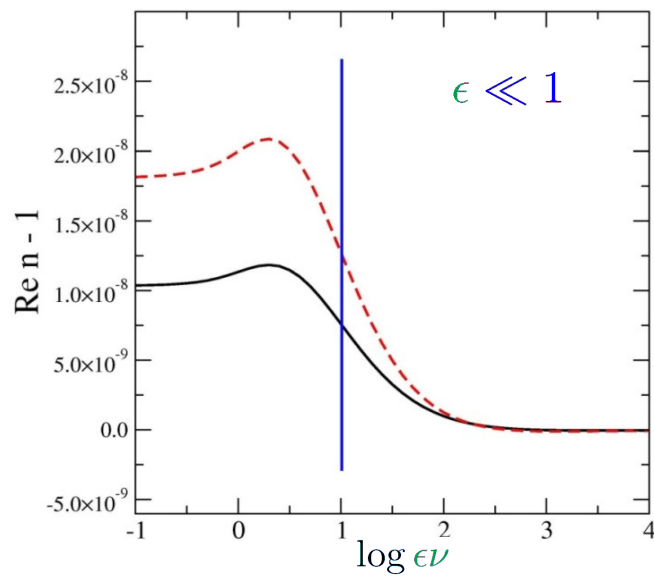
Beyond Heisenberg-Euler II

- Energy intensity diagram

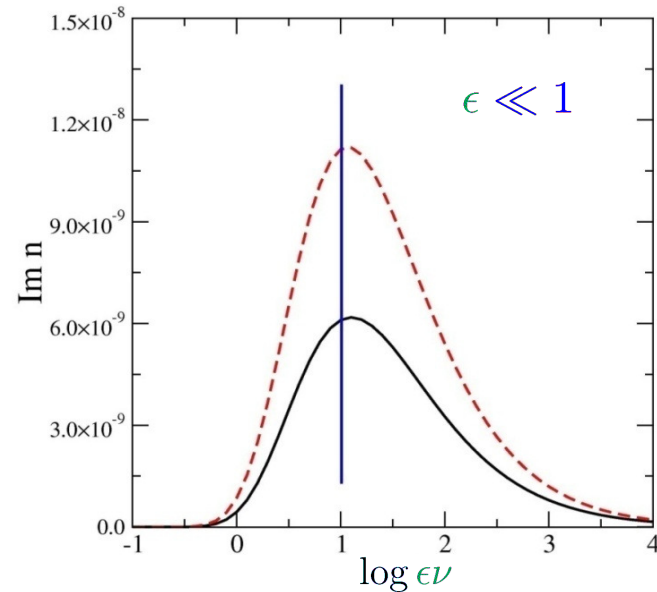


Beyond Heisenberg-Euler III

- e.g. $\epsilon V \simeq 3$ from Compton backscattering (few GeV e^-)



Anomalous dispersion



Absorption \rightarrow PP

(K. Langfeld)

Conclusion

- Laser power approaching exawatt regime
- Extreme field physics
 - ▣ **High-intensity QED** → Sauter-Schwinger limit
 - ▣ Needs to be **tested** experimentally for $a_0 > 1$!
 - ▣ **New physics** ('hidden sector': WISPs, ALPs,...)
- Theory (→ a_0 dependence)
 - ▣ Challenges:
 - Finite size effects: Pulse duration/extension
 - Beyond plane waves: Gaussian beams, ...
 - Beyond ext. field approx: Back reaction (→ C. Harvey)



Thank you very much...

...for your attention



Appendix



Field Invariants (cf. Schwinger 1951)

- Lorentz and gauge invariant characterisation of vacuum fields using field strength tensor \mathbb{F}

$$\mathcal{S} \equiv \frac{1}{4} \text{tr} \mathbb{F}^2 = \frac{1}{2} (E^2 - B^2) \quad (\text{scalar})$$

$$\mathcal{P} \equiv \frac{1}{4} \text{tr} \mathbb{F} \tilde{\mathbb{F}} = \mathbf{E} \cdot \mathbf{B} \quad (\text{pseudoscalar})$$

- Combine into invariant field variables

$$a \equiv \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} + \mathcal{S} \right)^{1/2} \stackrel{\text{SF}}{=} E$$

$$b \equiv \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} - \mathcal{S} \right)^{1/2} \stackrel{\text{SF}}{=} B$$



4 cases of fields (Taub 1948)

□ Table:

Name	Special frame (SF)	Invariants	
Electromagnetic	$E \parallel B$	$\mathcal{P} \neq 0$	$\mathcal{S} \neq 0$
Magnetic	B	$\mathcal{P} = 0$	$\mathcal{S} < 0$
Electric	E	$\mathcal{P} = 0$	$\mathcal{S} > 0$
Null	$E \perp B, E = B$	$\mathcal{P} = 0$	$\mathcal{S} = 0$

Modelling a laser: Gaussian beam

- Finite geometry parameter:

$$\kappa \equiv w/z_0 \lesssim 1/2\pi$$

- PW limit: $\kappa \rightarrow 0$

- Transverse fields:

$$E_T = B_T \equiv E$$

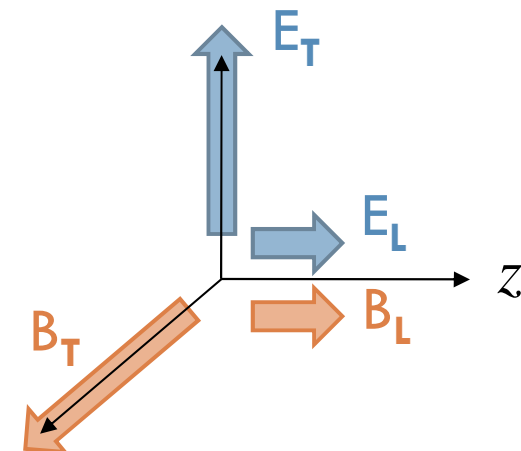
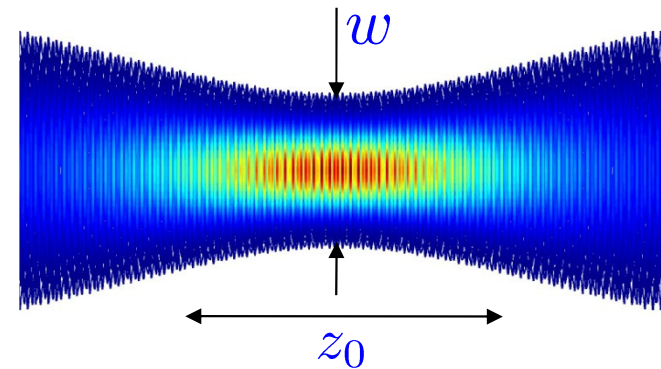
- Longitudinal fields:

$$E_L, B_L \sim \kappa E$$

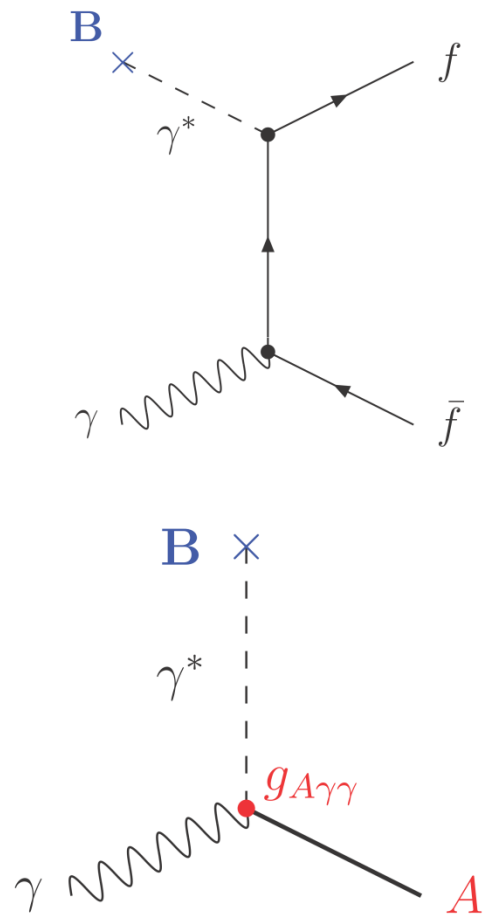
- Invariants **not null** but $O(\kappa^2)$:

$$S = (E_L^2 - B_L^2)/2, \quad \mathcal{P} = \mathbf{E}_L \cdot \mathbf{B}_L$$

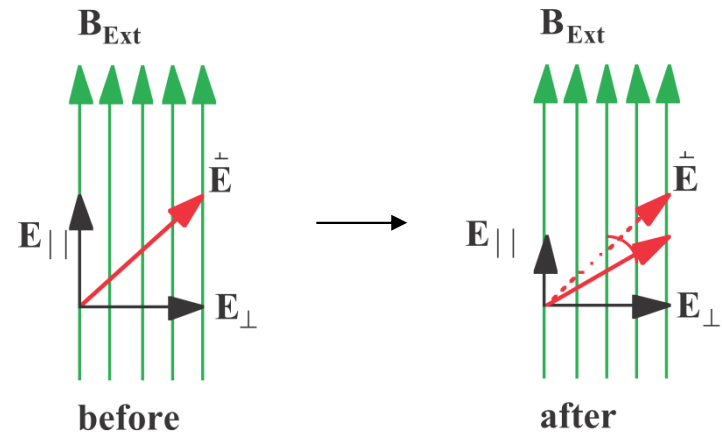
(Davis 1978, Narozhny et al. 2004)



Light fermions, ALPs



- 'disappearing' photons
- Absorption
- Coeff^s κ_{\pm}



- Rotation: $\Delta\theta \sim |\kappa_+ - \kappa_-|$

(A. Ringwald, hep-ph/07043195)