Magnetic Flux Tubes in Neutron Stars

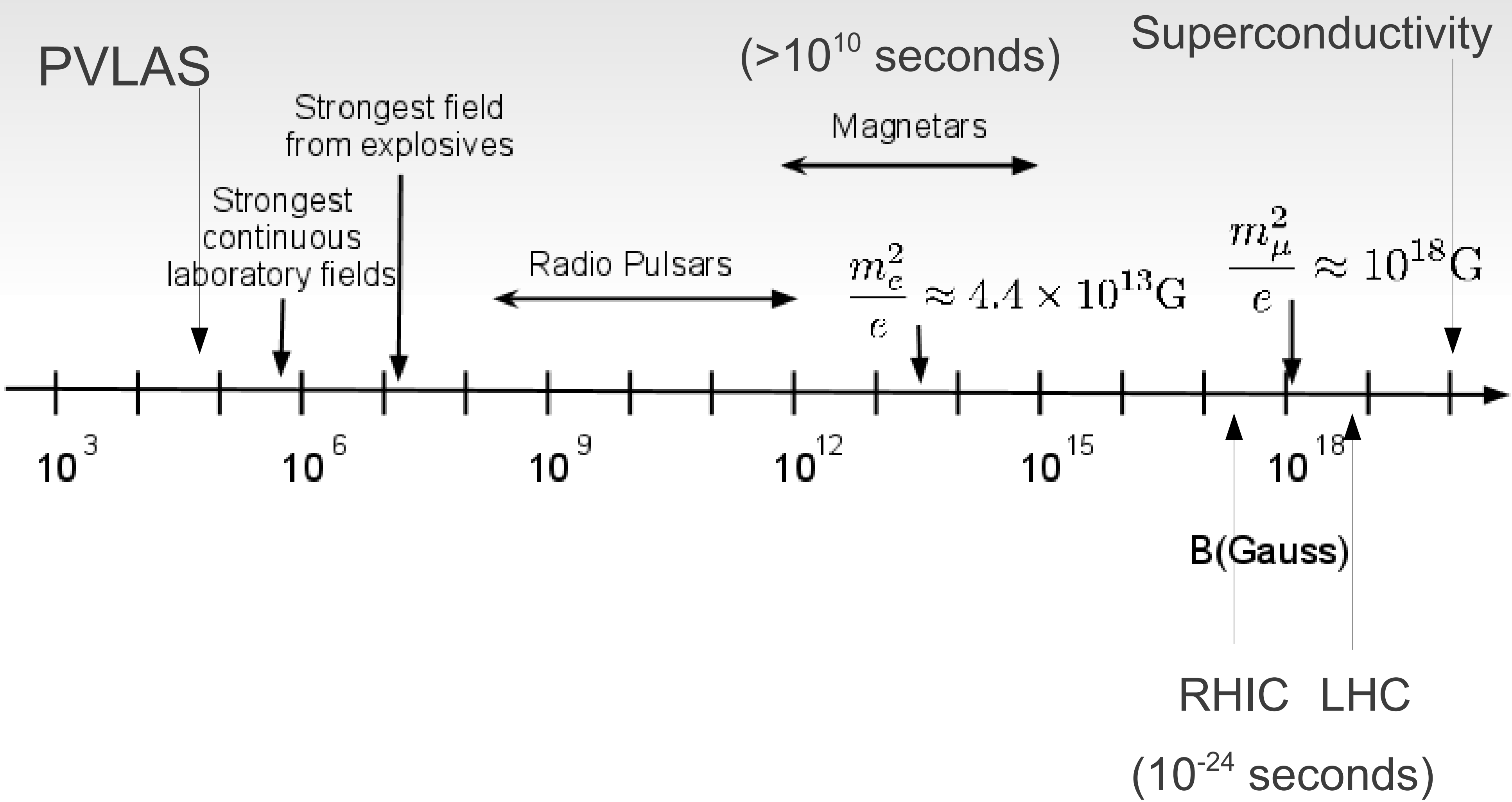
Dan Mazur
w/ Jeremy S. Heyl
University of British Columbia
QFEXT 2011
Bénasque, Spain

Image: christianjoore

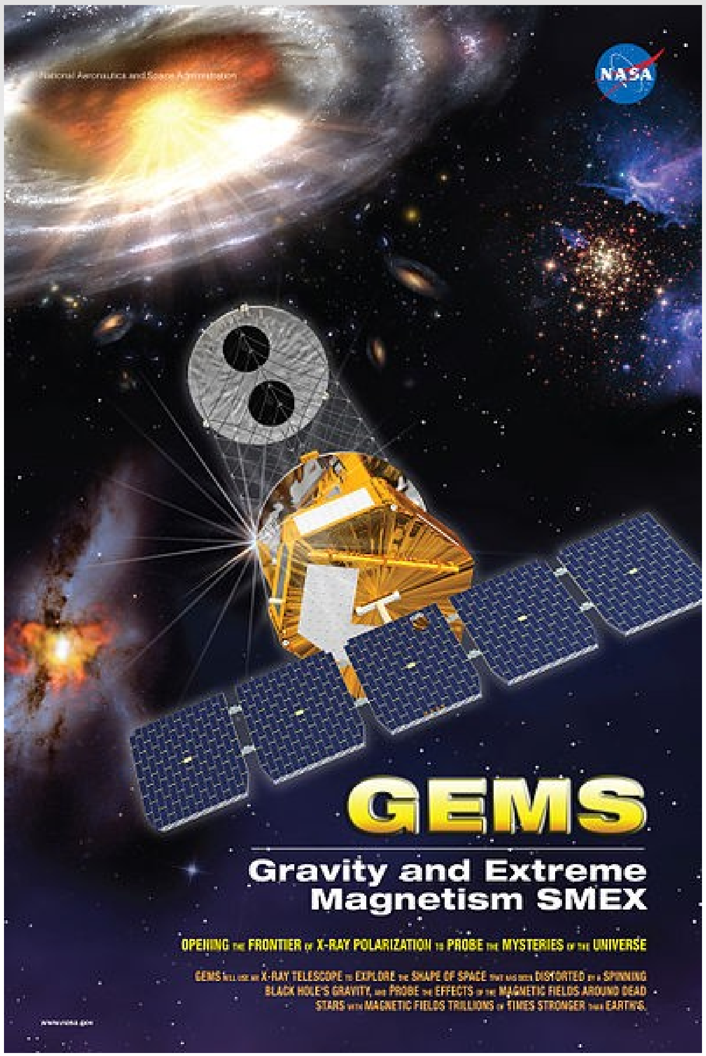
Magnetic Flux Tubes in Neutron Stars

- Neutron Stars
  - Magnetic fields
  - Superconductivity
- Worldline numerics
  - Brief Overview
  - GPGPU parallelization
- Effective action for magnetic flux tubes
  - Wide tubes, constant field approx.
  - Narrow tubes

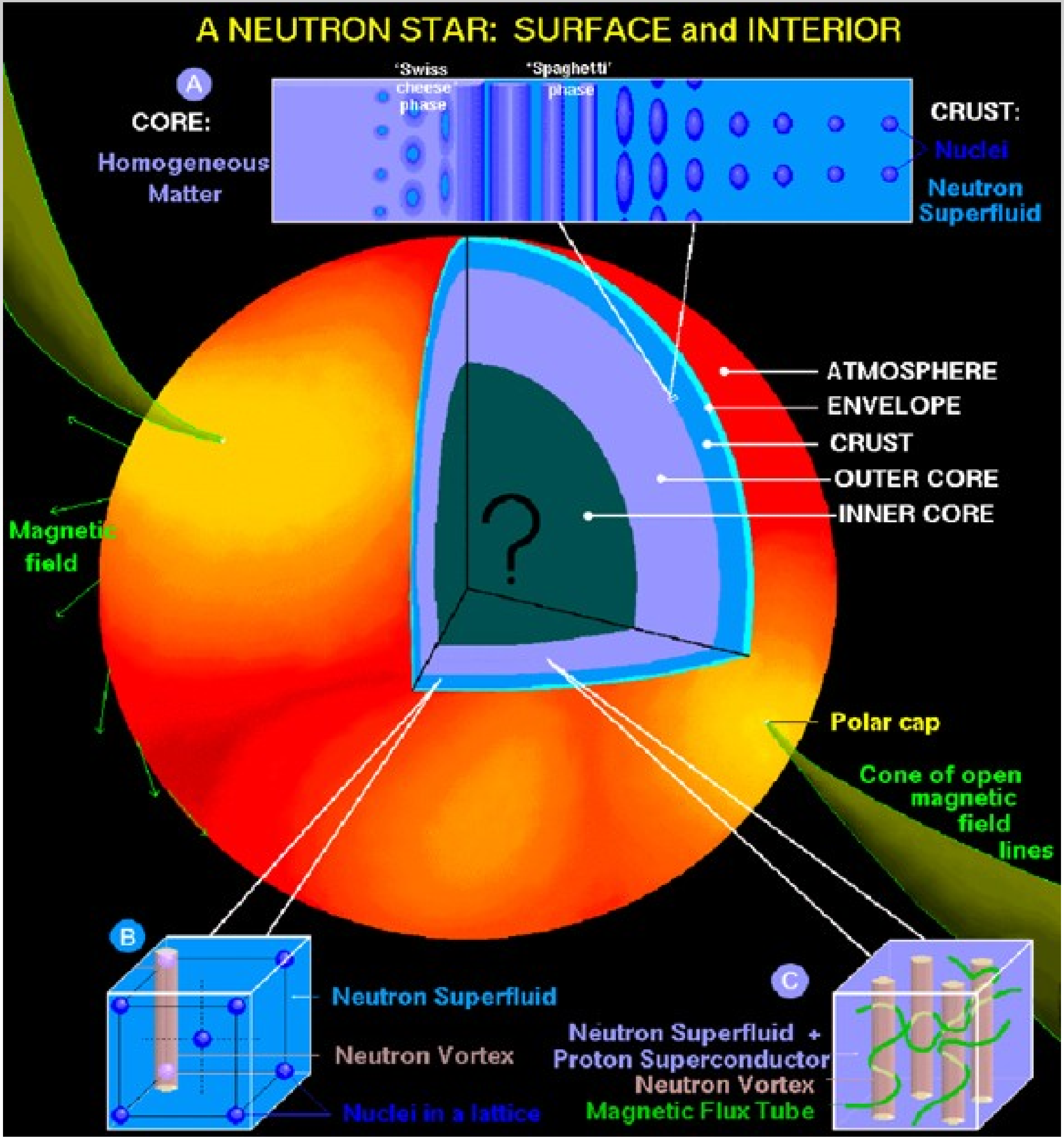
Magnetic Field Scales



Gravity and Extreme Magnetism SMEX



- Launch July 2014
- High angular resolution polarization measurements of neutron stars
- Probe of QED effects from neutron star magnetospheres



A NEUTRON STAR: SURFACE and INTERIOR
A
CORE:
Homogeneous Matter
'Swiss cheese' phase
'Spaghetti' phase
CRUST:
Nuclei
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic field
Polar cap
Cone of open magnetic field lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid +
Proton Superconductor
Neutron Vortex
Magnetic Flux Tube

Neutron Stars

- Compact stars with magnetic fields ranging from 10^8-10^{15} Gauss
- Proton superconductivity and neutron superfluidity in the core
- Likely Type-II superconductivity with large magnetic fields
  - Flux tube lattice with mean field (~10^{15} Gauss) exceeding the quantum critical field (4.4×10^{13} Gauss)

Comparing Superconductors

Laboratory superconductors

- Magnetic field strength varies over a few microns
- Background fields are small
- QED corrections small

Neutron Stars

- Magnetic field strength varies over fraction of a Compton wavelength (10^{-6} microns)
- Background fields are large
- QED corrections larger?

1-loop Effective Action

- Average quantum correction to the classical action at the 1 fermion loop level
- Must compute the fermion determinant

$$\Gamma[A^0_\mu] = \int d^4x \left(-\frac{1}{4} F^0_{\mu\nu} F^{0,\mu\nu} \right) - \frac{i\hbar}{2} \ln \mathrm{Det} \left[\frac{(\not{p} + \mathrm{e} \not{A}^0)^2 - \mathrm{m}^2}{\not{p}^2 - \mathrm{m}^2} \right]$$

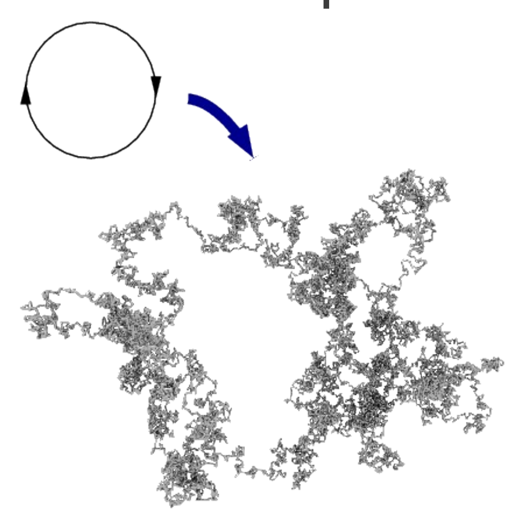
Worldline Numerics

Express the Euclidean 1-loop effective action
in the Schwinger proper time formalism

$$\frac{\Gamma^{(1)}_{\mathrm{ferm}}}{TL_z} = \frac{1}{4\pi}\int_0^\infty \rho_{\mathrm{cm}}\left[\int_0^\infty \frac{dT}{T^3}e^{-m^2T}\left\{\langle W[A_\rho(\rho(T))]\rangle_{\rho_{\mathrm{cm}}} - 1 - \frac{1}{3}(eB_{\mathrm{cm}}T)^2\right\}\right]d\rho$$

Approximate the weighted average over an infinite
ensemble of closed, continuous paths with
a sum over a finite ensemble of discrete loops

$$\langle \mathcal{O}[(\tau)]\rangle \approx \frac{1}{N_l}\sum_{i=1}^{N_l}\mathcal{O}[(\tau)]$$



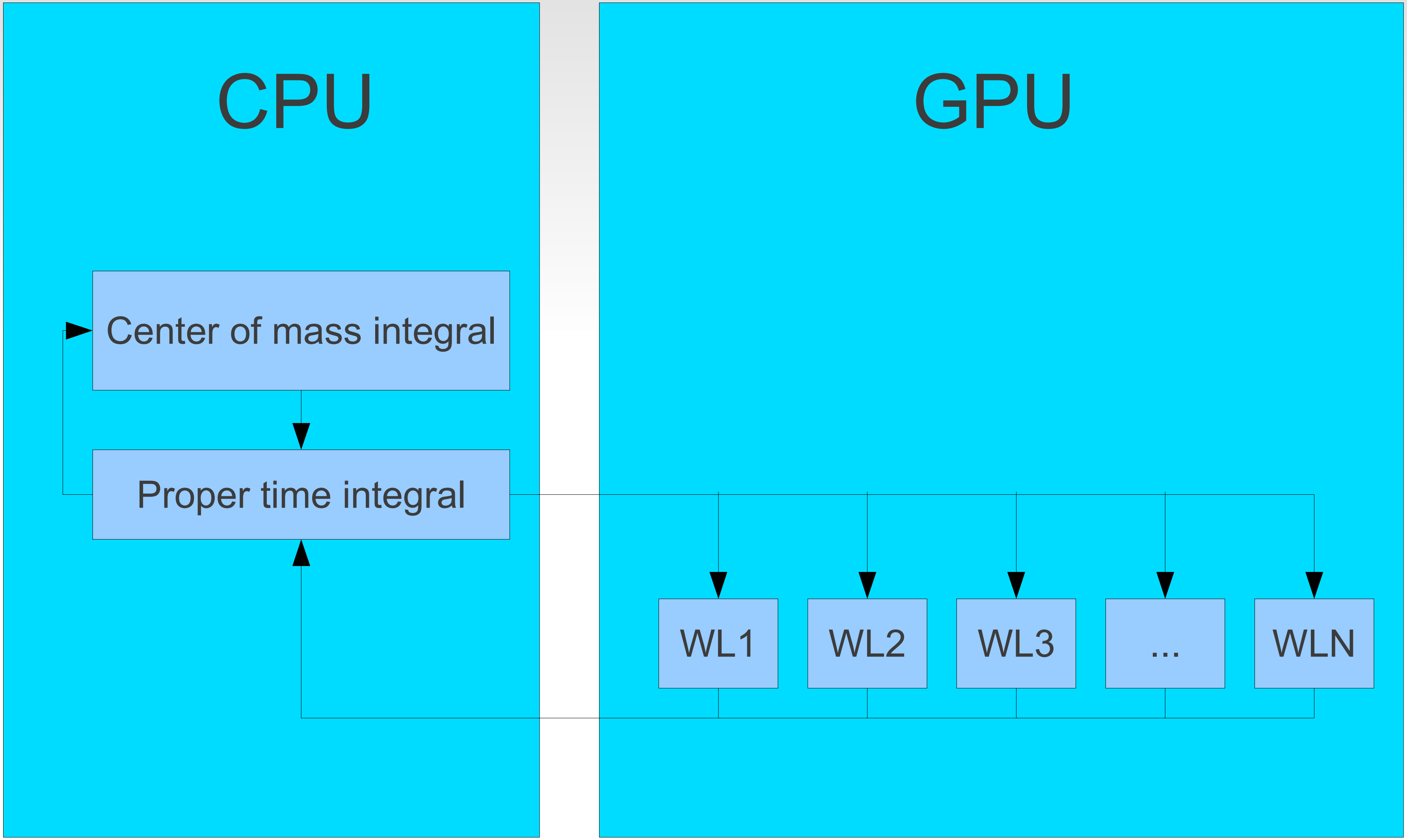
GPU Worldlines

- The Worldline technique is embarrassingly parallel
- GPUs support 1000s/10000s of parallel threads with very little overhead
- Ideal for this application
  - Very large number of lightweight threads

GPU Worldlines

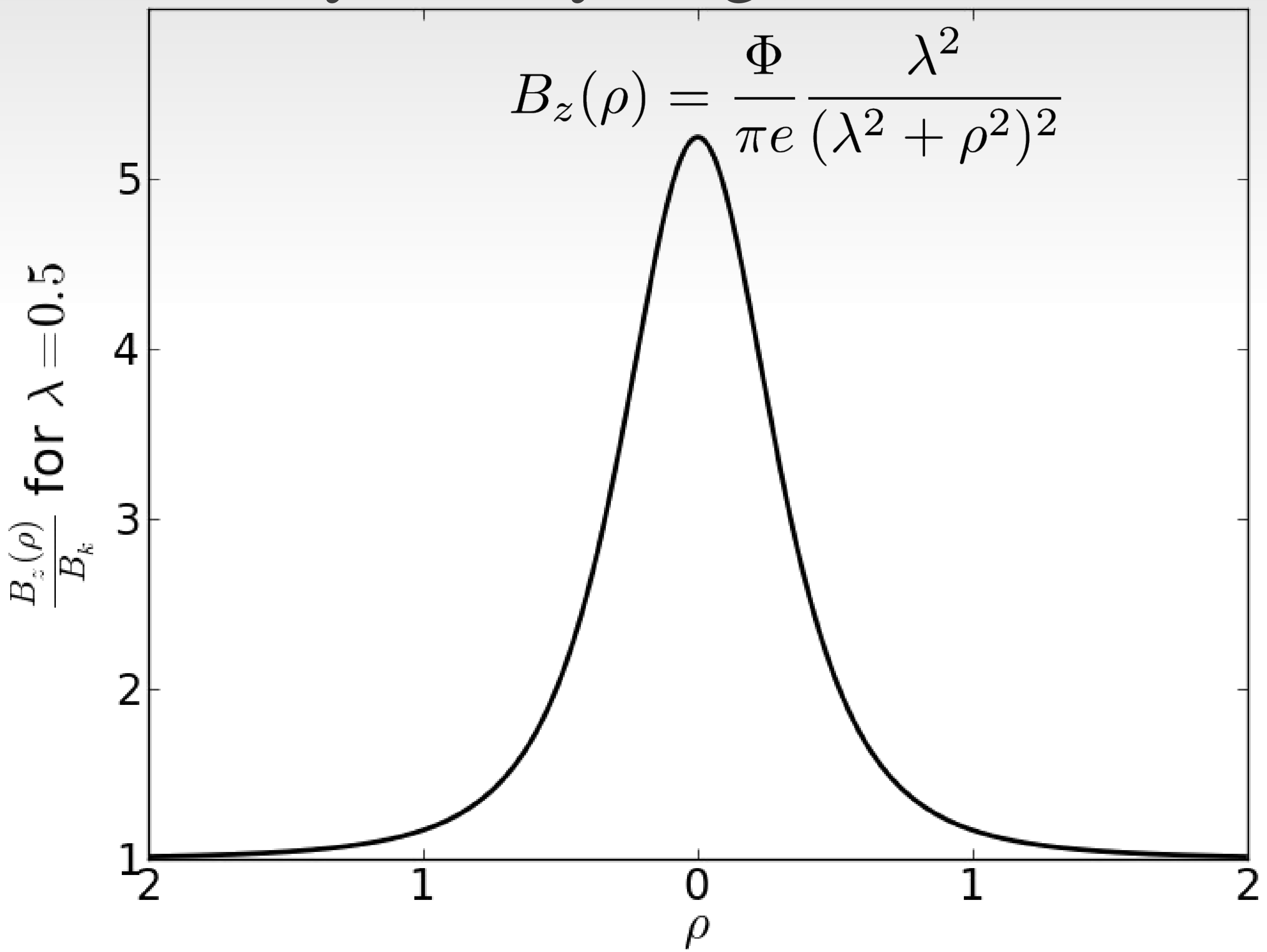
- e.g.) <Wilson Loop> w/ 5000 loops
    - 4.8s on serial MATLAB
    - 0.0013s on CUDA (Nvidia GPU language)
- Speedup of 3600x
- ~1200€ Nvidia Tesla C1060
    - 30,720 simultaneous parallel threads
    - Power ~ 1-2 CPUs

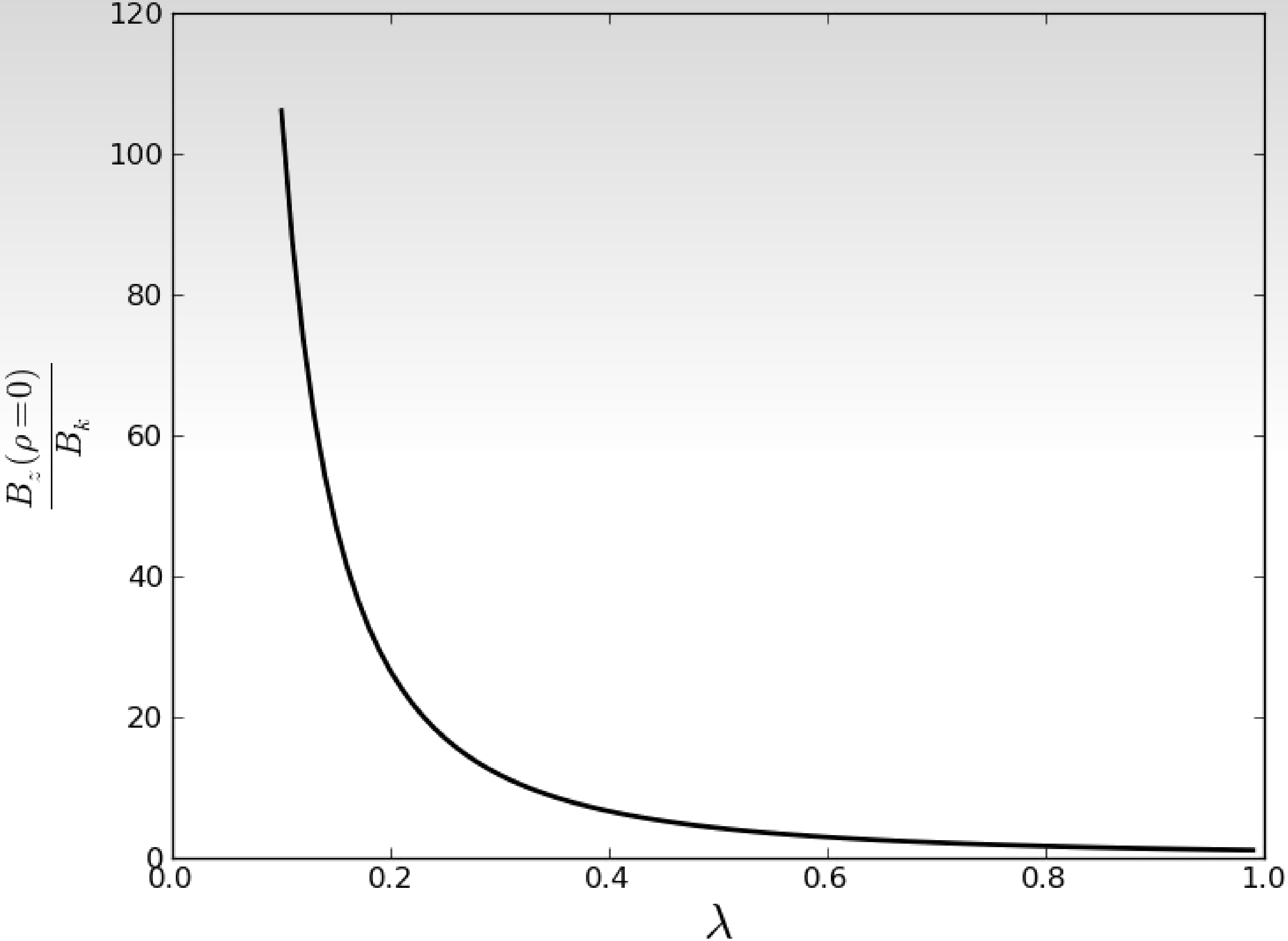
Co-processing Worldlines



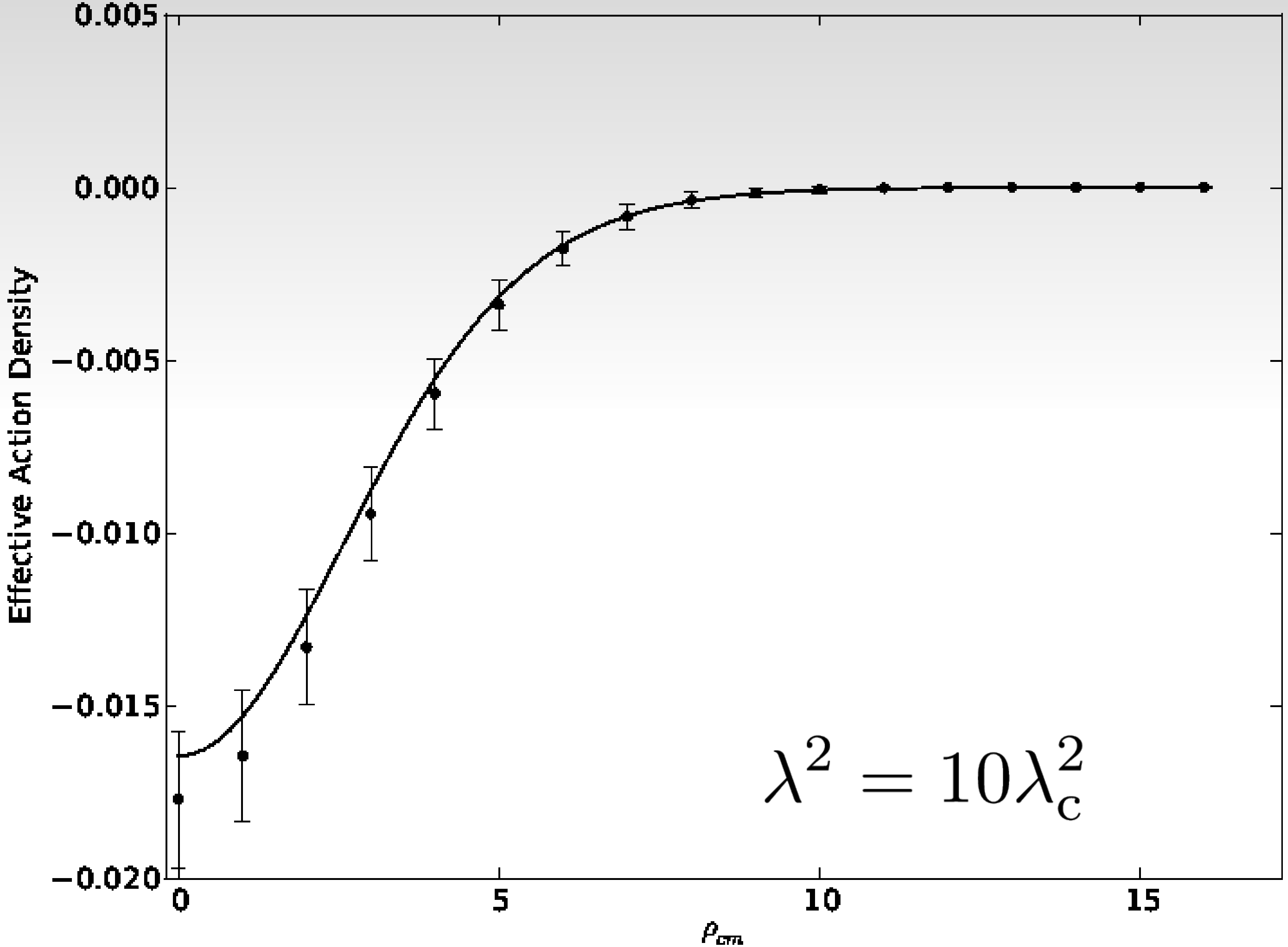
Flux Tube Model

- Cylindrical Symmetry. e.g.)

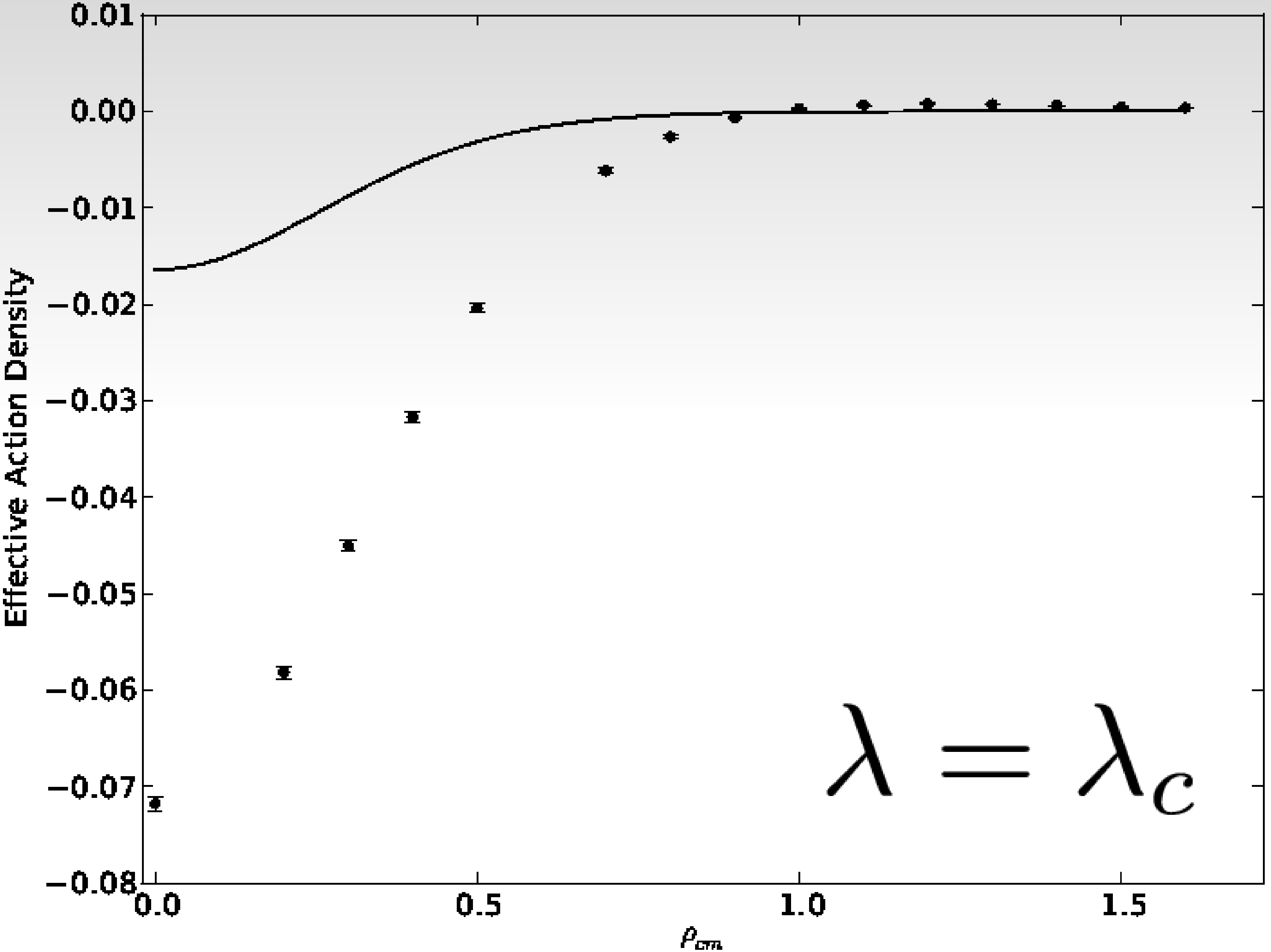



120
100
80
60
40
20
0
0.0
0.2
0.4
0.6
0.8
1.0
λ
$\frac{B_z(\rho=0)}{B_k}$

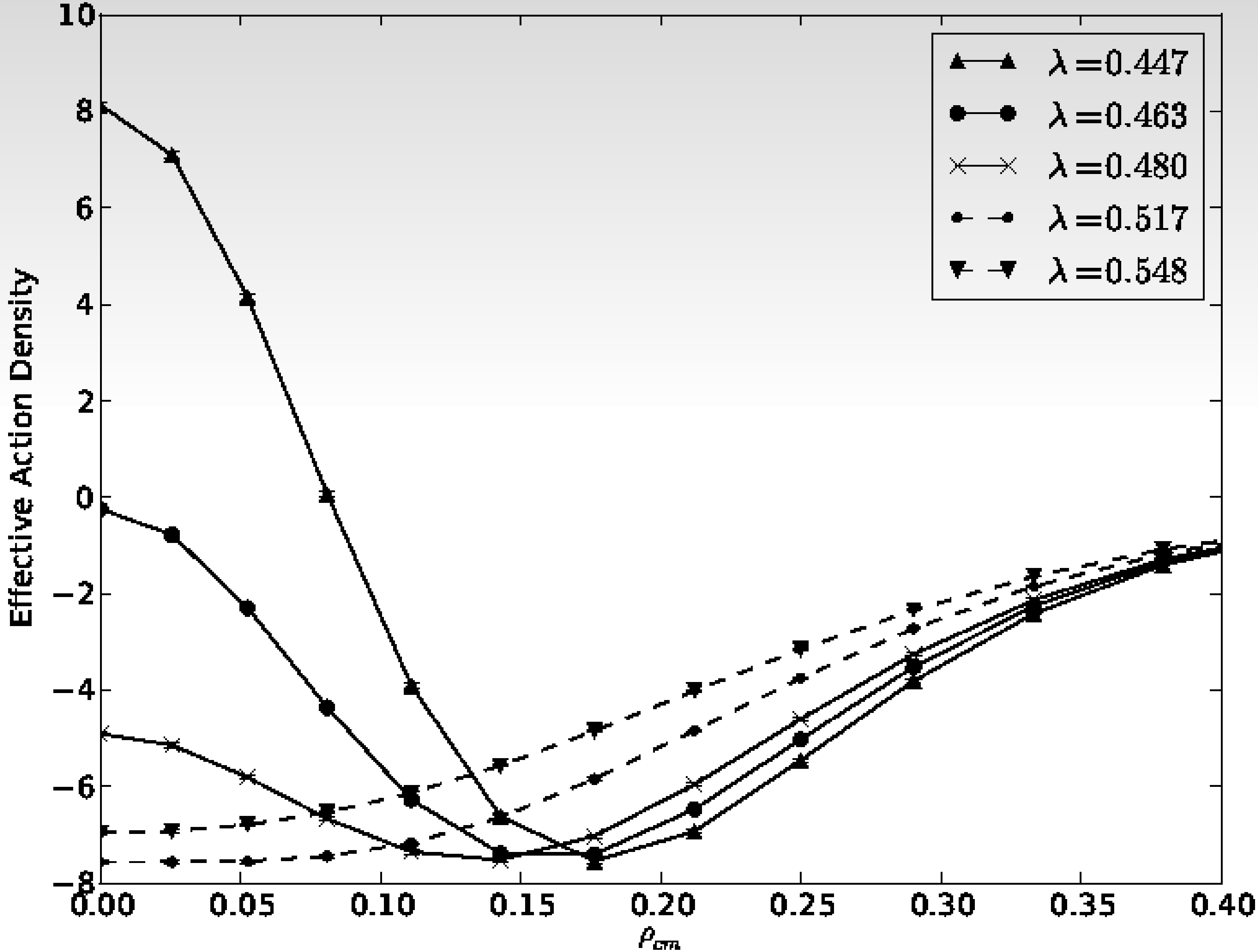
Wide Flux Tubes



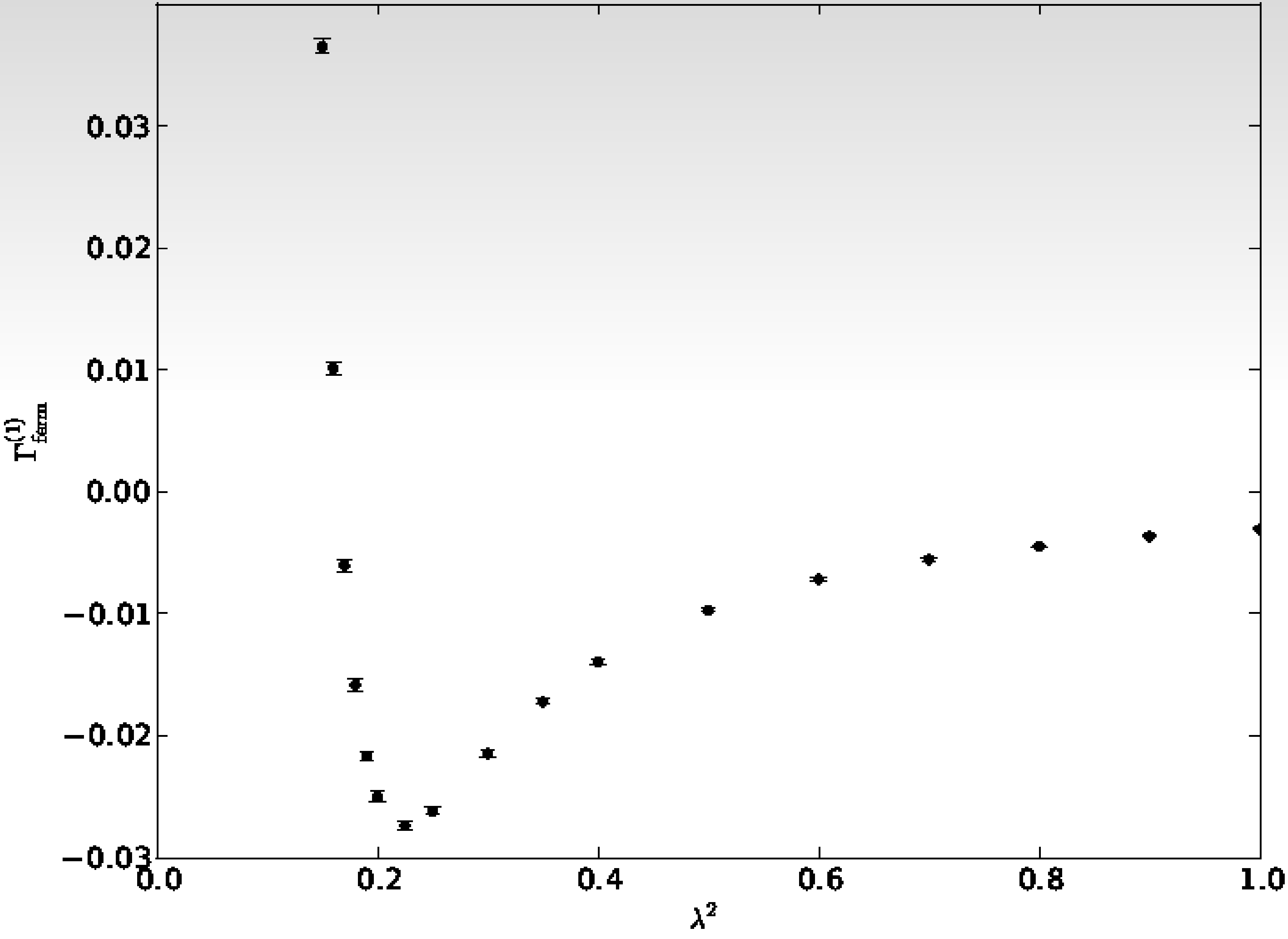
Intermediate Flux Tubes



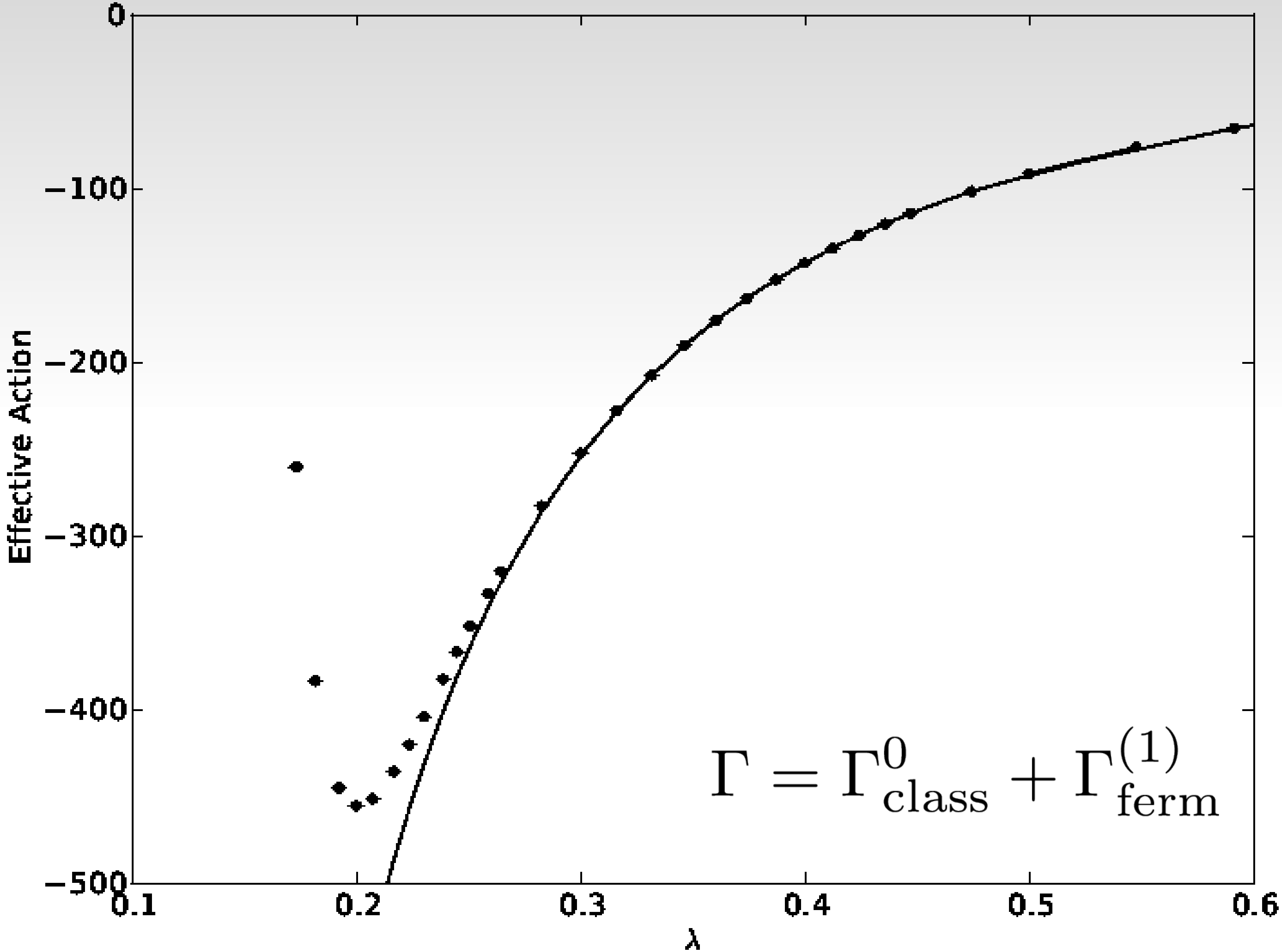
Narrow Flux Tubes



The Fermion Term



Total Action



Implications

- Quantum effects are large contribution to the action
- Instanton-like effect
  - Tunnelling to narrow flux tubes?

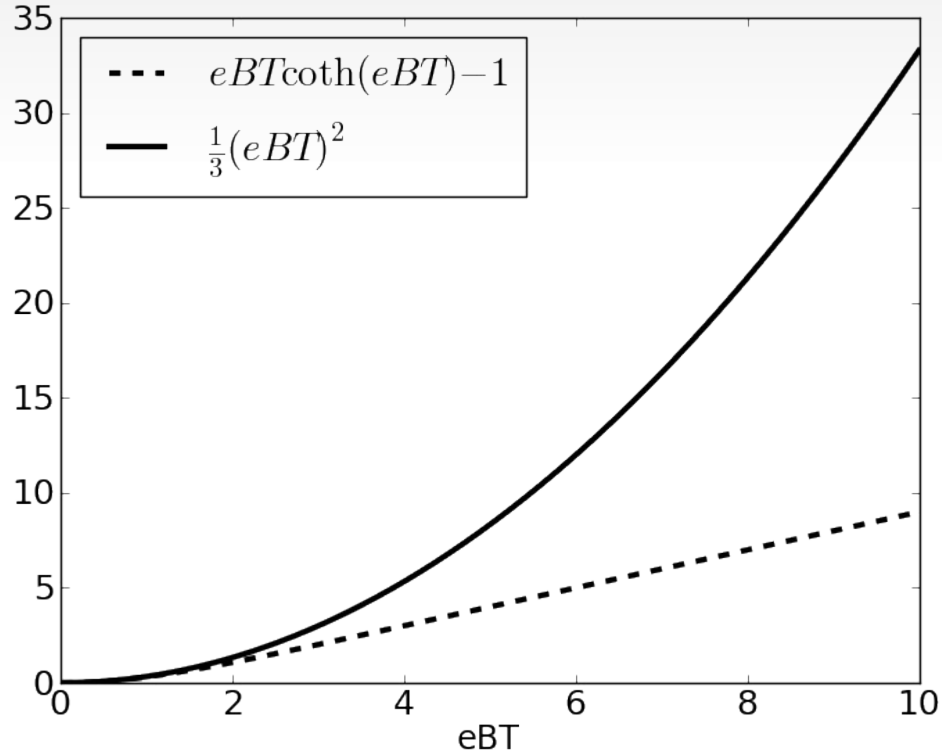
Notes

- Effect is robust to changes in profile shape: confirmed also for Gaussian profiles
- Effect occurs for both spinor and scalar electrons
- Bordag & Kirsten (1998)
  - Homogeneous Step-Function profile
  - "From this it is clear that **the complete energy, remaining** a **monotone decreasing function of the radius**, deviates only slighly from the classical energy for all values of the radius R except for very small ones." (i.e. $\lambda = 10^{-1122}$)

Signs and Locality

$$(\langle W \rangle - 1) - \frac{(eB_{\rm cm}T)^2}{3}$$

- Negative 1-loop contribution implies the local counter-term (T=0) is dominant (e.g. Constant field)



- Positive 1-loop contribution implies non-local contributions are dominant

Open Questions

- Worldline Method lends itself to arbitrary field configurations
  - Perform calculation for step-function flux tube
  - Perform calculation for a flux tube lattice
- What other physics is important for narrow flux tubes
  - 2-loop EA is small for constant fields
  - Worldline method can be expanded to include 2-loop effects
  - Other standard model fields

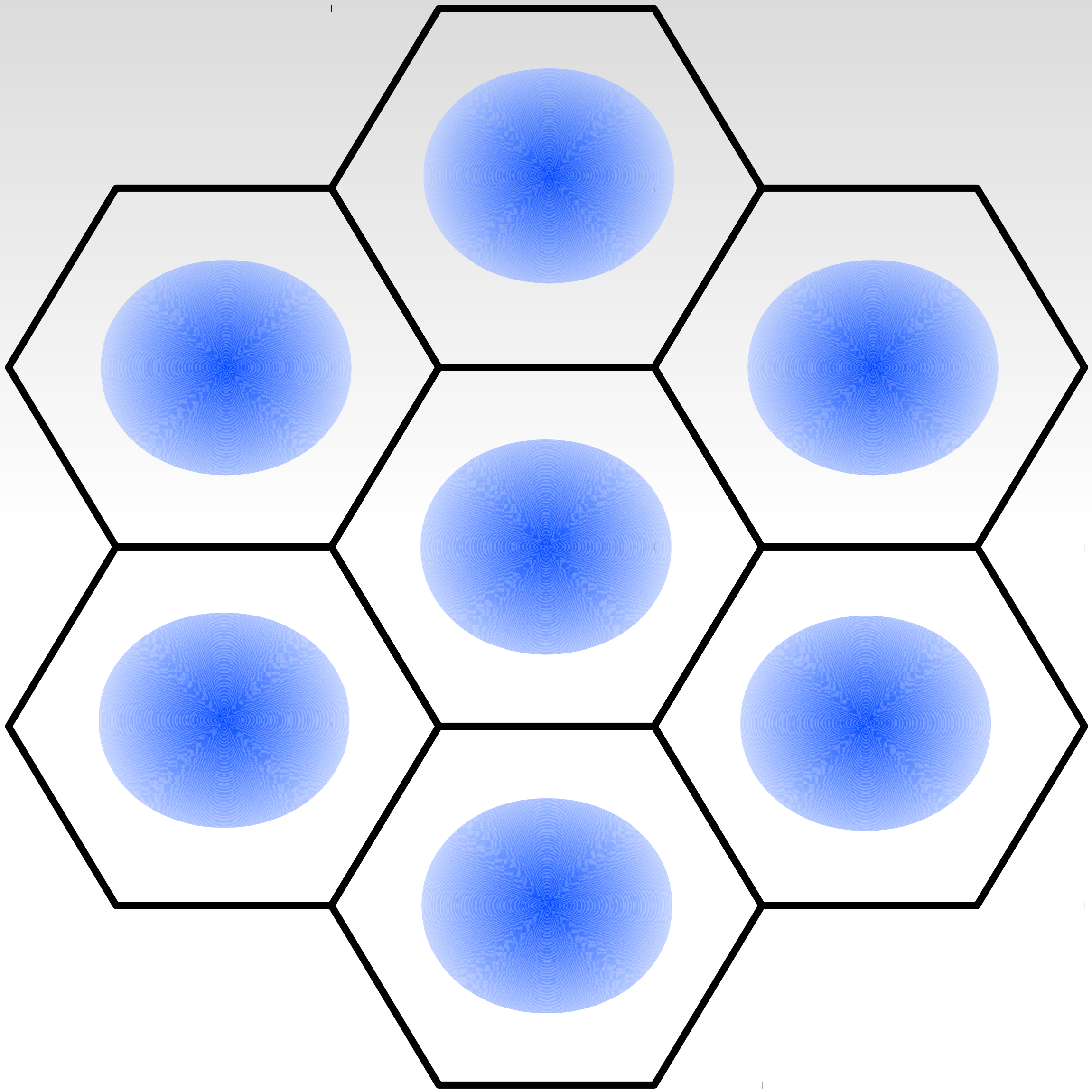
Summary

- Neutron Stars are a great arena for exploring vacuum effects in large magnetic fields
- GPU Worldline numerics can quickly calculate effective actions of arbitrary field configurations
- Quantum term can exceed the classical action due to strong non-local effects in rapidly changing fields
- Work still to be done on understanding these results

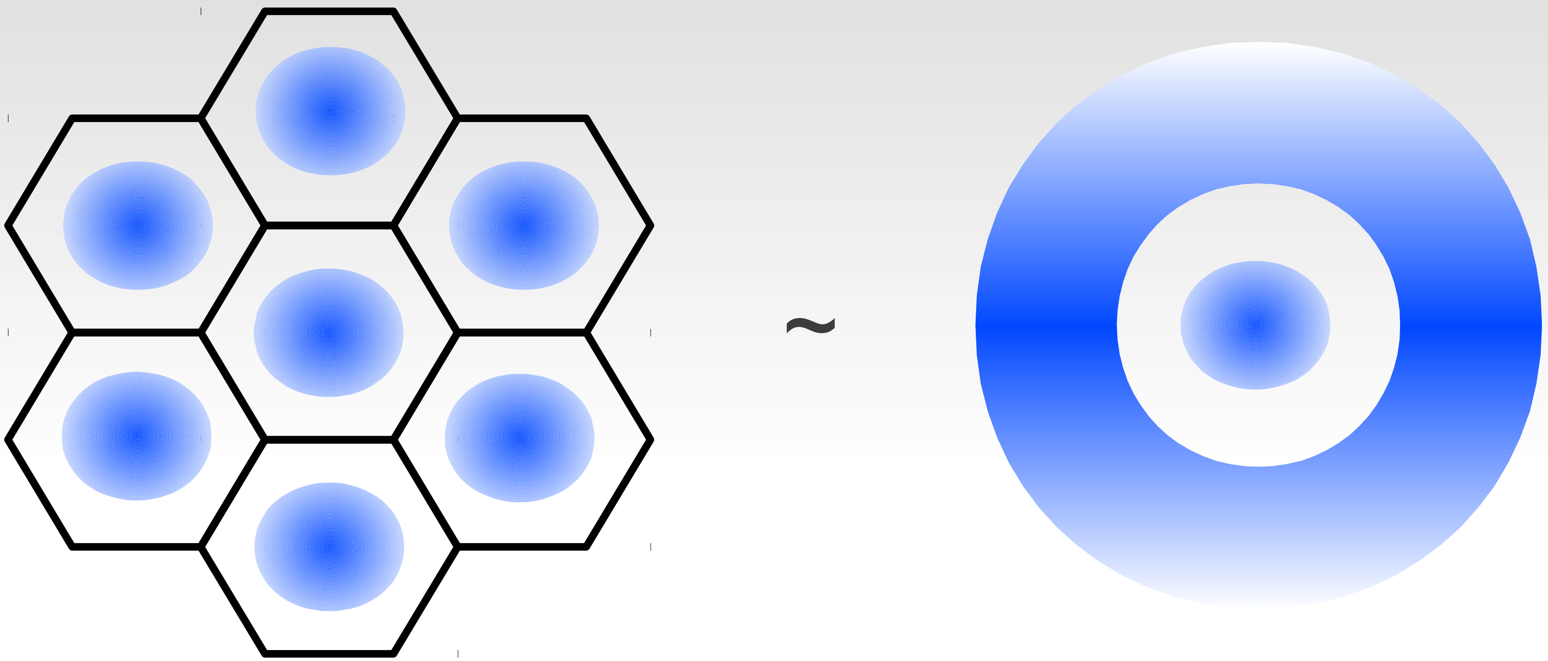
Flux Tube Lattice

- Nature is unlikely to squeeze an isolated flux tube to such small sizes
- In a lattice, flux tubes can't expand to infinity
  - Strong uniform field is the limiting case
- Is it possible for flux tubes to collapse in a strong lattice?

Flux Tube Lattice

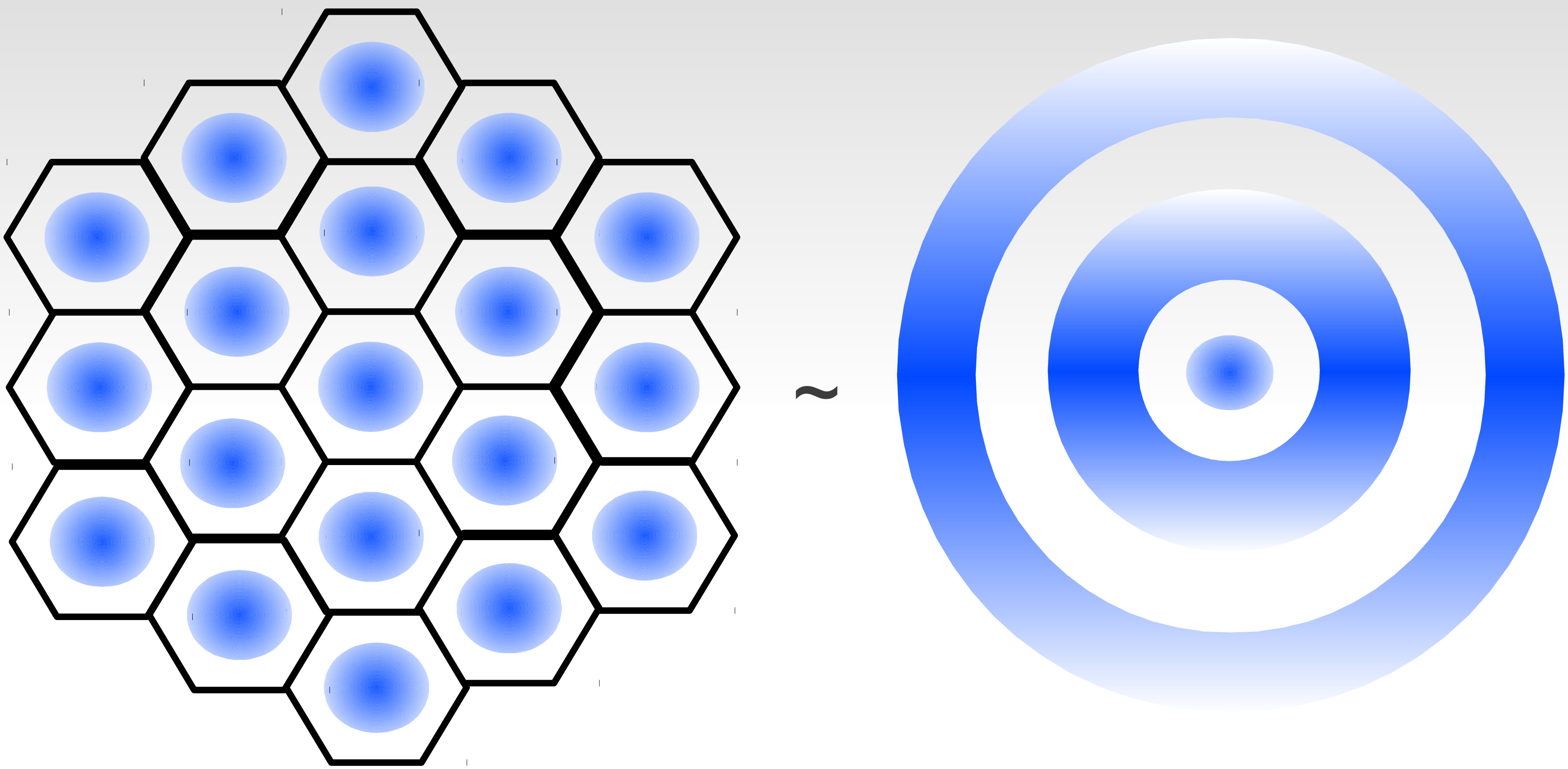


Flux Tube Lattice

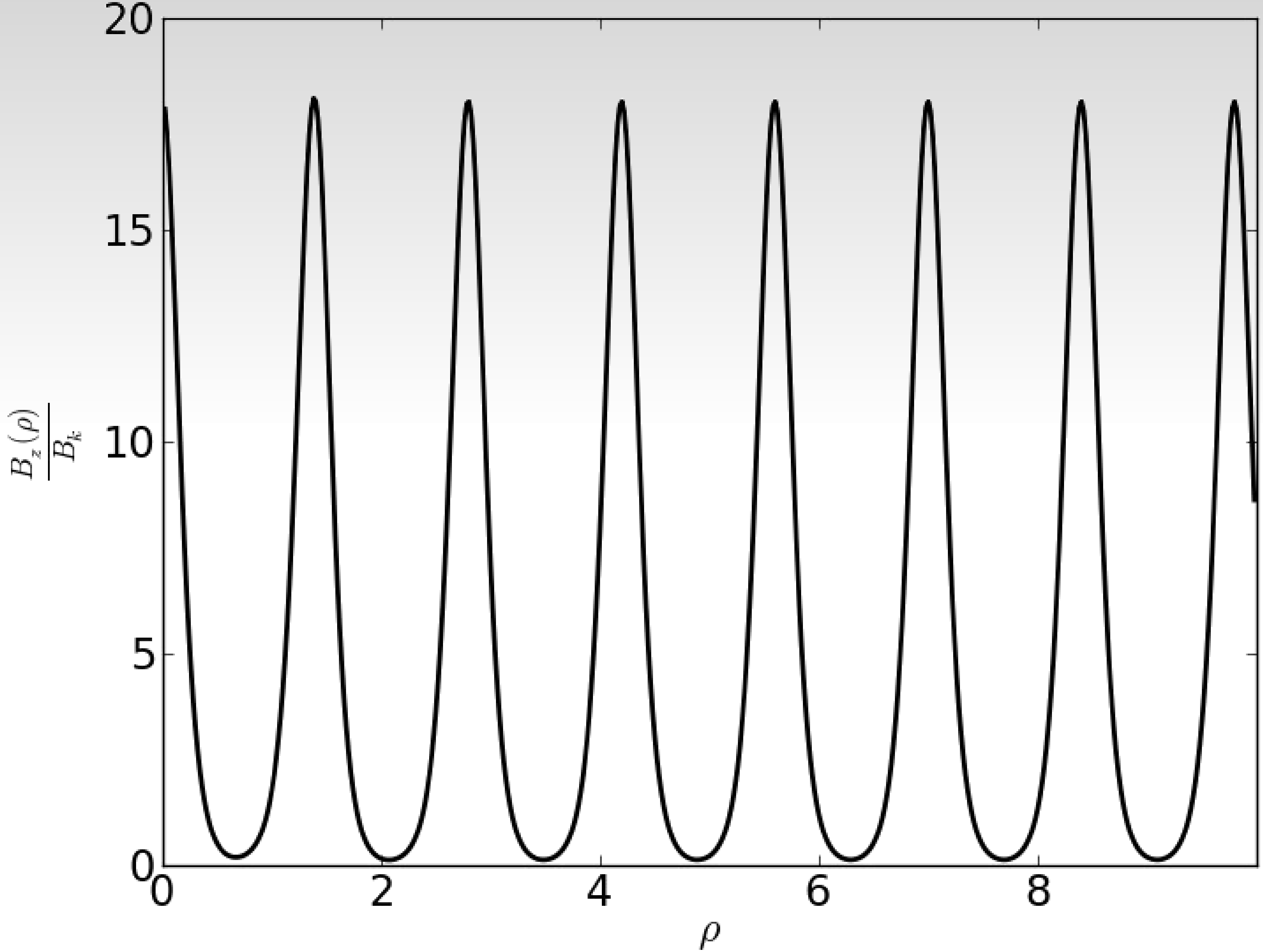


Consider the effective action per cell

Flux Tube Lattice



Consider the effective action per cell


20
15
10
5
0
0
2
4
6
8
$\frac{B_z(\rho)}{B_k}$
ρ