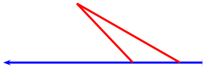
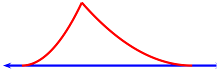
Significance of many-body contributions to Casimir energy

K. V. Shajesh

Rutgers, The State University of New Jersey,
Newark, New Jersey - 07102, USA.





with Martin Schaden

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Event: Quantum field theory under the influence of external conditions-2011 (QFEXT11)
Venue: Centro de Ciencias de Benasque Pedro Pascual, Benasque, Spain.

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- $$\Delta E_{12} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d\zeta}{2\pi} \, \mathrm{Tr} \ln \left[1 - G_0 T_1 G_0 T_2 \right]$$

  G_0 – free Green's function, $e^{-\zeta r_{12}}/4\pi r_{12}$, divergent for $r_{12} \to 0$.
  T_i's – single-body transition operators, effective potential.
  Log expansion, multiple scattering, closed paths.

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- Significance: Two-body interaction energy for disjoint bodies is free of divergence from the start. Explicit calculation for various non-trivial geometries has been achieved in the last decade. General statements possible without detailed calculation.

Irreducible three-body Casimir energies

Irreducible three-body contribution to (scalar) Casimir energy is finite and positive even as two-body contributions diverge.

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A formal expression for ΔE_{123} is

$$\Delta E_{123} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{d\zeta}{2\pi} \,\mathrm{Tr}\ln\Big[1 + X_{12}\Big\{\tilde{T}_1\tilde{T}_2\tilde{T}_3 X_{23} + \tilde{T}_1\tilde{T}_3\tilde{T}_2 X_{32} - \tilde{T}_1\tilde{T}_2\tilde{T}_1\tilde{T}_3 - \tilde{T}_1\tilde{T}_3\tilde{T}_2\tilde{T}_3 X_{23} - \tilde{T}_1\tilde{T}_2\tilde{T}_3\tilde{T}_2 X_{32}\Big\} X_{13}\Big],$$

where $\tilde{T}_i = G_0 T_i$, and X_{ij}'s are solutions to the integral equations,

$$\big[1 - \tilde{T}_i\tilde{T}_j\big] X_{ij} = 1.$$

Outline

1 Many-body Green's functions

2 Examples
- Catalytic dissociation (scalar analog)
- Weak triangular-wedge on a Dirichlet plate
- Weak parabolic-wedge on a Dirichlet plate

3 Casimir hammock

4 Cutting and pasting objects

5 Anomalous repulsion

Many-body Green's functions

The free Green's function of a massless scalar field satisfies

$$[-\boldsymbol{\nabla}^2 + \zeta^2] G_0(\mathbf{x}, \mathbf{x}') = \delta^{(3)}(\mathbf{x} - \mathbf{x}').$$

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One obtains a formal solution in the form

$$G_i = G_0 - G_0 T_i G_0,$$

where the transition matrix T_i is given by

$$T_i = V_i(1 + G_0 V_i)^{-1} = (1 + V_i G_0)^{-1} V_i = V_i - V_i G_0 V_i + V_i G_0 V_i G_0 V_i - \ldots.$$

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We practically set $G_0 = 1$ using the notation:

$$\tilde{G}_i \to G_i G_0^{-1}, \qquad \tilde{V}_i \to G_0 V_i, \qquad \text{and} \qquad \tilde{T}_i \to G_0 T_i.$$

Many-body Green's functions: Transition matrix

N-body Green's function satisfies the equation

$$\left[1 + \tilde{V}_1 + \tilde{V}_2 + \ldots + \tilde{V}_N\right] \tilde{G}_{1\ldots N} = 1.$$

The solution may again be written in the form

$$\tilde{G}_{1\ldots N} = 1 - \tilde{T}_{1\ldots N},$$

where the N-body transition matrix $\tilde{T}_{1\ldots N}$ satisfies the equation

$$\left[1 + (\tilde{V}_1 + \tilde{V}_2 + \cdots + \tilde{V}_N)\right] \tilde{T}_{1\ldots N} = (\tilde{V}_1 + \tilde{V}_2 + \cdots + \tilde{V}_N).$$

$$\tilde{T}_{1\ldots N} = \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{T}^{ij}_{1\ldots N} = \text{Sum}\left[\tilde{\mathbf{T}}_{1\ldots N}\right]; \quad \tilde{\mathbf{T}}_{1\ldots N} = \begin{pmatrix} \tilde{T}^{11}_{1\ldots N} & \tilde{T}^{12}_{1\ldots N} & \cdots & \tilde{T}^{1N}_{1\ldots N} \\ \tilde{T}^{21}_{1\ldots N} & \tilde{T}^{22}_{1\ldots N} & \cdots & \tilde{T}^{2N}_{1\ldots N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{T}^{N1}_{1\ldots N} & \tilde{T}^{N2}_{1\ldots N} & \cdots & \tilde{T}^{NN}_{1\ldots N} \end{pmatrix}$$

Many-body Green's functions: Faddeev's equations

In matrix notation the transition matrix satisfies

$$\left[\mathbf{1} + \tilde{\mathbf{V}}_{\text{diag}} + \tilde{\mathbf{\Theta}}^{V}_{1\ldots N}\right] \cdot \tilde{\mathbf{T}}_{1\ldots N} = \tilde{\mathbf{V}}_{\text{diag}},$$

where we have introduced general matrix symbols

$$\mathbf{\Theta}^{A}_{1\ldots N} = \begin{pmatrix} 0 & A_1 & A_1 & \cdots & A_1 \\ A_2 & 0 & A_2 & \cdots & A_2 \\ A_3 & A_3 & 0 & \cdots & A_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_N & A_N & A_N & \cdots & 0 \end{pmatrix}, \qquad \mathbf{A}_{\text{diag}} = \begin{pmatrix} A_1 & 0 & 0 & \cdots & 0 \\ 0 & A_2 & 0 & \cdots & 0 \\ 0 & 0 & A_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_N \end{pmatrix}.$$

Using these definitions we have

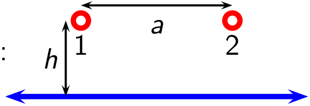
$$\left[\mathbf{1} + \tilde{\mathbf{V}}_{\text{diag}}\right] \cdot \tilde{\mathbf{T}}_{\text{diag}} = \tilde{\mathbf{V}}_{\text{diag}} \qquad \text{and} \qquad \left[\mathbf{1} + \tilde{\mathbf{V}}_{\text{diag}}\right] \cdot \tilde{\mathbf{\Theta}}^{T}_{1\ldots N} = \tilde{\mathbf{\Theta}}^{V}_{1\ldots N},$$

which leads to the Faddeev's equation:

$$\left[\mathbf{1} + \tilde{\mathbf{\Theta}}^{T}_{1\ldots N}\right] \cdot \tilde{\mathbf{T}}_{1\ldots N} = \tilde{\mathbf{T}}_{\text{diag}}.$$

Catalytic dissociation (scalar analog)

Consider atom-like potentials on a Dirichlet plate:



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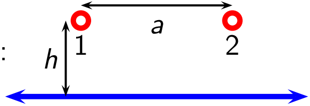

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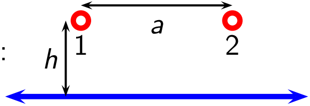

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$$g(\beta) = \tfrac{2}{\beta(1+\beta)} - \tfrac{1}{\beta^3}$$

$$\beta = \sqrt{1 + (2h/a)^2}$$

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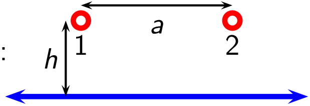

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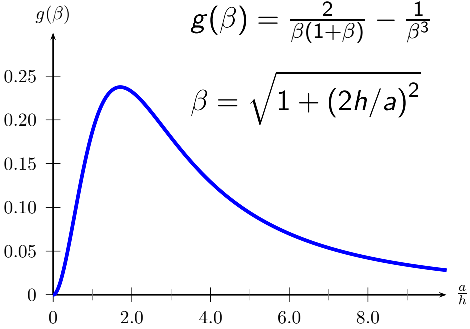

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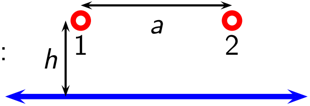

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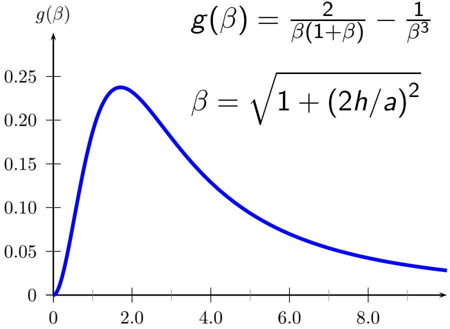
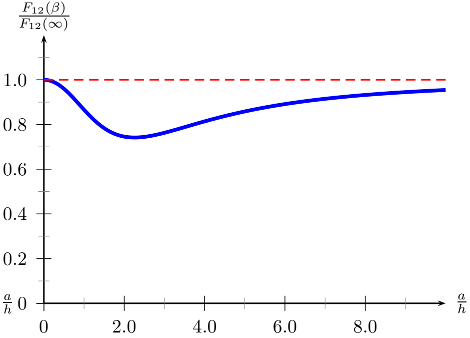
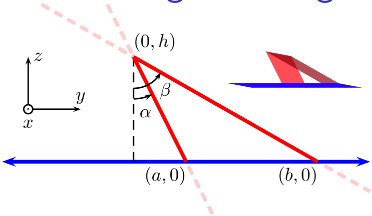




Weak triangular-wedge on a Dirichlet plate



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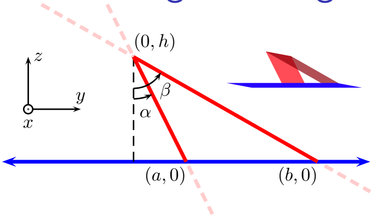

$$\mathcal{E}(\alpha,\beta) = \frac{\Delta E^W_{123}}{L_x}\left[\frac{\lambda_1\lambda_2}{32\pi^3}\right]^{-1}$$

$$= |\tilde{a}\,\tilde{b}| \int_0^1 \int_0^1 \frac{du_1 du_2}{\bar{u}^2_{12}}\, Q\left(\frac{u^2_{12}}{\bar{u}^2_{12}}\right),$$

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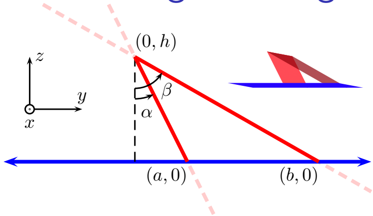

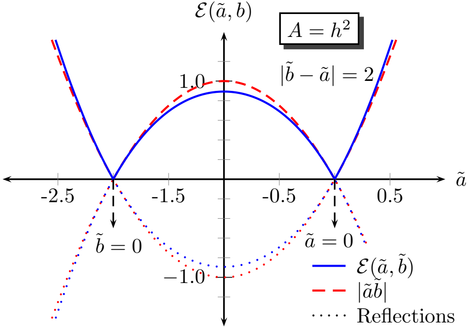

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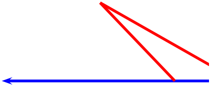

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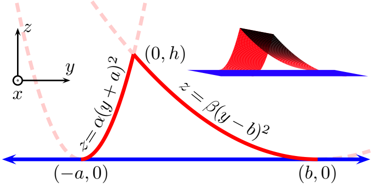

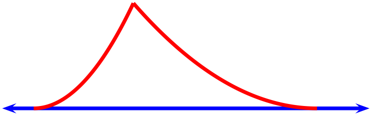

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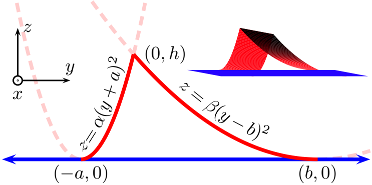

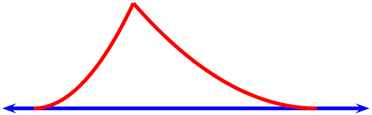

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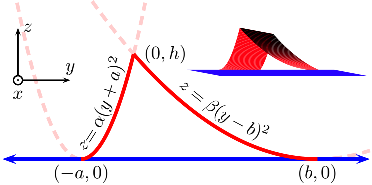

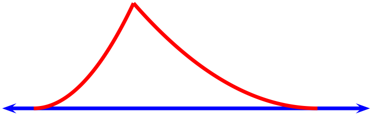

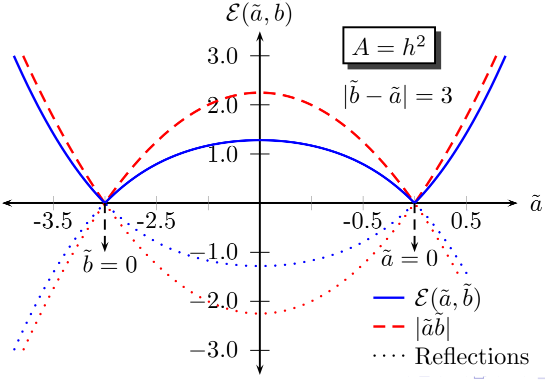

Casimir hammock

Two-body Casimir energy of a surface weakly interacting with a Dirichlet plate:

$$\frac{\Delta E_{i3}^{W}}{L_x} = -\frac{1}{32\pi^2} \int d^2 r \frac{V_i(\mathbf{r})}{|z|^2}.$$

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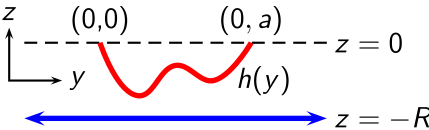

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$$V_i(\mathbf{r}) = \lambda_i \delta(z - h(y)), \qquad h(0) = h(a) = 0.$$

Dirichlet plate at $z = -R$.

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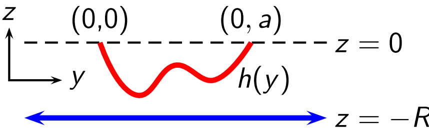

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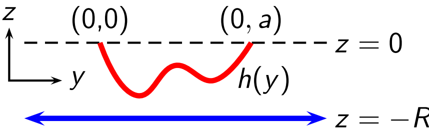

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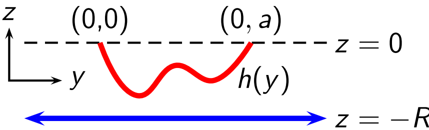

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Equation:

$$d\tilde{y} = \frac{d\tilde{h}}{\sqrt{\frac{\tilde{p}^2}{\left(1 - \frac{E_0}{|1-\tilde{h}|^2}\right)^2} - 1}}$$

$$\tilde{h} = \frac{h(y)}{R} \qquad \tilde{p} = \frac{P}{a}$$

$$\tilde{y} = \frac{y}{R} \qquad E_0 = \frac{\lambda}{32\pi^2 R^2}\frac{1}{\tilde{p}}\frac{1}{\gamma}$$

Solution close to a parabola.

Two-body → One-body

$$E_{12}(\lambda_1, \lambda_2, a_{12}) = E_0 + \Delta E_1(\lambda_1) + \Delta E_2(\lambda_2) + \Delta E_{12}(\lambda_1, \lambda_2, a_{12}),$$

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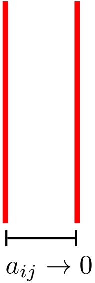
Limit $a_{12} \to 0$

$$E_{12}(\lambda_1, \lambda_2, a_{12} \to 0) = E_0 + \Delta E_{(1+2)}(\lambda_1 + \lambda_2)$$



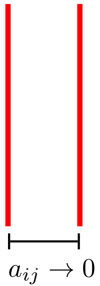
Two-body → One-body

$$E_{12}(\lambda_1, \lambda_2, a_{12}) = E_0 + \Delta E_1(\lambda_1) + \Delta E_2(\lambda_2) + \Delta E_{12}(\lambda_1, \lambda_2, a_{12}),$$

Limit $a_{12} \to 0$

$$E_{12}(\lambda_1, \lambda_2, a_{12} \to 0) = E_0 + \Delta E_{(1+2)}(\lambda_1 + \lambda_2)$$

$$\Delta E_{(1+2)}(\lambda_1 + \lambda_2) = \Delta E_1 + \Delta E_2 + \Delta E_{12}$$



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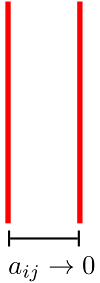
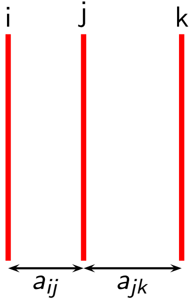


Limits: $a_{12} \to 0$ and $\lambda_i \to \infty$

$$\Delta E_{(1+2)}(\infty) = \Delta E_1 + \Delta E_2 + \lim_{\lambda_i \to \infty} \lim_{a_{12} \to 0} \Delta E_{12}$$

Note: Order of the limits matters.

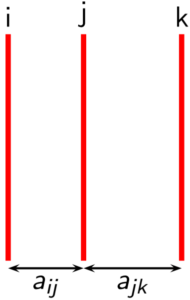
Three-body interaction



$$E_{123} = E_0 + (\Delta E_1 + \Delta E_2 + \Delta E_{12}) + \Delta E_3 + (\Delta E_{23} + \Delta E_{13} + \Delta E_{123})$$

ΔE_{ij}: Irreducible two-body part of energy.
ΔE_{123}: Irreducible three-body part of energy.

Three-body interaction



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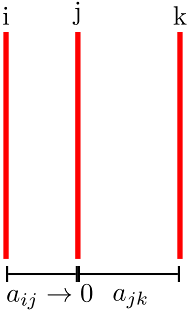
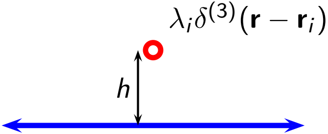


Limit $a_{12} \to 0$

$$E_{123} = E_0 + \Delta E_{(1+2)} + \Delta E_3 + \Delta E_{(1+2)3}$$

$$\Delta E_{123} = \Delta E_{(1+2)3} - \Delta E_{23} - \Delta E_{13}$$

(Scalar) atom on a Dirichlet plate

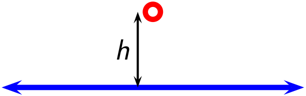


(Scalar) atom on a Dirichlet plate

Interaction energy between a (scalar) atom and a Dirichlet plate is

$$E_{i3} = \frac{1}{2}\int \frac{d\zeta}{2\pi} \text{Tr} \ln \left[1 - G_0 T_i G_0 T_3\right]$$

$$\lambda_i \delta^{(3)}(\mathbf{r} - \mathbf{r}_i)$$



$$[G_0 T_3 G_0](\mathbf{r}_i, \mathbf{r}_i) = \frac{e^{-2h|\zeta|}}{8\pi h}$$

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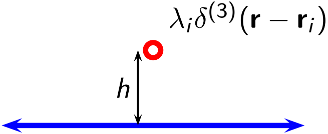

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$$
\begin{aligned}
E_{i3} &= \frac{1}{2}\int \frac{d\zeta}{2\pi} \text{Tr} \ln \left[1 - G_0 T_i G_0 T_3\right] \\
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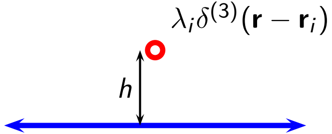
&= \frac{1}{4\pi h} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(-\frac{\lambda_i}{8\pi h} \right)^n \\
&= \frac{1}{4\pi h} \text{Li}_2 \left(-\frac{\lambda_i}{8\pi h} \right)
\end{aligned}
$$



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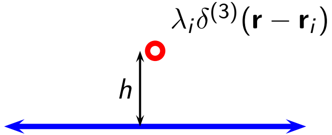

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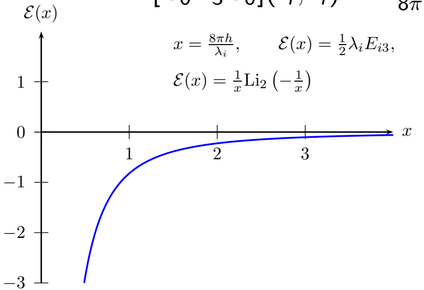
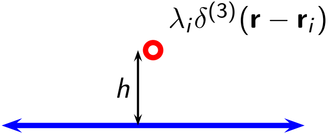


Transition from Casimir-Polder to London

$$\mathcal{E}(x) = \frac{1}{x}\mathrm{Li}_2\left(-\frac{1}{x}\right) \qquad x = \frac{8\pi h}{\lambda_i}$$

$$= \begin{cases} 1 - \frac{1}{2x}\left[\frac{\pi^2}{3} + \ln^2 x\right] & (x \ll 1), \\ -\frac{1}{x^2} & (x \gg 1). \end{cases}$$



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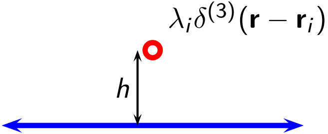
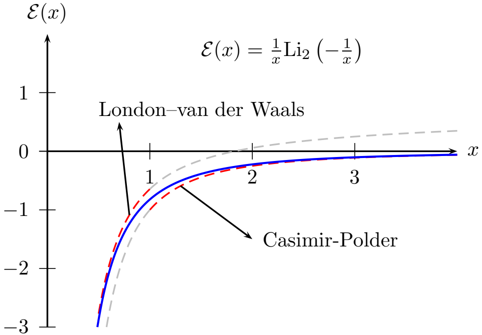
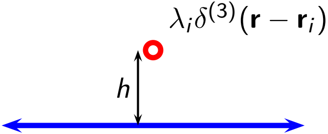




Anomalous repulsion

$$\mathcal{E}(x) = \frac{1}{x}\mathrm{Li}_2\left(-\frac{1}{x}\right) \qquad x = \frac{8\pi h}{\lambda_i}$$

$$\sim -\sum_{n=1}^{N}\left(-\frac{1}{x}\right)^{n+1}\frac{1}{n^2}$$



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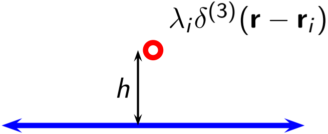

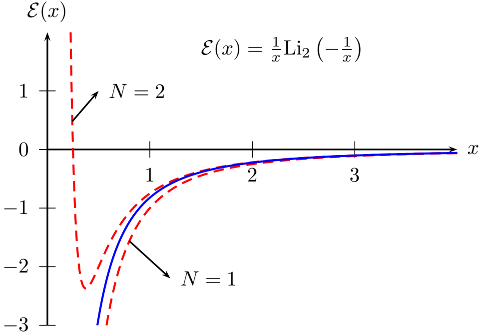

References

M. Schaden, "Irreducible many-body Casimir energies of intersecting objects," EPL **94 (4)** 41001 (2011).

K. V. Shajesh and M. Schaden, "Many-Body Contributions to Green's Functions and Casimir Energies," Phys. Rev. D **83**, 125032 (2011), arXiv:1103.3048 [hep-th].