Interference effects in Heisenberg-Schwinger Vacuum Pair Production for Time Dependent Laser Pulses

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CD & G.Dunne, PRL **104** (2010), PRD **83** (2011), arXiv:1110.xxxx

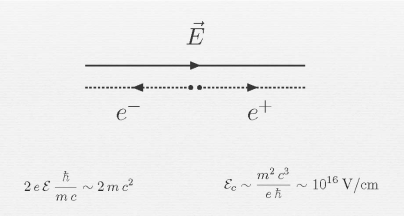
Vacuum Decay

Schwinger's Result

- Vacuum Decay Rate:
  $1 - |< 0|0 >|^2 \approx \text{Im}\,[\Gamma_{\text{eff}}]$
- Interpretation: External field induces real particle-antiparticle current.
- Schwinger's Result:

$$N = \frac{e^2 E_0^2}{4\pi^3} \sum_{n=1} \frac{1}{n^2} \text{Exp}\left[-\frac{nm^2\pi}{eE_0}\right]$$

(constant field)

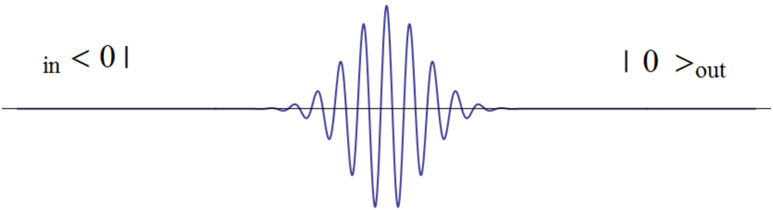


- Virtual Pairs are separated by the External Electric Field.
- Non-perturbative effect in field strength

Ongoing Research

- Decay rates i.e spectrum of produced particles for realistic fields
- Backreaction Effects, Finite Size Effects
- Pulse Optimizations

Vacuum Decay



Effective Action

- Vacuum Persistence Amplitude: $_{\rm in} < 0|0 >_{\rm out} = $ Exp $[i\Gamma_{\rm eff}]$
- Euler-Heisenberg Effective Action (one-loop,constant field)

  $$\Gamma_{\rm eff} = \frac{e^4}{360\pi^2 m^4} \int d^4x \left[\left(\vec{E}^2 - \vec{B}^2\right)^2 + 7\left(\vec{E}.\vec{B}\right)^2 + ...\right]$$

  Heisenberg-Euler 1936, Weisskopf 1936

Numerical Computation Methods For Fields Fields Varying In Time

Quantum Kinetic Equation (QKE)

- Distribution function: $f_{\pm}\left(\vec{k},t\right) =< 0|a^{\dagger}_{\vec{k}}(t)a_{\vec{k}}(t)|0 >\equiv \text{Im}\left[S_{eff}\right]$
- $\dot{f}_{\pm}\left(\vec{k},t\right) = \frac{W_{\pm}(t)}{2}\int_{-\infty}^{t} dt'\, W_{\pm}(t)\left[1 \pm 2 f_{\pm}\left(\vec{k},t\right)\right] cos\left(2\int_{t'}^{t}\omega(\tau)\right)$

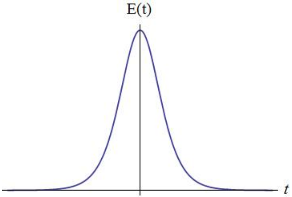
  $\omega(t) = m^2 + k_{\perp}^2 + (k_3 - eA(t))^2\,,\; W_{+}(t) = \frac{\dot{\omega}(t)}{\omega(t)},\; W_{-}(t) = \frac{\dot{\omega}(t)\sqrt{m^2+k_{\perp}^2}}{\omega(t)(k_3 - eA(t))}$
- $f_{\pm}(\vec{k},\infty)$: number of produced pairs per unit volume

Riccati Equation

- $\dot{R}_{\pm}\left(\vec{k},t\right) = \frac{\pm W_{\pm}(t)}{2}\left(e^{-2i\phi(t)} \mp R_{\pm}^2\left(\vec{k},t\right)e^{2i\phi(t)}\right)\,,\quad \phi(t) = \int_{-\infty}^{t}\omega(t')dt'$
- $\left|R_{\pm}\left(\vec{k},\infty\right)\right|^2 = \frac{f_{\pm}(\vec{k},\infty)}{1\pm f_{\pm}(\vec{k},\infty)}$: Asymptotic reflection probability for scattering problem

Exactly Soluble Case and Comparison

- $A(t) = E_0 \tau tanh(\frac{t}{\tau}), \quad E(t) = E_0 sech(\frac{t}{\tau})^2$



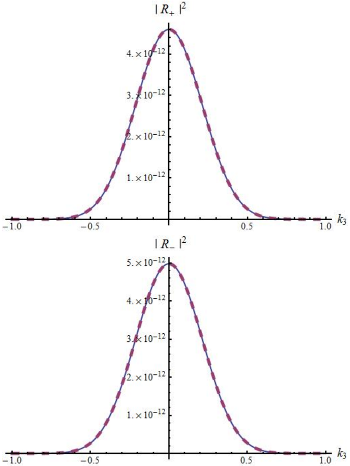
- $|R_+(\infty)|^2 = \frac{cosh(\alpha-\beta)+cos\lambda}{cosh(\alpha+\beta)+cos\lambda}$

$$|R_-(\infty)|^2 = \frac{sinh(\frac{\alpha-\beta+\lambda'}{2})sinh(\frac{-\alpha+\beta+\lambda'}{2})}{sinh(\frac{\alpha+\beta+\lambda'}{2})+sinh(\frac{\alpha+\beta-\lambda'}{2})}$$

$$\alpha = \pi\tau\sqrt{m^2 + k_\perp^2 + (k_3 + eE_0)^2}$$

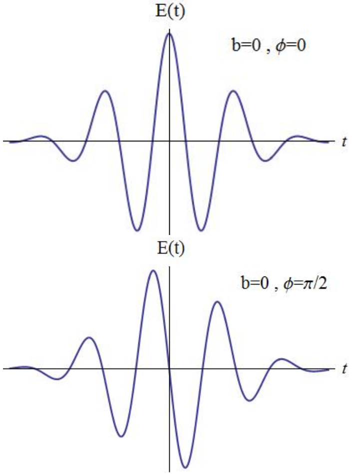
$$\beta = \pi\tau\sqrt{m^2 + k_\perp^2 + (k_3 - eE_0)^2}$$

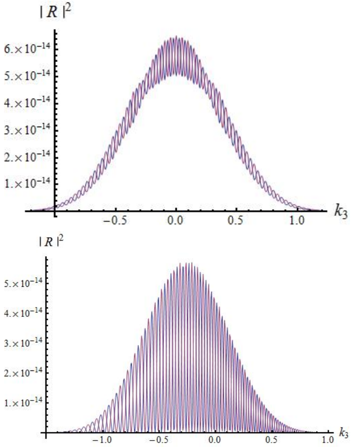
$$\lambda = \pi\tau\sqrt{\frac{1}{\tau^2} - (2eE_0\tau)^2}, \lambda' = 2E_0\pi\tau^2$$



Realistic Laser Pulses and Interference

$$E(t) = E_0 e^{-\frac{t^2}{2\tau^2}} cos(\omega t + bt^2 + \phi), \quad b\text{: chirp parameter}, \quad \phi\text{: phase}$$





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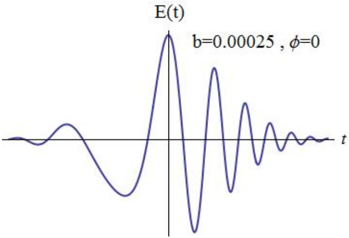
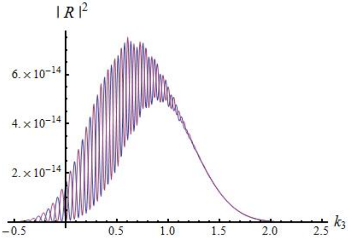




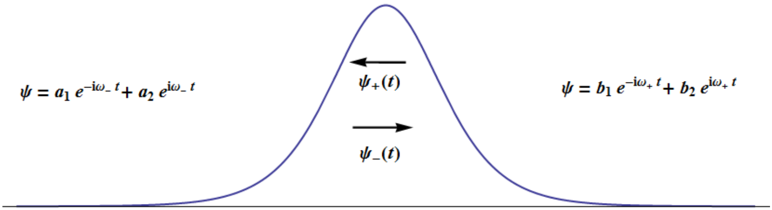
- Momentum spectra show off phase oscillations between scalar and spinor QED
- Semiclassical methods can be used to explain how oscillations arise

WKB Method

SCATTERING PROBLEM IN 1-D

Relativistic wave equations can be reduced to Schrodinger-like equation in time:

$$\ddot{\Psi}(t) + \omega(t)^2\Psi(t) = 0, \;\; \ddot{\Psi}(t) + \left(\omega(t)^2 + iE(t)\right)\Psi(t) = 0$$



WKB ansatz: $\Psi(t)_\pm = \frac{exp\left[\pm i \int^t \omega(t')dt'\right]}{\sqrt{\omega(t)}}, \qquad \Psi(t)_\pm = \frac{exp\left[\pm i \int^t \omega(t')dt'\right]}{\sqrt{\omega(t)(\omega(t) \pm k_3(t))}}$

$$k_3(t) = k_3 - eA(t), \quad \lim_{t\to\pm\infty} \omega(t) = \omega_\pm$$

Motivation: Reflected waves propagate backward in time admitting anti-particle interpretation

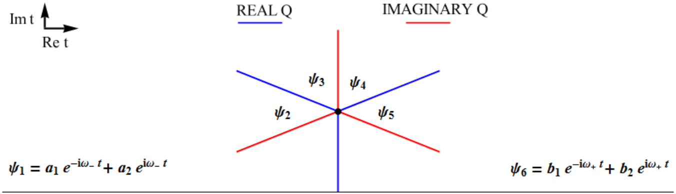
- Relative Reflection Probability: $|R(\infty)|^2 = \left|\frac{b2}{b1}\right|^2 \sim$ anti particle flux

Analytic Continuation

SEMICLASSICAL APPROXIMATION

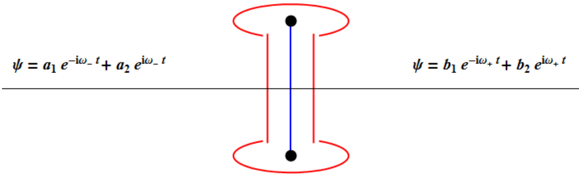
- Coefficients a_1, a_2 are related to b_1 and b_2 via analytic continuation
- Observation : WKB ansatz have branch points at $\omega(t_p) = 0$,
  Time dependent pulses: $\Rightarrow$ t_ps form complex conjugate pairs
  (Over the barrier scattering)

$$\Psi(t)_\pm = \frac{exp[\pm i\int^t \omega(t')dt']}{\sqrt{\omega(t)}}, \quad \Psi(t)_\pm = \frac{exp[\pm i\int^t \omega(t')dt']}{\sqrt{\omega(t)(\omega(t)\pm k_3(t))}}, \quad Q = i\int_{t_p}^{t}\omega$$



- Coefficients of $\Psi_\pm(t)$ and phase integrals are defined locally in each section using Monodrumy rules.
- Unitarity is preserved: $|a_1|^2 \mp |a_2|^2 = |b_1|^2 \mp |b_2|^2$

One Pair of Turning Points

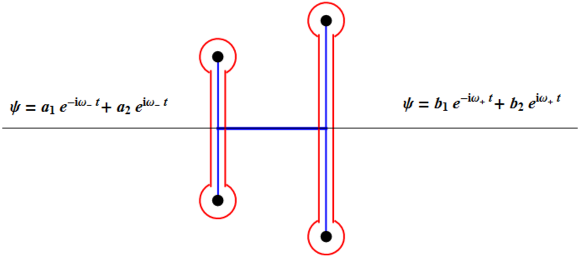


- Initial Condition: $a_2 \to 0$
- $|R_\pm(\infty)|^2 = \left|\frac{b_2}{b_1}\right|^2 \approx \frac{e^{-2K}}{1 \pm e^{-2K}} \approx e^{-2K}$

$$K = i\int_{t_1}^{t_2} \omega(t), \quad K \in \mathbb{R} > 0, \quad t_1,\ t_2 : \text{critical points}$$

- No oscillatory behavior

Two Pairs of Turning Points

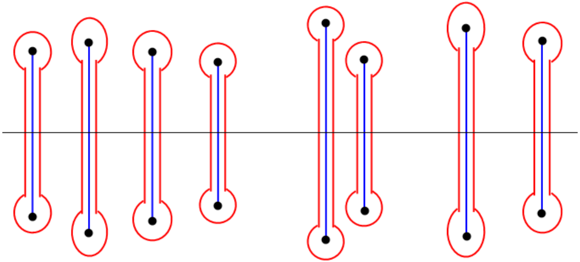


- $|R_\pm(\infty)|^2 \approx e^{-2K_1} + e^{-2K_2} \pm 2e^{-K_1-K_2}\cos 2\alpha$

  $$K_1 = i\int_{t_1}^{t_2}\omega(t) \quad K_2 = i\int_{t_3}^{t_4}\omega(t), \quad \alpha = \int_{Re[t_1]}^{Re[t_3]}\omega(t)$$
- Oscillations are due to interference between pairs of critical points.
- Spin statistics effect: Interference is out of phase

$$\Psi(t)_\pm = \frac{exp[\pm i\int^t \omega(t')dt']}{\sqrt{\omega(t)}}, \quad \Psi(t)_\pm = \frac{exp[\pm i\int^t \omega(t')dt']}{\sqrt{\omega(t)(\omega(t)\pm k_3(t))}}$$

N Pairs of Turning Points

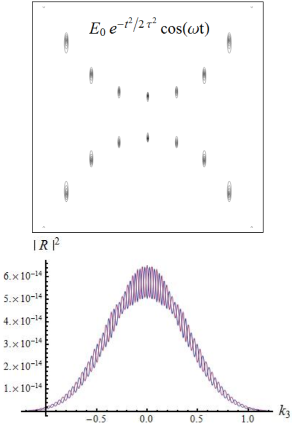


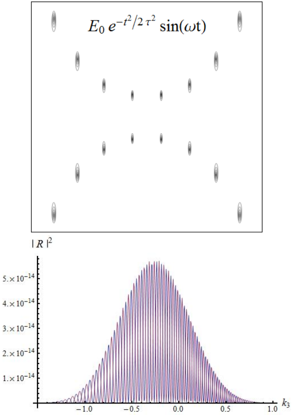
- $|R_\pm(\infty)|^2 \approx \sum_i e^{-2K_i} + \sum_{i<j} (\pm)^{i-j} e^{-K_i - K_j} \cos 2\alpha_{ij} +$ Higher order interference terms

$$K_i = i \int_{t_i}^{t_i^*} \omega(t), \quad \alpha_{ij} = \int_{Re[t_i]}^{Re[t_j]} \omega(t)$$

- Pair closest to real axis contributes the most; interference effect becomes visible when multiple pairs dominate.

N Pairs of Turning Points





- $|R_\pm(\infty)|^2 \approx \sum_i e^{-2K_i} + \sum_{i<j} (\pm)^{i-j} e^{-K_i-K_j} \cos 2\alpha_{ij} + \ldots$
- Pair closest to real axis contributes the most; interference effect becomes visible when multiple pairs dominate.

Wordline Formalism

$$\Gamma_{\text{eff}}^{\text{scalar}} = i\int_0^\infty \frac{dT}{T}\int d^4x \int \mathcal{D}^4x\,\mathcal{D}^4p \exp\left[i\int_0^T \Pi^2 - m^2 - p_\mu \dot{x}^\mu\right]$$

$$\Gamma_{\text{eff}}^{\text{spinor}} = \frac{-i}{2}\int_0^\infty \frac{dT}{T}\int d^4x \int \mathcal{D}^4x\,\mathcal{D}^4p \exp\left[i\int_0^T \Pi^2 - m^2 - p_\mu \dot{x}^\mu\right] \text{Det}\left[\delta_{\mu\nu}\frac{d}{d\tau} - \frac{ie}{mc}F(x(\tau))_{\mu\nu}\right]^{\frac{1}{2}}$$

$$\Pi_\mu = p_\mu - \frac{e}{c}A_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Stationary Phase Approximation to T integral
- Saddle Point of Path integral $\Rightarrow$ Classical Solutions: $\ddot{x}_\mu = F_\mu^\nu \dot{x}_\nu$ (closed paths)
- Time Dependent Fields

$$\Gamma_{\text{eff}}^{\text{scalar}} \approx i\int dk_3 \int dx_0\, e^{-i\int_0^T \dot{x}_0^2 d\tau}$$

$$\Gamma_{\text{eff}}^{\text{spinor}} \approx i\int dk_3 \int dx_0\, e^{-i\int_0^T \dot{x}_0^2 d\tau} cos\left(\frac{1}{2}\int_0^T \partial_0 A_3(x_0) d\tau\right)$$

Initial Conditions and Complex Trajectories

Initial Conditions:

$\dot{x}_0(0) = \dot{x}_0(T_s) = 0, \quad x_0(0) = x_0(T_s) = t_p,$ Motivation: $\int_{t_p}^{t_p^*} \omega(t)dt \equiv \int_0^T \dot{x}_0(\tau)^2 d\tau$

Trajectories:

Type I : $\ddot{x}_\mu = \pm i F_\mu^\nu \dot{x}_\nu$ $(\tau \to \pm i\tau)$, Connects t_p to t_p^*: decaying instanton trajectories

$\pm : x_0(0) = \text{Re}\,[t_p] \pm \text{Im}\,[t_p]$

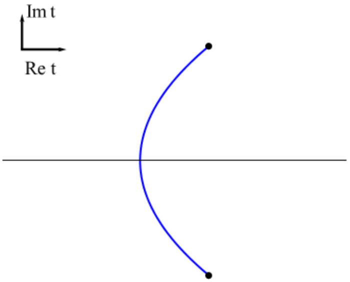
Type II: $\ddot{x}_\mu = F_\mu^\nu \dot{x}_\nu$, Connects t_p to neighboring t_p': results in interference.

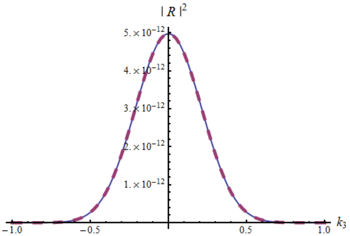
$$\Gamma_{\text{eff}}^{\text{scalar}} \approx i \int dk_3 \sum_{t_p} e^{-i\int_0^T \dot{x}_0^2 d\tau}$$

$$\Gamma_{\text{eff}}^{\text{spinor}} \approx i \int dk_3 \sum_{t_p} e^{-i\int_0^T \dot{x}_0^2 d\tau} cos\left(\frac{1}{2}\int_0^T \partial_0 A_3(x_0) d\tau\right)$$

- **Equations of motion are manifestly complex; behavior of complex solutions determine semiclassical scattering amplitudes.**
- **Wordlines start on t_ps, all *distinct*, periodic paths that can be formed must taken into account.**

One Pair of Turning Points



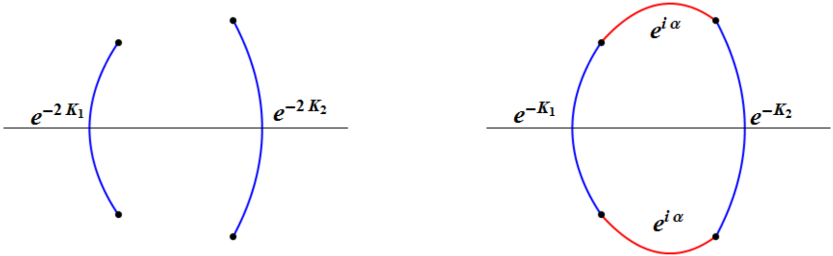


$$\mathrm{Im}\left[\Gamma_{\mathrm{eff}}^{\pm}\right] \approx \int dk_3\, e^{\int_0^{T_s} \dot{x}_0^2 d\tau}, \quad |R(\infty)|^2 = e^{\int_0^{T_s} \dot{x}_0^2 d\tau}$$

One (Dominating) Pair of Turning Points:

$$A(t) = E_0 \tau tanh(\tfrac{t}{\tau}),\ E(t) = E_0 Sech(\tfrac{t}{\tau})^2, \quad E_0 = 0.1,\ \tau = 10,\ m = 1 \equiv .5\,\mathrm{MeV}$$

Two Pair of Turning Points

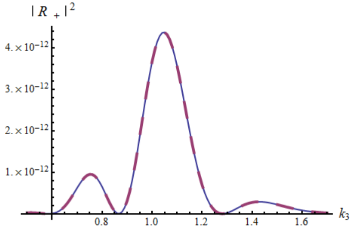


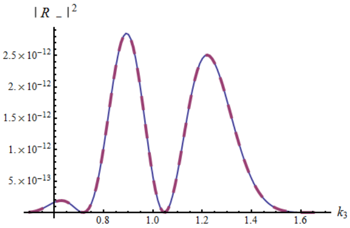
$$\mathrm{Im}\left[\Gamma^{\pm}_{\mathrm{eff}}\right] \approx \int dk_3 \left(e^{\int_0^{T_s^1} \dot{x}_0^2 d\tau} + e^{\int_0^{T_s^2} \dot{x}_0^2 d\tau} \pm 2\exp\left[\int_0^{T_s^1/2} \dot{x}_0^2 d\tau + \int_0^{T_s^2/2} \dot{x}_0^2 d\tau\right] \cos\left(2\int_0^{T_s^{12}/2} \dot{x}_0^2 d\tau\right)\right)$$

$$|R_{\pm}(\infty)|^2 \approx e^{-2K_1} + e^{-2K_2} \pm 2e^{-K_1-K_2}\cos 2\alpha$$

$$K_1 = i\int_{t_1}^{t_2}\omega(t) \quad K_2 = i\int_{t_3}^{t_4}\omega(t), \quad \alpha = \int_{Re[t_1]}^{Re[t_3]}\omega(t)$$

Two Pair of Turning Points





$$A(t) = \frac{E_0 \tau}{1+\frac{t}{\tau}^2},\ E(t) = -\frac{2E_0 t}{\tau\left(1+\frac{t}{\tau}^2\right)^2},\qquad E_0 = 0.1,\ \tau = 10$$

$\omega(t)$ has symmetric *tps* : $t_1 = t_3,\ t_2 = t_4 \Rightarrow T_s^1 = T_s^2$

$$\mathrm{Im}\left[\Gamma_{\mathrm{eff}}^{\pm}\right] \approx 2\int dk_3\, e^{\int_0^{T_s} \dot{x}_0^2 d\tau}\left(1 \pm cos\left(2\int_0^{T_s^{12}/2} \dot{x}_0^2 d\tau\right)\right)$$

Summary and Outlook

- **WKB: Oscillations in spectra are due to interference between multiple turning point pairs.**
- **Complex Worldlines: Complex, closed trajectories passing through multiple turning point pairs results in interference.**
- **Interference(resonance) in spectra might be utilized as a guideline for pulse optimization.**
- **Spacetime dependent fields: WKB method in higher dimensions is unknown, wordline formalism might provide more insight.**

THANK YOU.