Creation of Neutral Fermions by Magnetic Barriers

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Overview and Motivation

1. An inhomogeneous magnetic field (MF) can produce work on neutral fermions with *anomalous magnetic moment* $\Longrightarrow$ They can create neutral fermions from vacuum.
2. Examples: Neutron $(\mu_n = -1.9130427(5)\mu_N, \mu_N = e/2m_N)$ and possible Neutrinos†,
3. MF up to 10^{18}G can be generated during a supernova explosion (**can create neutrino-antineutrino pairs**),
4. MF around $B \sim 10^{19}$G, can be generated in heavy-ion collisions at RHIC and LHC,
5. Superconducting cosmic string could generate fields more then 10^{30}G in their vicinities. $B^n_{cr} \sim 10^{20}$G can create neutron-antineutron pairs.

†C. Giunti, A. Studenikin, (2009), M. Dvornikov, arXiv:1011.4300, M. Deniz, et al, (2010)and A.G. Beda, et al (2007).

Contend

1. Exact solutions of Dirac-Pauli equation
2. General aspects of the QFT description of the problem
3. Specific examples of inhomogeneous external MF
4. Conclusions

Dirac-Pauli equation†

Relativistic description of spin 1/2 neutral fermions

$$\left(\gamma^{\mu}\hat{p}_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}(x)\right)\psi(x) = 0\,,$$
$$\hat{p}_{\mu} = i\partial_{\mu}\,,\quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}]_{-}\,.$$

External magnetic field: Nonuniform and constant in z-direction: $\mathbf{B}(y) = (0,0,B_z(y))\,,\; B_z(y) = F_{21}(y)\,.$
The complete set of solutions is

$$\psi_n(t,\mathbf{r}) = \exp(-ip_0 t + ip_x x + ip_z z)\,\psi_n(y)\,,$$

where* $\psi_n(y)$: eigenvectors of mutually commuting of $\hat{p}^0$, $\hat{p}^1$, $\hat{p}^3$, R and $\hat{\Pi}_z = \hat{\pi}_z - \mu B_z(y)\,,$

$$R = \left[1 + (p_z/\omega)^2\right]^{-1/2}\left(s\gamma^0\gamma^3 p_z/\omega + \gamma^0\Sigma_z\right),$$
$$\hat{\pi}_z = \Sigma_z\left(\gamma^1 p_x + \gamma^2\hat{p}^2\right) + m\Sigma_z\,,$$

†W. Pauli, Rev. Mod. Phys. 13 (1941). *S. P. Gavrilov and D. M. Gitman, arXiv:1101.4243.

Eigenvalue and quadratic equation

$$R\psi_n(y) = s\psi_n(y)\,,\ s = \pm 1,\ p_0 = \omega\sqrt{1 + (p_z/\omega)^2},$$
$$[\hat{\pi}_z - \mu B_z(y) - s\omega]\,\psi_n(y) = 0\ ,\quad n = (p_x, p_z, \omega, s)\,.$$

$\psi_n(y)$ can be presented as

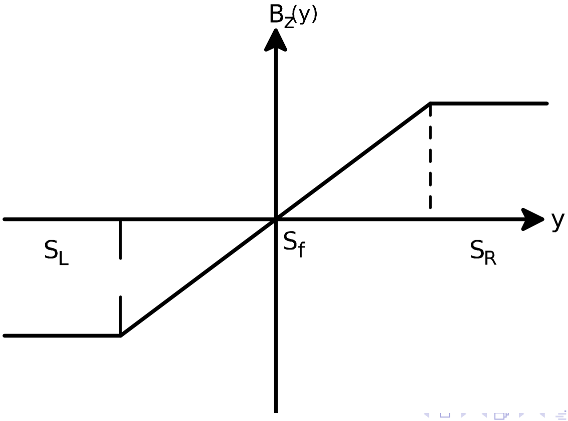
$$\psi_n(y) = \frac{1}{4}(1 + sR)\,[\hat{\pi}_z + \mu B_z(y) + s\omega]\,\varphi_{n,\chi}(y)\left(1 + i\chi\gamma^1\right)v\,,$$
$$\left\{-\partial_y^2 + m^2 + p_x^2 + i\chi\mu\partial_y B_z(y) - [\omega + s\mu B_z(y)]^2\right\}\varphi_{n,\chi}(y) = 0.$$

1. **The problem is technically reduced to the problem of charged particle in a field given by a nonlinear scalar potential.** Thus we can use results elaborated for the pair creation by electric barriers (Klein effect)[†].

[†] A. I. Nikishov, Nucl. Phys. B 21 (1970).

Structure of the external magnetic fields

1. We will consider external magnetic fields which are homogeneous at asymptotic regions $y \longrightarrow \pm\infty$.
2. The fields and its derivative obeys $B_z(\pm\infty) \gtrless 0$ and $\partial_y B_z(y) > 0$, respectively.



Asymptotic conditions and inner product

1)At $\underline{y \to \pm\infty}$ the MF is homogeneous: Thus we construct two sets $\left\{ {}_{\zeta}\psi_{n}\left(t,\mathbf{r}\right)\right\}$ and $\left\{ {}^{\zeta}\psi_{n}\left(t,\mathbf{r}\right)\right\}$ of independent solutions,

$$
\begin{aligned}
-i\partial_{y}\ {}_{\zeta}\psi_{n}\left(t,\mathbf{r}\right) &= {}_{\zeta}p_{y}\ {}_{\zeta}\psi_{n}\left(t,\mathbf{r}\right) , \quad \zeta = \mathrm{sign}\ {}_{\zeta}p_{y} , \quad y \to -\infty ; \\
-i\partial_{y}\ {}^{\zeta}\psi_{n}\left(t,\mathbf{r}\right) &= {}^{\zeta}p_{y}\ {}^{\zeta}\psi_{n}\left(t,\mathbf{r}\right) , \quad \zeta = \mathrm{sign}\ {}^{\zeta}p_{y} , \quad y \to +\infty .
\end{aligned}
$$

2)MF exist during a suff. large time $T \to \infty$. Then **one can ignore effects of its switching on and off**

$$
\begin{aligned}
\left(\psi_{n},\psi'_{n'}\right)_{\sigma} &= \int_{\sigma}\overline{\psi}_{n}\left(t,\mathbf{r}\right)\gamma^{\mu}\psi'_{n'}\left(t,\mathbf{r}\right)d\sigma_{\mu} \longrightarrow \left(\psi_{n},\psi_{n'}\right)_{y} , \\
\left(\psi_{n},\psi_{n'}\right)_{y} &= \int_{V_{y}=T.L_{x}.L_{z}}\psi_{n}^{\dagger}\left(t,\mathbf{r}\right)\gamma^{0}\gamma^{2}\psi_{n'}\left(t,\mathbf{r}\right)dtdxdz ,
\end{aligned}
$$

3)$\psi_{n}\left(t,\mathbf{r}\right)$ satisfy the periodic conditions in t, x, z with sufficiently large periods L_{x}, $L_{z} \to \infty$.

Orthonormality relations

By using the asymptotic conditions, we satisfy the orthonormality relations

$$\left({}_{\zeta}\psi_{n},{}_{\zeta'}\psi_{n'}\right)_{y}=\zeta\eta_{L}\delta_{\zeta,\zeta'}\delta_{n,n'};\ \ \left({}^{\zeta}\psi_{n},{}^{\zeta'}\psi_{n'}\right)_{y}=\zeta\eta_{R}\delta_{\zeta,\zeta'}\delta_{n,n'},$$

where

$$\eta_{L}=\mathrm{sign}\pi_{s}\left(L\right),\ \ \eta_{R}=\mathrm{sign}\pi_{s}\left(R\right);$$
$$\pi_{s}\left(L/R\right)=\omega-sU_{L/R}\ ;\ \pi_{s}\left(L\right)=\pi_{s}\left(R\right)+sU;$$
$$U=U_{R}-U_{L}>0\,,$$
$$U_{L}=-\mu B_{z}\left(-\infty\right)<0,\ \ U_{R}=-\mu B_{z}\left(+\infty\right)>0\,.$$

Orthonormality relations

Two complete sets ${}_\zeta\psi_n(t,\mathbf{r})$ and ${}^\zeta\psi_n(t,\mathbf{r})$ are related as follows:

$$\begin{aligned} \eta_L\ {}^\zeta\psi_n(t,\mathbf{r}) &= {}_+\psi_n(t,\mathbf{r})\, g\left({}_+\middle|{}^\zeta\right) - {}_-\psi_n(t,\mathbf{r})\, g\left({}_-\middle|{}^\zeta\right); \\ \eta_R\ {}_\zeta\psi_n(t,\mathbf{r}) &= {}^+\psi_n(t,\mathbf{r})\, g\left({}^+\middle|{}_\zeta\right) - {}^-\psi_n(t,\mathbf{r})\, g\left({}^-\middle|{}_\zeta\right), \end{aligned}$$

where

$$\left({}_\zeta\psi_n, {}^{\zeta'}\psi_{n'}\right)_y = \delta_{nn'}\, g\left({}_\zeta\middle|{}^{\zeta'}\right), \quad g\left({}^{\zeta'}\middle|{}_\zeta\right) = g\left({}_\zeta\middle|{}^{\zeta'}\right)^*$$

Unitarity relations:

$$\begin{aligned} g\left({}^{\zeta'}\middle|{}_+\right) g\left({}_+\middle|{}^\zeta\right) - g\left({}^{\zeta'}\middle|{}_-\right) g\left({}_-\middle|{}^\zeta\right) &= \zeta\eta_L\eta_R\delta_{\zeta,\zeta'}; \\ g\left({}_{\zeta'}\middle|{}^+\right) g\left({}^+\middle|{}_\zeta\right) - g\left({}_{\zeta'}\middle|{}^-\right) g\left({}^-\middle|{}_\zeta\right) &= \zeta\eta_L\eta_R\delta_{\zeta,\zeta'}. \end{aligned}$$

Then $g\left({}_+\middle|{}^+\right)$ and $g\left({}_+\middle|{}^-\right)$ are independent.

General aspects of the QFT description†

Classical description: Classification of particle/antiparticle states

1. The classical energy on the xy plane is

$$\left|\omega_{cl}^{\pm}\right| = \sqrt{p_x^2 + p_y^2\left(y\right) + m^2} \pm s\left|\mu\right| B_z\left(y\right) \, , \quad B_z\left(\pm\infty\right) \gtrless 0 \, ,$$

   1. $s = +1$: $B_z(y)$ accelerates antiparticle along the axis y, particles - in the opposite direction.
   2. $s = -1$: $B_z(y)$ accelerates particle along the axis y, antiparticles - in the opposite direction.
   3. It is enough to consider only one case, let say $s = +1$.

†S. P. Gavrilov and D. M. Gitman, (in preparation).

General aspects of the QFT description

Constraints on the external MF for the particle creation case

1. If,

$$U = -\mu \left[B_z(+\infty) - B_z(-\infty)\right] > 2\pi_x, \; \left(\pi_x = \sqrt{p_x^2 + m^2}\right),$$
$$-\mu B_z(-\infty) + \pi_x \leq \omega \leq -\mu B_z(+\infty) - \pi_x$$

final particles are situated in the range S_L and final antiparticles in the range S_R $\Longrightarrow$ Particle creation range.

2. Inner product on hyperplane $t = const$

$$\left(\psi, \psi'\right)_t = \int_t \psi^\dagger(t, \mathbf{r})\, \psi'(t, \mathbf{r})\, d\mathbf{r}\,.$$

and standard QFT particle-antiparticle classification.

Orthogonality relation on hyperplane t=const

$$\left(\psi_{n},\psi'_{n'}\right)_{t}=\delta_{n,n'}L_xL_z\mathcal{R},\ \ \mathcal{R}=\int_{-L_1}^{L_2}Qdy,$$

$$Q=\varphi^{*}_{n,\chi}\left(y\right)\left[\pi_x^2+\left(\omega+s\mu B_z\left(y\right)+s\chi i\overrightarrow{\partial}_y\right)^2\right]\varphi'_{n,\chi}\left(y\right),$$

where $L_1,\ L_2\rightarrow\infty$ are parameters of volume regularization. Integral $\mathcal{R}$ can be represented as

$$\mathcal{R}=\int_{-L_1}^{y_L}Qdy+\int_{y_L}^{y_R}Qdy+\int_{y_R}^{L_2}Qdy$$

The leading contribution of $\mathcal{R}$ are $\mathcal{R}_L\sim L_1$ and $\mathcal{R}_R\sim L_2$. Both are from asymptotic ranges,

$$\mathcal{R}=\mathcal{R}_L+\mathcal{R}_R,\ \ \mathcal{R}_L=\int_{-L_1}^{y_L}Q_Ldy,\ \ \mathcal{R}_R=\int_{y_R}^{L_2}Q_Rdy,$$

$$Q_{L/R}=\left(\varphi_{n,\chi}\left(y\right)\right)^{*}\left[\pi_x^2+\left(\pi_s\left(L/R\right)+s\chi i\overrightarrow{\partial}_y\right)^2\right]\varphi'_{n,\chi}\left(y\right).$$

Orthogonality of the wave functions

We can show that the orthogonality relation,

$$\left({}_{\zeta}\psi_{n'}, {}^{\zeta}\psi_{n}\right)_{t} = 0\,,$$

holds if L_1 and L_2 satisfy the condition

$$L_1 \left| \frac{\pi_s(L)}{{}_{\zeta}p_y} \right| - L_2 \left| \frac{\pi_s(R)}{{}^{\zeta}p_y} \right| = 0\,,$$

where

$$\left({}_{\zeta}p_y\right)^2 = \left[\pi_s(L)\right]^2 - \pi_x^2 \geq 0,\ \left({}^{\zeta}p_y\right)^2 = \left[\pi_s(R)\right]^2 - \pi_x^2 \geq 0.$$

This relation express the ortogonality of particle and antiparticle wave functions at given time instant.

- **We relate standard picture of stationary scattering by one-dim. potential barrier, which is normally used in QM, with standard QFT description that based on an inner product on hyperplane $t = const$.**

In- and out- states

1. Sign of the physical momentum of particles obeys $p_y^{ph} = p_y$. For antiparticles $p_y^{ph} = -p_y$
2. ${}^{+}\psi_n(t,\mathbf{r})$ and ${}_{+}\psi_n(t,\mathbf{r})$ **describe outgoing particles and antiparticles**
3. ${}^{-}\psi_n(t,\mathbf{r})$ and ${}_{-}\psi_n(t,\mathbf{r})$ **describe incoming particles and antiparticles**
4. IN vacuum: absence of incoming (anti)particles$\Longrightarrow$Presence of outgoing (anti)particles indicates stationary creation of pairs from the vacuum.

By decomposing the component of QF operator with $n \in \Omega$, we define **creation and annihilation operators,**

$$\Psi_n(t,\mathbf{r}) = \mathcal{M}^{-1/2}\left[a_n(out)\ {}^{+}\psi_n(t,\mathbf{r}) + b_n^{\dagger}(out)\ {}_{+}\psi_n(t,\mathbf{r})\right],$$

$$\Psi_n(t,\mathbf{r}) = \mathcal{M}^{-1/2}\left[a_n(in)\ {}^{-}\psi_n(t,\mathbf{r}) + b_n^{\dagger}(in)\ {}_{-}\psi_n(t,\mathbf{r})\right],$$

$$\mathcal{M} = 2(L_2/T)\left|\pi_s(R)/{}^{\zeta}p_y\right|\left|g\left({}_{+}|{}^{-}\right)\right|^2,$$

Bogolyubov transformation

Nonzero canonical anticommutation relations:

$$[a_n(in), a_k^\dagger(in)]_+ = [a_n(out), a_k^\dagger(out)]_+ = \delta_{nk}.$$
$$[b_n(in), b_k^\dagger(in)]_+ = [b_n(out), b_k^\dagger(out)]_+ = \delta_{nk}.$$

The *in*-vacuum $|0, in\rangle$ and *out*-vacuum $|0, out\rangle$ definitions:

$$a_n\left(in\right)|0, in\rangle = b_n\left(in\right)|0, in\rangle = 0,\ \forall n;$$
$$a_n\left(out\right)|0, out\rangle = b_n\left(out\right)|0, out\rangle = 0,\ \forall n.$$

(Bogolyubov transformation):

$$a_n\left(out\right) = g\left({}^-|_+\right)^{-1} g\left({}^+|_+\right) a_n\left(in\right) - g\left({}_-|^+\right)^{-1} b_n^\dagger\left(in\right),$$
$$b_n^\dagger(out) = g\left({}^-|_+\right)^{-1} a_n\left(in\right) + g\left({}_-|^+\right)^{-1} g\left({}_+|^+\right) b_n^\dagger(in).$$

Mean number of created pairs

Differential mean number of created particles and antiparticles

$$N_n^{(+)} = \langle 0, in | a_n^\dagger (out) a_n (out) | 0, in \rangle = \left| g \left({}_- \middle| {}^+ \right) \right|^{-2},$$
$$N_n^{(-)} = \langle 0, in | b_n^\dagger (out) b_n (out) | 0, in \rangle = \left| g \left({}_+ \middle| {}^- \right) \right|^{-2}.$$

Mean number of created pairs:

$$N_n^{(+)} = N_n^{(-)} = N_n,$$

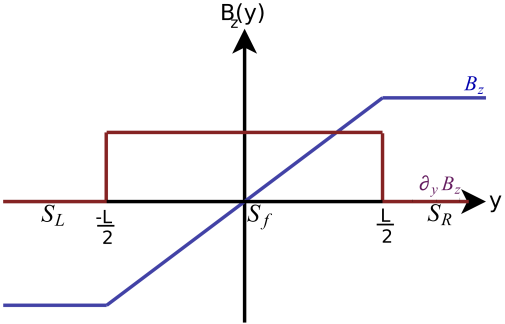
Total number of created pairs:

$$N = \sum_n N_n.$$

Probability for the vacuum to remain a vacuum:

$$P_v = |c_v|^2 = \exp\left\{ \sum_n \ln(1 - N_n) \right\}.$$

Linearly growing magnetic field



The quadratic equation can be written as WPC diff. eq.,

$$\left\{-\frac{d^2}{dy^2} - \left[\omega + s\mu B_z(y)\right]^2 + m^2 + p_x^2 + i\chi\mu\frac{d}{dy}B_z(y)\right\}\varphi_{n,\chi}(y) = 0\,,$$

Linearly growing magnetic field

$$\left[\frac{d^2}{d\xi^2} - \xi^2 - \lambda + i\chi\right] \varphi_{n,\chi}(\xi) = 0\,, \quad \xi(y) = \frac{|\mu|(B'y + B_0) - \omega}{\sqrt{|\mu| B'}}$$

then the coefficient $g\left({}_+|{}^-\right)$ is,

$$g\left({}_+|{}^-\right) = e^{\frac{i\pi}{4}} e^{\frac{i\tilde{\nu}\pi}{2}} \sqrt{\frac{|p_y(y_1)|\,\tilde{q}_1\,|p_y(y_2)|}{8|\mu| B'\tilde{q}_2}} \left[\tilde{f}_2(y_2)\tilde{f}_1(y_1) - \tilde{f}_1(y_2)\tilde{f}_2(y_1)\right]\,,$$

$$\tilde{f}_1(y) = \left(1 - \frac{i}{|p_y(y)|}\partial_y\right) D_{-\tilde{\nu}-1}\left[(1+i)\xi(y)\right]\,, \quad y_1 = -\frac{L}{2}\,,$$

$$\tilde{f}_2(y) = \left(1 - \frac{i}{|p_y(y)|}\partial_y\right) D_{\tilde{\nu}}\left[(1-i)\xi(y)\right]\,, \quad y_2 = +\frac{L}{2}\,,$$

$$\lambda = \frac{p_x^2 + m^2}{|\mu| B'}\,, \quad \tilde{\nu} = -\frac{i\lambda}{2}\,, \quad \tilde{q}_i = |p_y(y_i)| - (-1)^i\left[\omega + \mu B_z(y_i)\right]\,.$$

Linearly growing magnetic field

Asymptotic expression for the mean and total number of fermions created

For $\sqrt{|\mu|\,B'}L \gg 1,\ m^2/\,|\mu|\,B'$ we have,

$$\begin{aligned} N_n &= \left|g\left({}_+|^-\right)\right|^{-2} \simeq e^{-\lambda\pi}\left\{1+\frac{1}{2}\sqrt{\lambda\left(1-e^{-\lambda\pi}\right)}\times\right. \\ &\times \left.\left[\frac{1}{|\xi_1|^3}\sin\phi_1+\frac{1}{\xi_2^3}\sin\phi_2\right]\right\}, \\ \phi_i &= \left(\frac{\pi}{4}+\arg\Gamma\left(\tilde{\nu}\right)+\lambda\ln\sqrt{2}\,|\xi_i|-\xi_i^2\right), \quad i=(1,2)\,. \end{aligned}$$

The total number is calculated as

$$N=\frac{L_xL_zT}{(2\pi)^3}\sum_{s=\pm1}\int dp_xdp_z\int_0^{\omega_{\max}^2}\frac{N_nd\omega^2}{\sqrt{\omega^2+p_z^2}},$$

where is $\omega_{\max}=|\mu B'|\,L/2$.

Linearly growing magnetic field

Asymptotic expression for the total number of fermions created

Final expressions: Total mean number

$$N = \frac{(1+\ln 4)\, T L_x L_z L^2 \left|\mu B'\right|^{5/2}}{16\pi^3} \exp\left(-\frac{\pi m^2}{|\mu B'|}\right).$$

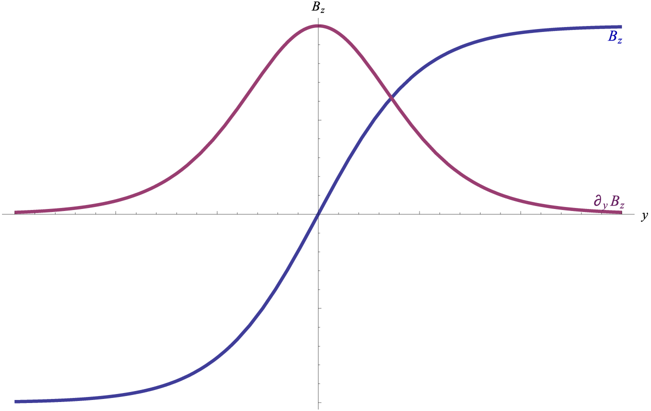
Unlike the case of electric field, mean density $N/V \sim L$. Then the total quanities are very sensitive to the length L of magnetic field inhomogeneity.

Vacuum-to-vacuum transition probability

$$P_v = \exp\left(-\beta N\right), \quad \beta = \sum_{l=0}^{\infty} (l+1)^{-3/2} \exp\left(-\frac{l\pi m^2}{|\mu B'|}\right).$$

Sauter type magnetic field

$$B_z(y) = \alpha B \tanh\left(\frac{y}{\alpha}\right) , \quad \partial_y B_z(y) = B \cosh^{-2}\left(\frac{y}{\alpha}\right) .$$



Sauter type magnetic field

Mean number of fermions created

$$N_n = \left| g\left({}_{-}|^{+} \right) \right|^{-2}$$

$$= \frac{\sinh(\pi\alpha\beta_-)\sinh(\pi\alpha\beta_+)}{\sinh\left[\frac{\pi\alpha}{2}\left(\beta_- - \beta_+ + 2|\mu|\chi B\right)\right]\sinh\left[\frac{\pi\alpha}{2}\left(\beta_+ - \beta_- + 2|\mu|\chi B\right)\right]}$$

$$\beta_\pm = \sqrt{(\omega \mp |\mu| B)^2 - p_x^2 - m^2},$$

For $\alpha \gg \left(1 + \sqrt{\lambda}\right) / \sqrt{|\mu| B'}$,

$$N_n = e^{-\pi\lambda}.$$

Conclusions

- **Consistent QFT description of neutral particle creation (due to their magnetic moments) by magnetic barriers is formulated**
- **Problem is technically reduced to the problem of charge particle creation by an electric barrier**
- **Features of neutral particle creation from the vacuum by a growing magnetic field is studied**
- **This mechanism may be responsible for the creation of neutrino-antineutrino pair during the formation of a neutron star.**

THANKS !