Generalizations of Heisenberg-Euler Energy to Strong Electric Fields

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External field in QFT

QFT in an external background is an effective **model** when a part of a quantized field is strong enough to be treated as a given (external) classical one. E.g., QED with an external electromagnetic field formally arises as

$$j^{\mu}A_{\mu} \rightarrow j^{\mu}\left(A_{\mu}+A_{\mu}^{\mathrm{ext}}\right).$$

This is naturally implied as a certain approximation. In fact, it is supposed that a quantum processes under consideration does not affect significantly the external field (**back-reaction is supposed to be small**). Due to new challenges in astrophysics, discovery of quark-gluon plasma, heavy ions collisions, and creation of such materials as graphene **the problem of strong external field impact on the physical vacuum and the corresponding backreaction** has already become of practical interest.
Back-reaction has to be calculated to answer **the question of a consistency of theories with external backgrounds**.
First of all: A **constant uniform electromagnetic field**.

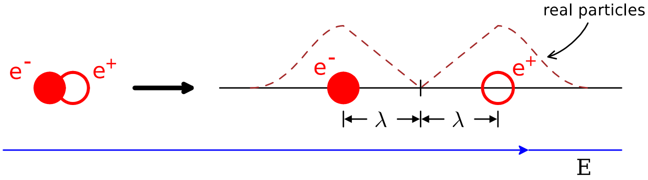
Particle creation from the vacuum

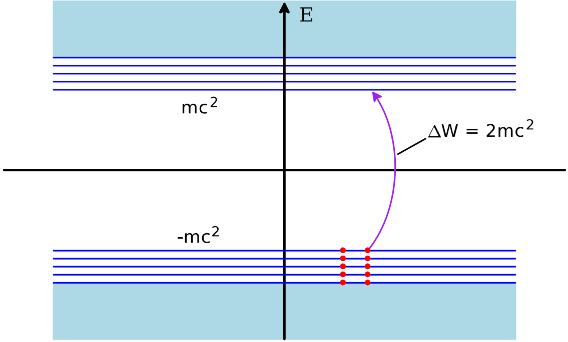
Both magnetic and electric fields polarize the vacuum, but **only the electric field** can produce the work acting on charged particles, even virtual ones, then destroying the physical vacuum.
Qualitatively: if an electric field is strong enough, it can pass to a virtual electron-positron pair energy greater than its rest mass $2mc^2$ to transform them into real particles. **The vacuum is practically stable if** $E \ll E_c$,

$$eE_c \cdot 2\lambda = 2mc^2 \overset{\lambda = \hbar/mc}{\Longrightarrow} E_c = \frac{m^2c^3}{e\hbar} \approx 1,3 \cdot 10^{16} \mathrm{V/cm} \, .$$

Creating pairs from the vacuum, **the super strong electric field loses its energy and thus destroys itself**. That is why the principal part of the backreaction is due to the particle creation but not to the vacuum polarization.

Fradkin,Gitman,Shvartsman, **QED with Unstable Vacuum** (Springer 1991).


real particles
e-
e+
e-
e+
λ
λ
E


E
mc2
ΔW = 2mc2
-mc2

Heisenberg-Euler Lagrangian and maximal magnetic field

Heisenberg, Euler (1936): change of vacuum energy in QED with constant $E \ll E_c$ and $B \parallel E$ fields.

$$\mathcal{E}_0 = -\sum_{\mathbf{p},r} \left(\varepsilon_{\mathbf{p},r}^{(-)} - \varepsilon_{\mathbf{p},r}^{(-)} \Big|_{B=0} \right) \varepsilon_{\mathbf{p},r}^{(-)}, \quad \varepsilon_{\mathbf{p},r}^{(-)} = \sqrt{m^2 + \mathbf{p}_\perp^2 + (p_3)^2}$$

Thus, they obtained a change $\Delta\mathcal{L}_{\mathrm{M}}$ of M. L. $\mathcal{L}_{\mathrm{M}}$. For $E = 0$, $B \gg B_c = m^2 c^3 / e\hbar = 4,4 \cdot 10^{13}\mathrm{G}$:

$$\Delta\mathcal{L}_{\mathrm{M}} = -\left(\frac{\alpha}{3\pi} \ln \frac{B}{B_c} \right) \mathcal{L}_{\mathrm{M}}, \quad \mathcal{L}_{\mathrm{M}} = \left(E^2 - B^2 \right) / 8\pi, \quad \alpha = e^2 / \hbar c.$$

However, in such calculations: the loop expansion makes sense only for the magnetic fields $B \ll B_{\max}$,

$$B_{\max} = B_c \exp\left(\frac{3\pi}{\alpha} \right) \approx B_c 10^{560} \quad \left(\frac{B_c}{4} \exp\left(\frac{\pi^{3/2}}{\sqrt{\alpha}} + 1,2 \right) \approx B_c 10^{28} \right)$$

Ritus (Sov.Phys.JETP,1975;77); Shabad,Usov, taking into acc. the interaction of virtual particle interaction (PRL, 2006).

Schwinger effective Lagrangian

$$c_v = \langle 0, out | 0, in \rangle = \exp\left(\frac{i}{\hbar} \int dx \mathcal{L}_{\mathrm{eff}} \right), \quad P^v = |c_v|^2 .$$

For $E = \mathrm{const.}$, the probability P^v is related to $\mathrm{Im}\,\mathcal{L}_{\mathrm{eff}}$ as follows (Schwinger, Phys.Rev.82 (1951) 664):

$$P^v = |c_v|^2 = \exp\left\{ -\frac{VTc}{\hbar} 2\,\mathrm{Im}\,\mathcal{L}_{\mathrm{eff}} \right\} = \exp\{-\mu N\},$$

$$\mu = \sum_{l=0}^{\infty} (l+1)^{-d/2} \exp\left(-\pi l \frac{E_c}{E} \right) \quad \text{in } d-\text{dim}$$

Total number of created particles for T and V (Nikishov, 1970; Gavrilov, Gitman, PRD**53** (1996) 7162):

$$N = \frac{VTcJ}{(2\pi)^{d-1} \lambda^d} \left(\frac{E}{E_c} \right)^{d/2} \exp\left(-\pi \frac{E_c}{E} \right), \quad J = 2^{\left[\frac{d}{2}\right]-1}$$

Thus, the vacuum instability is essential for $E \geq E_c$!

Restrictions on electric field from Schwinger EL

$\Delta\mathcal{L}_{\mathrm{M}} = \mathcal{L}_{\mathrm{eff}}$. In the case $E \gg E_{\mathrm{c}}$, $B = 0$, $d = 4$

$$\mathrm{Re}\ \mathcal{L}_{\mathrm{eff}} = -\frac{\alpha E^2}{24\pi^2} \ln \frac{E}{E_{\mathrm{c}}} \Longrightarrow \Delta\mathcal{E}_{\mathrm{M}} = \mathrm{Re}\ \mathcal{L}_{\mathrm{eff}} < 0 \ .$$

Thus, the total energy density of the electric field vanishes at $E_{\max}$,

$$\mathcal{E} = \mathcal{E}_{\mathrm{M}} + \Delta\mathcal{E}_{\mathrm{M}} = 0 \text{ at } E \sim E_{\max} = E_{\mathrm{c}} \exp\left(\frac{3\pi}{\alpha}\right) \approx E_{\mathrm{c}} \exp\left(10^3\right) \ .$$

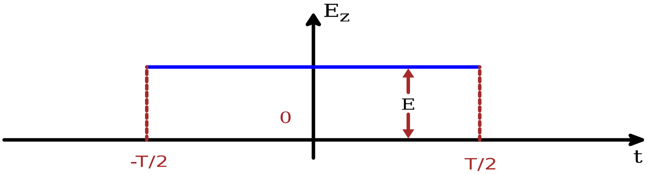
However, $E_{\max}$ cannot be considered a maximal strength of external constant electric field.

$$E\frac{\partial\,\mathrm{Re}\,\mathcal{L}_{\mathrm{eff}}}{\partial E} - \mathrm{Re}\,\mathcal{L}_{\mathrm{eff}} = w^{c}(t) = \mathrm{Re}\langle 0, out|\, T_{00}(t)\, |0, in\rangle c_{v}^{-1} \ \ \text{local},$$
$$w(t, T) = \langle 0, in|\, T_{00}|0, in\rangle, \quad w - w^{c} = w^{p}(T) \neq 0 \ \ \text{total}$$

$\mathrm{Re}\ \mathcal{L}_{\mathrm{eff}}$ describs effects of vacuum polarization only. **The complete back-reaction can be obtained by nonperturbative calculating the mean value of the energy-momentum tensor of matter, $T_{\mu\nu}$.**

T-constant electric field

In QED, it was caculated the mean value of the energy density of matter in one-loop approximation, $w(t, T)$, **taking an exact account of the interaction with** T-const. **electric field** (Gavrilov, Gitman (PRD78, 2008; PRL 101, 2008)
T-const. EF turns on to E at $-T/2 = t_1$ and turns off to 0 at $T/2 = t_2$.

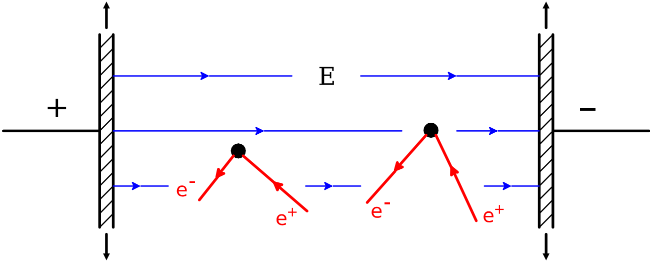


$A_1(t) = -Et$, $t \in [t_1, t_2]$, being constant for $t \in (-\infty, t_1)$ and $t \in (t_2, +\infty)$.

L-constant electric field

In recent modification of QED, it was caculated the mean value of the energy density $w(t, L)$**, taking an exact account of the interaction with electric potential barrier in the form of** L-const. **electric field** (Gavrilov, Gitman (paper in preparation).
L-const. EF turns on to E at $-L/2 = x_L$ and turns off to 0 at $L/2 = x_R$.
$A_0(x) = Ex$, $x \in [x_L, x_R]$, being uniform for $x \in (-\infty, x_L)$ and $x \in (x_R, +\infty)$.
It is a time-independent nonuniform electric field that vanishes at the spatial infinity

T- and L-constant electric field



In case $cT,\ L \gg L_{\text{stab}}$,

$$L_{\text{stab}} = \sqrt{\frac{c\hbar}{eE}}\left[1 + E_{\text{c}}/E\right],$$

$$\frac{dL_{\text{stab}}}{dE} < 0, \quad L_{\text{stab}}\big|_{E=E_c} = 2\lambda \approx c \cdot 1,3 \cdot 10^{-21}\text{s} \approx 4 \cdot 10^{-13}\text{m}\ ,$$

all finite effects of particle-creation reach their asymptotic values, whereas the details of switching the field on and off can be neglected (Gavrilov, Gitman, PRD**53** (1996) 7162).

Vacuum energy change in T-const. EF

$$w=\langle 0,in|\,T_{00}\,|0,in\rangle=\frac{1}{2}\langle 0,in|\left[\hat{\psi}(x)^{\dagger},\hat{H}_{\mathrm{D}}\hat{\psi}(x)\right]|0,in\rangle\Big|_{t=T/2-0},$$

$w=\frac{\langle\hat{H}_{QED}\rangle}{V}$, $\hat{H}_{\mathrm{D}}$ is Dirac Hamiltonian; $\hat{\psi}(x)$ obey Dirac Eq. with $A_{\mu}(x)$.

We have a complete account of vacuum polarization as well as pair-creation during the entire time T.

$$w=\frac{1}{4}\left(\lim_{t'\to t-0}+\lim_{t'\to t+0}\right)\mathrm{tr}\left(\partial_{t'}-\partial_{t}\right)S_{in}(x,x')\Big|_{\mathbf{x}=\mathbf{x}',t=t_{2}-0}.$$

Here $S_{in}(x,x')$ is $\mathrm{in}-\mathrm{in}$ GF,

$$\begin{aligned}S_{in}(x,x') &= i\langle 0,in|\,T\psi(x)\bar{\psi}(x')|0,in\rangle=S^{c}(x,x')+S^{p}(x,x'),\\ S^{c}(x,x') &= i\langle 0,out|\,T\psi(x)\bar{\psi}(x')|0,in\rangle\langle 0,out|0,in\rangle^{-1},\end{aligned}$$

$S^{c}(x,x')$ is Feynman causal GF, and $S^{p}(x,x')$ obeys Dirac Eq.

$|0,in\rangle$ - initial free particle vacuum at $t\to-\infty$;

Vacuum energy change in L-const. EF

$$w = \langle 0, in| T_{00} |0, in\rangle = \frac{1}{2}\langle 0, in| \left[\hat{\psi}(x)^{\dagger}, \hat{H}_{\mathrm{D}}\hat{\psi}(x)\right] |0, in\rangle\Big|_{x=L/2-0} ,$$

In the idealized picture, which we consider here, the external field acts during a large time $T \rightarrow \infty$, beeng time independent within this time interval T, such that one can ignore effects of its switching on and and off. This allows us to believe that time dependence of wave functions is trivial. Considering the one-particle mean energy and electric current through surface $x = const$, we can identify initial and final particle and antiparticle states then to show that $|0, in\rangle$ - initial free particle vacuum at $t \rightarrow -\infty$; $|0, out\rangle$ - final free particle vacuum at $t \rightarrow +\infty$.

Vacuum polarization

The separation of S_{in} into the c- and p-parts is responsible for the separation of w into the two respective summands $w = w^c + w^p$.

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- w^c can be calculated with S^c in constant electric field at $T \to \infty$.
- w^c is expressed in terms of the real-valued part of HEL (at $B = 0$). This contribution is due to vacuum polarization. In a superstrong electric field, $d = 4$, it has the form

$$w^c = E \frac{\partial \,\mathrm{Re}\, \mathcal{L}_{\mathrm{eff}}}{\partial E} - \mathrm{Re}\, \mathcal{L}_{\mathrm{eff}} \approx - \left(\frac{\alpha}{3\pi} \ln \frac{eE}{m^2} \right) \mathcal{L}_{\mathrm{M}}.$$

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- The contribution w^p due to particle-creation, it is nonlocal and is calculated via $S^p(x, x')$.

Back reaction due to vacuum instability

In a T-const. EF, quantum numbers of particles are $n = (\mathbf{p}, r)$, $r = (r_1, \ldots, r_{[d/2]-1})$, $r_j = \pm 1$.

$$S^p(x, x') = i \sum_n {}_-\psi_n(x) \left[G({}_+|^-) G({}_-|^-)^{-1} \right]^\dagger_{nn} {}_+\bar{\psi}_n(x') .$$

$\{{}_\pm\psi_n(x)\}$-in-solutions in T-const. EF, their asymptotics at $t \leq t_1$ are free stationary states of Dirac Hamiltonian with $A_1 = ET/2$. $G\left({}_\pm|^\pm\right)$ (Bogolyubov coefficients) are defined as

$$^\pm\psi(x) = {}_+\psi(x) G\left({}_+|^\pm\right) + {}_-\psi(x) G\left({}_-|^\pm\right) .$$

In particular, diff. mean numbers of electrons created are $\aleph_n = \left|G\left({}_-|^+\right)\right|^2_{nn}$.
$\{^\pm\psi_n(x)\}$-out-solutions in T-const. EF, their asymptotics at $t \geq t_2$ are free stationary states of Dirac Hamiltonian with $A_1 = -ET/2$.

Back reaction due to vacuum instability

For calculating w^p one needs $S^p(x, x')$ at $x \approx x'$, which is:

$$S^p = -i \int d\mathbf{p} \sum_r \aleph_{\mathbf{p},r} \left[{}^+\psi_{\mathbf{p},r}(x)\ {}^+\bar{\psi}_{\mathbf{p},r}(x') - {}^-\psi_{\mathbf{p},r}(x)\ {}^-\bar{\psi}_{\mathbf{p},r}(x') \right] ,$$

then we obtain

$$w^p = \frac{2}{(2\pi)^{d-1}} \int d\mathbf{p} \sum_r \aleph_{\mathbf{p},r} \varepsilon_{\mathbf{p},r} \ .$$

$\varepsilon_{\mathbf{p},r} = \sqrt{m^2 + \mathbf{p}_\perp^2 + (eET/2 - p_1)^2}$ are energies of out-particles in a T-const. EF.

Back reaction due to vacuum instability

In a T-const. EF, differential mean numbers $\aleph_{\mathbf{p},r}$ have the form

$$\aleph_{\mathbf{p},r} = \exp\left(-\pi\frac{m^2+\mathbf{p}_\perp^2}{eE}\right)$$

for $|p_1| \leq \sqrt{eE}\left(\sqrt{eE}T/2 - K\right)$,
K a large arbitrary $\sqrt{eE}T \gg K \gg 1 + m^2/eE$.
$\aleph_{\mathbf{p},r}$ is fast-decreasing out of the interval T.
Calculating w^p (total quantity), we obtain T-leading terms in the T-dependent form

$$w^p = eETn^{cr}, \quad n^{cr} = \frac{N}{V} = J\frac{(eE)^{d/2}T}{(2\pi)^{d-1}}\exp\left(-\pi\frac{m^2}{eE}\right).$$

(Gavrilov, Gitman, Yokomizo (paper in preparation).

EMT of created pairs

In a T-const. EF, at $t > t_2$

$$\begin{aligned} \langle 0, in| T_{11} |0, in\rangle &= \langle 0, in| T_{00} |0, in\rangle = eETn^{cr}, \\ \langle 0, in| T_{22} |0, in\rangle &= \langle 0, in| T_{33} |0, in\rangle = \widetilde{n} \ln \left(\sqrt{|qE|\, T} \right). \end{aligned}$$

Back reaction due to vacuum instability

In a L-const. EF, differential mean numbers $\aleph_{\mathbf{p},r}$ have the same form

$$\aleph_{\mathbf{p},r} = \exp\left(-\pi\frac{m^2+\mathbf{p}_\perp^2}{eE}\right)$$

for $|p_0| \leq \sqrt{eE}\left(\sqrt{eE}L/2 - K\right)$.

$\aleph_{\mathbf{p},r}$ is fast-decreasing out of the interval L.

Thus, in the limit T, $L \to \infty$, $\aleph_{\mathbf{p},r}$ and n^{cr} represents the result for constant uniform field.

Calculating w^p, we obtain L-leading terms in the the L- and T-dependent form

$$w^p = eELn^{cr}\ , \quad n^{cr} = J\frac{(eE)^{d/2}\,T}{(2\pi)^{d-1}}\exp\left(-\pi\frac{m^2}{eE}\right)\ .$$

w^p are different functions of and for T-const. and L-const. EF.

Consistency restrictions on electric field and its duration

Particle-creation is the main reason for the change of matter energy. The smallness of back-reaction in $d = 3 + 1$, $w^p \ll E^2/8\pi$, implies a restriction on dim.less parameter $(eEc/\hbar)\, T^2$:

$$(ceE/\hbar)\, T^2 \ll \frac{\pi^2}{2\alpha} \exp\left(\pi \frac{E_c}{E}\right) \quad \text{for } T - \text{const.EF}$$

or on the electric field strength for a given time T,

$$eE \ll \frac{\pi^2 \hbar}{2\alpha c T^2} \exp\left(\pi \frac{E_c}{E}\right).$$

For L-const. EF

$$eE \ll \frac{\pi^2 \hbar}{2\alpha L T} \exp\left(\pi \frac{E_c}{E}\right).$$

Consistency restrictions on electric field and its duration

Graphene. Example of setting the problem.
The smallness of back-reaction in , $w^p V_{(2)} \ll \frac{E^2}{8\pi} V_{(3)}$, $V_{(3)} = V_{(2)} L_z$ implies

$$\sqrt{eE} \ll \frac{\pi \hbar L_z}{8\alpha L T} .$$

Consistency restrictions at finite temperature

If the initial state is in thermal equilibrium at high temperatures $\theta \gg ecET$, we have a weaker restriction for fermions:

$$eE \ll \frac{\pi}{c}\sqrt{\frac{3\hbar\theta}{\alpha T^3}}\exp\left(\frac{\pi}{2}\frac{E_c}{E}\right);$$

stronger restriction for bosons:

$$T\ln\left(\sqrt{ceE/\hbar}T\right) \ll \frac{\pi^2\hbar}{2J\alpha\theta}\exp\left(\pi\frac{E_c}{E}\right).$$

$J = 1$ for scalar particles, $J = 3$ for vector particles

Initial state as thermal equilibrium

at temperature θ

$$w = \underset{(\sim T^0)}{w^c} + \underset{(\sim T^1)}{w^c_\theta} + \underset{(\sim T^2)}{\tau^p_\theta},$$

w^c_θ - from work of the field on particles from the many-particle initial state,
$\tau^p_\theta = w^p + w^p_\theta$ - energy density of pairs created from the many-particle initial state,

$$w^p_\theta = -\frac{1}{4\pi^3}\int_D d\mathbf{p} \sum_{r=\pm 1} \aleph_{\mathbf{p},r} n_{\mathbf{p},r}(in)\, \varepsilon_{\mathbf{p},r} \ ,$$

where $n_{\mathbf{p},r}(in) = \left[\exp\left(\tilde{\varepsilon}_{\mathbf{p},r}/\theta\right) + 1\right]^{-1}$, $\tilde{\varepsilon}_{\mathbf{p},r}$ is the energy of a free *in*-particle.

Soft parton production by SU(3) chromoelectric field

$$n_{p_\perp}^{gluon} \gg n_{p_\perp}^{quark},$$

then only the energy density of gluons created is important. Total energy density of gluons created at low temperature, $\theta \ll q\sqrt{C_1}T$,

$$w \simeq w^p \lesssim q\sqrt{C_1}T\aleph^{gluon}, \; \aleph^{gluon} = \frac{3Tq^2C_1}{8\pi^3},$$

at high temperature, $\theta \gg q\sqrt{C_1}T$,

$$w \lesssim \frac{3\theta Tq^2C_1}{8\pi^3}\ln\left(q\sqrt{C_1}T^2\right).$$

$C_1 = E^aE^a$ is Casimir invariants for $SU(3)$.

$$1 \ll q\sqrt{C_1}\,T^2,$$

At low temperature, $\theta \ll q\sqrt{C_1}\,T$,

$$q\sqrt{C_1}\,T^2 \ll \frac{\pi^2}{3q^2}\,,$$

at high temperature, $\theta \gg q\sqrt{C_1}\,T$,

$$\theta T \ln\left(q\sqrt{C_1}\,T^2\right) \ll \frac{\pi^2}{3q^2}\,.$$

Conclusion

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- **Consistency restrictions on electric field and its duration for given model depend on space dimension and temperature**

The end

The $\mathbf{p}_\perp$-distribution densities of gluons $n^{gluon}_{\mathbf{p}_\perp}$ and quarks $n^{quark}_{p_\perp}$ produced from vacuum:

$$n^{gluon}_{\mathbf{p}_\perp} = \frac{1}{4\pi^3}\sum_{j=1}^{3} Tq\tilde{E}_{(j)}\tilde{\aleph}^{(j)}_{\mathbf{p}},$$

$$n^{quark}_{p_\perp} = \frac{1}{4\pi^3}\sum_{j=1}^{3} TqE_{(j)}\aleph^{(j)}_{\mathbf{p}},$$

$$\tilde{\aleph}^{(j)}_{\mathbf{p}} = \exp\left(-\frac{\pi\mathbf{p}^2_\perp}{q\tilde{E}_{(j)}}\right), \quad \aleph^{(j)}_{\mathbf{p}} = \exp\left(-\pi\frac{M^2+\mathbf{p}^2_\perp}{qE_{(j)}}\right),$$

where $E_{(j)}$ are the eigenvalues of the matrix iT^aE^a for the fundamental representation of $SU(3)$; $\tilde{E}_{(j)}$ are the positive eigenvalues of the matrix $if^{abc}E^c$ for the adjoint representation of $SU(3)$;

$$\left|E_{(j)}\right| \leq \sqrt{C_1/3} \text{ and } \left|\tilde{E}_{(j)}\right| \leq \sqrt{C_1},$$

$C_1 = E^aE^a$ is Casimir invariants for $SU(3)$. (Gavrilov, Gitman, Tomazelli, NPB, 2008)

Total energy density of gluons created from vacuum:

$$w^p = \sum_{j=1}^{3} w^{p(j)}, \quad w^{p(j)} = \frac{1}{4\pi^3} \int_{D_{(j)}} d\mathbf{p} \aleph_{\mathbf{p}}^{(j)} \varepsilon_{\mathbf{p}}^{(j)} \ ,$$

from many-particle state at finite temperature:

$$\begin{aligned} w &= w^p + w_\theta^p, \quad w_\theta^p = \sum_{j=1}^{3} w_\theta^{(j)}, \\ w_\theta^{(j)} &= \frac{1}{4\pi^3} \int_{D_{(j)}} d\mathbf{p} \aleph_{\mathbf{p}}^{(j)} n_{\mathbf{p}}^{(j)} (in) \, \varepsilon_{\mathbf{p}}^{(j)}, \\ n_{\mathbf{p}}^{(j)} (in) &= \left[\exp \left(\tilde{\varepsilon}_{\mathbf{p}} / \theta \right) - 1 \right]^{-1} \ . \end{aligned}$$

(Gavrilov, Gitman, PRL, 2008)