Negative Casimir entropy between spheres

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[P. Rodriguez–Lopez, PRB 84, 075431 (2011).]

Collaborators

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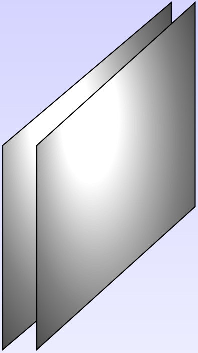
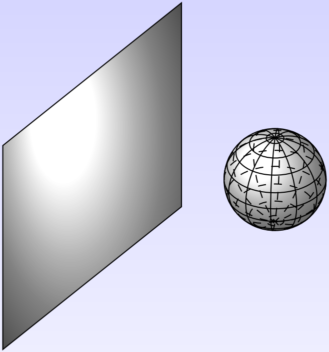
Outline

Introduction



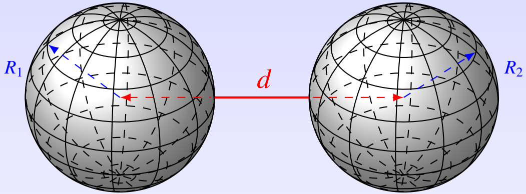
- Negative entropy found in a system of Drude parallel plates [V. B. Bezerra *et. al.* PRA 65, 052113 (2002).] [Gert-Ludwig Ingold *et. al.* PRE 80, 041113 (2009).]



- Also found in perfect metal sphere–plate system. [A. Canaguier-Durand, *et. al.* PRA 82, 012511 (2010).]

2 spheres

- Geometry under consideration:

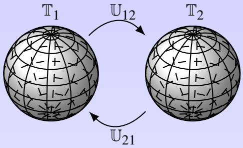


[P. Rodriguez–Lopez, PRB 84, 075431 (2011).]

- $R_1 = R_2 = R$
- Asymptotic limit: $\frac{R}{d} \ll 1$

Multiscattering Formalism

$$\mathcal{F} = k_B T \sum_{n=0}^{\infty}{}' \log |\mathbb{1} - \mathbb{N}(\kappa_n)|$$



$$S = -\partial_T \mathcal{F} \qquad F = -\partial_d \mathcal{F}$$

[T. Emig, *et. al.* PRL 99(17), 170403 (2007).] [S. J. Rahi, *et. al.* PRD 80, 085021 (2009).]

[O. Kenneth and I. Klich. PRB 78(1), 014103 (2008).] [A. Lambrecht, *et. al.* NJP 8(10), 243 (2006).]

1. $\mathbb{N} = \mathbb{T}_1 \mathbb{U}_{12} \mathbb{T}_2 \mathbb{U}_{21}$
2. $\mathbb{T}_i(\kappa) =$ Scattering matrix of each i^{th} object, for spheres $(P = E, M)$:

$$\mathbb{T}^{PP'}_{\ell m, \ell' m'}(\kappa) = -\frac{i_\ell(\kappa R)\partial_R(R i_\ell(\tilde{n}\kappa R)) - \alpha_P \partial_R(R i_\ell(\kappa R)) i_\ell(\tilde{n}\kappa R)}{k_\ell(\kappa R)\partial_R(R i_\ell(\tilde{n}\kappa R)) - \alpha_P \partial_R(R k_\ell(\kappa R)) i_\ell(\tilde{n}\kappa R)} \delta_{\ell\ell'}\delta_{mm'}\delta_{PP'}$$

$$\alpha_E = \epsilon \qquad \alpha_M = \mu \qquad \tilde{n} = \sqrt{\epsilon\mu}$$

3. $\mathbb{U}_{ij}(\kappa) =$ Translation matrix from object i to object j.
4. $\kappa_n = n/\lambda_T =$ Matsubara frequency.

Perfect metal Spheres

- The susceptibilities are:

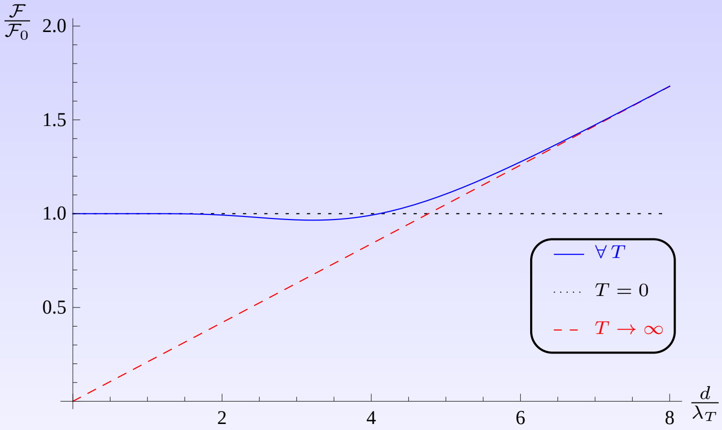
$$\epsilon_i(ic\kappa) \to \infty \qquad \mu_i(ic\kappa) = \mu_0$$

- Dominant contributions of $\mathbb{T}$ matrix in the Far Distance Limit:

$$\mathbb{T}^{MM}_{1m,1m'} = -\frac{q^3}{3}\left(\frac{R}{d}\right)^3 \qquad \mathbb{T}^{EE}_{1m,1m'} = \frac{2q^3}{3}\left(\frac{R}{d}\right)^3$$

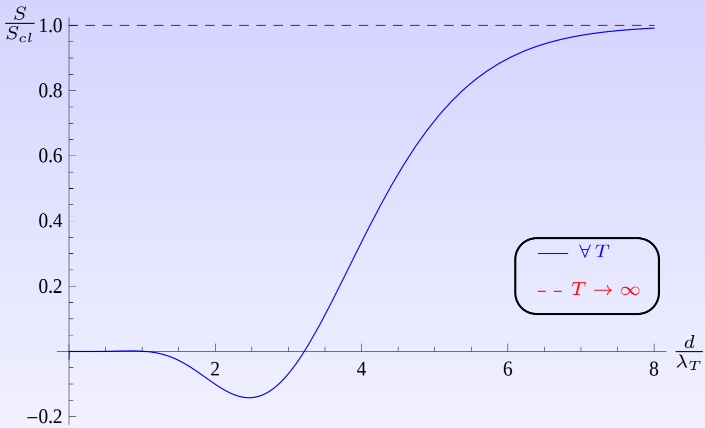
- $q = \kappa\, d$ is the adimensional frequency.
- Same power exponent.
- Different signs.

Perfect metal spheres - Helmholtz Free Energy



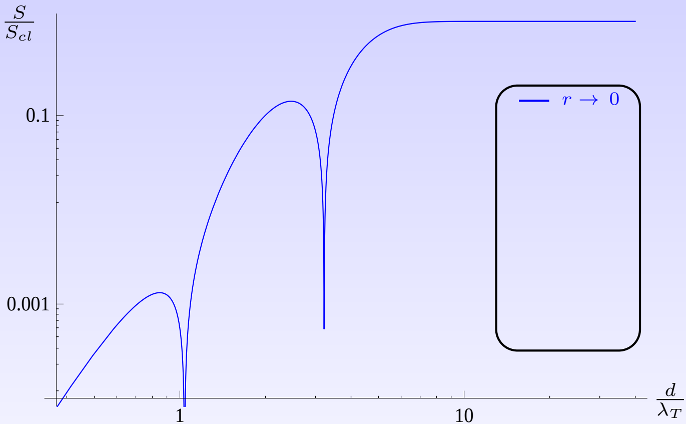
$$\mathcal{F}_0 = -\frac{143\hbar c R^6}{16\pi d^7} \qquad \lambda_T = \frac{\hbar c}{2\pi k_B T}$$

Perfect metal spheres - Entropy



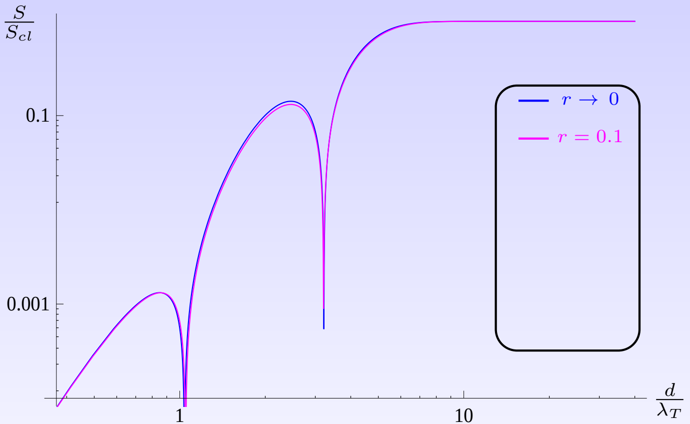
$$S_{cl} = \frac{15 k_B R^6}{4 d^6} \qquad \lambda_T = \frac{\hbar c}{2\pi k_B T}$$

Perfect metal spheres - Entropy



$$r = \frac{R}{d} \qquad \lambda_T = \frac{\hbar c}{2\pi k_B T}$$

Perfect metal spheres - Entropy



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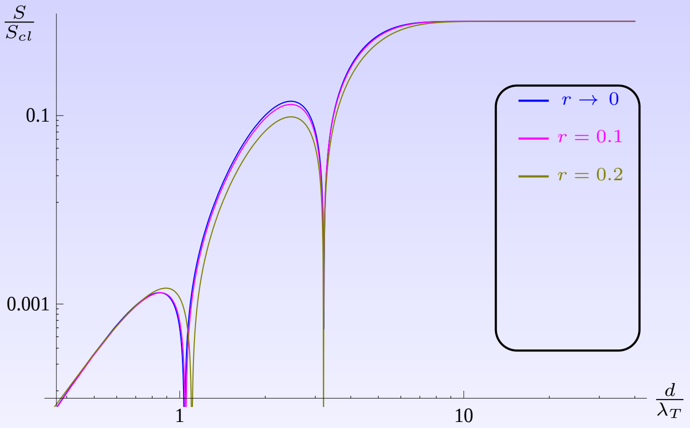

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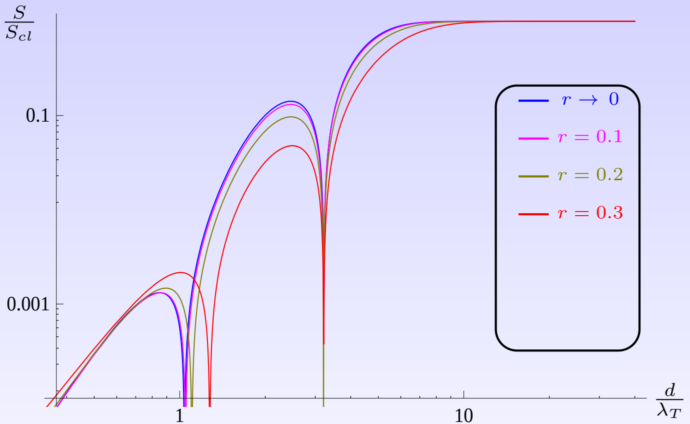

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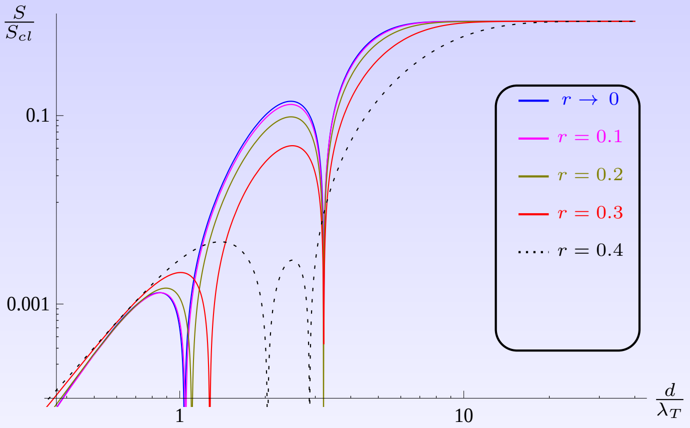
Perfect metal spheres - Entropy



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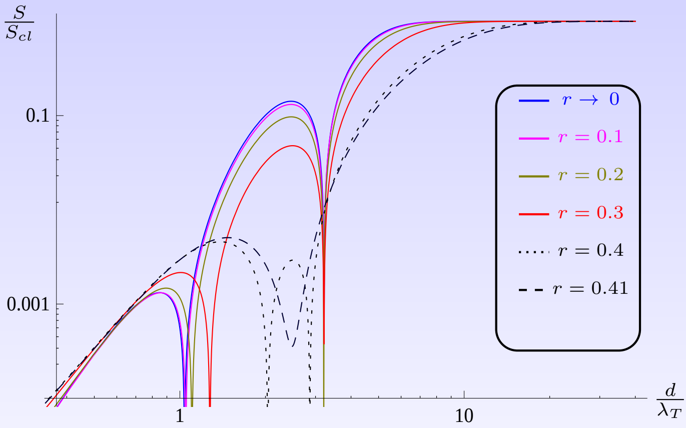
Perfect metal spheres - Entropy



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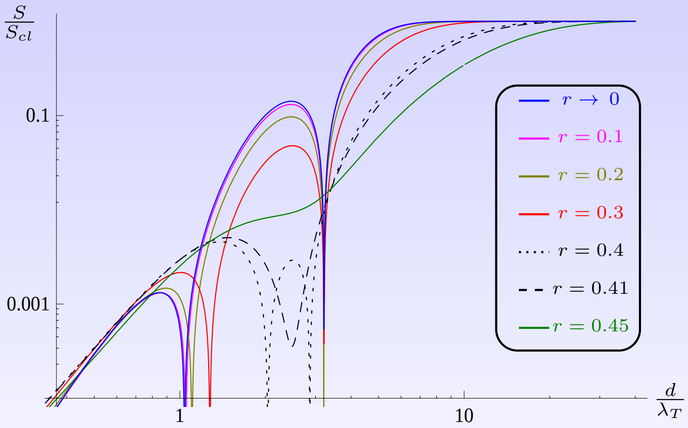
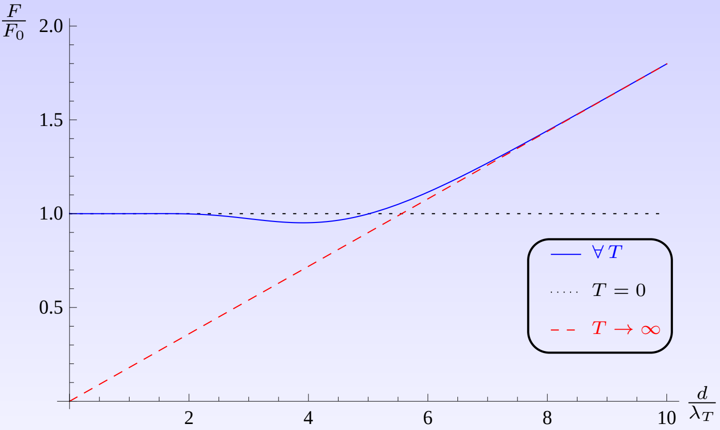
Perfect metal spheres - Entropy



$$r = \frac{R}{d} \qquad \lambda_T = \frac{\hbar c}{2\pi k_B T}$$

Perfect metal spheres - Force



$$F_0 = -\frac{1001\hbar c R^6}{16\pi d^8} \qquad \lambda_T = \frac{\hbar c}{2\pi k_B T}$$

Plasma Model Spheres

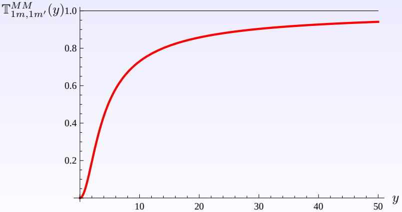
- The susceptibilities are:

$$\epsilon_i(ic\kappa) = \epsilon_0 + \frac{4\pi^2}{\lambda_P \kappa^2} \qquad \mu_i(ic\kappa) = \mu_0$$

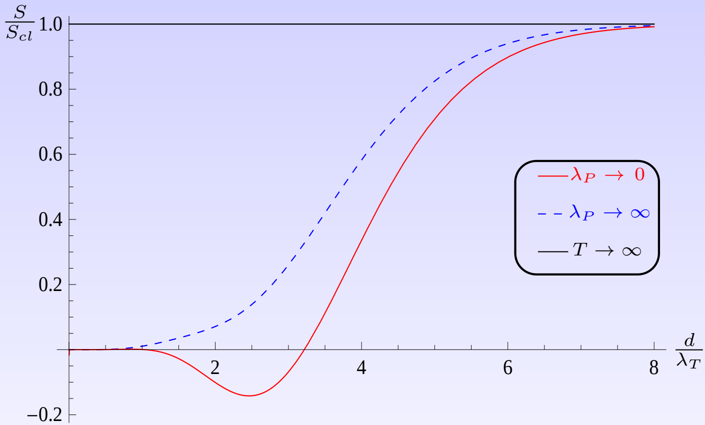
- Dominant contributions of $\mathbb{T}$ matrix in the Far Distance Limit:

$$\mathbb{T}^{MM}_{1m,1m'} = -\frac{q^3}{3}\left(\frac{R}{d}\right)^3 \left(1 - 3y^{-1}\coth(y) + 3y^{-2}\right) \qquad \mathbb{T}^{EE}_{1m,1m'} = \frac{2q^3}{3}\left(\frac{R}{d}\right)^3$$

- $y = 2\pi \frac{R}{\lambda_P}$
- Same power exponent.
- Different signs.



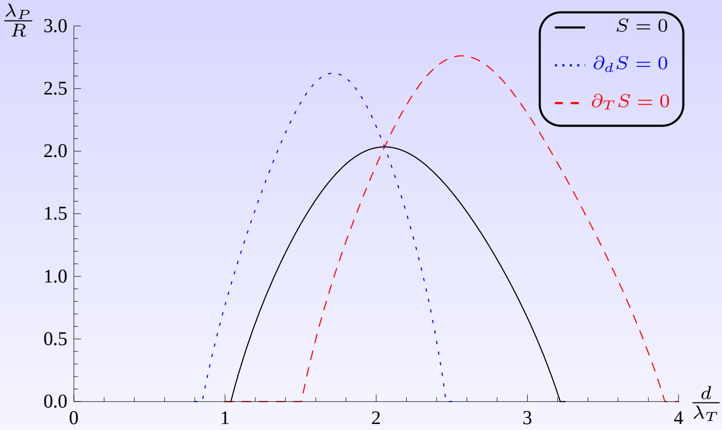
Plasma Model Spheres



$$\lim_{\lambda_P \to 0} \mathbb{T}^{MM}_{1m,1m'} = -\frac{q^3}{3}\left(\frac{R}{d}\right)^3$$

$$\lim_{\lambda_P \to \infty} \mathbb{T}^{MM}_{1m,1m'} = 0$$

Plasma Model Spheres



Drude Model Spheres

- The susceptibilities are:

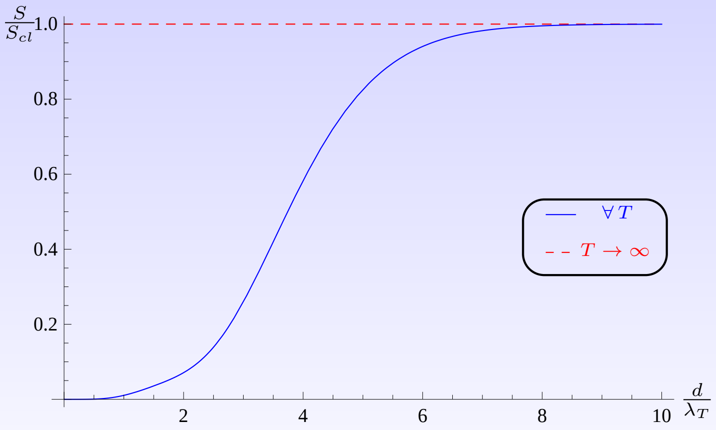
$$\epsilon_i(ic\kappa) = \epsilon_0 + \frac{4\pi^2}{\lambda_P^2\kappa^2 + \frac{\pi c\kappa}{\sigma}} \qquad \mu_i(ic\kappa) = \mu_0$$

- Dominant contributions of $\mathbb{T}$ matrix in the Far Distance Limit:

$$\mathbb{T}^{MM}_{1m,1m'} = -\frac{4\pi q^4}{45}\frac{R\sigma}{c}\left(\frac{R}{d}\right)^4 \qquad \mathbb{T}^{EE}_{1m,1m'} = \frac{2q^3}{3}\left(\frac{R}{d}\right)^3$$

- $\mathbb{T}^{MM}_{1m,1m'}$ subdominant compared with $\mathbb{T}^{EE}_{1m,1m'}$.
- Different signs.
- When $\sigma \to \infty \Rightarrow$ Plasma Model, but it is a singular limit.

Drude Model Spheres

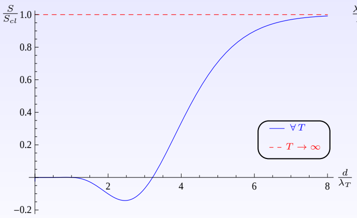


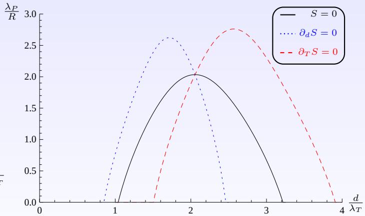
Discussion

1. Why intervals of negative entropy appear?
2. Is $3^{\underline{rd}}$ Law of Thermodynamics verified?

$$\min S(T) = \lim_{T\to 0} S(T)$$

3. Is $2^{\underline{nd}}$ Law of Thermodynamics verified? There exist processes whose dynamics tends to decrease the entropy of the system?





Why intervals of negative entropy appear?

$$\mathbb{U} = \left(\begin{array}{c|c} \mathbb{U}^{EE} & \mathbb{U}^{EM} \\ \hline \mathbb{U}^{ME} & \mathbb{U}^{MM} \end{array} \right) \qquad \mathbb{T} = \left(\begin{array}{c|c} \mathbb{T}^{EE} & \mathbb{O} \\ \hline \mathbb{O} & \mathbb{T}^{MM} \end{array} \right)$$

$$\text{sgn}\left[\mathbb{T}^{EE}\right] = -\text{sgn}\left[\mathbb{T}^{MM}\right]$$

- Then, in the Far Distance Limit

$$\log|\mathbb{1} - \mathbb{N}| \approx -\text{Tr}\left[\mathbb{N}\right] = E_{EE} + E_{EM} + E_{ME} + E_{MM}$$

- with $E_{ij} = -\text{Tr}\left[\mathbb{T}_1^{ii}\mathbb{U}_{12}^{ij}\mathbb{T}_2^{jj}\mathbb{U}_{21}^{ji}\right]$

$$\begin{cases} E_{EE} = -\text{Tr}\left[\mathbb{T}_1^{EE}\mathbb{U}_{12}^{EE}\mathbb{T}_2^{EE}\mathbb{U}_{21}^{EE}\right] < 0 & \Rightarrow \{E_{EE}, E_{MM}\} < 0 \\ E_{EM} = -\text{Tr}\left[\mathbb{T}_1^{EE}\mathbb{U}_{12}^{EM}\mathbb{T}_2^{MM}\mathbb{U}_{21}^{ME}\right] > 0 & \Rightarrow \{E_{EM}, E_{ME}\} > 0 \end{cases}$$

- When $\partial_T|E_{EE} + E_{MM}| < \partial_T|E_{EM} + E_{ME}|$, intervals of $S < 0$ appear.

Is 3rd Law verified?

$$\text{min } S(T) = \lim_{T\to 0} S(T)$$

- Krein Formula

$$\beta\mathcal{F} = -\sum_{n=0}^{\infty}{}' \left[\log\left|\Delta + \kappa_n^2\right| + \sum_{\alpha=1}^{n}\log|\mathbb{T}_\alpha| + \log|\mathbb{1} - \mathbb{N}|\right]$$

- $\mathcal{F}_V = -k_B T \sum_{n=0}^{\infty}{}' \log\left|\Delta + \kappa_n^2\right| \propto V$
- $\mathcal{F}_i = -k_B T \sum_{n=0}^{\infty}{}' \log|\mathbb{T}_i| \propto V_i$
- $\mathcal{F}_C = -k_B T \sum_{n=0}^{\infty}{}' \log|\mathbb{1} - \mathbb{N}| \propto \frac{R^6}{d^7}$

$$\frac{S_C}{S_V} = 0 \Rightarrow \text{min } S(T) = \text{min } S_V(T) = \lim_{T\to 0} S(T) = 0$$

Is $2^{\underline{nd}}$ Law verified?

- In an isolated system, for any process:

$$dS \geq 0$$

  Microcanonical ensemble.
- However, $T = cte \Rightarrow$ Canonical ensemble
- $2^{\underline{nd}}$ Law in Canonical ensemble: $d\mathcal{F} \leq 0$
- S can verify either $dS > 0$ or $dS < 0$, but $d\mathcal{F} \leq 0$.
- The unique consequence of $S < 0$ interval is a nonmonotonicity of the force with T

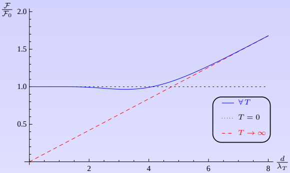
$$\frac{\partial F}{\partial T} = -\frac{\partial^2 \mathcal{F}}{\partial T \partial d} = \frac{\partial S}{\partial d}$$

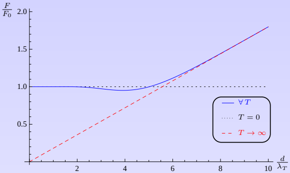
Conclusions

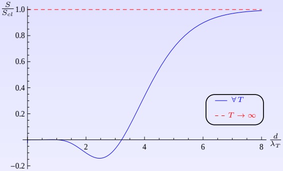
1. An interval of negative entropy appears
   - Perfect Metal spheres
   - Low penetration lenght Plasma model spheres.

   in an interval of distances and temperatures in the far distance limit.
2. This interval appears because of the cross polarization terms (E_{EM}, E_{ME}) of EM Casimir energy. It does not have a scalar analogous.
3. Third Law is verified (Krein formula).
4. Second Law is verified, we study the Canonical ensemble, not the Microcanonical.
5. Negative entropy interval produces nonmonotonicies of the Casimir force with the temperature.

THANK YOU FOR YOUR ATTENTION







$$\frac{\partial F}{\partial T} = -\frac{\partial^2 \mathcal{F}}{\partial T \partial d} = \frac{\partial S}{\partial d}$$