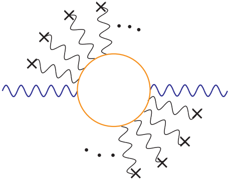
Optical Probes of the Quantum Vacuum:

The Photon Polarization Tensor in External Fields



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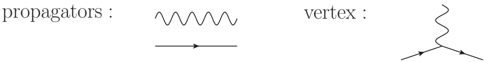
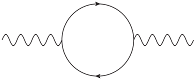
A. Introduction

A first look at the photon polarization tensor

consider quantum electrodynamics (QED):

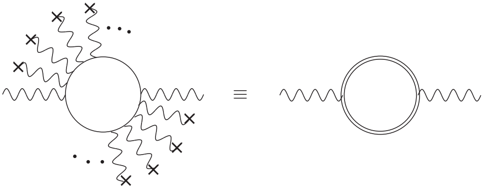


- 1-loop polarization tensor (in the absence of external fields)

$$\Pi^{\mu\nu}(k) \ =$$ 

- in the presence of an external field

  $\rightarrow$ 1-loop polarization tensor



Photon propagation in the quantum vacuum

$\Pi^{\mu\nu}$ is the central input to an effective theory for photon propagation in the quantum vacuum

$$\mathcal{L}_{\rm eff}[A] = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\int_{x'} A_\mu(x)\,\Pi^{\mu\nu}(x,x')\,A_\nu(x')$$

$\uparrow$

vacuum fluctuations

(here A_μ denotes a classical, macroscopic field)

without external fields: $\Pi^{\mu\nu}$ easily evaluated in momentum space

$\leftrightarrow$ in the presence of (constant) external fields: rather involved

- gives rise to modified speeds of light in external fields
- accounts for pair creation effects $\sim$ imaginary part

Our agenda

$\Pi^{\mu\nu}$ for arbitrarily oriented, constant external e.m. fields is

- conveniently evaluated in momentum space,
- known in terms of a double integral expression

  [I. A. Batalin & A.E.Shabad; Sov. Phys. JETP **33**, 483 (1971)]
  [W. Dittrich & H. Gies; Springer Tracts Mod. Phys. **166**, 1 (2000)]
  [C. Schubert; Nucl. Phys. B **585**, 407-428 (2000)]

  within the propertime formalism [J. S. Schwinger; Phys. Rev. **82**, 664 (1951)].

We aim at

- maximum, in particular non-perturbative insights,
- retaining the full momentum dependence.

This is

- important whenever transforming to position space,
- necessary when boundary conditions are set in position space.

B. The photon polarization tensor

The basic structure of the photon polarization tensor

constant magnetic field: metric $(-,+,+,+)$ $\rightarrow$ $k^2 = \mathbf{k}^2 - \omega^2$

[L. F. Urrutia; Phys. Rev. D **17**, 1977 (1978)]

It is convenient to decompose the four-momentum k^μ in components $\parallel$ and $\perp$ to $\mathbf{B} = B\mathbf{e}_1$,

$$k^\mu = k^\mu_\parallel + k^\mu_\perp \,, \qquad k^\mu_\parallel = (k^0, k^1, 0, 0)\,, \qquad k^\mu_\perp = (0, 0, k^2, k^3)\,. \tag{1}$$

Tensors can be decomposed analogously, $g^{\mu\nu} = g^{\mu\nu}_\parallel + g^{\mu\nu}_\perp$.

Then

$$\Pi^{\mu\nu}(k) = \frac{\alpha}{2\pi}\int_0^\infty \frac{\mathrm{d}s}{s}\int_{-1}^{+1}\frac{\mathrm{d}\nu}{2}\left\{\mathrm{e}^{-\mathrm{i}\Phi_0 s}\,\frac{z}{\sin(z)}\left[N_0\left(g^{\mu\nu}k^2 - k^\mu k^\nu\right) + (\tilde{N}_1 - N_0)\left(g^{\mu\nu}_\parallel k^2_\parallel - k^\mu_\parallel k^\nu_\parallel\right)\right.\right.$$
$$\left.\left. + (\tilde{N}_2 - N_0)\left(g^{\mu\nu}_\perp k^2_\perp - k^\mu_\perp k^\nu_\perp\right)\right] + \text{c.t.}\right\}, \tag{2}$$

with $z = eBs$, and

$$\Phi_0 = m^2 - \mathrm{i}\epsilon + \frac{1-\nu^2}{4}k^2_\parallel + \frac{\cos\nu z - \cos z}{2z\sin z}k^2_\perp\,, \tag{3}$$

$$\begin{aligned} N_0 &= \cos\nu z - \nu\sin\nu z\cot z\,,\\ \tilde{N}_1 &= (1-\nu^2)\cos z\,,\\ \tilde{N}_2 &= 2\frac{\cos\nu z - \cos z}{\sin^2 z}\,. \end{aligned} \tag{4}$$

The basic structure of the photon polarization tensor

constant magnetic field: [W. Dittrich & H. Gies; Springer Tracts Mod. Phys. **166**, 1 (2000)]

With projectors onto photon modes polarized $\parallel$ and $\perp$ to the plane spanned by $\mathbf{k}$ and $\mathbf{B}$,

$$P_{\parallel}^{\mu\nu} = g_{\parallel}^{\mu\nu} - \frac{k_{\parallel}^{\mu} k_{\parallel}^{\nu}}{k_{\parallel}^2}, \qquad \text{and} \qquad P_{\perp}^{\mu\nu} = g_{\perp}^{\mu\nu} - \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2}. \tag{5}$$

and a third projector,

$$P_0^{\mu\nu} = g^{\mu\nu} - \frac{k^{\mu} k^{\nu}}{k^2} - P_{\parallel}^{\mu\nu} - P_{\perp}^{\mu\nu}, \tag{6}$$

we obtain

$$\Pi^{\mu\nu}(k) = \Pi_0 P_0^{\mu\nu} + \Pi_{\parallel} P_{\parallel}^{\mu\nu} + \Pi_{\perp} P_{\perp}^{\mu\nu}, \tag{7}$$

with

$$\left\{ \begin{array}{c} \Pi_0 \\ \Pi_{\parallel} \\ \Pi_{\perp} \end{array} \right\} = \frac{\alpha}{2\pi} \int_0^{\infty} \frac{\mathrm{d}s}{s} \int_{-1}^{+1} \frac{\mathrm{d}\nu}{2} \left[\mathrm{e}^{-\mathrm{i}\Phi_0 s} \frac{z}{\sin z} \left(\left\{ \begin{array}{c} N_0 \\ \tilde{N}_1 \\ N_0 \end{array} \right\} k_{\parallel}^2 + \left\{ \begin{array}{c} N_0 \\ N_0 \\ \tilde{N}_2 \end{array} \right\} k_{\perp}^2 \right) + \text{c.t.} \right]. \tag{8}$$

The three projectors span the transverse subspace.

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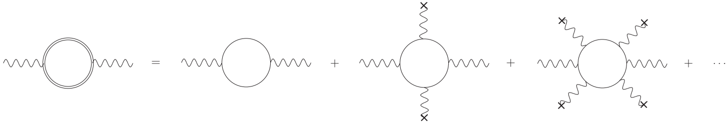
Available insights and limitations

constant magnetic field:

- $\frac{eB}{m^2} \ll 1 \;\leftrightarrow\;$ perturbative expansion in # of field insertions



- $\frac{eB}{m^2}$ arbitrary: limited insights

  [W. y. Tsai & T. Erber; Phys. Rev. D **10**, 492 (1974) & Phys. Rev. D **12**, 1132 (1975)]

  but: "on-the-light-cone" $\leftrightarrow$ $k^2 = 0$, and for $\frac{\mathbf{k}_\perp^2}{eB} \gg 1$ only

- $\frac{eB}{m^2} \gg 1$ & $k^2 > -4m^2 \;\leftrightarrow\;$ restriction to lowest Landau level, i.e., below pair-creation threshold

  [A. E. Shabad; Annals Phys. **90**, 166 (1975) & arXiv:hep-th/0307214]

Towards a special alignment

here we want to elaborate on the latter point:

- we aim at insights beyond the pair-creation threshold, and beyond "on-the-light-cone"
- we claim that the special alignment $\mathbf{k} \parallel \mathbf{B}$ is the simplest case,

$$\Phi_0 = m^2 + \frac{1-\nu^2}{4}\left(\mathbf{k}_\parallel^2 - \omega^2\right) + \frac{\cos \nu z - \cos z}{2z \sin z}\,\mathbf{k}_\perp^2$$

- in this limit, $\Pi^{\mu\nu}$ has the following structure:

$$\Pi^{\mu\nu}(k) \;=\; \Pi_\parallel(k)\, P_\parallel^{\mu\nu} + \Pi_\pm(k)\, \underbrace{\left(P_+^{\mu\nu} + P_-^{\mu\nu}\right)}_{\text{circular polarization } (\pm)}$$

with $\{P_+^{\mu\nu},\, P_-^{\mu\nu}\} \equiv \{P_0^{\mu\nu},\, P_\perp^{\mu\nu}\}$

- even though, the $\parallel$-component is of particular interest

A special alignment

consider $\mathbf{k} \parallel \mathbf{B} \quad \leftrightarrow \quad \mathbf{k}_\perp = 0$:

[R. A. Cover & G. Kalman; Phys. Rev. Lett. **33**, 1113 (1974)]
[W. y. Tsai & T. Erber; Act. Phys. Austr. **45**, 245 (1976)]

- propertime integration can be performed explicitly:

$$\int_0^\infty ds \quad \to \quad \lim_{\tilde{\epsilon}\to 0} \int_{0-\mathrm{i}\tilde{\epsilon}}^{\infty-\mathrm{i}\tilde{\epsilon}} ds \,,$$

  and analytical continuation in eB, consistent with the electric-magnetic duality: $B \leftrightarrow \mathrm{i}E$, and $k_\parallel^\mu \leftrightarrow k_\perp^\mu$

- in the full momentum regime
- focus on the $\parallel$ - component: $\left(\Phi_0 = m^2 - \mathrm{i}\epsilon + \frac{1-\nu^2}{4} k_\parallel^2\right)$

$$\Pi_\parallel(k) = k_\parallel^2 \frac{\alpha}{2\pi} \int_0^1 d\nu \, (1-\nu^2) \left[\ln\left(\frac{m^2}{2eB}\right) - \Psi\left(\frac{\Phi_0}{2eB}\right) - \frac{eB}{\Phi_0} \right].$$

A special alignment

- the Digamma function has an exact series representation,

$$\Psi(\xi) = -\gamma - \frac{1}{\xi} + \sum_{n=1}^{\infty} \frac{\xi}{n(\xi + n)},$$

where γ denotes the Euler-Mascheroni constant

- therewith: $\left(\Phi_0 = m^2 - \mathrm{i}\epsilon + \frac{1-\nu^2}{4} k_\parallel^2\right)$

$$\Pi_\parallel(k) = k_\parallel^2 \frac{\alpha}{2\pi} \int_0^1 \mathrm{d}\nu \, (1 - \nu^2) \left[\frac{eB}{\Phi_0} - \sum_{n=1}^{\infty} \frac{\Phi_0}{n\,(\Phi_0 + 2eBn)} + \gamma \right]$$

$$= k_\parallel^2 \frac{\alpha}{2\pi} \int_0^1 \mathrm{d}\nu \, (1 - \nu^2) \left[\sum_{n=0}^{\infty} \frac{c_n eB}{\Phi_0 + 2eBn} + \gamma - \sum_{n=1}^{\infty} \frac{1}{n} \right],$$

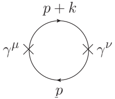
with $c_0 = 1$, $c_{n\in\mathbb{N}} = 2$.

$$\xrightarrow{\frac{eB}{\Phi_0} \gg 1} k_\parallel^2 \frac{\alpha eB}{2\pi} \int_0^1 \mathrm{d}\nu \, \frac{(1 - \nu^2)}{\Phi_0}\,.$$

A special alignment

We briefly outline an alternative way to obtain the result for $\Pi_{\|}$:

- via Landau levels
- in the absence of external fields:



In $D = d + 1$ space-time dimensions, this yields (not yet renormalized):

$$\Pi^{\mu\nu}(k) = \mathrm{i}(\mathrm{i}e)^2 \,\mathrm{tr}\left\{ \int \frac{d^D p}{(2\pi)^D}\, \gamma^\mu \frac{\mathrm{i}}{\not{p} - m - \mathrm{i}\epsilon} \gamma^\nu \frac{\mathrm{i}}{\not{p} + \not{k} - m - \mathrm{i}\epsilon} \right\}$$

$$= \left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) \frac{\alpha D}{2} \frac{\Gamma\left(\frac{4-D}{2}\right)}{(4\pi)^{\frac{D-2}{2}}} \int_0^1 d\nu\, (1-\nu^2) \left[\frac{1}{m^2 - \mathrm{i}\epsilon + k^2 \frac{1-\nu^2}{4}}\right]^{\frac{4-D}{2}} . \quad (1)$$

A special alignment

turning on a magnetic field $\mathbf{k} \parallel \mathbf{B}$:

We rewrite

$$\int \frac{d^4p}{(2\pi)^4} = \int \frac{dp^0 dp_x}{(2\pi)^2} \int \frac{dp_y dp_z}{(2\pi)^2} = \int \frac{d^2p_\parallel}{(2\pi)^2} \int \frac{dp_\perp^2}{4\pi} \,. \tag{2}$$

In a magnetic field, we encounter Landau level quantization, implying

$$p_\perp^2 = 2eBn, \quad \text{with } n \in \mathbb{N}_0. \tag{3}$$

Accordingly,

$$\int \frac{d^4p}{(2\pi)^4} \quad \to \quad \frac{eB}{2\pi} \sum_{n=0}^{\infty} c_n \int \frac{d^2p_\parallel}{(2\pi)^2} \,, \tag{4}$$

Focusing on the $\parallel$-component,

$$m^2 \quad \to \quad m_n^2 \equiv m^2 + 2eBn. \tag{5}$$

The integral to be performed is in $D = 2$ dimensions.

$\to$ after renormalization:

$$\Pi_\parallel(k) = k_\parallel^2 \frac{\alpha}{2\pi} \int_0^1 d\nu \, (1-\nu^2) \left[\ln\left(\frac{m^2}{2eB}\right) - \Psi\left(\frac{\Phi_0}{2eB}\right) - \frac{eB}{\Phi_0} \right] .$$

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Back to the general situation

now, we have:

- identified the correct propertime integration contour,
- non-perturbative insights in the full momentum regime by means of "large z" expansion $\quad \leftrightarrow \quad \frac{eB}{\Phi_0} \gg 1$

the result is:

- only $\parallel$ - component does not vanish in this limit,

$$\Pi_0 \approx 0, \qquad \Pi_\perp \approx 0,$$

$$\Pi_\parallel \approx \mathrm{e}^{-\frac{\mathbf{k}_\perp^2}{2eB}} \, k_\parallel^2 \, \frac{\alpha eB}{2\pi} \int_0^1 \mathrm{d}\nu \, \frac{1-\nu^2}{m^2 - \mathrm{i}\epsilon + k_\parallel^2 \frac{1-\nu^2}{4}}$$

- allows for a Fourier transformation

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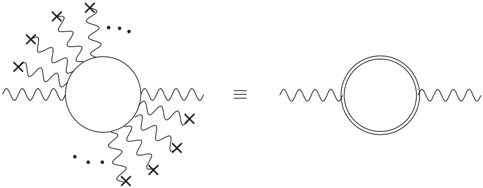
C. An exemplary application

Beyond QED - Minicharges

- the photon polarization tensor accounts for the vacuum fluctuations of the underlying theory
- perhaps there are so far undetected particles (e.g., fermions) around - who knows? $\leftrightarrow$ beyond QED / standard model $\rightarrow$ Minicharges: tiny coupling ϵe, mass m_ϵ
- if they are there, they contribute to the polarization tensor

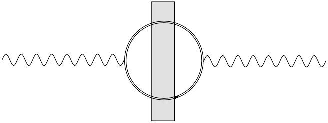


question: how can these effects be separated/$\rightarrow$ detected?

Light-shining-through-walls

answer: by shining light through walls! $\rightarrow$ there are experiments(!)

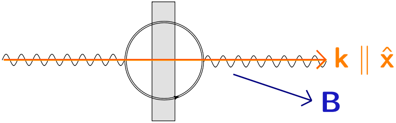
- basic idea: "virtual tunneling" or "tunneling of the 3rd kind"



- problem formulated in position space
  $\rightarrow$ the wall imposes a boundary condition
- without magnetic field [H. Gies & J. Jaeckel; JHEP 0908:063 (2009)]
- in the presence of an external magnetic field
  $\rightarrow$ need full momentum dependence
  $\rightarrow$ if m_ϵ is tiny as well, we are in the non-perturbative regime

The particular scenario

Consider the following scenario:



theoretical treatment (very schematic):

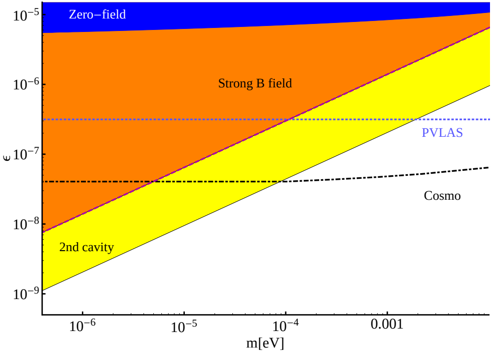
- partial Fourier transformation
- $$(\omega^2 + \partial_x^2)\, A_{\parallel}(x,\omega) \;= \underbrace{\int \mathrm{d}x'\, \Pi(x - x',\omega)\, A_{\parallel}(x',\omega)}_{=:j(x,\omega)}$$
- $$j(x>0,\omega) = \int_{-\infty}^{0} \mathrm{d}x''\, \Pi(x - x'',\omega)\, a(\omega)\, \sin(\omega x'')$$
  with incident photon amplitude $a(\omega)$.
- transition probability: $P_{\gamma\to\gamma} = \lim_{x\to\infty} \left| \frac{A_{\mathrm{out}}(x,\omega)}{a(\omega)} \right|^2$

The particular scenario

We find: $P^{(strong)}_{\gamma\to\gamma} \simeq \alpha^2 \frac{\epsilon^4}{36\pi^2}\left(\frac{\epsilon e B \cos^2\theta}{m^2}\right)^2 e^{-\frac{\omega^2 \tan^2\theta}{\epsilon e B}}$.

With parameters $B = 5\mathrm{T}$, $\omega = 532\mathrm{nm}$, $n_{\mathrm{in}} = 10^{25}$, $\mathcal{N} = 10^5$,

and using $n_{\mathrm{out}} = \mathcal{N}\ n_{\mathrm{in}}\ P_{\gamma\to\gamma}$, we obtain:



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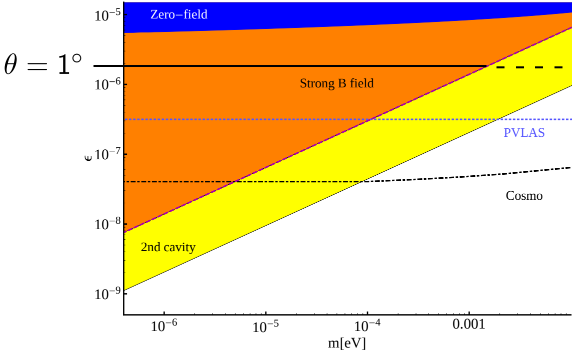

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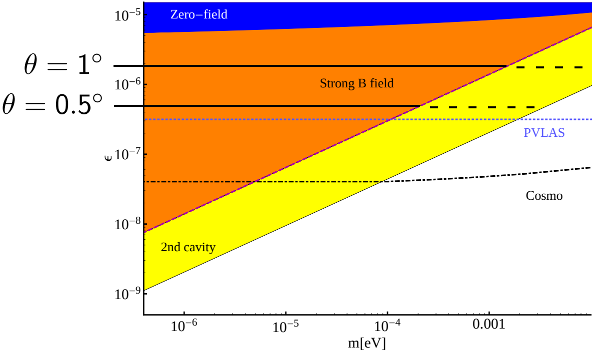

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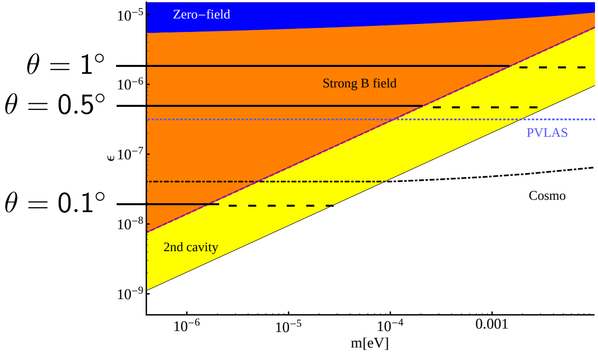

D. Conclusions & Outlook

Conclusions & Outlook

The photon polarization tensor is the central quantity, when aiming at investigating & understanding vacuum polarization effects in intense fields.

We have obtained:

- non-perturbative insights,
- retaining the full momentum dependence,
- i.e., in particular beyond the pair-creation threshold,

in case of a constant external magnetic field.

This is

- important whenever transforming to position space,
- necessary when boundary conditions are set in position space.

It hopefully will be published soon.

Thank you for your attention!