QUANTUM FIELD THEORY in GRAPHENE

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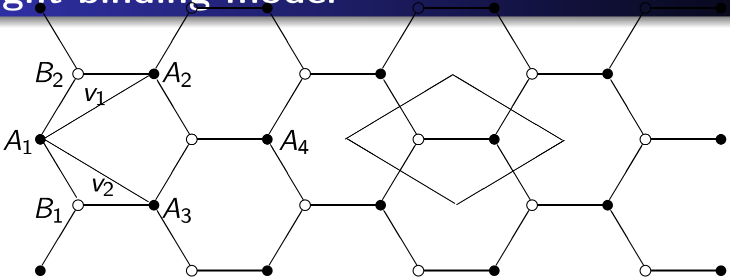
The Dirac Model

The Dirac model for graphene was proposed in 1984 by G. Semenoff and by DiVincenzo and Mele — 20 years before the experimental discovery of graphene by Geim, Novoselov and others!
The Dirac model can be derived from the Tight binding model.
Graphene is a one-atom-thick (planar) system with a hexagonal lattice:

Tight binding model



$$H = -t \sum_{\alpha \in A} \sum_{i=1}^{3} (a^\dagger(\mathbf{r}_\alpha) b(\mathbf{r}_\alpha + \mathbf{u}_i) + b^\dagger(\mathbf{r}_\alpha + \mathbf{u}_i) a(\mathbf{r}_\alpha)),$$

Eigenvectors (numbered by the momentum $\mathbf{k}$)

$$|\psi\rangle = C_A \sum_{\alpha \in A} e^{i\mathbf{k}\mathbf{r}_\alpha} a^\dagger(\mathbf{r}_\alpha)|0\rangle + C_B \sum_{\alpha \in B} e^{i\mathbf{k}\mathbf{r}_\alpha} b^\dagger(\mathbf{r}_\alpha)|0\rangle.$$

Eigenvalues:

$$E = \pm t \sqrt{1 + 4\cos(\frac{\sqrt{3}}{2}k_y d)[\cos(\frac{3}{2}k_x d) + \cos(\frac{\sqrt{3}}{2}k_y d)]}.$$

(with d being the lattice spacing).
Spectrum: two surfaces, $E > 0$ and $E < 0$, which touch each other at 6 Fermi points where $E = 0$. Among these 6 points only two are inequivalent:

$$K_\pm = \big(0, \pm 4\pi/(3\sqrt{3}d)\big)$$

Next: take each one of these Fermi points, expand in momenta in the limit $d \to 0$:

$$H_{\pm} = \frac{3}{2} td \begin{pmatrix} 0 & iq_x \pm q_y \\ -iq_x \pm q_y & 0 \end{pmatrix} = v_F(-q_x\sigma_2 \pm q_y\sigma_1),$$

where $v_F \simeq 1/300$ is the Fermi velocity. Summing up two Fermi point contributions:

$$H = -iv_F(\gamma^x\partial_x + \gamma^y\partial_y), \;\; \gamma^x = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, \;\; \gamma^y = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}.$$

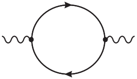
This is the Dirac Hamiltonian. Taking into account spin variables makes the spinors 8-component.

Generalizations:
– Add an electromagnetic field by $\partial \to \partial + ieA$, that can be (i) an external electromagnetic radiation, (b) an external magnetic field, (ii) a fluctuating electromagnetic field.
– Add a temperature.
– Add a chemical potential and a mass.
– Impurities.

The Dirac model is expected to be valid up to the energies $\sim$ 2eV. "Characteristic" energies of (most) current experiments are of order of fractions of eV (or less).

Polarization Tensor

QFT with planar fermions:
in 1980's: Appelquist, Chodos, Semenoff, Niemi, Reddlich, Jackiw, Deser, ...
in XXI Century: Gusynin, Sharapov, Miransky, Gorbar, Shovkovy, Pyatkovskiy, Khveshchenko,....
The most relevant quantity one can calculate here by the QFT methods is the polarization tensor Π defined through the effective action for planar fermions in the presence of an external magnetic field:

$S_{\mathrm{eff}}(A) = A$  A

$$= \frac{1}{2}\int \frac{d^3p}{(2\pi)^3} A_j(p)\Pi^{jl}(p)A_l(p).$$

Example: two-component massive fermions.

$$\Pi^{mn} = \frac{\alpha}{v_F^2}\,\eta^m_j \left[\Psi(\tilde{p})\left(g^{jl} - \frac{\tilde{p}^j\tilde{p}^l}{\tilde{p}^2}\right) + i\phi(\tilde{p})\epsilon^{jkl}\tilde{p}_k\right]\eta^n_l$$

where $\eta = \mathrm{diag}\,(1, v_F, v_F)$, $\tilde{p}^m \equiv \eta^m_n p^n$.

Suppose, the graphene sample is flat, occupying the plane $x^3 = a$. The effective equations of motion for the electromagnetic field read

$$\partial_\mu F^{\mu\nu} + \delta(x^3 - a)\Pi^{\nu\rho} A_\rho = 0,$$

which is equivalent to a free propagation of light outside the surface and the matching conditions on the surface

$$\begin{aligned} A_\mu|_{x^3=a+0} &= A_\mu|_{x^3=a-0}, \\ (\partial_3 A_\mu)|_{x^3=a+0} - (\partial_3 A_\mu)|_{x^3=a-0} &= \Pi_\mu^{\ \nu} A_\nu|_{x^3=a}. \end{aligned}$$

(Here $\Pi^{3\mu} = 0$)

Results: Quantum Hall Effect

External conditions: a strong magnetic field ($1 - 10$ Tesla) perpendicular to the surface of graphene; varying chemical potential μ controlled by the gate potential; zero temperature.

Quantity of interest: zero-frequency off-diagonal real part of the polarization tensor (=dc Hall conductivity).

Calculations: Beneventano, Santangelo, ...

Big success of the Dirac model: prediction/explanation of the anomalous Hall conductivity in graphene:

$$\sigma_{xy} \sim (n + \frac{1}{2})$$

(that is observed on experiment).

Still to be done:
– role of the phase of quantum determinant;
– more sophisticated external conditions.

Results: Optical absorption

Setup: absorption of light by suspended graphene, no magnetic field, arbitrary temperature, arbitrary, but small mass in and μ.
Quantity of interest: imaginary part of the diagonal polarization tensor at non-zero frequencies.
Theory: universal absorption rate of about 2% – Enormous!
Experiment: Nair et al (2008); Kuzmenko et al (2008) - wonderful confirmation.

Results: The Faraday Effect

Setup: Polarization rotation of EM radiation passing through graphene in a strong external magnetic field perpendicular to the surface of graphene.

On the Dirac model side: polarization tensor for non-zero frequencies, external magnetic field, and impurities (!). Fortunately, at zero temperature.

General theoretical discussion: Volkov & Mikhailov (1985); Fialkovsky & D.V. (2009) ...

Experiment: Kuzmenko (2010): Giant Faraday rotation in graphene. (up to 0,1 rad!).

IF & DV (2011): Dirac model is in an agreement with the experiment. It also predicts other peaks at different frequencies. The Faraday effect instrumental for measuring parameters of the Dirac model.

Results: The Casimir Effect

Setup: Graphene layer parallel to another graphene/metal/dielectric. No magnetic field, but other parameters are variable.
Dirac model – gives the polarization tensor which defines reflection coefficients to be substituted in the Lifshitz formula.
Most spectacular prediction: temperature enhancement of the Casimir interaction (Fialkovsky, Marachevsky and DV (2011); in agreement with Gomez-Santos (2009)).
Experiment: NO EXPERIMENT.

Conclusions

The Dirac model of graphene

- has solid theoretical grounds
- confirmed by experiments whenever tested
- deserves more attention from both theoretical and experimental sides