The Casimir effect in the sphere-plane geometry

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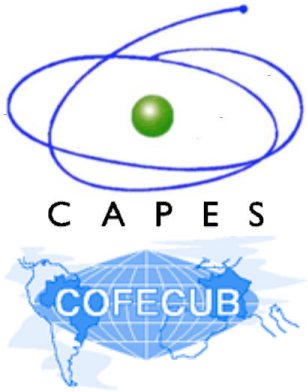

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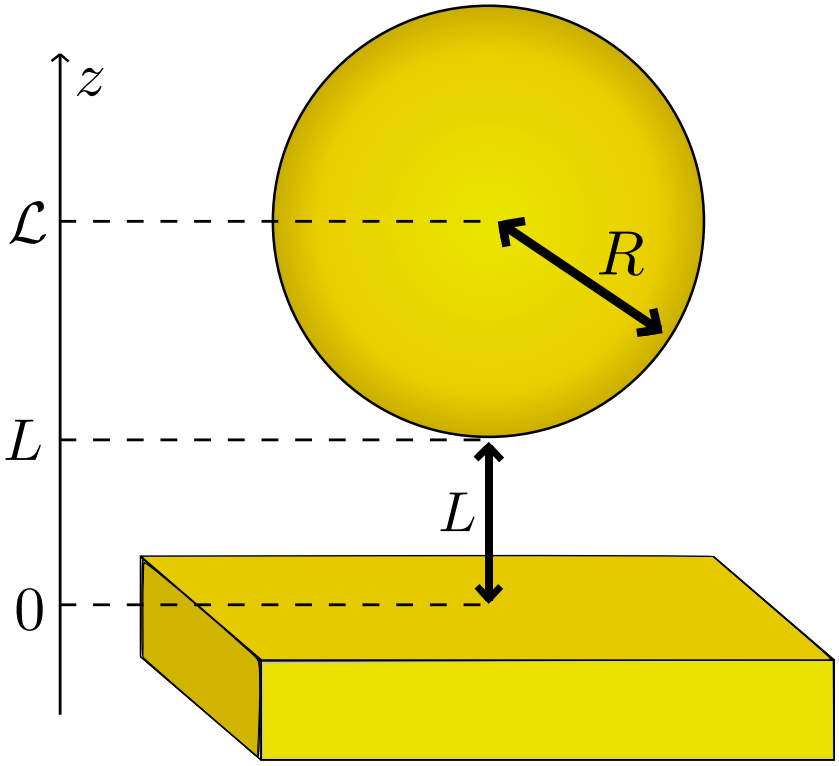
Physique quantique et applications

Introduction

Motivation: study the non-trivial dependence of the Casimir interaction on the geometry.

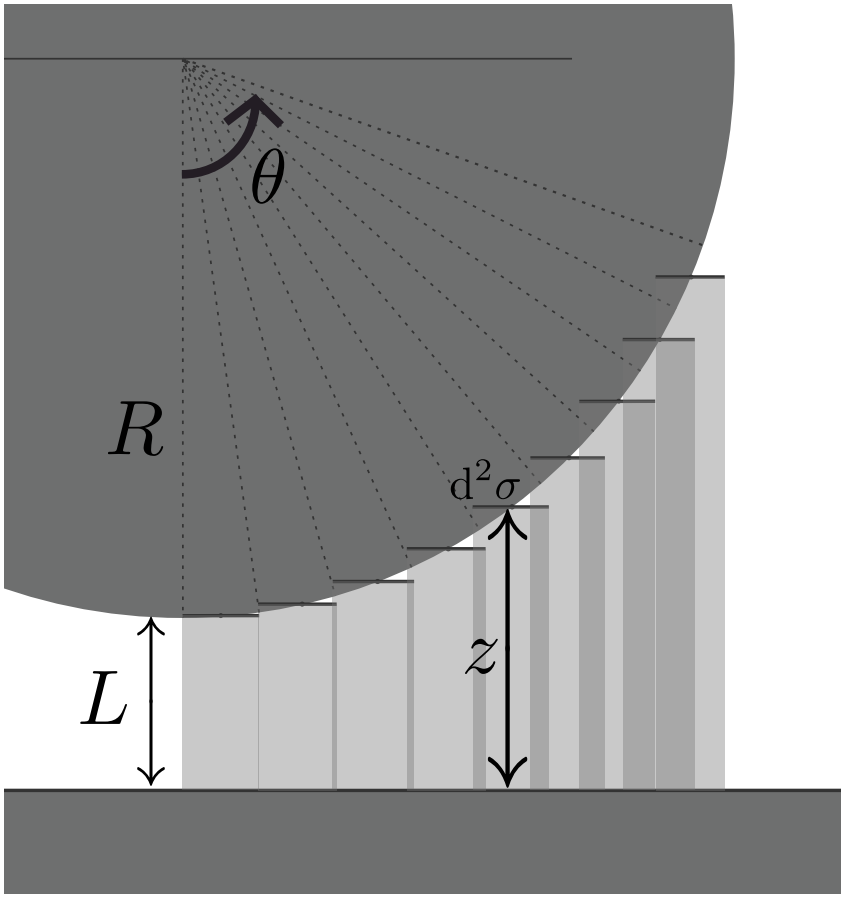
L: distance of closest approach.
R: radius of the sphere.

$\mathcal{L} = L + R$:
center-to-plate distance.
$\frac{L}{R}$: aspect ratio.



$\Rightarrow$ interesting geometry with curvature, non-specular reflection, coupling of polarizations, finite size ($\neq$ plane-plane).

The Proximity Force Approximation (PFA)



$$F_{\mathrm{PFA}} = 2\pi R \frac{E_{\mathrm{PP}}}{A}$$

$$G_{\mathrm{PFA}} = 2\pi R \frac{F_{\mathrm{PP}}}{A}$$

Domain of validity is ($L \ll R$) but the error is uncotrolled for $\frac{L}{R} > 0$.

$$\rho_E = \frac{E}{E_{\mathrm{PFA}}}$$
$$= 1 + \beta_E \frac{L}{R} + \gamma_E \left(\frac{L}{R}\right)^2 + \cdots$$

- Estimation of the linear correction terms β_E, β_F and β_G (experimental prescription[1] $|\beta_G| < 0.4$).
- PFA uncouples geometry from other effects.

[1] D.E. Krause, R.S. Decca, D. Lopez and E. Fishbach, *PRL* **8**, 243 (2007)

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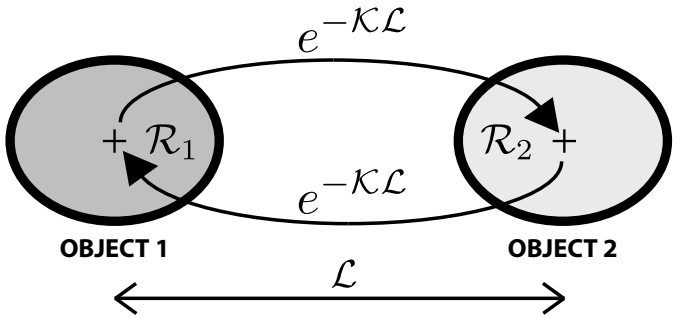
The scattering formula

$\mathcal{R}$: Scattering operator $\qquad |\,\mathrm{out}\rangle = \mathcal{R}\,|\,\mathrm{in}\rangle$

Round-trip operator ($\omega = \imath\xi$):

$$\mathcal{M}(\imath\xi) = \mathcal{R}_1(\imath\xi)e^{-\mathcal{K}\mathcal{L}}\mathcal{R}_2(\imath\xi)e^{-\mathcal{K}\mathcal{L}}$$



Scattering formula[2] at $T = 0$ and $T > 0$:

$$\boxed{E = \frac{\hbar}{2\pi}\int_0^\infty d\xi\,\ln\det\left[\mathcal{I} - \mathcal{M}(\imath\xi)\right]}$$

$$\boxed{\mathcal{F} = k_B T {\sum_n}' \ln\det\left[\mathcal{I} - \mathcal{M}(\imath\xi_n)\right]} \quad \text{with} \quad \xi_n = n\frac{2\pi k_B T}{\hbar}$$

All information on geometry and optical properties is encoded in $\mathcal{M}(\imath\xi)$.

[2] A. Lambrecht, P.A. Maia Neto, and S. Reynaud, New J. Phys. **8**, 243 (2006).

Multipolar expansion in the sphere-plane geometry

$$\begin{cases} \mathcal{F} = k_B T \sum_n{}' \ln\det\left(\mathcal{I} - \mathcal{M}(\imath\xi_n)\right) \quad ; \quad F = -\dfrac{\partial \mathcal{F}}{\partial L} \quad ; \quad G = \dfrac{\partial F}{\partial L} \\ \mathcal{M} = \mathcal{R}_S e^{-\mathcal{K}\mathcal{L}} \mathcal{R}_P e^{-\mathcal{K}\mathcal{L}} \quad ; \quad \xi_n = 2\pi n k_B T/\hbar \end{cases}$$

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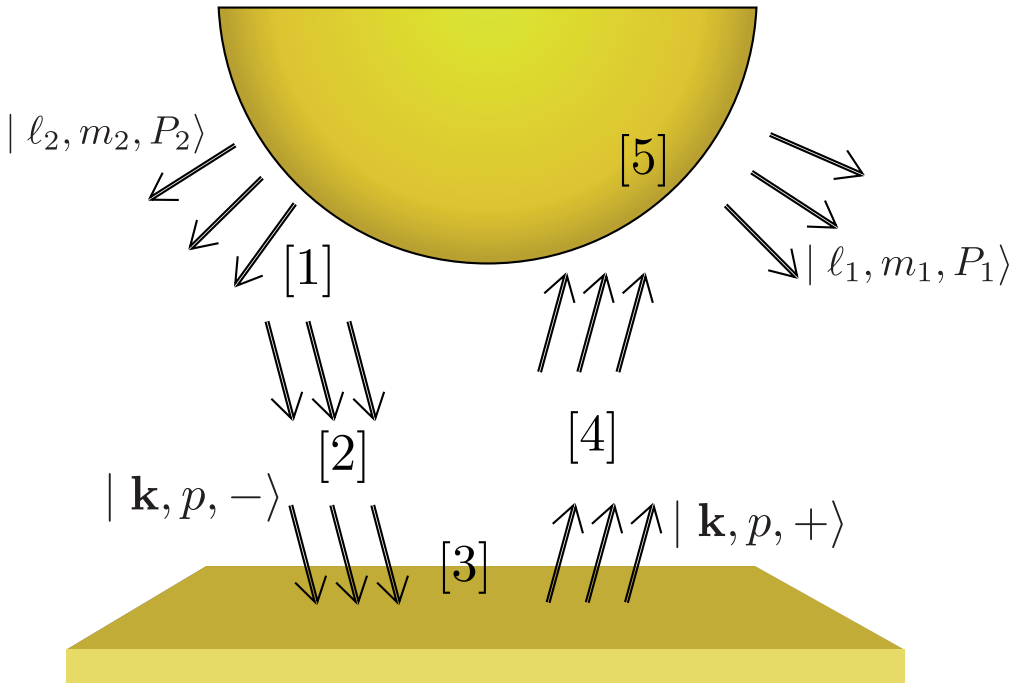


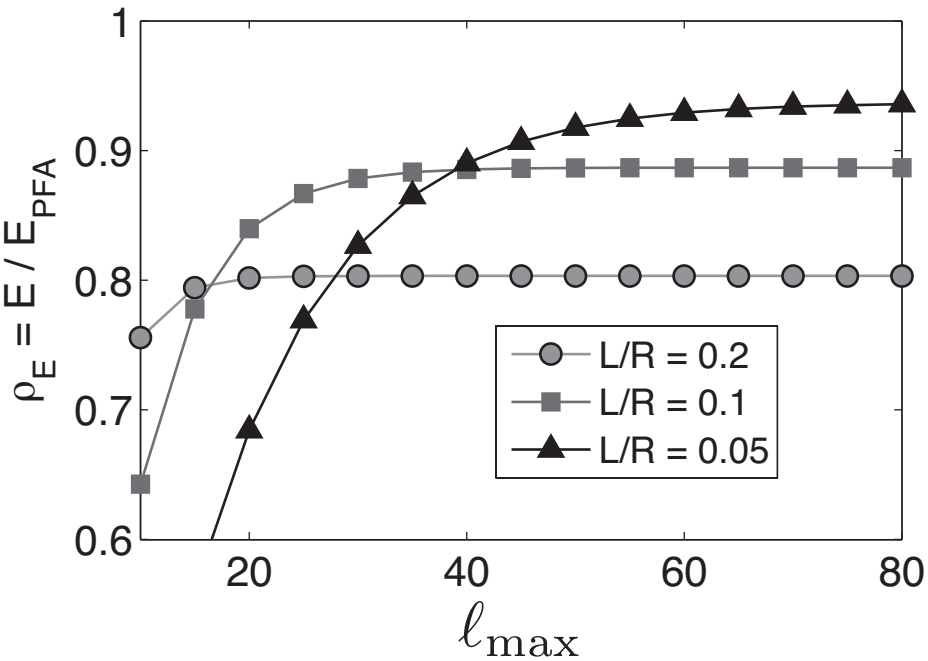
[1]: projection from spherical to planar modes.
[2], [4]: translations.
[3]: Fresnel coefficients.
[5]: Mie coefficients.

$$\mathcal{M}_{1;2} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_{p=TE,TM} \underbrace{\langle l_1 m_1 P_1|\ \overset{[5]}{\mathcal{R}_S}\ |\mathbf{k},+,p\rangle\ e^{\overset{[4]}{-\kappa\mathcal{L}}}\ \overset{[3]}{r_p}(k)\ e^{\overset{[2]}{-\kappa\mathcal{L}}}\ \langle \mathbf{k},-,p\ \overset{[1]}{|}\ l_2 m_2 P_2\rangle}_{\text{cavity operator}}$$

$\ell_{\max}$ cut-off

But numerics need $dim(\mathcal{M}) < \infty$!
$\Rightarrow$ truncation in the spherical modes at $|m| \leq \ell \leq \ell_{\max}$

- smaller $L/R \Rightarrow$ more modes ($\ell_{\max} \gtrsim \frac{4}{L/R}$ needed).
- for a given $\ell_{\max}$, numerical results are accurate for large enough $\frac{L}{R}$.



$\Rightarrow$ Method well-adapted for $\frac{L}{R} \gtrsim 1$ ($\rightarrow$nanospheres[3]).
$\Rightarrow$ High truncation $\ell_{\max}$ allows for low values of $\frac{L}{R}$.

[3] A. Canaguier-Durand, A. Gérardin, R. Guérout, P. A. Maia Neto, V. Nesvizhevsky, Alexei Yu. Voronin, A. Lambrecht, S. Reynaud, *Phys. Rev. A* **83**, 032508 (2011).

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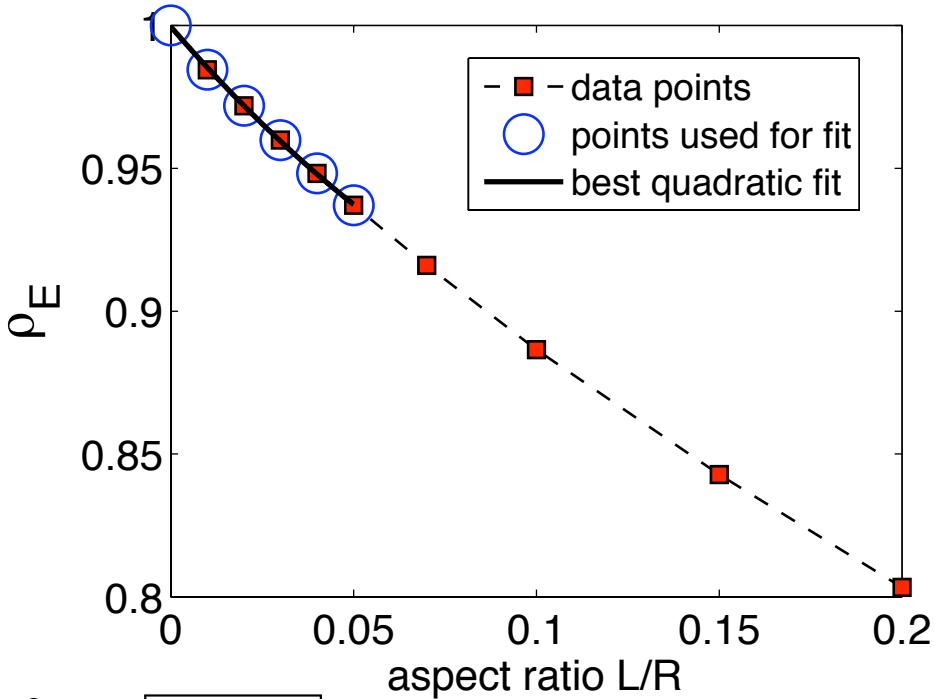
Comparison with the Proximity Force Approximation

Estimation of β:

$$\rho_E = \frac{E}{E_{\mathrm{PFA}}} = 1 + \beta_E \frac{L}{R} + \gamma_E \left(\frac{L}{R}\right)^2 + \cdots$$

$\Rightarrow$ extrapolation to small $\frac{L}{R}$ in order to get the β_E.



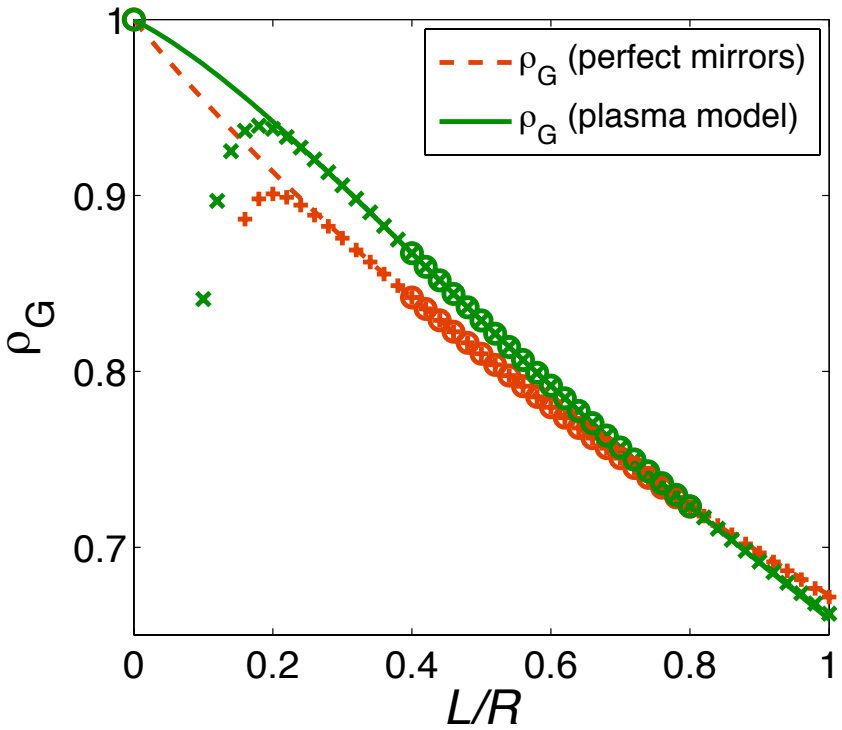
From the fit, $\beta_E \simeq -1.47$ and $\beta_G = \frac{\beta_E}{3} = \boxed{-0.49}$.

- Differs from scalar computations (8 times bigger) $\Rightarrow$ coupling of polarizations.
- In contradiction with experimental $|\beta_G| < 0.4$ (Krause et al. 2007).
- In good agreement with other teams[4].

[4] T. Emig, *J. Stat. Mech.: Theory Exp.*, **P**04007 (2008).

Influence of the material on the accuracy of PFA

Same procedure with plasma model[5] for the mirrors $\rho_G = \frac{G^{\mathrm{plas}}}{G^{\mathrm{plas}}_{\mathrm{PFA}}}$:



- $R = 100$ nm ; $\lambda_P = 136$ nm.
- Accuracy of PFA is affected by imperfect reflection.
- Bending of the curve for ρ_G.
- $\beta_G^{\mathrm{plas}} \simeq -0.2$ back inside the experimental bound.

$\Rightarrow$ Correlation between the effects of geometry and finite conductivity.

[5] A. Canaguier-Durand, P. A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud, *Phys. Rev. Lett.* **102**, 230404 (2009).

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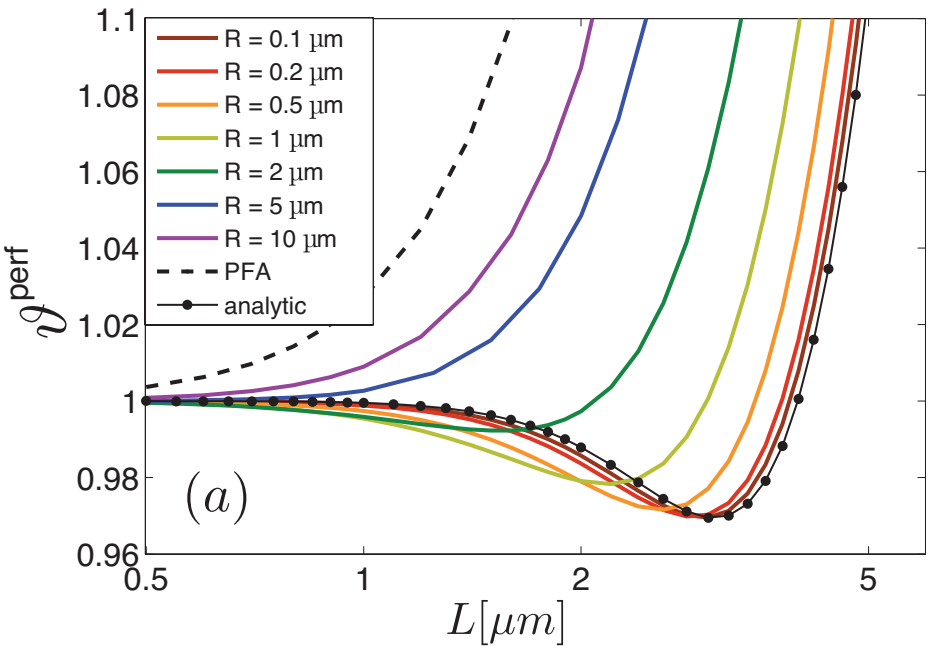
Temperature and geometry (1/2)

$(L, R, \lambda_T = \frac{\hbar c}{k_B T})$: inclusion of $T = 300$ K through the Matsubara sum.

Change in the Casimir force due to the temperature[6] $\vartheta = \frac{F(T)}{F(0)}$:



- $\vartheta < \vartheta_{\mathrm{PFA}}$: lower thermal increase.
- depends on R.
- convergence to analytical result ($R \ll L$).
- $\vartheta_F < 1$: repulsive contribution of thermal photons to the force.

$\Rightarrow$ Correlations between the effects of geometry and temperature (already observed for scalar fields[7]).

[6] A. Canaguier-Durand, P. A. Maia Neto, A. Lambrecht, S. Reynaud, *Phys. Rev. Lett.* **104**, 040403 (2010).

[7] K. Klingmüller and H. Gies, *J. Phys. A* **41** 164042 (2008).

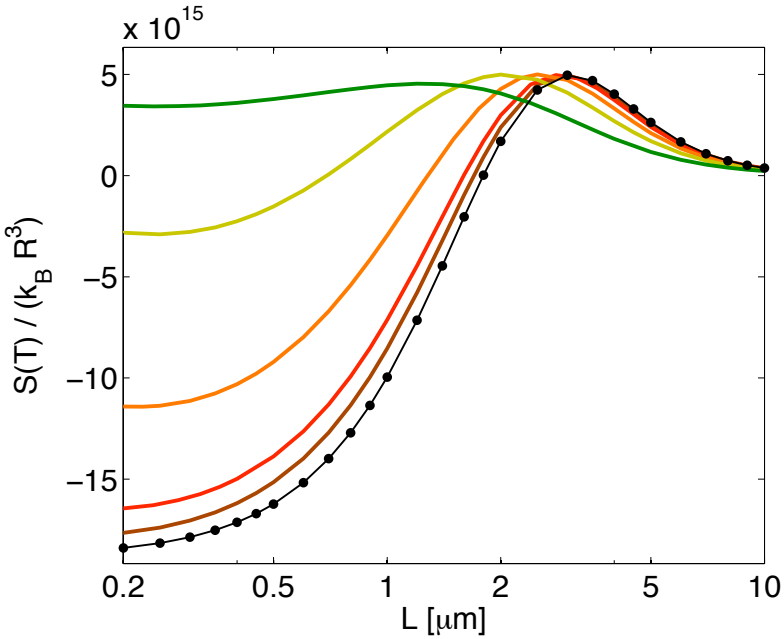
Temperature and geometry (2/2)

- $(L \gg R)$-limit can computed analytically for any temperature:

$$\boxed{\mathcal{F}^{perf} \simeq -\frac{3\hbar c R^3}{4\lambda_T \mathcal{L}^3}\phi(\nu)} \quad \text{with} \quad \left\{ \begin{array}{rcl} \phi(\nu) & = & \frac{\nu^2 \cosh\nu + \nu \sinh\nu + \cosh\nu \sinh^2\nu}{2\sinh^3\nu} \\ \nu & = & 2\pi \frac{\mathcal{L}}{\lambda_T} \end{array} \right. .$$

- its $(T \to 0)$-limit in agreement with corresponding limit in a different derivation[8].

- $\vartheta < 1$ can be related to the appearance of negative values for the Casimir entropy[9] $S = -\frac{\partial \mathcal{F}}{\partial L}$.

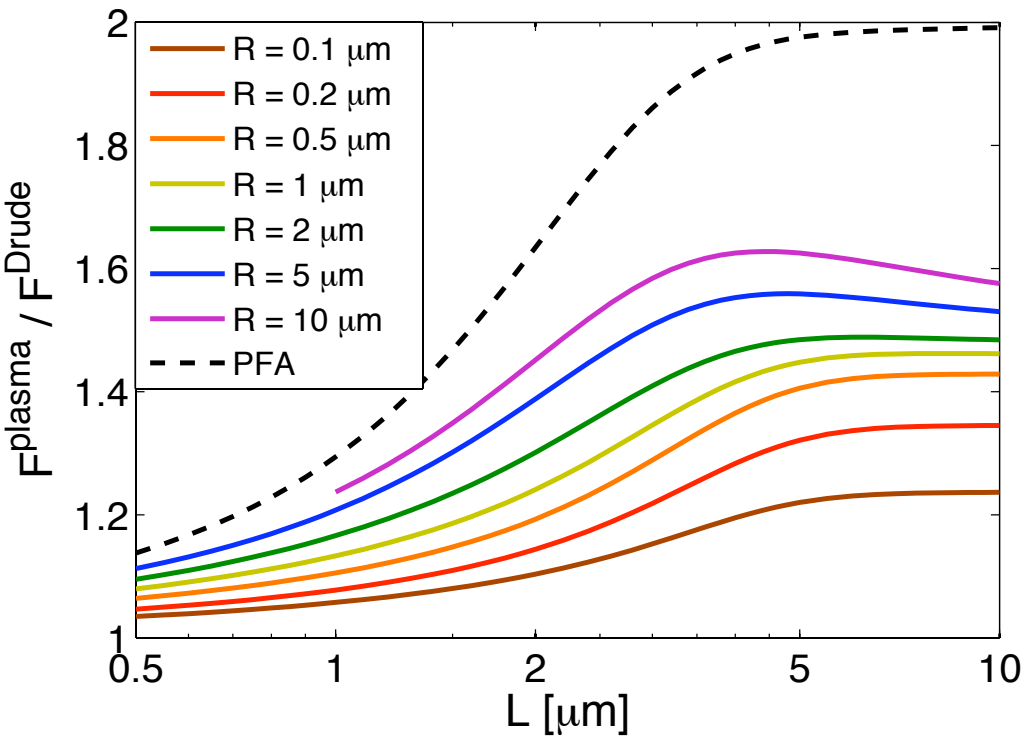


[8] M. Bordag, I. Pirozhenko, *Phys. Rev. D* **81** 085023 (2010).

[9] A. Canaguier-Durand, P.A. Maia Neto, A. Lambrecht, S. Reynaud, *Phys. Rev. A* **82**, 012511 (2010).

Interplay between the effect of dissipation and geometry

We compare the results of plasma and Drude models by taking the ratio $\frac{F^{plasma}}{F^{Drude}}$ at non-zero temperature.



- strong interplay between geometry, temperature and dissipation.
- the gap between plasma and Drude is always smaller.
- large distance analytical limit:

$$\frac{F^{plasma}}{F^{Drude}}(L \to \infty) \leq \frac{3}{2}$$

.

Conclusions

Scattering formalism allows to treat exactly the sphere-plane geometry, with a full EM multipolar treatment.

The sphere-plane configuration is a simple example to study the dependance of the Casimir effect on the geometry:

- coupling of the polarizations $\Rightarrow$ full EM treatment necessary.
- correlations between the geometry and the effects of finite conductivity $\Rightarrow$ β_G is no more in contradiction with experimental prescription.
- correlations between the geometry and temperature $\Rightarrow$ temperature can reduce $|F|$.
- correlations between the geometry and the effects of dissipation and temperature $\Rightarrow$ $\frac{F^{\mathrm{plas}}}{F^{\mathrm{Drud}}}$ goes to a value $\leq \frac{3}{2}$ at large distances (instead of 2 for plane-plane).

PhD defense September 30th in Paris (french). Manuscript online (soon).

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