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Casimir and Casimir-Polder forces in chiral and non-reciprocal media

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Outline

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  - Dispersion forces
  - Repulsive forces?
  - Unusual material properties
- **QED in nonlocal, nonreciprocal media**
  - Ohm's law
  - Field quantisation
- **QED in local bianisotropic media**
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- **Casimir–Polder potential of chiral molecules**
  - Perturbation theory
  - Perfect mirror

Introduction:

Dispersion forces

and unusual materials

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Dispersion force: Effective electromagnetic force between neutral, polarizable objects

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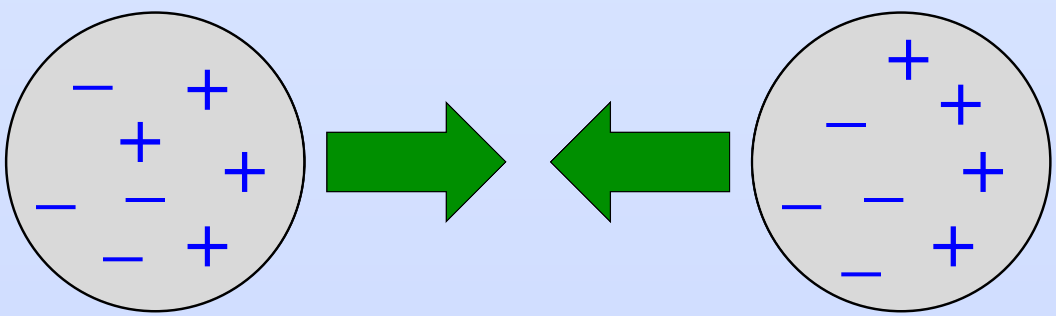

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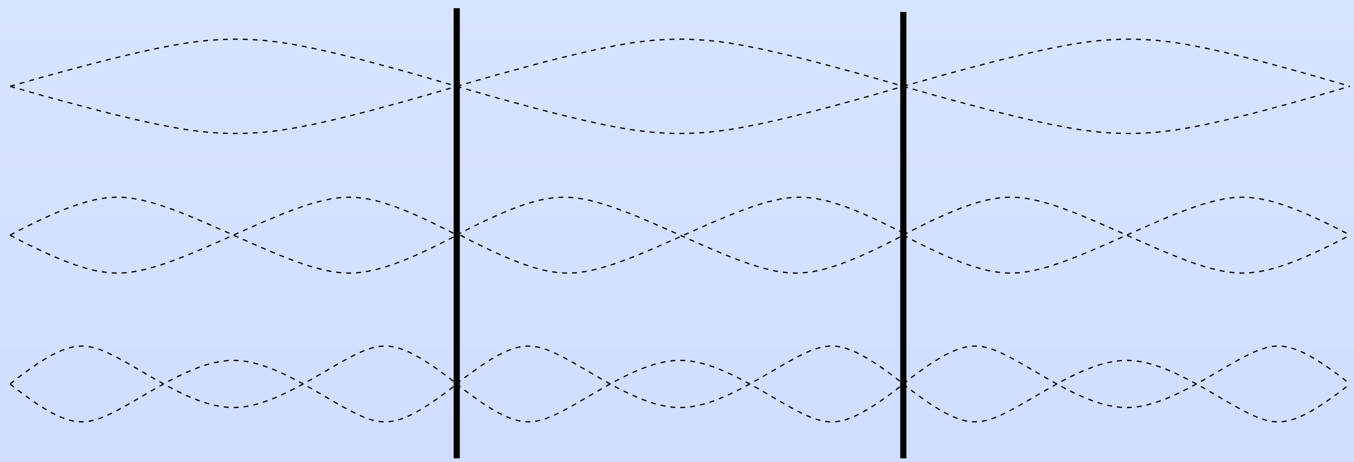

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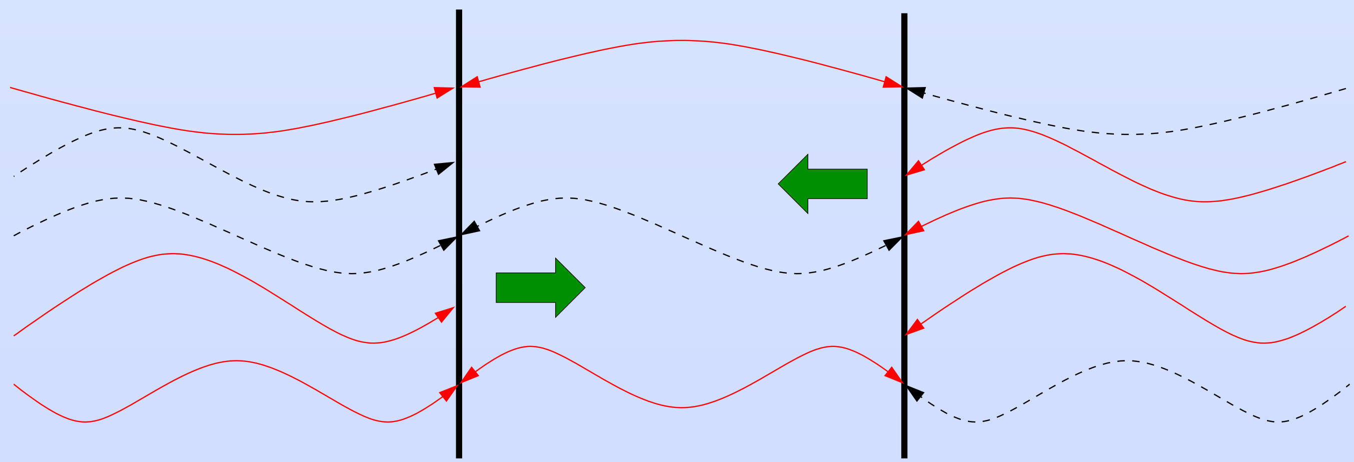

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Physical origin: Correlated charge and/or field fluctuations

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Direction of ground-state dispersion forces between electric objects in vacuum ⇒ **typically attractive!**

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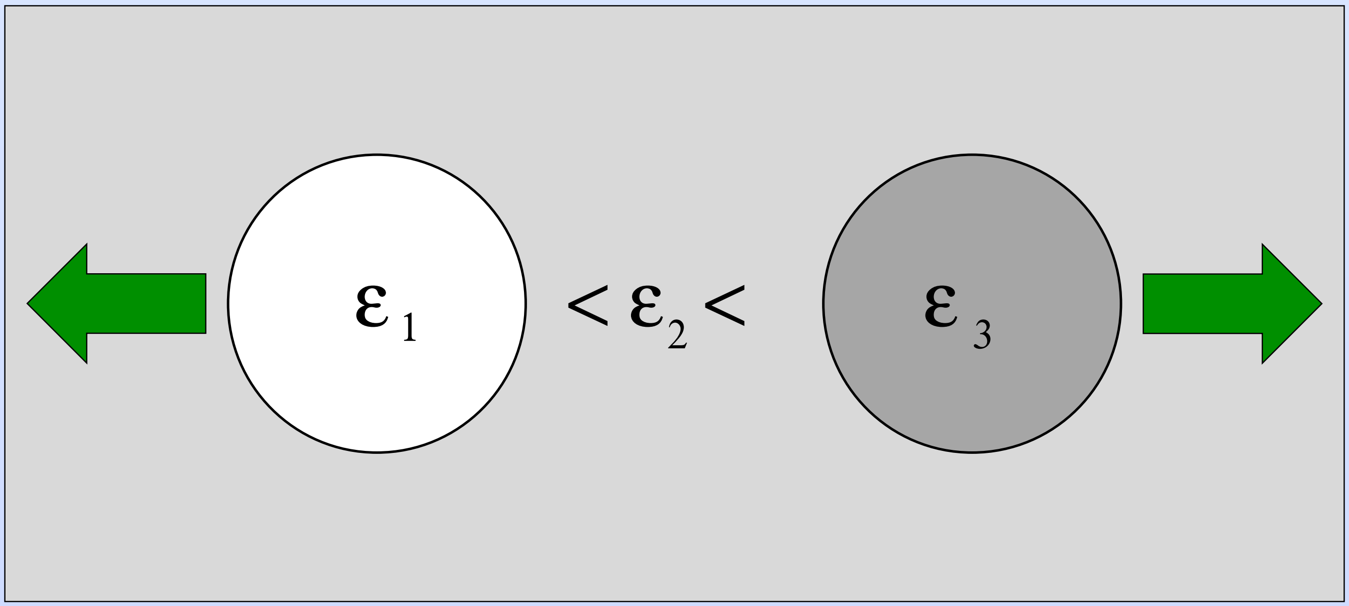

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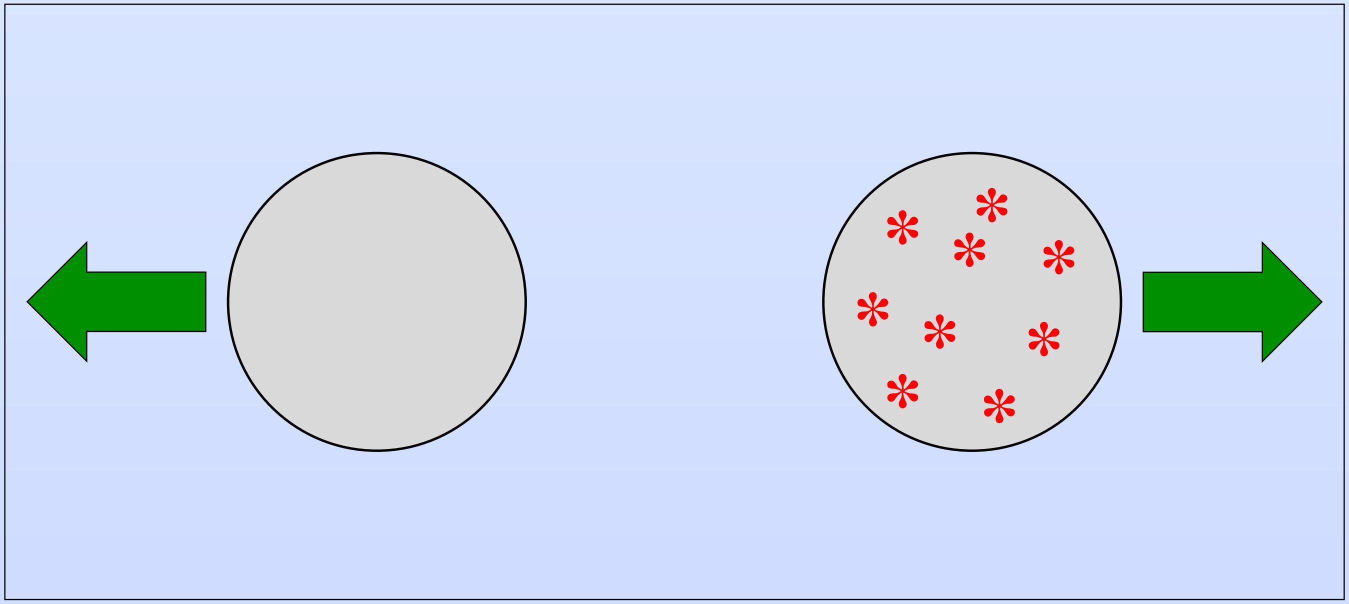

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Repulsive dispersion forces:

- Objects in a medium
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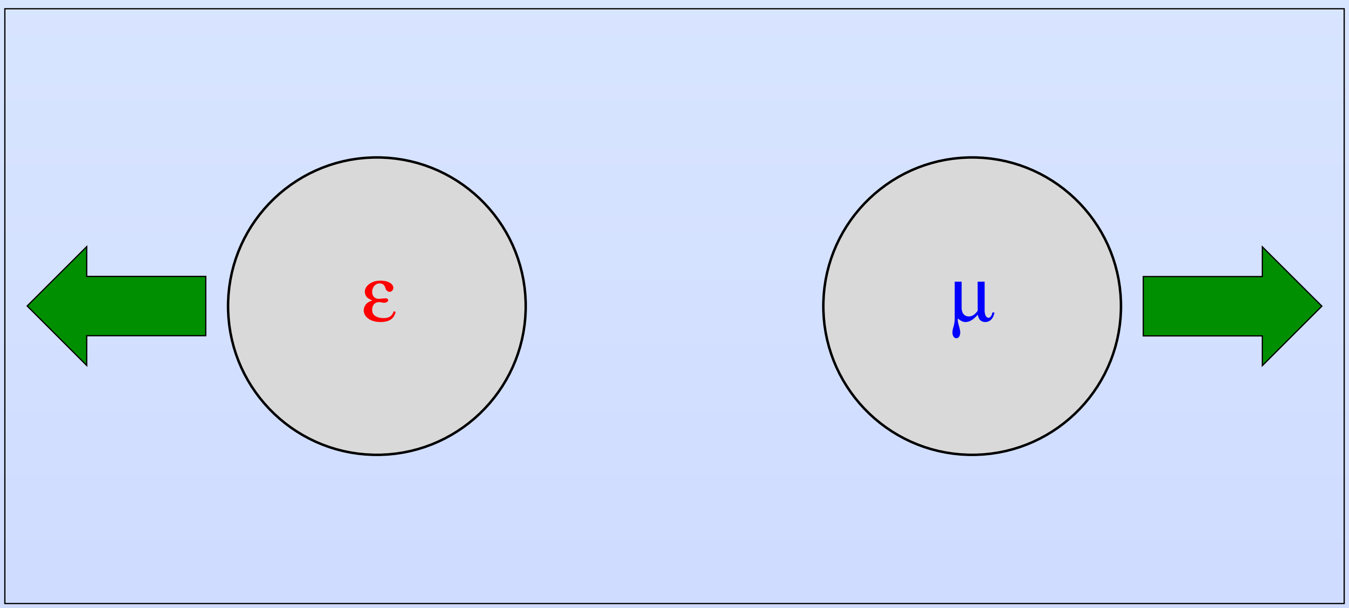
- Magnetoelectric objects



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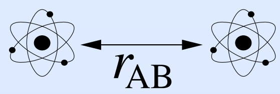
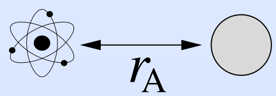
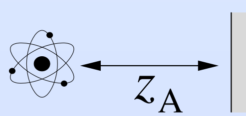
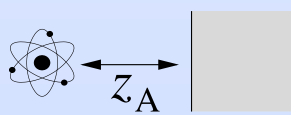
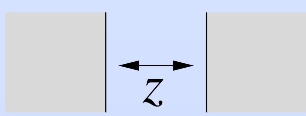
Electric vs magnetic objects

Distance →	Retarded		Nonretarded	
Objects →	e ↔ e	e ↔ m	e ↔ e	e ↔ m
 r_{AB}	$-\frac{1}{r_{AB}^8}$	$+\frac{1}{r_{AB}^8}$	$-\frac{1}{r_{AB}^7}$	$+\frac{1}{r_{AB}^5}$
 r_A	$-\frac{1}{r_A^8}$	$+\frac{1}{r_A^8}$	$-\frac{1}{r_A^7}$	$+\frac{1}{r_A^5}$
 z_A	$-\frac{1}{z_A^6}$	$+\frac{1}{z_A^6}$	$-\frac{1}{z_A^5}$	$+\frac{1}{z_A^3}$
 z_A	$-\frac{1}{z_A^5}$	$+\frac{1}{z_A^5}$	$-\frac{1}{z_A^4}$	$+\frac{1}{z_A^2}$
 z	$-\frac{1}{z^4}$	$+\frac{1}{z^4}$	$-\frac{1}{z^3}$	$+\frac{1}{z}$

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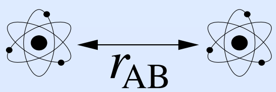
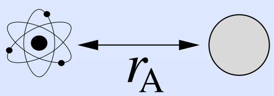
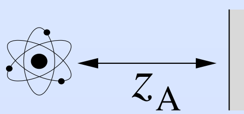
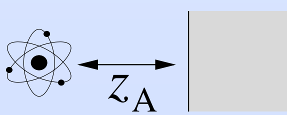
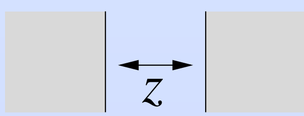
	$-\frac{1}{r_{\mathrm{AB}}^8}$	$+\frac{1}{r_{\mathrm{AB}}^8}$	$-\frac{1}{r_{\mathrm{AB}}^7}$	$+\frac{1}{r_{\mathrm{AB}}^5}$
	$-\frac{1}{r_{\mathrm{A}}^8}$	$+\frac{1}{r_{\mathrm{A}}^8}$	$-\frac{1}{r_{\mathrm{A}}^7}$	$+\frac{1}{r_{\mathrm{A}}^5}$
	$-\frac{1}{z_{\mathrm{A}}^6}$	$+\frac{1}{z_{\mathrm{A}}^6}$	$-\frac{1}{z_{\mathrm{A}}^5}$	$+\frac{1}{z_{\mathrm{A}}^3}$
	$-\frac{1}{z_{\mathrm{A}}^5}$	$+\frac{1}{z_{\mathrm{A}}^5}$	$-\frac{1}{z_{\mathrm{A}}^4}$	$+\frac{1}{z_{\mathrm{A}}^2}$
	$-\frac{1}{z^4}$	$+\frac{1}{z^4}$	$-\frac{1}{z^3}$	$+\frac{1}{z}$

Duality invariance: $\alpha \leftrightarrow \beta/c^2$, $\varepsilon \leftrightarrow \mu$ (e → m, m → e)

S.Y.B., S. Scheel, Phys. Rev. Lett. **102**, 140404 (2009)

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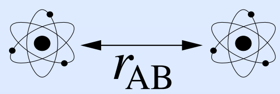
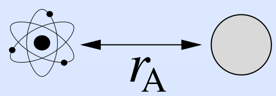
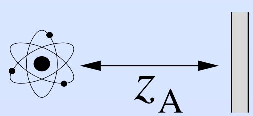
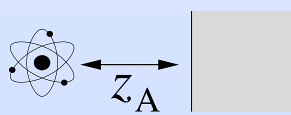
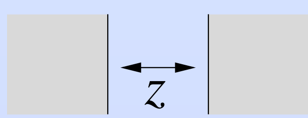
 r_{AB}	$-\frac{1}{r_{\mathrm{AB}}^8}$	$+\frac{1}{r_{\mathrm{AB}}^8}$	$-\frac{1}{r_{\mathrm{AB}}^7}$	$+\frac{1}{r_{\mathrm{AB}}^5}$
 r_A	$-\frac{1}{r_{\mathrm{A}}^8}$	$+\frac{1}{r_{\mathrm{A}}^8}$	$-\frac{1}{r_{\mathrm{A}}^7}$	$+\frac{1}{r_{\mathrm{A}}^5}$
 z_A	$-\frac{1}{z_{\mathrm{A}}^6}$	$+\frac{1}{z_{\mathrm{A}}^6}$	$-\frac{1}{z_{\mathrm{A}}^5}$	$+\frac{1}{z_{\mathrm{A}}^3}$
 z_A	$-\frac{1}{z_{\mathrm{A}}^5}$	$+\frac{1}{z_{\mathrm{A}}^5}$	$-\frac{1}{z_{\mathrm{A}}^4}$	$+\frac{1}{z_{\mathrm{A}}^2}$
 z	$-\frac{1}{z^4}$	$+\frac{1}{z^4}$	$-\frac{1}{z^3}$	$+\frac{1}{z}$

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⇒ **Opposites repel!**

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Unusual material properties

Simplest magnetoelectric: permittvity $\varepsilon(\boldsymbol{r},\omega)$,
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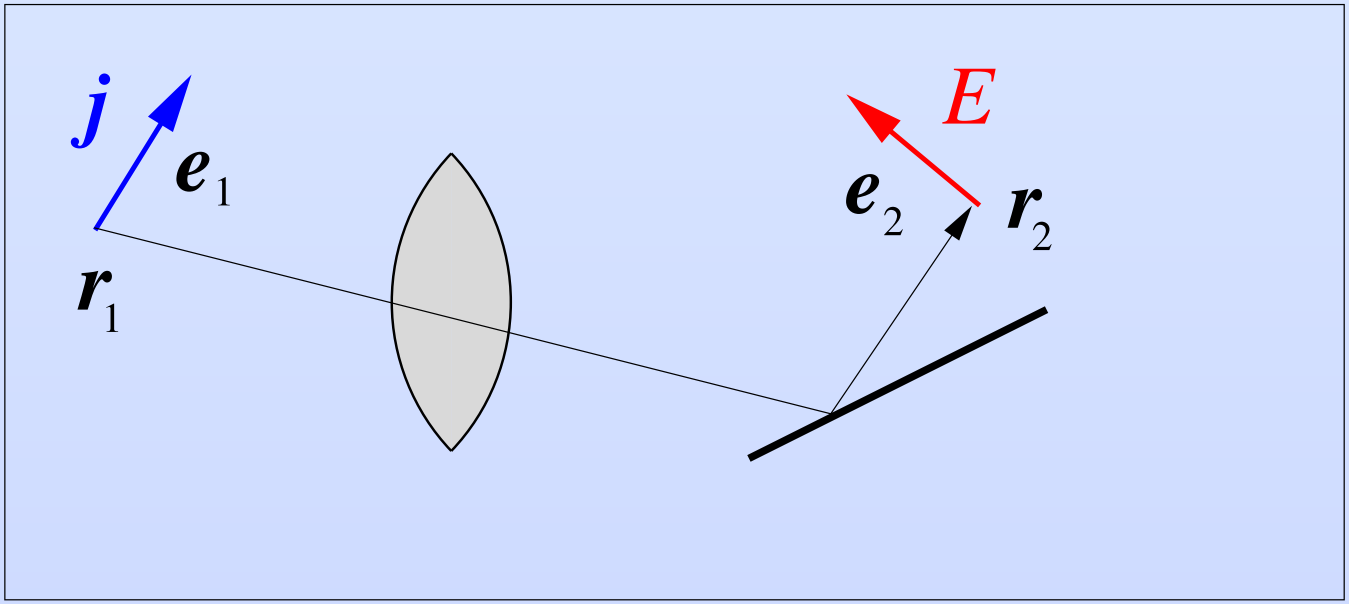
  ($\rightarrow$ chiral media)
- *nonreciprocal media:* violation of Onsager theorem
  ($\rightarrow$ external magnetic fields, spontaneous symmetry breaking, moving media)

Onsager theorem

Onsager reciprocity:

current j along $\boldsymbol{e}_1$ at $\boldsymbol{r}_1$ causes field E along $\boldsymbol{e}_2$ at $\boldsymbol{r}_2$



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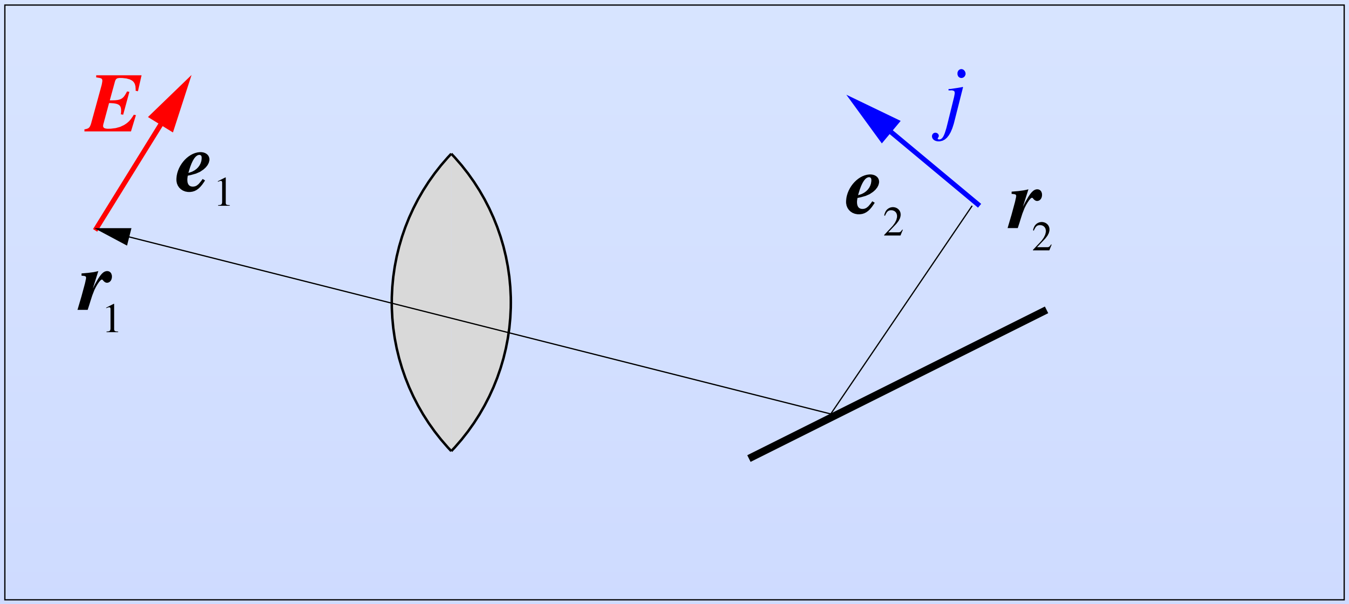

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Formally:

$$\underline{\boldsymbol{E}}(\boldsymbol{r},\omega) = i\mu_0\omega \int \mathrm{d}^3r' \, \mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\boldsymbol{j}}(\boldsymbol{r}',\omega),$$

$$\mathbf{G}^{\top}(\boldsymbol{r}',\boldsymbol{r},\omega) = \mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)$$

QED in nonlocal,

nonreciprocal media

Maxwell equations + Ohm's law

Maxwell equations:

$$\boldsymbol{\nabla}\cdot\hat{\boldsymbol{B}} = 0, \qquad \boldsymbol{\nabla}\times\hat{\boldsymbol{E}} + \dot{\hat{\boldsymbol{B}}} = \mathbf{0},$$

$$\boldsymbol{\nabla}\cdot\hat{\boldsymbol{E}} = \frac{\hat{\rho}_{\mathsf{in}}}{\varepsilon_0}, \qquad \boldsymbol{\nabla}\times\hat{\boldsymbol{B}} - \frac{1}{c^2}\dot{\hat{\boldsymbol{E}}} = \mu_0\hat{\boldsymbol{j}}_{\mathsf{in}}$$

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Linear constitutive relations:

$$\hat{\boldsymbol{j}}_{\mathsf{in}}(\boldsymbol{r},t) = \int_{-\infty}^{\infty} \mathrm{d}\tau \int \mathrm{d}^3 r' \, \mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\tau)\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}',t-\tau) + \hat{\boldsymbol{j}}_{\mathsf{N}}(\boldsymbol{r},t),$$

$$\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\tau) = \mathbf{0} \text{ for } c\tau < |\boldsymbol{r}-\boldsymbol{r}'| \text{ (causality)}$$

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Frequency components:

$$\underline{\hat{\boldsymbol{j}}}_{\text{in}}(\boldsymbol{r},\omega) = \int\mathrm{d}^3r'\,\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{E}}}(\boldsymbol{r}',\omega) + \underline{\hat{\boldsymbol{j}}}_{\text{N}}(\boldsymbol{r},\omega)$$

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Solution:

$$\underline{\hat{\boldsymbol{E}}}(\boldsymbol{r},\omega) = i\mu_0\omega\int\mathrm{d}^3r'\,\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot\underline{\hat{\boldsymbol{j}}}_{\text{N}}(\boldsymbol{r}',\omega)$$

Green tensor

Helmholtz equation:

$$\left[\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\; - \frac{\omega^2}{c^2}\right]\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega) - i\mu_0\omega\int \mathrm{d}^3 s\,\mathbf{Q}(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathbf{G}(\boldsymbol{s},\boldsymbol{r}',\omega) = \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')$$

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General properties: Reciprocal media $[\mathbf{Q}^{\top}(\boldsymbol{r}',\boldsymbol{r},\omega)=\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)]$

- *Schwarz reflection principle:* $\mathbf{G}^*(\boldsymbol{r},\boldsymbol{r}',\omega)=\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',-\omega^*)$
- *Onsager reciprocity:* $\mathbf{G}^{\top}(\boldsymbol{r}',\boldsymbol{r},\omega)=\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)$
- *Integral relation:*

$$\mu_0\omega\int \mathrm{d}^3s\int \mathrm{d}^3s'\,\mathbf{G}(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathrm{Re}\,\mathbf{Q}(\boldsymbol{s},\boldsymbol{s}',\omega)\cdot\mathbf{G}^*(\boldsymbol{s}',\boldsymbol{r}',\omega)$$
$$= \mathrm{Im}\,\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)$$

Green tensor

General properties: **Non**reciprocal media $[\mathbf{Q}^{\top}(\boldsymbol{r}',\boldsymbol{r},\omega)\neq\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)]$

- *Schwarz reflection principle:* $\mathbf{G}^{*}(\boldsymbol{r},\boldsymbol{r}',\omega)=\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',-\omega^{*})$
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- *Integral relation:*

$$\mu_0\omega\int \mathrm{d}^3s\int \mathrm{d}^3s'\,\mathbf{G}(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathcal{R}\mathrm{e}\mathbf{Q}(\boldsymbol{s},\boldsymbol{s}',\omega)\cdot\mathbf{G}^{*}(\boldsymbol{s}',\boldsymbol{r}',\omega)=\mathcal{I}\mathrm{m}\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega),$$

$$\mathcal{R}\mathrm{e}\mathbf{T}(\boldsymbol{r},\boldsymbol{r}')=\frac{1}{2}\Big[\mathbf{T}(\boldsymbol{r},\boldsymbol{r}')+\mathbf{T}^{\dagger}(\boldsymbol{r}',\boldsymbol{r})\Big],$$

$$\mathcal{I}\mathrm{m}\mathbf{T}(\boldsymbol{r},\boldsymbol{r}')=\frac{1}{2i}\Big[\mathbf{T}(\boldsymbol{r},\boldsymbol{r}')-\mathbf{T}^{\dagger}(\boldsymbol{r}',\boldsymbol{r})\Big]$$

Field quantisation

Commutation relations:

$$\left[\hat{\underline{\boldsymbol{j}}}_{\mathsf{N}}(\boldsymbol{r},\omega),\hat{\underline{\boldsymbol{j}}}_{\mathsf{N}}^{\dagger}(\boldsymbol{r}',\omega')\right]=\frac{\hbar\omega}{\pi}\mathcal{R}\mathrm{e}\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)\delta(\omega-\omega')$$

Field quantisation

Commutation relations:

$$\left[\underline{\hat{\boldsymbol{j}}}_{\mathsf{N}}(\boldsymbol{r},\omega),\underline{\hat{\boldsymbol{j}}}^{\dagger}_{\mathsf{N}}(\boldsymbol{r}',\omega')\right]=\frac{\hbar\omega}{\pi}\mathcal{R}\mathrm{e}\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)\delta(\omega-\omega')$$

Fundamental variables: $\left[\hat{\boldsymbol{f}}(\boldsymbol{r},\omega),\hat{\boldsymbol{f}}^{\dagger}(\boldsymbol{r}',\omega')\right]=\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')$

$$\hat{\boldsymbol{j}}_{\mathsf{N}}(\boldsymbol{r},\omega)=\sqrt{\frac{\hbar\omega}{\pi}}\int\mathrm{d}^3r'\,\mathbf{R}(\boldsymbol{r},\boldsymbol{r}',\omega)\hat{\boldsymbol{f}}(\boldsymbol{r}',\omega),$$

with $\int\mathrm{d}^3s\,\mathbf{R}(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathbf{R}^{\dagger}(\boldsymbol{r}',\boldsymbol{s},\omega)=\mathcal{R}\mathrm{e}\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)$

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$$\text{with } \int \mathrm{d}^3s\,\mathbf{R}(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathbf{R}^{\dagger}(\boldsymbol{r}',\boldsymbol{s},\omega)=\mathcal{R}\mathrm{e}\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega)$$

Medium–field Hamiltonian:

$$\hat{H}_{\mathsf{F}}=\int \mathrm{d}^3r\int_0^{\infty}\mathrm{d}\omega\,\hbar\omega\,\hat{\boldsymbol{f}}^{\dagger}(\boldsymbol{r},\omega)\cdot\hat{\boldsymbol{f}}(\boldsymbol{r},\omega)$$

Field quantisation

Commutation relations:

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Consistency: Maxwell equations $\surd$ free-space QED $\surd$
fluctuation–dissipation theorem $\surd$

Local bianisotropic media

Local bianisotropic media

Maxwell equations:

$$\nabla \cdot \underline{\hat{B}} = 0, \qquad \nabla \times \hat{E} + -i\omega \underline{\hat{B}} = \mathbf{0},$$

$$\nabla \cdot \underline{\hat{D}} = 0, \qquad \nabla \times \underline{\hat{H}} + i\omega \underline{\hat{D}} = \mathbf{0}$$

Local bianisotropic media

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$$\nabla\cdot\underline{\hat{D}}=0, \qquad \nabla\times\underline{\hat{H}}+i\omega\underline{\hat{D}}=\mathbf{0}$$

Constitutive relations: $\hat{D}=\varepsilon_0\hat{E}+\hat{P}$, $\hat{H}=\hat{B}/\mu_0-\hat{M}$

$$\underline{\hat{P}} = \varepsilon_0(\boldsymbol{\varepsilon}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}-\mathbf{I})\cdot\underline{\hat{E}}+\frac{1}{Z_0}\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\underline{\hat{B}}+\underline{\hat{P}}_{\mathrm{N}},$$

$$\underline{\hat{M}} = \frac{1}{Z_0}\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}\cdot\underline{\hat{E}}+\frac{1}{\mu_0}(\mathbf{I}-\boldsymbol{\mu}^{-1})\cdot\underline{\hat{B}}+\underline{\hat{M}}_{\mathrm{N}}$$

Local bianisotropic media

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$$\nabla\cdot\underline{\hat{D}} = 0, \qquad \nabla\times\underline{\hat{H}} + i\omega\underline{\hat{D}} = \mathbf{0}$$

Constitutive relations: $\hat{D} = \varepsilon_0\hat{E} + \hat{P}$, $\hat{H} = \hat{B}/\mu_0 - \hat{M}$

$$\underline{\hat{P}} = \varepsilon_0(\boldsymbol{\varepsilon} - \boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta} - \mathbf{I})\cdot\underline{\hat{E}} + \frac{1}{Z_0}\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\underline{\hat{B}} + \underline{\hat{P}}_{\mathrm{N}},$$
$$\underline{\hat{M}} = \frac{1}{Z_0}\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}\cdot\underline{\hat{E}} + \frac{1}{\mu_0}(\mathbf{I} - \boldsymbol{\mu}^{-1})\cdot\underline{\hat{B}} + \underline{\hat{M}}_{\mathrm{N}}$$

Alternative form:

$$\underline{\hat{D}} = \varepsilon_0\boldsymbol{\varepsilon}\cdot\underline{\hat{E}} + \frac{1}{c}\boldsymbol{\xi}\cdot\underline{\hat{H}} + \underline{\hat{P}}_{\mathrm{N}} + \frac{1}{c}\boldsymbol{\xi}\cdot\underline{\hat{M}}_{\mathrm{N}},$$
$$\underline{\hat{B}} = \frac{1}{c}\boldsymbol{\zeta}\cdot\underline{\hat{E}} + \mu_0\boldsymbol{\mu}\cdot\underline{\hat{H}} + \mu_0\boldsymbol{\mu}\cdot\underline{\hat{M}}_{\mathrm{N}}$$

Relation to conductivity

Conductivity:

$$\mathbf{Q}(\boldsymbol{r},\boldsymbol{r}',\omega) = \frac{1}{i\mu_0\omega}\boldsymbol{\nabla}\times(\boldsymbol{\mu}^{-1}-\mathbf{I})\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\times\overleftarrow{\boldsymbol{\nabla}}' + \frac{1}{Z_0}\boldsymbol{\nabla}\times\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}') + \frac{1}{Z_0}\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\times\overleftarrow{\boldsymbol{\nabla}}' - i\varepsilon_0\omega(\boldsymbol{\varepsilon}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}-\mathbf{I})\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')$$

Noise terms: $\underline{\hat{\boldsymbol{j}}}_{\mathrm{N}} = -i\omega\underline{\hat{\boldsymbol{P}}}_{\mathrm{N}} + \boldsymbol{\nabla}\times\underline{\hat{\boldsymbol{M}}}_{\mathrm{N}}$

Green tensor:

$$\left[\boldsymbol{\nabla}\times\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\nabla}\times - \frac{i\omega}{c}\boldsymbol{\nabla}\times\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}\cdot + \frac{i\omega}{c}\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\nabla}\times - \frac{\omega^2}{c^2}(\boldsymbol{\varepsilon}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta})\cdot\right]\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega) = \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')$$

Commutation relations

$$\left[\underline{\hat{\boldsymbol{P}}}_{\mathsf{N}}(\boldsymbol{r},\omega),\underline{\hat{\boldsymbol{P}}}_{\mathsf{N}}^{\dagger}(\boldsymbol{r}',\omega')\right]=\frac{\varepsilon_0\hbar}{\pi}\mathcal{I}\mathsf{m}(\boldsymbol{\varepsilon}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta})\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega'),$$

$$\left[\underline{\hat{\boldsymbol{M}}}_{\mathsf{N}}(\boldsymbol{r},\omega),\underline{\hat{\boldsymbol{M}}}_{\mathsf{N}}^{\dagger}(\boldsymbol{r}',\omega')\right]=-\frac{\hbar}{\pi\mu_0}\mathcal{I}\mathsf{m}\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega'),$$

$$\left[\underline{\hat{\boldsymbol{P}}}_{\mathsf{N}}(\boldsymbol{r},\omega),\underline{\hat{\boldsymbol{M}}}_{\mathsf{N}}^{\dagger}(\boldsymbol{r}',\omega')\right]=\frac{\hbar}{2\pi iZ_0}(\boldsymbol{\zeta}^{\dagger}\cdot\boldsymbol{\mu}^{-1\dagger}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1})\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')$$

Commutation relations

$$\left[\underline{\hat{\boldsymbol{P}}}_{\mathrm{N}}(\boldsymbol{r},\omega),\underline{\hat{\boldsymbol{P}}}_{\mathrm{N}}^{\dagger}(\boldsymbol{r}',\omega')\right]=\frac{\varepsilon_0\hbar}{\pi}\mathcal{I}\mathrm{m}(\boldsymbol{\varepsilon}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta})\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega'),$$

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$$\left[\underline{\hat{\boldsymbol{P}}}_{\mathrm{N}}(\boldsymbol{r},\omega),\underline{\hat{\boldsymbol{M}}}_{\mathrm{N}}^{\dagger}(\boldsymbol{r}',\omega')\right]=\frac{\hbar}{2\pi i Z_0}(\boldsymbol{\zeta}^{\dagger}\cdot\boldsymbol{\mu}^{-1\dagger}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1})\cdot\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')$$

Basic variables: $\left[\hat{\boldsymbol{f}}_{\lambda}(\boldsymbol{r},\omega),\hat{\boldsymbol{f}}_{\lambda'}^{\dagger}(\boldsymbol{r}',\omega')\right]=\delta_{\lambda\lambda'}\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')$

$$\begin{pmatrix}\hat{\boldsymbol{P}}_{\mathrm{N}}(\boldsymbol{r},\omega)\\ \hat{\boldsymbol{M}}_{\mathrm{N}}(\boldsymbol{r},\omega)\end{pmatrix}=\sqrt{\frac{\hbar}{\pi}}\mathcal{R}\begin{pmatrix}\hat{\boldsymbol{f}}_{e}(\boldsymbol{r},\omega)\\ \hat{\boldsymbol{f}}_{m}(\boldsymbol{r},\omega)\end{pmatrix}$$

with $\mathcal{R}\cdot\mathcal{R}^{\dagger}=\begin{pmatrix}\varepsilon_0\mathcal{I}\mathrm{m}(\boldsymbol{\varepsilon}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}) & \dfrac{\boldsymbol{\zeta}^{\dagger}\cdot\boldsymbol{\mu}^{-1\dagger}-\boldsymbol{\xi}\cdot\boldsymbol{\mu}^{-1}}{2iZ_0}\\ -\dfrac{\boldsymbol{\mu}^{-1}\cdot\boldsymbol{\zeta}-\boldsymbol{\mu}^{-1\dagger}\cdot\boldsymbol{\xi}^{\dagger}}{2\pi i Z_0} & -\dfrac{\mathcal{I}\mathrm{m}\boldsymbol{\mu}^{-1}}{\mu_0}\end{pmatrix}$

Duality invariance

Maxwell equations in dual-pair notation:

$$\boldsymbol{\nabla}\cdot\begin{pmatrix} Z_0\underline{\hat{\boldsymbol{D}}} \\ \underline{\hat{\boldsymbol{B}}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$\boldsymbol{\nabla}\times\begin{pmatrix} \underline{\hat{\boldsymbol{E}}} \\ Z_0\underline{\hat{\boldsymbol{H}}} \end{pmatrix} - i\omega\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} Z_0\underline{\hat{\boldsymbol{D}}} \\ \underline{\hat{\boldsymbol{B}}} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix},$$

$$\begin{pmatrix} Z_0\underline{\hat{\boldsymbol{D}}} \\ \underline{\hat{\boldsymbol{B}}} \end{pmatrix} = \frac{1}{c}\begin{pmatrix} \boldsymbol{\varepsilon} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} \end{pmatrix}\begin{pmatrix} \underline{\hat{\boldsymbol{E}}} \\ Z_0\underline{\hat{\boldsymbol{H}}} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & \boldsymbol{\mu} \end{pmatrix}\begin{pmatrix} Z_0\underline{\hat{\boldsymbol{P}}}_{\mathrm{N}} \\ \mu_0\underline{\hat{\boldsymbol{M}}}_{\mathrm{N}} \end{pmatrix}$$

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Duality transformation:

$$\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}^{\circledast} = \mathcal{D}(\theta)\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}, \qquad \mathcal{D}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Transformation of response functions

$$\begin{pmatrix}\varepsilon\\ \xi\\ \zeta\\ \mu\end{pmatrix}^{\circledast} = \begin{pmatrix}\cos^2\theta & \sin\theta\cos\theta & \sin\theta\cos\theta & \sin^2\theta\\ -\sin\theta\cos\theta & \cos^2\theta & -\sin^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos\theta & -\sin^2\theta & \cos^2\theta & \sin\theta\cos\theta\\ \sin^2\theta & -\sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta\end{pmatrix}\begin{pmatrix}\varepsilon\\ \xi\\ \zeta\\ \mu\end{pmatrix}$$

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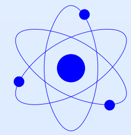
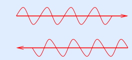
Discrete vs continuous symmetry:

- *Isotropic media* ($\boldsymbol{\varepsilon}=\varepsilon\mathbf{I}$, $\boldsymbol{\mu}=\mu\mathbf{I}$, $\boldsymbol{\xi}=\boldsymbol{\zeta}=\mathbf{0}$): discrete
- *Biisotropic media* ($\boldsymbol{\varepsilon}=\varepsilon\mathbf{I}$, $\boldsymbol{\mu}=\mu\mathbf{I}$, $\boldsymbol{\xi}=\xi\mathbf{I}$, $\boldsymbol{\zeta}=\zeta\mathbf{I}$): continuous
- *Anisotropic media* ($\boldsymbol{\xi}=\boldsymbol{\zeta}=\mathbf{0}$): discrete
- *Reciprocal media* ($\boldsymbol{\varepsilon}^{\mathsf{T}}=\boldsymbol{\varepsilon}$, $\boldsymbol{\xi}^{\mathsf{T}}=-\boldsymbol{\zeta}$, $\boldsymbol{\mu}^{\mathsf{T}}=\boldsymbol{\mu}$): discrete

Casimir–Polder potential of chiral molecules

Perturbation theory

Idea: System in ground state $|g\rangle = |0\rangle$  $\otimes\ |\{0\}\rangle$ 

Interaction $\hat{H}_{AF} = -\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}_A) - \hat{\boldsymbol{m}}\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}_A) \Rightarrow$ energy shift ΔE

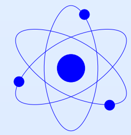
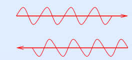
$\Rightarrow$ Casimir-Polder potential $U(\boldsymbol{r}_A) = \Delta E(\boldsymbol{r}_A)$

2nd-order perturbation theory:

$$\Delta E = \sum_\psi \frac{|\langle g|\hat{H}_{AF}|\psi\rangle|^2}{E_g - E_\psi}$$

H.B.G. Casimir, D. Polder, PRA **73** 360 (1948)

Perturbation theory

Idea: System in ground state $|g\rangle = |0\rangle$  $\otimes\ |\{0\}\rangle$ 

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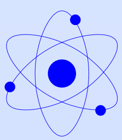
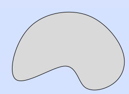
$\Rightarrow$ Casimir-Polder potential $U(\boldsymbol{r}_A) = \Delta E(\boldsymbol{r}_A)$

$$U(\boldsymbol{r}_A) = -\frac{\hbar\mu_0}{\pi}\int_0^\infty \mathrm{d}\xi\, \xi\chi(i\xi)\,\mathrm{Tr}\Big[\boldsymbol{\nabla}\times\mathbf{G}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi)\Big]$$

 *Chiral polarisability:*

$$\chi(\omega) = \frac{2}{3\hbar}\lim_{\epsilon\to 0}\sum_k \frac{i\omega R_{0k}}{\omega_k^2-\omega^2-i\omega\epsilon},$$

$$R_{0k} = \mathrm{Im}(\boldsymbol{d}_{0k}\cdot\boldsymbol{m}_{k0})$$

 *Scattering Green tensor:*

$$\mathbf{G}^{(1)}(\boldsymbol{r},\boldsymbol{r}',\omega)$$

H.B.G. Casimir, D. Polder, PRA **73** 360 (1948)

Example: Atom near perfect mirror

Curie principle: need chiral body to detect chiral molecule

Perfect chiral mirror: $r_{s\to p} = \mp 1$, $r_{p\to s} = \pm 1$

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Chiral Casimir–Polder potential:

$$U(z_A) = \pm \frac{\hbar}{8\pi^2 \varepsilon_0 z_A^3} \int_0^\infty \mathrm{d}\xi \, \frac{\chi(i\xi)}{c} \, \mathrm{e}^{-2\xi z_A/c} \left(1 + 2\,\frac{\xi z_A}{c}\right)$$

Example: Atom near perfect mirror

Curie principle: need chiral body to detect chiral molecule

Perfect chiral mirror: $r_{s\to p} = \mp 1$, $r_{p\to s} = \pm 1$

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Long-/short-distance limits:

$$U(z_A) = \begin{cases} \mp\dfrac{1}{16\pi^2\varepsilon_0 z_A^5}\displaystyle\sum_k \frac{R_{0k}}{\omega_k^2}\,, \\ \pm\dfrac{1}{12\pi^2\varepsilon_0 c z_A^3}\displaystyle\sum_k R_{0k}\ln(\omega_k z_A/c) \end{cases}$$

Summary

Macroscopic quantum electrodynamics

- *Nonlocal, nonreciprocal media:* conductivity
- *Local bianisotropic media:* cross-susceptibilities
- *Duality:* continuous (if Onsager violation allowed)

Chiral Casimir-Polder potential

- *Perfectly chiral plate:* left- vs right-handed molecules

Outlook

- *Local duality:* force maximal for opposite dual pairs?
- *Nonreciprocal media:* CP-violating systems?
- *Moving systems:* symmetrise Green tensor?

Further Reading:

S.Y.B., *Dispersion Forces I + II*, Springer Tracts in Modern Physics, to appear spring 2012