Temperature independent Casimir-Polder forces: thermal non-equilibrium and arbitrary geometry

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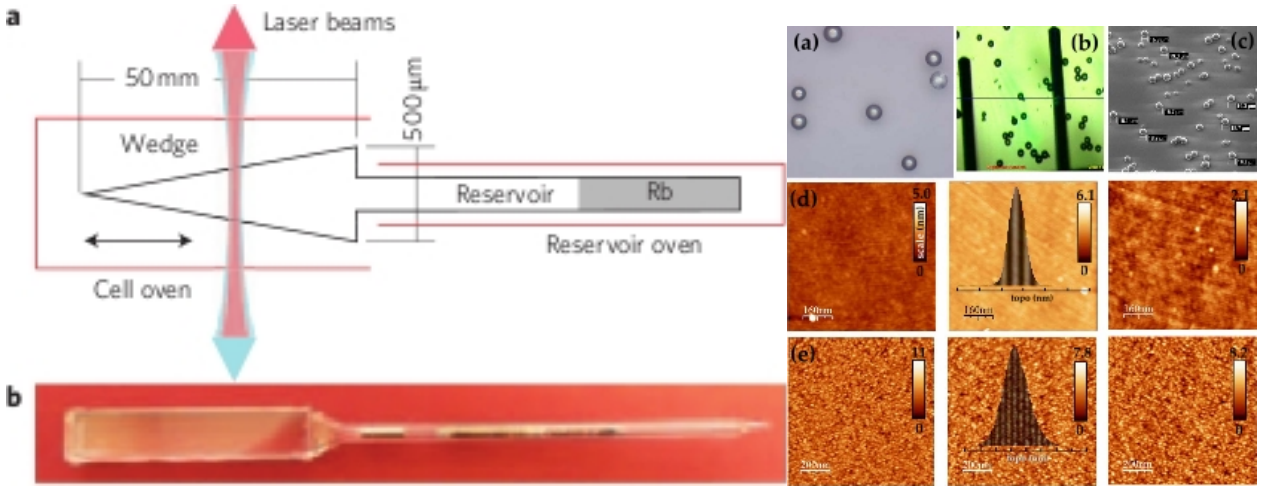
Outline

- Casimir–Polder forces at finite temperature: equilibrium
- thermal nonequilibrium: dynamical approach
- temperature-invariance: an example
- arbitrary geometry and temperature corrections



Field quantization in media

Helmholtz equation for electric field in media

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) = \frac{\omega^2}{c^2\varepsilon_0}\mathbf{P}_N(\mathbf{r},\omega)$$

solved by dyadic Green function $\mathbf{E}(\mathbf{r},\omega) = \frac{\omega^2}{c^2\varepsilon_0}\int d^3s\, \mathbf{G}(\mathbf{r},\mathbf{s},\omega)\cdot\mathbf{P}_N(\mathbf{s},\omega)$

- need to know field excitations in the presence of dielectrics
- Helmholtz equation with noise $\Rightarrow$ no field expansion into modes
- promote noise polarisation to operator-valued vector field

$$\hat{\mathbf{P}}_N(\mathbf{r},\omega) = i\sqrt{\tfrac{\hbar\varepsilon_0}{\pi}\operatorname{Im}\varepsilon(\mathbf{r},\omega)}\,\hat{\mathbf{f}}(\mathbf{r},\omega)$$

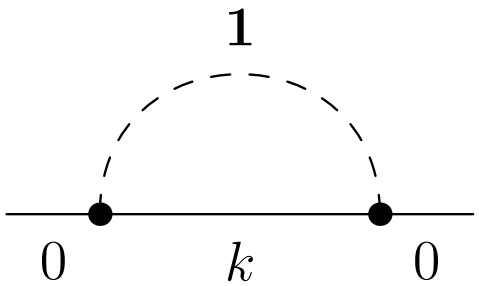
Casimir–Polder potential from perturbation theory

Hamiltonian in dipole approximation

$\hat{H}_{AF} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_A)$

linear in both atomic and field variables $\Rightarrow$ leading-order energy shift given in second order perturbation theory

$$\Delta E = \sum_{I \neq G} \frac{\langle G|\hat{H}_{\mathrm{AF}}|I\rangle\langle I|\hat{H}_{\mathrm{AF}}|G\rangle}{E_G - E_I}$$



$|G\rangle = |0\rangle|\{0\}\rangle$: ground state of atom and field
$|I\rangle = |k\rangle|\mathbf{1}(\mathbf{r},\omega)\rangle$: intermediate state with atomic excitation $|k\rangle$ and single field excitation $|\mathbf{1}(\mathbf{r},\omega)\rangle = \hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)|\{0\}\rangle$

Casimir–Polder potential from perturbation theory

matrix elements in perturbative energy shift:

$$\langle 0|\langle\{0\}|\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\mathbf{r}_A)|k\rangle|\mathbf{1}(\mathbf{r},\omega)\rangle = i\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\varepsilon_0}\operatorname{Im}\varepsilon(\mathbf{r},\omega)}\,\mathbf{d}_{0k}\cdot\mathbf{G}(\mathbf{r}_A,\mathbf{r},\omega)$$

- energy difference: $E_G - E_I \equiv -\hbar(\omega_{k0}+\omega)$
- sum over intermediate states: $\displaystyle\sum_{I\neq G} \equiv \sum_k \int_0^\infty d\omega \int d^3r$

use integral relation for dyadic Green function

$$\frac{\omega^2}{c^2}\int d^3s\,\varepsilon_I(\mathbf{s},\omega)\mathbf{G}(\mathbf{r},\mathbf{s},\omega)\cdot\mathbf{G}^+(\mathbf{r}',\mathbf{s},\omega) = \operatorname{Im}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)$$

Casimir–Polder potential from perturbation theory

$$\Delta E = -\frac{\mu_0}{\pi} \sum_k \int_0^\infty \frac{d\omega}{\omega_{k0} + \omega} \omega^2 \mathbf{d}_{0k} \cdot \mathrm{Im}\, \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{k0}$$

rotation to imaginary frequency axis $\omega \mapsto i\xi$

$$U(\mathbf{r}_A) = \frac{\hbar\mu_0}{2\pi} \int_0^\infty d\xi\, \xi^2 \mathrm{Tr}\left[\boldsymbol{\alpha}(i\xi) \cdot \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, i\xi)\right]$$

with atomic polarisability

$$\boldsymbol{\alpha}(\omega) = \frac{2}{\hbar} \sum_k \frac{\omega_{k0} \mathbf{d}_{k0} \otimes \mathbf{d}_{0k}}{\omega_{k0}^2 - \omega^2 - i\omega\epsilon}$$

Dispersion forces in thermal equilibrium

in thermal equilibrium: use fluctuation-dissipation theorem with thermal expectation values

$$\langle \hat{\mathbf{E}}(\mathbf{r},\omega) \otimes \hat{\mathbf{E}}^{\dagger}(\mathbf{r}',\omega') \rangle = \frac{\hbar}{\pi\varepsilon_0} \left[\bar{n}_{\mathrm{th}}(\omega)+1\right] \frac{\omega^2}{c^2} \mathrm{Im}\, \mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')$$

with $\bar{n}_{\mathrm{th}}(\omega) = \left[e^{\hbar\omega/(k_B T)} - 1\right]^{-1}$

But: in most experiments involving ultracold atoms they are not in thermal equilibrium with the environment (plate, radiation field)

$\Rightarrow$ cannot use equilibrium theory, need dynamical theory

$\Rightarrow$ expect force dynamics due to absorption/emission of thermal photons

Dispersion forces in thermal nonequilibrium

CP force in perturbative limit [$\xi_N = 2\pi k_B T N/\hbar$: Matsubara freq.][1]

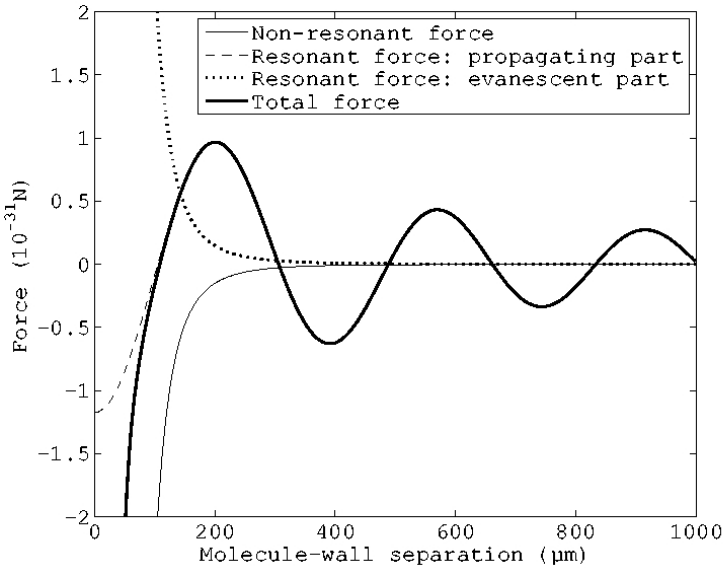
$$\mathbf{F}_n(\mathbf{r}_A) = -\mu_0 k_B T \sum_{N=0}^{\infty} \left(1 - \tfrac{1}{2}\delta_{N0}\right) \xi_N^2 \alpha_n(i\xi_N) \boldsymbol{\nabla}_A \mathrm{Tr}\, \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, i\xi_N) +$$
$$\frac{\mu_0}{3} \sum_k \omega_{nk}^2 \left\{\Theta(\omega_{nk}) \left[\bar{n}_{\mathrm{th}}(\omega_{nk}) + 1\right] - \Theta(\omega_{kn}) \bar{n}_{\mathrm{th}}(\omega_{kn})\right\} |\mathbf{d}_{nk}|^2 \boldsymbol{\nabla}_A \mathrm{Tr}\, \mathrm{Re}\, \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, \omega_{nk})$$

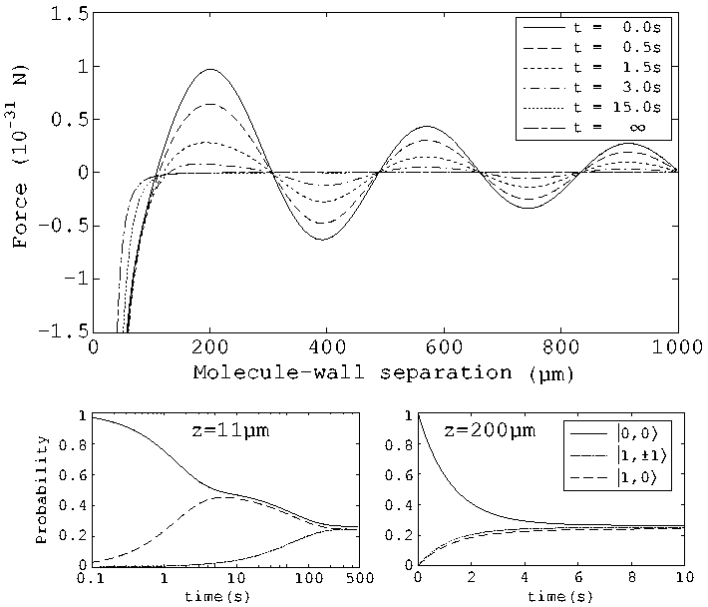
- first term: nonresonant (Lifshitz-like) force component, all (Matsubara) frequencies involved
- second term: resonant force components at atomic transition frequencies
  - further division into evanescent and propagating parts
  - due to absorption and emission of thermal photons
  - for ground-state atoms: resonant force components visible on time scales smaller than inverse ground-state heating rate Γ_{0k}^{-1}

[1] S.Y. Buhmann and S. Scheel, Phys. Rev. Lett. **100**, 253201 (2008).

Dispersion forces in thermal nonequilibrium

ground-state LiH near a gold surface at room temperature[2]:





[2] S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. A **79**, 052903 (2009).

Casimir–Polder interaction with Rydberg atoms

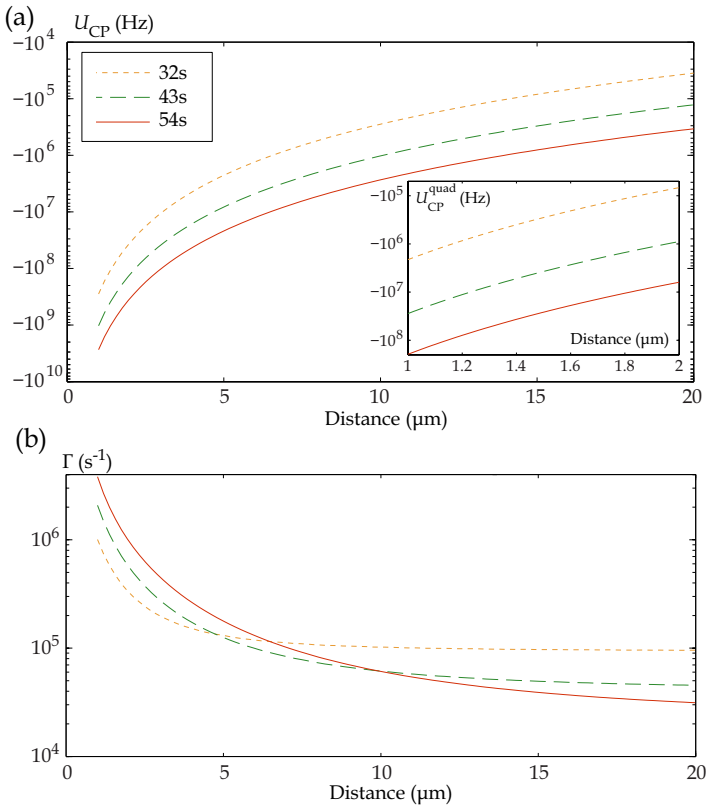
Rydberg atoms

radius of atom in Rydberg state with principal quantum number n:
$\langle r \rangle \simeq n^2 a_0$, e.g. $n = 32 \Rightarrow 2\langle r \rangle \simeq 100\text{nm}$

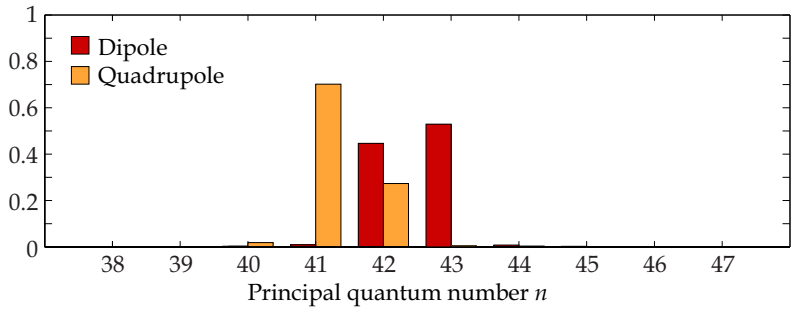
dipole matrix elements grow as $\propto n^2$ transition frequencies to neighboring energy levels at long wavelengths ($\simeq 100\mu\text{m}$)

- size of Rydberg atoms implies that dipole approximation is not good enough $\Rightarrow$ quadrupole moments contribute at short distances
- expect extremely large Casimir–Polder interaction
- temperature-dependent dispersion potential?

Casimir–Polder interaction with Rydberg atoms



Rb with dominant transitions $ns \longrightarrow (n-2)d$, $n = 32, 43, 54$



contributions to level shifts of 43s

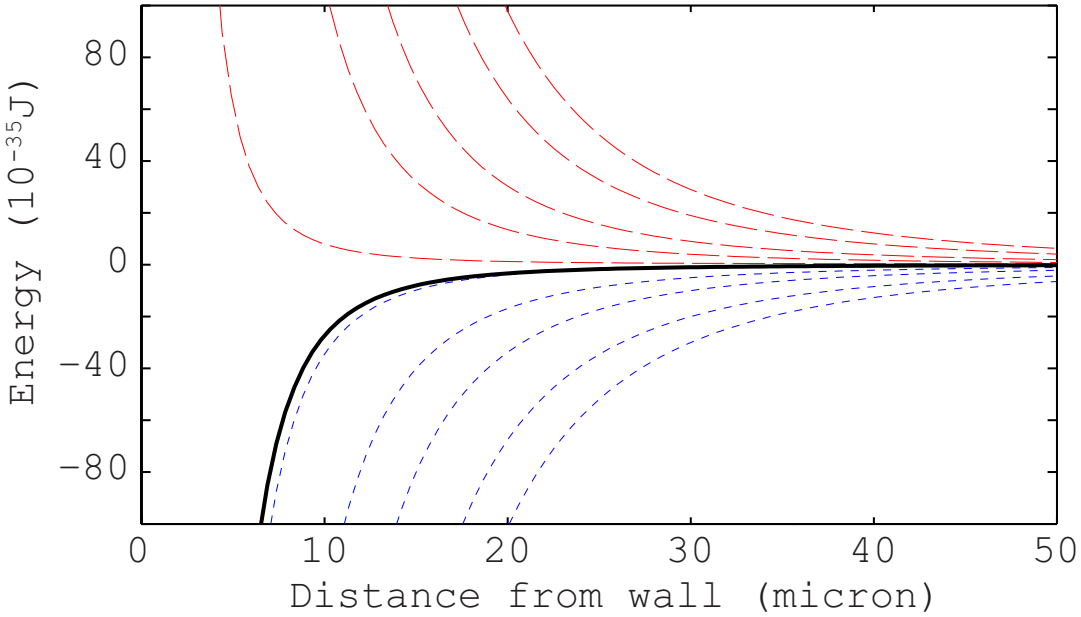
$\Rightarrow$ Casimir–Polder shifts on the order of GHz at micrometer distances[a]

$\Rightarrow$ lifetime decreases with increasing n rather than increases as in free space

[a] J.A. Crosse *et al.*, Phys. Rev. A **82**, 010901(R) (2010).

Dispersion forces in thermal nonequilibrium

ground-state CP potential of LiH in front of Au surface:



temperatures: 10K, 50K, 100K, 200K, 300K
dashed line: evanescent contribution
dotted line: nonresonant (Lifshitz) contribution
solid line: total potential is independent of temperature

Dispersion forces in thermal nonequilibrium

in nonretarded limit: propagating component negligible in favour of evanescent component; planar surface, perfect metal: $r_s \approx -1$, $r_p \approx 1$:

$$U_n^{\rm nr}(z_A) = -\frac{k_{\rm B}T}{12\pi\varepsilon_0\hbar z_A^3}\sum_k |\mathbf{d}_{nk}|^2 {\sum_{j=0}^{\infty}}' \frac{\omega_{kn} {\rm e}^{-2jz_A\xi/c}}{\omega_{kn}^2 + j^2\xi^2}\left[1 + 2j\,\frac{z_A\xi}{c} + 2j^2\,\frac{z_A^2\xi^2}{c^2}\right]$$

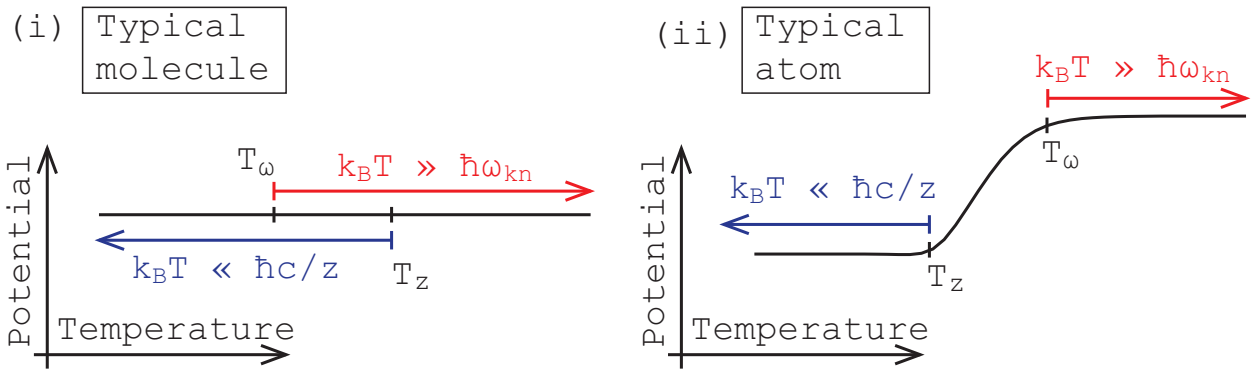
$$U_n^{\rm ev}(z_A) = \frac{1}{24\pi\varepsilon_0 z_A^3}\sum_k n(\omega_{kn})|\mathbf{d}_{nk}|^2$$

- define geometric temperature $T_z = \hbar c/(z_A k_{\rm B})$ (temperature of radiation whose wavelength is of order z_A)
- spectroscopic temperature $T_\omega = \hbar|\omega_{kn}|/k_{\rm B}$ (temperature necessary to noticably populate the upper level)[3]

[3] S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

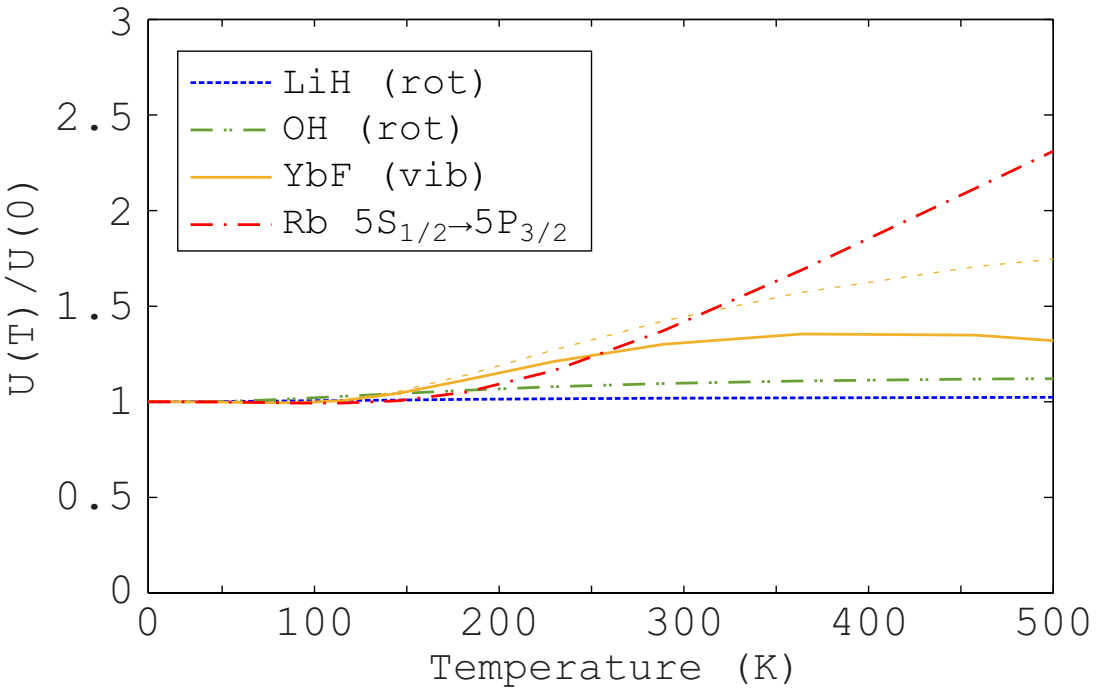
Temperature invariance despite large photon numbers

total potential becomes temperature-independent[4]
in the spectroscopic high-temperature limit $T \gg T_\omega = \hbar|\omega_{kn}|/k_B$:
$\xi/|\omega_{kn}| \gg 1 \Rightarrow$ lowest term in Matsubara sum dominates
and the geometric low-temperature limit $T \ll T_z = \hbar c/(z_A k_B)$:
$z_A \xi/c \ll 1 \Rightarrow$ exponential ≈ 1, sum can be performed



[4] S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

Temperature invariance despite large photon numbers



values of $z_A\omega_{kn}/c$:

0.046 (LiH)
0.26 (OH)
1.59 (YbF)
40.2 (Rb)

distance 5 μm from a planar Au surface[5]

[5] S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

Temperature invariance despite large photon numbers

temperature-invariance not restricted to planar surfaces

$$\mathbf{\Gamma}_\omega(\mathbf{r}) = \frac{\omega^2}{c^2}\mathbf{G}(\mathbf{r},\mathbf{r},\omega)$$

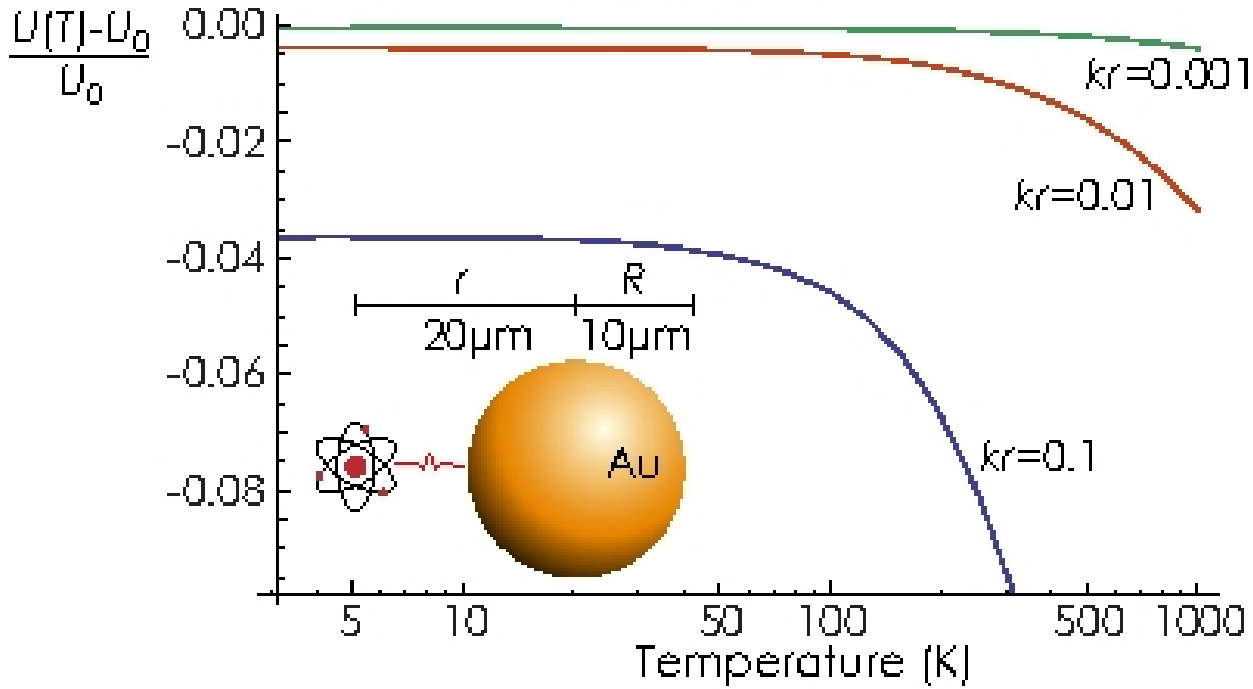
is always frequency-independent to leading order in nonretarded limit
$\Rightarrow$ Matsubara sum can be performed and the CP potential is always

$$U_n(\mathbf{r}) \simeq -\frac{1}{2\varepsilon_0}\sum_k \mathbf{d}_{nk}\cdot\mathbf{\Gamma}_0^{(S)}(\mathbf{r})\cdot\mathbf{d}_{kn}$$

which is manifestly independent of temperature and transition frequencies ω_{nk}
temperature correction resulting from slight non-cancellation:

$$U_n(\mathbf{r}) = \frac{1}{\varepsilon_0}\sum_k \mathbf{d}_{nk}\cdot\left\{\left[\mathrm{Re}\mathbf{\Gamma}^{(S)}_{\omega_{kn}}(\mathbf{r}) - \mathbf{\Gamma}_0^{(S)}(\mathbf{r})\right]\frac{k_B T}{\hbar\omega_{kn}} - \frac{1}{2}\mathrm{Re}\mathbf{\Gamma}^{(S)}_{\omega_{kn}}(\mathbf{r})\right\}\cdot\mathbf{d}_{kn} + \mathcal{O}(1/T)$$

Temperature invariance despite large photon numbers



exact CP potential $U(T)$ on a two-level atom (transition energy $\hbar ck$) outside a Au sphere, compared with U_0 for perfect conductor[6]

[6] S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, arXiv:1106.5015.

Summary

- description of thermal nonequilibrium forces using macroscopic QED
- dynamical theory of thermal nonequilibrium dispersion forces
- temperature-invariant dispersion forces in nonretarded regime near conducting bodies
- results from general low-frequency properties of the Green tensor, valid for arbitrary geometries

More on applications of macroscopic QED to dispersion forces:

- Stefan Buhmann: Tuesday, 16:00
- Agnes Sambale: Tuesday, 16:30