Awaking the vacuum in relativistic stars (and putting it to sleep)

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19th September 2011

10th Conference on Quantum Field Theory under the influence of external conditions
Benasque - Spain

Summary

A brief explanation about Quantum Field Theory in Curved Spaces

On the mechanism

Awaking the vacuum in relativistic stars

Putting the vacuum to sleep

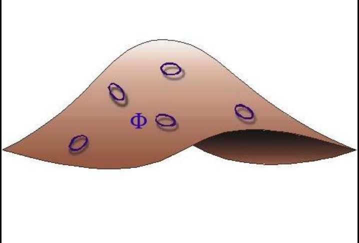
Conclusions and remarks

A brief explanation about Quantum Field Theory in Curved Spaces

Classical background spacetime $(\mathcal{M}, g_{ab})$

$$+$$

Quantized field Φ over $(\mathcal{M}, g_{ab})$



A brief explanation about Quantum Field Theory in Curved Spaces

In order to calculate the energy density in some state in QFTCS we must follow the steps below:

- Fix the background spacetime;
- Solve the field equation and find a complete set of modes;
- Expand the field operator in terms of negative and positive-norm modes;
- Formally substitute the field operator in the energy-momentum-tensor expression.

Due to the fact that the energy-momentum tensor depends quadratically on the field operator one need to employ some regularization and renormalization scheme to obtain meaningful physical results.

On the mechanism

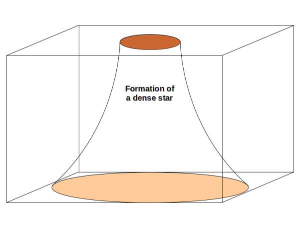
Hypothesis

The physical system:

Spherically symmetric star made of perfect fluid

+

A free quantum scalar field $\hat{\Phi}$



The space-time metric:

$$ds^2 \sim \begin{cases} -dt^2 + dx^2 + dy^2 + dz^2, & \text{past} \\ -f(\chi)\left(dt^2 - d\chi^2\right) + r^2(\chi)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), & \text{future} \end{cases} .$$

Field equation: the massless Klein-Gordon equation.

$$-\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\hat{\Phi}\right) + \xi R\hat{\Phi} = 0.$$

Note: for the mechanism in more general spacetimes see
WCCL and D.A.T. Vanzella, *Phys. Rev. Lett.*, **104**, 161102 (2010).

On the mechanism

In the past

Field equation:

$$\left(\partial_t^2 - \nabla^2\right)\hat{\Phi} = 0.$$

I will choose the modes that in the past are normalized plane-waves,

$$u_{\mathbf{k}} \overset{\text{past}}{\sim} \frac{1}{\sqrt{16\pi^3\omega_{\mathbf{k}}}} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}.$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2}$. The field $\hat{\Phi}$, can be written as

$$\hat{\Phi} = \int_{\mathbb{R}^3} d^3k\,(\hat{a}_{\mathbf{k}}u_{\mathbf{k}} + \text{h.c.}).$$

These modes define the following vacuum state, the usual Minkowski vacuum:

$$\hat{a}_{\mathbf{k}}|0\rangle_{\text{M}} = 0$$

for all $\mathbf{k} \in \mathbb{R}^3$.

On the mechanism

In the future

Field equation:

$$\frac{1}{f}\partial_t^2\hat{\Phi} - \frac{1}{fr^2}\partial_\chi\left(r^2\partial_\chi\hat{\Phi}\right) - \frac{1}{r^2\sin\theta}\left[\partial_\theta\left(\sin\theta\partial_\theta\hat{\Phi}\right) + \partial_\varphi^2\hat{\Phi}\right] + \xi R\hat{\Phi} = 0.$$

In the future the modes $u_{\mathbf{k}}$ are no longer plane-waves. Therefore, in order to learn about their behaviour in the future it is useful (but not necessary) to choose those modes that have the form

$$v_{\sigma l\mu} \overset{\text{future}}{\sim} T_\sigma \frac{F_{\sigma l}}{r} Y_{l\mu},$$

where $Y_{l\mu}$ are the spherical harmonics. The equations satisfied by T_σ and $F_{\sigma l}$ are

$$\frac{d^2}{dt^2}T_\sigma + \sigma T_\sigma = 0 \qquad -\frac{d^2}{d\chi^2}F_{\sigma l} + V_{\text{eff}}^{(l)}F_{\sigma l} = \sigma F_{\sigma l}.$$

On the mechanism

In the future

The equation for the radial part of the modes in the future is

$$-\frac{d^2}{d\chi^2}F_{\sigma l} + V_{\text{eff}}^{(l)}F_{\sigma l} = \sigma F_{\sigma l},$$

where I have defined the effective potential $V_{\text{eff}}^{(l)}$ as

$$V_{\text{eff}}^{(l)} := f\left[\xi R + \frac{l(l+1)}{r^2}\right] + \frac{1}{r}\frac{d^2 r}{d\chi^2}.$$

Certainly, these equations allow solutions with positive values of σ. It means that there is a set of solutions of the field equation with the form

$$v_{\varpi l\mu} \overset{\text{future}}{\sim} \frac{e^{-i\varpi t}}{\sqrt{2\varpi}}\frac{F_{\varpi l}(\chi)}{r(\chi)}Y_{l\mu}(\theta,\varphi),$$

where $\varpi = \sqrt{\sigma}$.

On the mechanism

In the future

Question: could $V_{\rm eff}^{(l)}$ be negative enough in order to allow solutions of the radial differential equation with $\sigma < 0$?

On the mechanism

In the future

Through the Einstein equations, it is possible to write $V_{\text{eff}}^{(0)}$ as

$$V_{\text{eff}}^{(0)} = f\left[8\pi G(\xi - 1/6)(\rho - 3P) + \frac{8\pi G}{3}(\bar{\rho} - \rho)\right],$$

where $\bar{\rho}(\chi) = \frac{3M(\chi)}{4\pi r^3(\chi)}$ and $M(\chi)$ is the mass of the object up to χ. From the experience with the time-independent Schrödinger equation

$$|V_{\text{eff}}^{(0)}|L^2 \ \sim \ 1$$

is an useful guide to search for bound solutions. From the previous expression for $V_{\text{eff}}^{(0)}$, it means that the condition for ρ is

$$\frac{G\rho L^2}{c^2} \approx \frac{\rho}{10^{15}\ \text{g/cm}^3}\left(\frac{L}{7\ \text{km}}\right)^2 \ \sim \ 1,$$

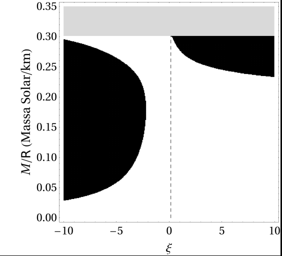
where I have used the typical density for neutron stars. The length-scale that appears in this case is consistent with the typical size of these objects.

On the mechanism

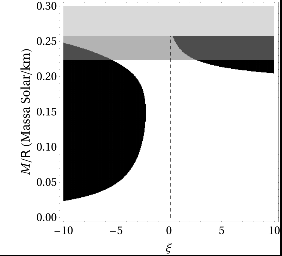
In the future: Numerical search for bound solutions

The results below show the existence of solutions for the radial equation with $\sigma < 0$ for different values of the mass-radius ratio of the star and ξ.

Uniform-density star



Parabolic density profile



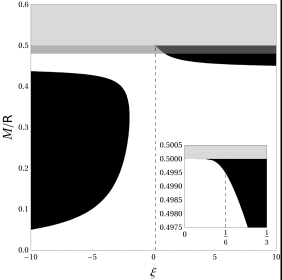
Note: these results can be found in
WCCL, G.E.A. Matsas and D.A.T. Vanzella, *Phys. Rev. Lett.*, **105**, 151102 (2010).

On the mechanism

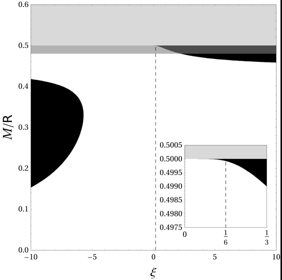
In the future: Bound solutions for a static spherical shell

The results below show the existence of solutions for the radial equation with $\sigma < 0$ for different values of the mass-radius ratio of the shell and ξ.

Modes with $l = 0$



Modes with $l = 1$



Note: these results will be presented in WCCL and D.A.T. Vanzella (in preparation).

On the mechanism

In the future

This result means that exist classically-stable stars for which the field equation also admits solutions with the following asymptotic form:

$$w_{\Omega l\mu} \overset{\text{future}}{\sim} \frac{e^{\Omega t - i\pi/12} + e^{-\Omega t + i\pi/12}}{\sqrt{2\Omega}} \frac{G_{\Omega l\mu}(\chi)}{r(\chi)} Y_{l\mu}(\theta,\varphi),$$

where $\Omega = \sqrt{|\sigma|}$. Since the $v_{\varpi l\mu}$ and $w_{\Omega l\mu}$ form a complete set of modes, it is possible to write

$$u_{\mathbf{k}} = \sum_{l\mu} \int d\varpi \left[\alpha_{\mathbf{k}\varpi l\mu} v_{\varpi l\mu} + \beta_{\mathbf{k}\varpi l\mu} v^*_{\varpi l\mu}\right] + \sum_{\Omega l\mu} \left[\alpha_{\mathbf{k}\Omega l\mu} w_{\Omega l\mu} + \beta_{\mathbf{k}\Omega l\mu} w^*_{\Omega l\mu}\right].$$

Therefore, the new base reveals that in the future some of the u-modes grow exponentially.

Awaking the vacuum in relativistic stars

Observable consequences

Let's consider the usual Minkowski vacuum in the asymptotic past as the initial state for the quantum field. Fluctuations:

$$ {}_{\mathrm{M}}\langle 0|\hat{\Phi}^2|0\rangle_{\mathrm{M}} \overset{\text{future}}{\sim} \frac{\kappa e^{2\bar{\Omega}t}}{2\bar{\Omega}} \left(\frac{\bar{G}}{r}\right)^2 \left[1 + \mathcal{O}\left(e^{-\epsilon t}\right)\right]. $$

The components of the energy-momentum tensor:

$$ \begin{aligned} {}_{\mathrm{M}}\langle 0|\hat{T}_{00}|0\rangle_{\mathrm{M}} \overset{\text{future}}{\sim} \; & {}_{\mathrm{M}}\langle 0|\hat{\Phi}^2|0\rangle_{\mathrm{M}} \left\{ \frac{(1-4\xi)}{2} \left(\bar{\Omega}^2 + \frac{(D\bar{G})^2}{\bar{G}^2}\right) \right. \\ & + (1-6\xi)\left(-\frac{2\xi D^2 r}{r} + \frac{(Dr)^2}{2r^2} \right. \\ & \left.\left. - \frac{D_i r D^i \bar{G}}{r\bar{G}}\right) + \mathcal{O}(e^{-\epsilon t}) \right\}, \end{aligned} $$

$$ \begin{aligned} {}_{\mathrm{M}}\langle 0|\hat{T}_{0i}|0\rangle_{\mathrm{M}} \overset{\text{future}}{\sim} \; & {}_{\mathrm{M}}\langle 0|\hat{\Phi}^2|0\rangle_{\mathrm{M}} \left\{ (1-4\xi)\frac{\bar{\Omega} D_i \bar{G}}{\bar{G}} - (1-6\xi)\frac{\bar{\Omega} D_i r}{r} \right. \\ & \left. + \mathcal{O}(e^{-\epsilon t}) \right\}. \end{aligned} $$

Awaking the vacuum in relativistic stars

An estimate of the asymptotic exponential growth of the energy density

Consider a star of size L and suppose that the typical mode wavelength is of order L.

$$\rho_{\text{vacuum}} \overset{\text{future}}{\sim} e^{\frac{2ct}{L}} \frac{\hbar c}{L^4} \sim \exp\left\{\frac{t/10^{-9}\ \text{s}}{L/1\ \text{m}}\right\} \frac{3\ \times\ 10^{-46}\ \text{g/cm}^3}{(L/1\ \text{m})^4}.$$

For $L \approx 10^4$ m and $\rho \approx 10^{15}$ g/cm^3 (typical size and density of a neutron star)

$$\frac{\rho_{\text{vacuum}}}{\rho} \overset{\text{future}}{\sim} 3 \times 10^{-77} \times \exp\left\{\frac{t}{10^{-5}\ \text{s}}\right\}.$$

Therefore

$$t \sim 2 \times 10^{-3}\ \text{s} \qquad \Rightarrow \qquad \frac{\rho_{\text{vacuum}}}{\rho} \sim 1.$$

Putting the vacuum to sleep

Probing the unstable phase with detectors

Let's consider a two-level Unruh-DeWitt detector coupled to the quantum field according to

$$\hat{S}_{\rm I} = \epsilon \int_{-\infty}^{+\infty} c(\tau)\hat{m}(\tau)\hat{\Phi}[x(\tau)]d\tau.$$

The excitation probability is given by

$$P_{\rm exc} = \epsilon^2 |\langle E_0|\hat{m}(0)|E_1\rangle|^2 \mathcal{F}(E_1 - E_0),$$

where

$$\mathcal{F}(E) := \int_0^T d\tau \int_0^T d\tau' e^{-iE(\tau-\tau')} \langle 0|\hat{\Phi}[x(\tau)]\hat{\Phi}[x(\tau')]|0\rangle,$$

the detector response function.

Putting the vacuum to sleep

Probing the unstable phase with detectors

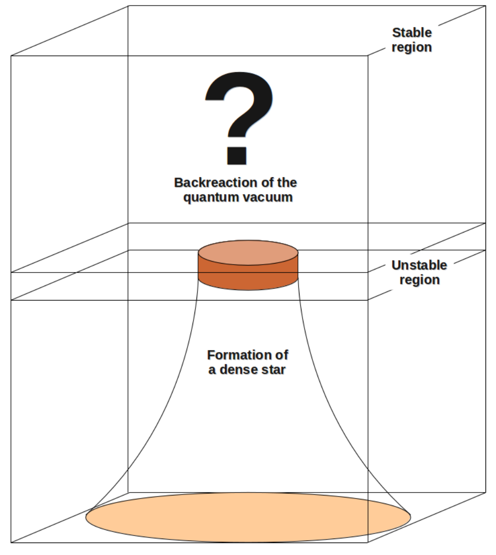
Suppose now that the detector is switched on when the vacuum is awakened. After a period of time T ($\bar{\Omega}T/\sqrt{-g_{00}} >> 1$), the dominant contribution to $P_{\rm exc}$ is

$$P_{\rm exc} \sim \epsilon^2 e^{\frac{2\bar{\Omega}T}{\sqrt{-g_{00}}}} \frac{|\langle E_0|\hat{m}(0)|E_1\rangle|^2}{2\bar{\Omega}\left[\left(\frac{\bar{\Omega}}{\sqrt{-g_{00}}}\right)^2 + \Delta E^2\right]} \times \int_{\mathbb{R}^3} d^3k \left|\alpha_{\mathbf{k}\bar{\Omega}} e^{i\pi/12} - \beta^*_{\mathbf{k}\bar{\Omega}} e^{-i\pi/12}\right|^2 \left[\frac{\bar{H}(\mathbf{x}_0)}{\Psi}\right]^2$$

Therefore, particle detectors will excite copiously during the unstable phase.

Putting the vacuum to sleep

Burst of particles



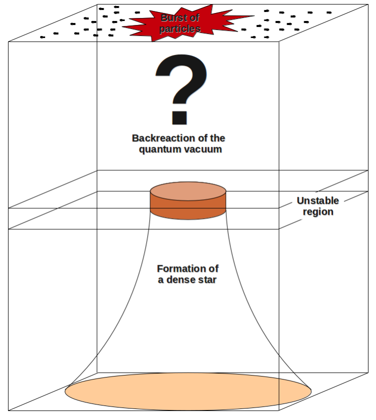
Putting the vacuum to sleep

Burst of particles

The calculation of the beta coefficient of the Bogoliubov transformation between the in and the out mode basis in this case leads to

$$|\beta_{l\mathbf{k}}|^2 \sim e^{2\bar{\Omega}T}$$

for $\bar{\Omega}T >> 1$. Here T is the duration of the unstable phase from the point of view of the static observers of that region of the background spacetime.



When the vacuum falls asleep a burst of particle is realised.

Note: these results will be presented in
A.G.S. Landulfo, WCCL, G.E.A. Matsas and D.A.T. Vanzella (in preparation).

Conclusions and remarks

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Conclusions and remarks

- In spite of the exponential growth, the energy-momentum tensor is covariantly conserved during the unstable phase.
- The hypothesis we have used, like a scalar field and an asymptotically static spacetime both in past and future, are just in order to simplify the arguments and put in evidence the main characteristics of the effect.
- The effect defines a time-scale for the backreaction. In the case of a neutron star with $\rho \approx 10^{15}\ \mathrm{g/cm^3}$ and $r_0 \approx 10^4\ \mathrm{m}$, the time that takes for $\rho_{\mathrm{vacuum}} \approx \rho$ is about $2 \times 10^{-3}\ \mathrm{s}$. Once the vacuum energy has become comparable to the star energy it is imperative taking it into account on the right-hand side of the Einstein equations. In this case, only by dealing with the vacuum backreaction one can decide the ultimate fate of the astrophysical object.

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- Although I have shown that the effect can be trigged for unusual values of ξ, the vacuum may be awaken for more natural values of ξ in more complicated space-times.

Conclusions and remarks

- The vacuum awakening mechanism provides an interesting interconnection between QFTCS and observational Astrophysics. The observation of a stable star could be used to rule out the existence of certain fields in Nature. Since 95% of the energy of the Universe is still unknown, such criterion is very welcome. On the other hand, the existence of fields with certain ξ may spoil the stability of some classically-stable stars.

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- The mechanism engendering the particle creation after the end of the vacuum instabilities is independent of the rate at which the background is changing. In this sense, this mechanism is different from the well-known particle creation phenomena in expanding universes and in black hole evaporation.

Thank you for your attention!