Fermion Determinant in non-Abelian radial backgrounds

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Introduction

- Recently, we developed a computational method (Hybrid of numerical and analytical) to evaluate functional determinant (mainly for boson). Dunne;Hur;Lee;M
- fermion effective action (formally)

$$\Gamma \sim -\ln\det(-i\gamma\cdot D + m) = -1/2\ln\det((\gamma\cdot D)^2 + m^2)$$

  using anti-hermitian gamma matrices $\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$
- Non-Abelian gauge field: $A_\mu = \eta^{\pm}_{\mu\nu a}\tau^a x_\nu f(r)$
  - Case I $\quad f(r) = \frac{1}{r^2}\frac{r^{2\alpha}}{\rho^2 + r^{2\alpha}}$
  - Case II $\quad f(r) = \frac{1}{\rho^2 + r^2} 1/2(1 + \tanh(\frac{r-R}{\beta\rho}))$
- Interesting Results
  - Exact results for the fermion determinants (when $m = 0$, in the case I)
  - Interaction energy between instanton and antiinstanton. (case II)
  - Numerical function $\Gamma_{\rm ren}(m,\alpha)$,$\Gamma_{\rm ren}(m,\beta,R)$

Boson Determinant(Review)

- Partial wave method is applicable to generic form of boson determinant

$$\det[-\partial^2 + V(r) + m^2]$$

in a radial background $V(r)$.

- 4D renormalized effective action

$$\Gamma_{\rm ren} = \ln \frac{\det(-\partial^2 + V(r) + m^2)}{\det(-\partial^2 + m^2)} + \Gamma_c$$

- sum of partial wave contributions

$$\Gamma_{\rm ren} = \sum_l g_l \ln \frac{\det(-\partial_r^2 + \mathcal{V}(r) + m^2)}{\det(-\partial_r^2 + \mathcal{V}_{\rm free}(r) + m^2)} + \Gamma_c$$

$$\mathcal{V}(r) = \mathcal{V}_{\rm free}(r) + V(r) = \frac{4l(l+1) + 3/4}{r^2} + V(r)$$

$$g_l = (2l+1)^2$$

Angular momentum cutoff L

- Introduce a cutoff L

$$\Gamma_{\rm ren} = \Gamma_L + \Gamma_H$$

$$\Gamma_L = \sum_{l=0}^{L} (\cdots); \qquad \Gamma_H = \sum_{l=L+1/2}^{\infty} (\cdots) + \Gamma_c$$

Gelfand-Yaglom theorem

$$\frac{\det(-d^2 + V_1)}{\det(-d^2 + V_2)} = \frac{\psi_1}{\psi_2}\bigg|_{\rm end}$$

$$(-d^2 + V_{1,2})\psi_{1,2} = 0$$

H-K (Schwinger) method

$$\ln\det(-d^2 + v)$$

$$= \int_0^\infty \frac{ds}{s} \mathrm{Tr}[e^{-s(-d^2+V+m^2)}]$$

High /Low sectors

Different strategies for two sectors

- Γ_L: Solve the ODE. analytically/numerically for each l
- Γ_H: Develop an asymptotic expansion of large l and perform the sum ℓ

$$\Gamma_H = Q_2 L^2 + Q_1 L + Q_{\log} + \sum_{n=1} Q_{-n} L^{-n}$$

- Analytically exact calculation of Γ_{ren} when it is possible

$$\lim_{L\to\infty} \left(\Gamma_L + Q_2 L^2 + Q_1 L + Q_{\log} \right)$$

- Numerical calculation of Γ_{ren}

$$\left(\Gamma_L + Q_2 L^2 + Q_1 L + Q_{\log} \right) + \sum_{n=1}^{N} Q_{-n} L^{-n}$$

with a finite L. The error is $1/L^{N+1}$.

QCD instanton determinant when $m = 0$

- Instanton determinant of scalar: same with the above except for using covariant derivative D_μ

$$D_\mu = \partial_\mu - iA_\mu; \quad A_\mu = \eta_{\mu\nu a}\tau^a x_\nu f(r)$$

with $f(r) = 1/(\rho^2 + r^2)$, $\alpha = 1$

- Exact solution of GY equation when $m = 0$

$$\Gamma_L = \sum_{l=0}^{L}(2l+1)(2l+2)\ln\frac{2l+1}{2l+2}$$

$$\Gamma_H = 2L^2 - 4L - 1/6\ln L + 172/72 - 1/3\ln 2 + \frac{1}{6}\ln\rho$$

- 'tHooft result:

$$\Gamma_{\text{ren}}(m=0) = 0.145873\ldots + 1/6\ln\mu\rho$$

Numerical works when $m \neq 0$, $f(r) = r^{2\alpha-2}/(1+r^{2\alpha})$

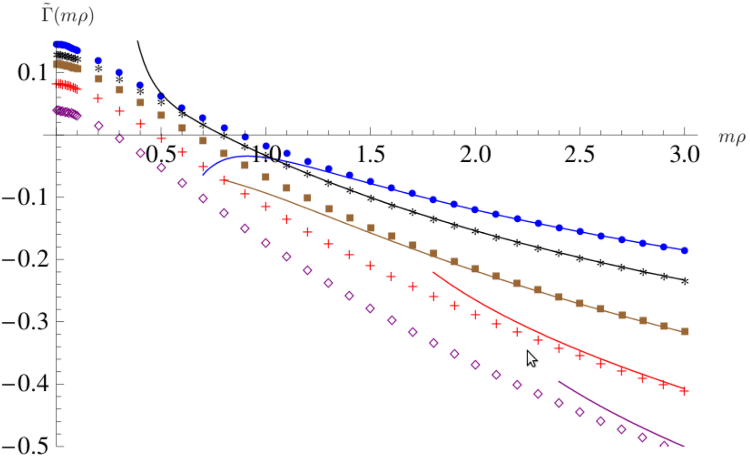


FIG. 5: Plots of the modified effective action as a function of $m\rho$. The (blue) dots, (black) stars, (brown) squares, (red) crosses and (purple) diamonds denote the values we get numerically for $\alpha = 1, 2, 3, 4, 5$ and the solid lines are for the associated large mass approximations.

Factorization

- representation of gamma matrix

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ -\bar{\sigma}_\mu & 0 \end{pmatrix}, \qquad (\text{with } \sigma_\mu = (\vec{\sigma}, i) \text{ and } \bar{\sigma}_\mu = (\vec{\sigma}, -i) = (\sigma_\mu^\dagger$$

- squared Dirac operator

$$(\gamma \cdot D)^2 = \begin{pmatrix} -D^2 - \frac{1}{2}\eta^{(-)}_{\mu\nu a}\sigma_a F_{\mu\nu} & 0 \\ 0 & -D^2 - \frac{1}{2}\eta^{(+)}_{\mu\nu a}\sigma_a F_{\mu\nu} \end{pmatrix}$$

- effective action

$$\begin{aligned} \Gamma_{\text{ren}} &= -\frac{1}{2}\ln\frac{\det[(\gamma\cdot D)^2 + m^2]}{\det[(\gamma\cdot\partial)^2 + m^2]} + \Gamma_c \\ &= \Gamma^{(+)}_{\text{ren}} + \Gamma^{(-)}_{\text{ren}} \end{aligned}$$

- It was shown that

$$\Gamma^{(+)}_{\text{ren}} - \Gamma^{(-)}_{\text{ren}} = \frac{1}{2}\frac{1}{(4\pi)^2}\ln\frac{m^2}{\mu^2}\int d^4x\, \mathrm{tr}(F_{\mu\nu}{}^*F_{\mu\nu}).$$

Factorization

- Simplified expression

$$\Gamma_{\rm ren} = 2\Gamma_{\rm ren}^{(\pm)} \mp \frac{1}{2} \ln \frac{m^2}{\mu^2} w$$

- winding number

$$w = \frac{1}{(4\pi)^2} \int d^4x \, \mathrm{tr}(F_{\mu\nu} {}^*F_{\mu\nu})$$

-

$$(\text{Case I}) \quad : \quad w = \begin{cases} 1 & , \quad \alpha > 1 \\ -1 & , \quad \alpha < -1 \end{cases} ,$$

$$(\text{Case II}) \quad : \quad w = \begin{cases} 1 & , \quad \beta > 0 \\ 0 & , \quad \beta < 0 \end{cases} .$$

Radial operator

- radial operator

$$-D^2 - \frac{1}{2}\eta^{(\pm)}_{\mu\nu a}\sigma_a F_{\mu\nu}$$
$$= -\frac{\partial^2}{\partial r^2} - \frac{3}{r}\frac{\partial}{\partial r} + \frac{4}{r^2}\vec{L}^2 + 8f(r)\vec{T}\cdot\vec{L}^{(\pm)} + 3r^2 f(r)^2$$
$$+4g_F(r)\vec{S}\cdot\vec{T},$$

with

$$\eta^{(\pm)}_{\mu\nu a}\sigma_a F_{\mu\nu} = -2\left[4f(r) + rf'(r) - 2r^2 f(r)^2\right]\sigma_a\tau_a \equiv -2g_F(r)\sigma_a\tau_a,$$

- 4D radial Laplacian:

$$\partial^2_{(l)} = \frac{\partial^2}{\partial r^2} + \frac{3}{r}\frac{\partial}{\partial r} - \frac{4l(l+1)}{r^2}$$

- addition of angular momentum

$$J^a = L^a + T^a + S^a = Q^a + S^a; \quad Q^a = L^a + T^a$$

$\vec{L} \cdot \vec{T}$ and $\vec{S} \cdot \vec{T}$

- good quantum numbers:
  $J^2 = j(j+1)$, $Q^2 = q(q+1)$, $S^2 = 1/2(1/2+1) = 3/4$

$$q = l \pm 1/2; \quad j = q \pm 1/2; \quad j = l+1,\ l,\ l,\ l-1$$

- Diagonalize $\vec{L} \cdot \vec{T}$

$$\begin{aligned}\vec{L} \cdot \vec{T} &= \frac{1}{2}(Q^2 - L^2 - S^2) \\ &= \frac{1}{2}(q(q+1) - l(l+1) - \frac{3}{4}) \to l \quad (\text{or } -(l+1))\end{aligned}$$

- but not $\vec{S} \cdot \vec{T}$

$$\begin{aligned}4g_F(r)\vec{S} \cdot \vec{T} &\to g_F(r), & (j = l \pm 1) \\ &\to \tfrac{g_F(r)}{2l+1}\begin{pmatrix} -2l-3 & 4\sqrt{l(l+1)} \\ 4\sqrt{l(l+1)} & -2l+1 \end{pmatrix}, & (j = l)\end{aligned}$$

Potentials for various sectors

- Classify the potential depending on l, J

$$\mathcal{V}_{l,l+1}(r) = 3r^2 f(r)^2 + 4lf(r) + g_F(r),$$

$$\mathcal{V}_{l,l-1}(r) = 3r^2 f(r)^2 - 4(l+1)f(r) + g_F(r),$$

$$\boldsymbol{\mathcal{V}}_{l,l}(r) = 3r^2 f(r)^2 + 4f(r) \begin{pmatrix} l & 0 \\ 0 & -l-1 \end{pmatrix} + \frac{g_F(r)}{2l+1} \begin{pmatrix} -2l-3 & 4\sqrt{l(l+1)} \\ 4\sqrt{l(l+1)} & -2l+1 \end{pmatrix}, \qquad (l \neq 0)$$

- when $l = 0$

$$\mathcal{V}_{0,0} = 3r^2 f(r)^2 - 3g_F(r).$$

- Task: solve the GY equations with these potential for each sector.

$$P_{l,j} \equiv \ln \frac{\det(-\partial_{(l)}^2 + \mathcal{V}_{l,j} + m^2)}{\det(-\partial_{(l)}^2 + m^2)} = \ln \frac{\psi_{l,j}}{\psi_l}$$

Rearrangement of the sum

- Low partial wave part $\Gamma_L^{(-)}$:

$$\Gamma_L^{(-)} = -\frac{1}{2}\Big(\sum_{l=\frac{1}{2}}^{L} \Big\{ (2l+1)^2 \Big(P_{l,l} + P_{l-\frac{1}{2},l+\frac{1}{2}} + P_{l+\frac{1}{2},l-\frac{1}{2}} \Big) - \Big(P_{l,l+1} + P_{l+\frac{1}{2},l-\frac{1}{2}} \Big) \Big\} + \big\{ P_{0,0} - P_{0,1} \big\} \Big),$$

- High angular momentum part $\Gamma_H^{(-)}$:

$$\Gamma_H^{(-)} = \frac{1}{2}\Big\{ \int_0^\infty \frac{ds}{s} \Big(e^{-m^2 s} \Big)_{\mathrm{reg}} \int_0^\infty dr \sum_{l=L+\frac{1}{2}}^{\infty} \mathcal{G}_l(r,r;s) + \text{c.t.} \Big\}$$

$$\mathcal{G}_l(r,r;s) = (2l+1)^2 \Big\{ \mathrm{tr}\,\mathbf{G}_{l,l}(r,r;s) + G_{l-\frac{1}{2},l+\frac{1}{2}} + G_{l+\frac{1}{2},l-\frac{1}{2}} - 2G_l^{\text{free}} - G_{l+\frac{1}{2}}^{\text{free}} - G_{l-\frac{1}{2}}^{\text{free}} \Big\} - \Big\{ G_{l,l+1} + G_{l+\frac{1}{2},l-\frac{1}{2}} - G_l^{\text{free}} - G_{l+\frac{1}{2}}^{\text{free}} \Big\}$$

Exact solutions to GY equation when $m = 0$

Factorization of squared Dirac Eq.(going back to first order Dirac)

$$-\left(\frac{\partial}{\partial r}+\frac{3}{r}+\frac{4}{r}\vec{L}^{(+)}\cdot\vec{S}+4rf(r)\vec{S}\cdot\vec{T}\right)\left(\frac{\partial}{\partial r}-\frac{4}{r}\vec{L}^{(+)}\cdot\vec{S}-4rf(r)\vec{S}\cdot\vec{T}\right)$$

- when $j = l + 1$:

$$\left(\frac{\partial}{\partial r}+\frac{3}{r}+\frac{2l}{r}+rf(r)\right)\left(\frac{\partial}{\partial r}-\frac{2l}{r}-rf(r)\right)\psi(r)=0.$$

- Easy to solve the first-order equation

$$\left(\frac{\partial}{\partial r}-\frac{2l}{r}-rf(r)\right)\psi(r)=0.$$

GY wave function: it has the correct small-r behaviour:

$$\psi_{l,j=l+1}(r)=r^{2l}e^{\int_0^r r_1 f(r_1)dr_1}.$$

- Direct application of the above method gives us (Not a GY sol)

$$\psi_1(r) = r^{-2(l+1)} e^{\int_0^r r_1 f(r_1) dr_1},$$

- independent solution

$$\psi_{l,j=l-1}(r) = 2(2l+1) r^{-2(l+1)} e^{\int_0^r r_1 f(r_1) dr_1} \int_0^r r_2^{4l+1} e^{-2\int_0^{r_2} r_1 f(r_1) dr_1} dr_2$$

- For the case I, it is possible to do the integrals:

$$P_{l,j=l+1} \sim \ln\left[\frac{\psi_{l,j=l+1}(R_e)}{\psi_l^{\text{free}}(R_e)}\right] \sim \ln R_e,$$

$$P_{l,j=l-1} \sim \ln\left[\frac{\psi_{l,j=l-1}(R_e)}{\psi_l^{\text{free}}(R_e)}\right] \sim -\ln R_e + \ln\left(\frac{2l+1}{2l}\right),$$

$j = l = 0$ sector and zero mode

- We turn to the sector $j = l = 0$.
- GY solution

$$\psi_1(r)|_{j=l=0} = e^{-3\int_0^r r_1 f(r_1) dr_1}.$$

It goes to zero when $r \to \infty$, because of zero mode.

- Introducing m, resolve GY eq:

$$P_{l=0,j=0} \sim \ln\left[\frac{\psi_{j=l=0}(R_e)}{\psi^{\text{free}}_{l=0}(R_e)}\right] = \ln m + \ln\left[\frac{\Gamma\left(1+\frac{1}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)}{2\Gamma\left(\frac{3}{\alpha}\right)}\right],$$

- From $l = 0, j = 1$

$$-P_{0,-1} = -\ln\left[\frac{\psi_{l=0,j=1}(R_e)}{\psi^{\text{free}}_{l=0}(R_e)}\right] = \ln m - \ln 4.$$

Hence,

$$P_{o,o} - P_{0,1} = 2\ln m + \ln\left[\frac{\Gamma\left(1+\frac{1}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)}{8\Gamma\left(\frac{3}{\alpha}\right)}\right], \qquad \text{(Case I)}.$$

Γ_L for small mass

- We can solve the 2×2 matrix GY eq. for the sector $(l, j = l)$ to find

$$P_{l,j=l} \sim \ln \frac{\det \mathbf{\Psi}_l(R_e)}{\psi_l^{\text{free}}(R_e)^2} = \ln \left[\frac{(2l+1)^3 \Gamma\left(\frac{2l+1}{\alpha}\right)^4}{8l(l+1)^2 \Gamma\left(\frac{2l}{\alpha}\right)^2 \Gamma\left(\frac{2l+2}{\alpha}\right)^2} \right].$$

- Sum of all results:

$$\Gamma^{(-)}_{l \le L}(A; m) = -\ln m - \frac{1}{2} \ln \left[\frac{\Gamma\left(1 + \frac{1}{\alpha}\right) \Gamma\left(\frac{2}{\alpha}\right)}{8 \Gamma\left(\frac{3}{\alpha}\right)} \right]$$
$$-\frac{1}{2} \sum_{l=\frac{1}{2}}^{L} \left\{ (2l+1)^2 \ln \left[\frac{(2l+1)^2 \Gamma\left(\frac{2l+1}{\alpha}\right)^4}{4l(l+1) \Gamma\left(\frac{2l}{\alpha}\right)^2 \Gamma\left(\frac{2l+2}{\alpha}\right)^2} \right] + \ln\left(\frac{2l+1}{2l+2}\right) \right.$$

Expressions for low partial waves

- For large enough L,

$$\Gamma^{(-)}_{l\le L} = \frac{2L^2}{\alpha} + \frac{3L}{\alpha} + [\ln(2L)+\gamma]\left(\frac{\alpha}{6} + \frac{1}{6\alpha} + \frac{1}{2}\right) - \ln m - \frac{1}{2}\ln\left[\frac{\Gamma\left(1+\frac{1}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)}{8\Gamma\left(\frac{3}{\alpha}\right)}\right] + C(\alpha) + O\left(\frac{1}{L}\right),$$

$$C(\alpha) = -\frac{1}{2}\sum_{l=\frac{1}{2},1,\cdots}^{\infty}\left\{(2l+1)^2\ln\left[\frac{(2l+1)^2\Gamma\left(\frac{2l+1}{\alpha}\right)^4}{4l(l+1)\Gamma\left(\frac{2l}{\alpha}\right)^2\Gamma\left(\frac{2l+2}{\alpha}\right)^2}\right] + \ln\left(\frac{2l+1}{2l+2}\right) + \frac{4l}{\alpha} + \frac{2}{\alpha} + \frac{\alpha^2+3\alpha+1}{6l\alpha}\right\}.$$

- Effective action of one chiral sector

$$\Gamma_{\text{ren}}^{(-)} = \left[\ln\left(\frac{\mu}{2}\right)+\gamma\right]\left(\frac{\alpha}{6}+\frac{1}{6\alpha}+\frac{1}{2}\right)-\ln m$$
$$-\frac{1}{2}\ln\left[\frac{\Gamma\left(1+\frac{1}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)}{8\Gamma\left(\frac{3}{\alpha}\right)}\right]+\frac{5\alpha}{36}-\frac{47}{72\alpha}+C(\alpha).$$

- Total effective action

$$\Gamma_{\text{ren}} = 2\Gamma_{\text{ren}}^{(-)}+\ln\left(\frac{m}{\mu}\right)$$
$$= -\ln(m\rho)+\frac{\alpha^2+1}{3\alpha}\ln(\mu\rho)+\tilde{C}(\alpha),$$

with

$$\tilde{C}(\alpha) = (\gamma-\ln 2)\left(\frac{\alpha}{3}+\frac{1}{3\alpha}+1\right)-\ln\left[\frac{\Gamma\left(1+\frac{1}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)}{8\Gamma\left(\frac{3}{\alpha}\right)}\right]$$
$$+\frac{5\alpha}{18}-\frac{47}{36\alpha}+2C(\alpha).$$

Table

- Values of $\tilde{C}(\alpha)$

α	1	2	3	4
$\tilde{C}(\alpha)$	-0.291747	-0.269189	-0.378112	-0.590437

α	5	10	20
$\tilde{C}(\alpha)$	-0.883495	-3.16105	-10.0277

massive case: numerical works

- ratio functions

$$\mathcal{R}_{l,j}(r) = \frac{\psi_{l,j}(r)}{\psi_l^{\text{free}}(r)},$$

- the GY equation for the ratio

$$\frac{d^2\mathcal{R}_{l,j}}{dr^2} + \left(\frac{1}{r} + 2m\frac{I'_{2l+1}(mr)}{I_{2l+1}(mr)}\right)\frac{d\mathcal{R}_{l,j}}{dr} - \mathcal{V}_{l,j}\mathcal{R}_{l,j} = 0,$$

- the initial value boundary conditions

$$\mathcal{R}_{l,j}|_{r=0} = 1, \qquad \mathcal{R}'_{l,j}|_{r=0} = 0.$$

- 2×2 matrix function and matrix GY eq.

$$\boldsymbol{\mathcal{R}}_l(r) = \frac{\boldsymbol{\Psi}_l(r)}{\psi_l^{\text{free}}(r)}.$$

$$\frac{d^2\boldsymbol{\mathcal{R}}_l}{dr^2} + \left(\frac{1}{r} + 2m\frac{I'_{2l+1}(mr)}{I_{2l+1}(mr)}\right)\frac{d\boldsymbol{\mathcal{R}}_l}{dr} - \boldsymbol{\mathcal{V}}_{l,l}\boldsymbol{\mathcal{R}}_l = 0,$$

initial boundary conditions

$$\mathcal{R}_l|_{r=0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathcal{R}'_l|_{r=0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

the functional determinant of matrix differential operator can be determined in terms of the ordinary determinant of the 2×2 matrix $\mathcal{R}_l(r = \infty)$.

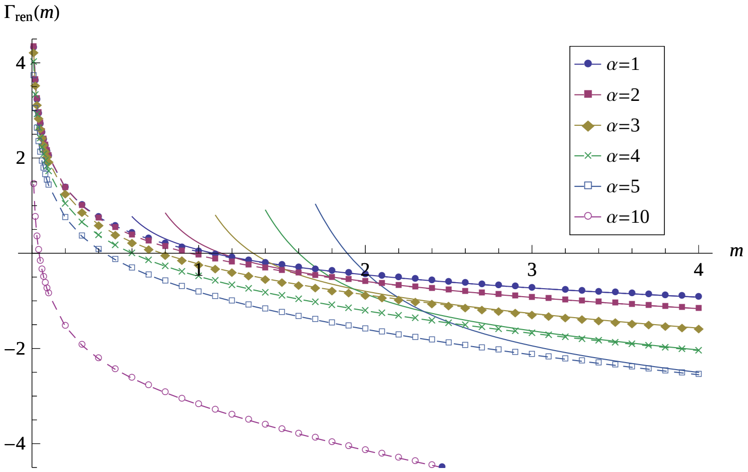
$$\Gamma^{(-)}_{l \le L} = -\frac{1}{2} \Bigg[\left(\ln \mathcal{R}_{0,0} - \ln \mathcal{R}_{0,1} \right) + \sum_{l=\frac{1}{2},1,\cdots}^{L} \Big\{ (2l+1)^2 \left(\ln \det \mathcal{R}_l \right.$$

$$\left. + \ln \mathcal{R}_{l-\frac{1}{2},l+\frac{1}{2}} + \ln \mathcal{R}_{l+\frac{1}{2},l-\frac{1}{2}} \right) - \ln \mathcal{R}_{l,l+1} - \ln \mathcal{R}_{l+\frac{1}{2},l-\frac{1}{2}} \Big\} \Bigg] \Bigg|_{r=}$$

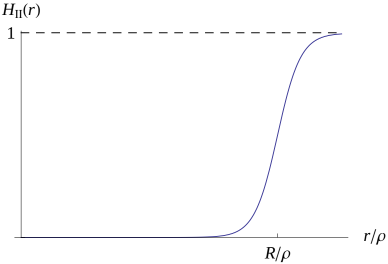
the high partial wave part (upto $O(\frac{1}{L^2})$)

$$\Gamma^{(-)}_{l > L} = \int_0^\infty dr \left[Q_2 L^2 + Q_1 L + Q_{\log} \ln \left(\frac{2L(u+1)}{\mu r} \right) + Q_0 + \frac{Q_{-1}}{L} + \frac{Q_{-2}}{L^2} + \cdots \right],$$

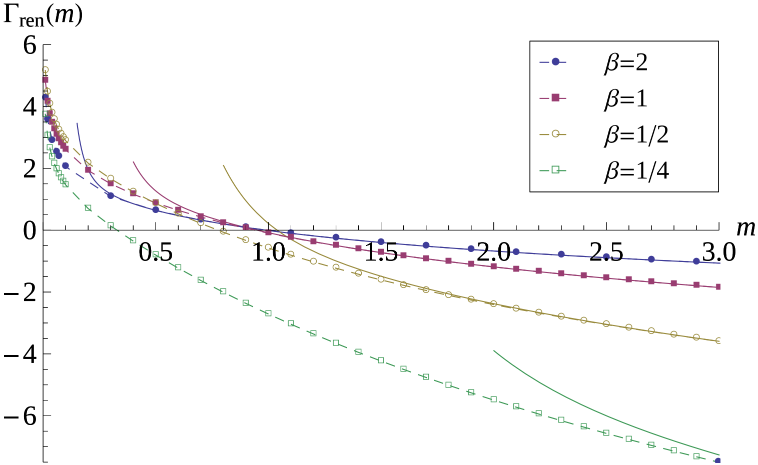
case I, $\alpha = 1, 2, 3, 4, 5, 10$



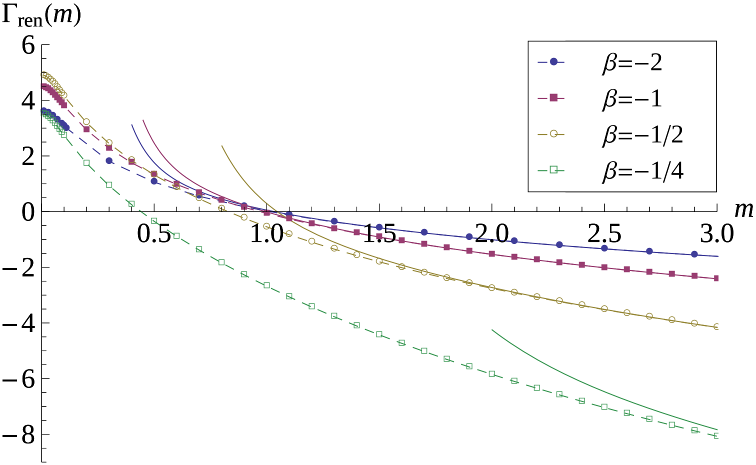
case II: potential



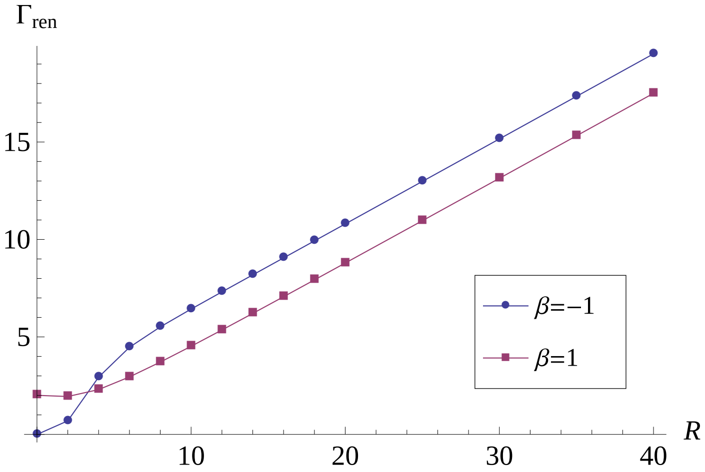
$\Gamma_{\text{ren}}(m)$ when $\beta > 0$



$\Gamma_{\text{ren}}(m), \beta < 0$

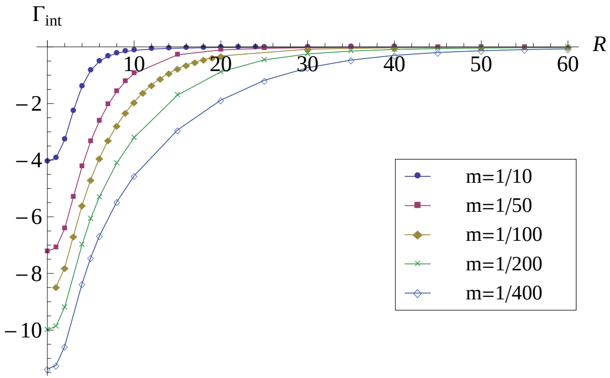


$\Gamma_{\text{ren}}(R)$ with $m = 1$



Interaction energy

$$\Gamma_{\mathrm{int}}(R;m) = \Gamma^{(\mathrm{II})}_{\mathrm{ren}}(R;m)\Big|_{\beta=-\beta_0} - \left[\Gamma^{(\mathrm{I})}_{\mathrm{ren}}(m)\Big|_{\alpha=1} + \Gamma^{(\mathrm{II})}_{\mathrm{ren}}(R;m)\Big|_{\beta=\beta_0}\right],$$

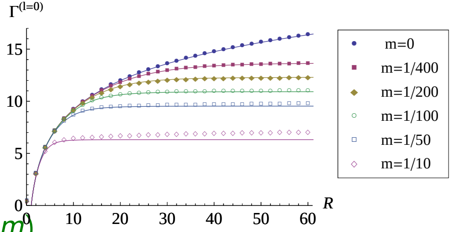


zero mode dominance when $m << 1$

“would-be” zero mode dominance

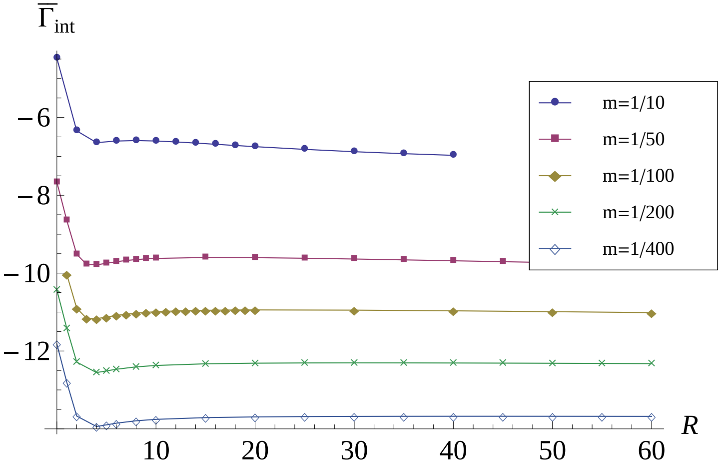
$$\Gamma^{(l=0)}_{\text{approx}}(R;m) = -\ln\left(\frac{m^2}{A} + \frac{1}{R^4}\right),$$

$$(A \approx 5.55).$$



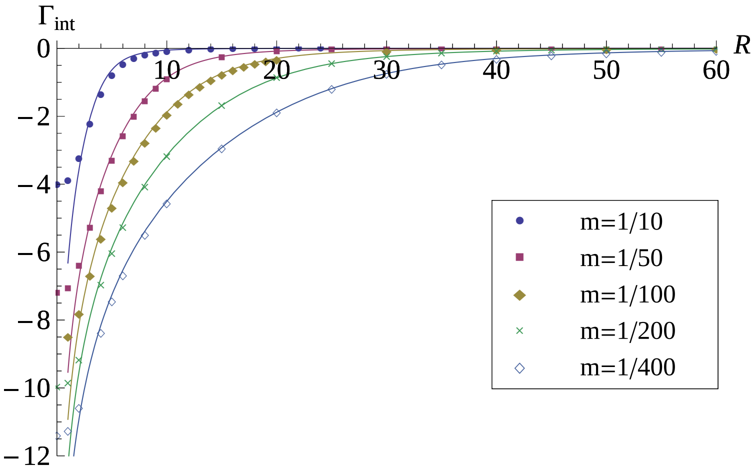
$$\bar{\Gamma}_{\text{int}}(R;m) = \Gamma_{\text{int}}(R;m) - \Gamma^{(l=0)}(R;m)$$

Interaction energy without zeromode



Interaction energy again

$$\Gamma_{\text{int}}^{(\text{approx})}(R;m) = -\ln\left(1 + \frac{A}{m^2 R^4}\right).$$



Remarks

- We have evaluated the 4-D spinor effective action in radial non-Abelian, gauge backgrounds, using a hybrid (numerical/analytical) scheme based on partial-wave cutoff method. (For the Abelian case, see DHHM)
- In the small mass limit, we get the log factor $-\ln(m\rho)$ (suppression of instanton effects by light fermions)
- We also get a negative contribution $-\frac{\alpha \ln \alpha}{3}$ in a background with $\alpha \gg 1$.
- We studied the interaction energy between one intanton and one antiinstanton.
- It is approximated by a simple formula

$$\Gamma_{\text{int}}^{(\text{approx})}(R; m) = -\ln\left(1 + \frac{A}{m^2 R^4}\right)$$

when m is small and R is large.