Time-dependent Robin boundary conditions in the dynamical Casimir effect

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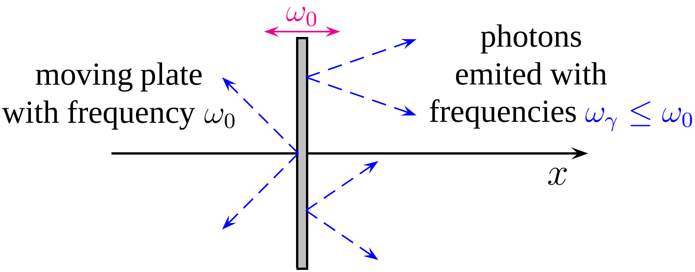
Plan of the talk:

1. Introduction: basic concepts and purposes
2. Experimental proposals
3. Robin boundary conditions
4. Modeling a moving mirror by a static one
5. Modified Robin boundary conditions
6. Final remarks and perspectives

Introduction: basic concepts and purposes

- The **dynamical Casimir effect** (DCE) consists, essentially, by
  - particle creation caused by moving boundaries;
  - radiation reaction forces on the moving boundaries.
- It already manifests for a unique moving plate:



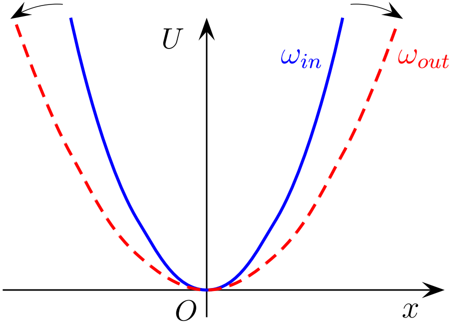
- For non-relativistic motion, we have $\omega_\gamma \leq \omega_0$.

Introduction: basic concepts and purposes

- **A quantum mechanical analog:** a harmonic oscillator (HO) with a time-dependent frequency

$$\omega(t) = cte \text{ for } t < t_i \ (\omega_{in}) \text{ and } t > t_f \ (\omega_{out})$$

- If the HO is in its ground state for $t < t_i$, there is a non zero probability of being found in an excited state for $t > t_f$
- A simple example: a **sudden** change from ω_{in} to ω_{out},



Introduction: basic concepts and purposes

- In this case, it can be shown that $a_{out} = \alpha a_{in} - \beta^* a_{in}^\dagger$ where

$$\alpha = \frac{1}{2}\left(\sqrt{\frac{\omega_{in}}{\omega_{out}}} + \sqrt{\frac{\omega_{out}}{\omega_{in}}}\right) \; ; \quad \beta = \frac{1}{2}\left(\sqrt{\frac{\omega_{in}}{\omega_{out}}} - \sqrt{\frac{\omega_{out}}{\omega_{in}}}\right)$$

  The final state is a squeezed state,

$$|0\rangle \longmapsto |\xi\rangle = S(\xi)|0\rangle \, , \quad \text{with} \quad S(\xi) = e^{\frac{1}{2}\left(\xi a^{\dagger 2} - \xi^* a^2\right)}$$

  and

$$\alpha = \cosh|\xi| \; ; \qquad \beta = \frac{\xi}{|\xi|} \sinh|\xi|$$

- Since moving mirrors $\longleftrightarrow$ time-dependent potentials, they may excite the field from its ground state (vacuum) to an excited state (with real quanta).
- From conservation energy arguments, there must appear a radiation reaction force on the moving boundaries.

Introduction: basic concepts and purposes

- The DCE can be understood in the opposite way: dissipative forces $\Longrightarrow$ particle creation

- Fluctuations of the static Casimir force - Barton - 1991;

- From the Fluctuation-Dissipation Theorem we expect dissipative forces on moving boundaries (Braginsky/Khalili - 1991 and Jaeckel/Reynaud - 1992);

- Again, invoking conservation of energy, the dissipative forces on the moving boundaries convert mechanical energy into field energy (real particle creation);

Introduction: basic concepts and purposes

First works:

- **Moore** (1970); *Quantum theory of the electromagnetic field in a variable cavity*; 1+1 model (scalar field); arbitrary motion;

$$Moore\ equation \implies R\Big(t - L(t)\Big) = R\Big(t + L(t)\Big) - 2$$

- **DeWitt** (1975); in the context of QFT in curved spacetimes
- **Fulling and Davies** (1976); $T_{\mu\nu}$ in $1+1$; conformal transf.; arbitrary motion of the mirror
- **Ford and Vilenkin** (1982); perturbative method in $3+1$; moving plate in non-relativistic motion.

- The particle creation phenomenon is strongly enhanced for a cavity in parametric resonance (**Dodonov-Klimov** (94))

Introduction: basic concepts and purposes

- Generalization for the **electromagnetic field** (**Maia Neto** *et al* - 94/96/98):
- **More recently:** threedimensional cavities, waveguides, DCE and quantum decoherence (**Dalvit, Maia Neto,..**); new methods of calculation (**Plunien, Elizalde,..**)
- **Experimental verification:** the DCE has been observed in the context of Circuit QED (**Wilson and collaborators** - 2011)

Main purposes of the present work

- To simulate (theoretically) a moving mirror by imposing on the field a time-dependent Robin BC at a **static** mirror.
- To consider slightly more general BC that can be useful in future experiments.

Experimental proposals

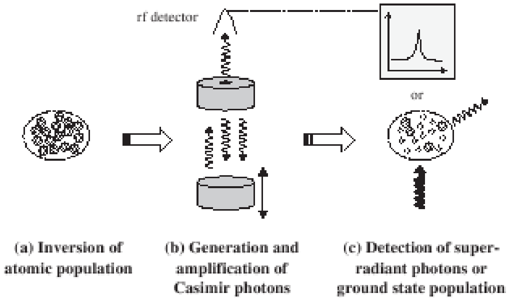
- **Static Casimir effect:** 1st measured in 1958 (bad accuracy); modern experiments (since 1997) $\Longrightarrow$ very good precision.
- **Dynamical Casimir effect:** observed $\approx$ 40 years after its theoretical prediction.
  - Dissipative effects are very small. A mirror in harmonic motion damped by the quantum vacuum flutuations:

$$\frac{d^2x}{dt^2} - \frac{\hbar}{6\pi Mc^2}\frac{d^3x}{dt^3} + \omega_0^2 x = 0 \quad \Longrightarrow \quad \frac{\Gamma}{\omega_0} = \frac{1}{12\pi}\frac{\hbar\omega_0}{Mc^2}$$

  - **best option:** to measure the created photons.
- Recent technological advances relevant to the DCE:
  - Possibility of rapid mechanical oscillations: $\sim 3,0\,GHz$;
  - Rapid changes of the reflecting properties of a semiconductor induced by the incidence of appropriate laser pulses.
  - Circuit QED.
- We shall describe briefly some experiments on DCE.

Experimental proposals

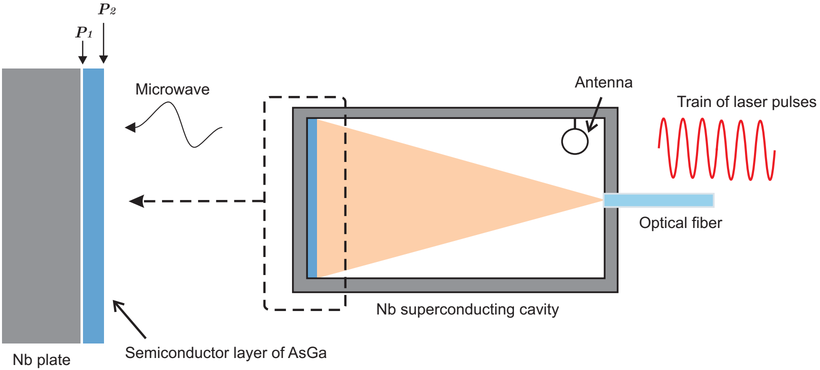
Superradiance proposal: Kim *et al* - 2006



- Oscillating cavity $(3GHz)$ $\longrightarrow$ parametric resonance;
- dynamical Casimir photons interact with excited Na atoms inside the cavity triggering a superradiant pulse;
- a detector can be coupled to the cavity and the time delay of the superradiant pulse is a signature of the Casimir photons.

Experimental proposals

Italian group: Braggio *et al* - 2005



- Cavity with a semiconductor layer $(\sim mm)$ in one of its walls;
- billions of laser pulses (with appropriate laser frequency) reach the semiconductor layer per second.

Experimental proposals

Italian group: Braggio *et al*

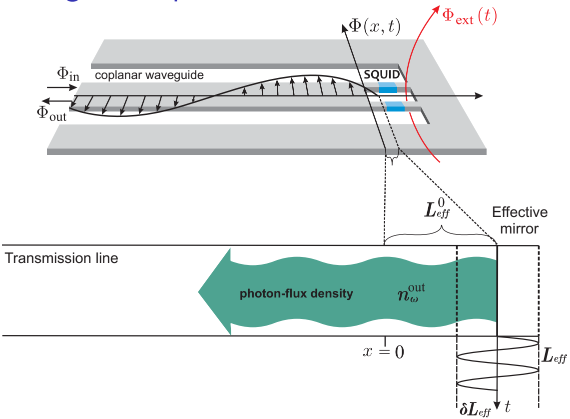
- The reflecting wall changes from a transparent one to a completely reflecting one billions of times per second ($\sim$ GHz)
- the changes $transparent \longleftrightarrow reflecting$, then, simulates an oscillatory motion of the mirror between surfaces P_1 and P_2
- important question: how fast does the effective reflecting surface go from P_1 to P_2 and vice versa?

$$P_1 \longleftrightarrow 10^{-15}s \qquad ; \qquad P_2 \longleftrightarrow 5 \times 10^{-12}s$$

- Reflectivity of $(AsGa) \sim Copper$ in the microwave range;
- $AsGa$ does not affect the cavity quality factor $Q\ (10^6)$
- detector sensibility: $\sim 10^4$ microwave photons $(2,5GHz)$, order of magnitude of the expected experimental signal.

Experimental proposals

Coplanar waveguide experiment



- Recently (2009), Johansson *et al* proposed an experiment on DCE in 1+1 in the context of **circuit QED**: *semi-infinite coplanar waveguide terminated with a SQUID*

Experimental proposals

Coplanar waveguide experiment

- The effective inductance of the SQUID can be tuned by an external time-dependent magnetic flux, $\Phi_{ext}(t)$, providing a tunable boundary condition;

- the set-up is equivalent to a one-dimensional transmission line with a tunable effective length, i.e., a *tunable mirror*;

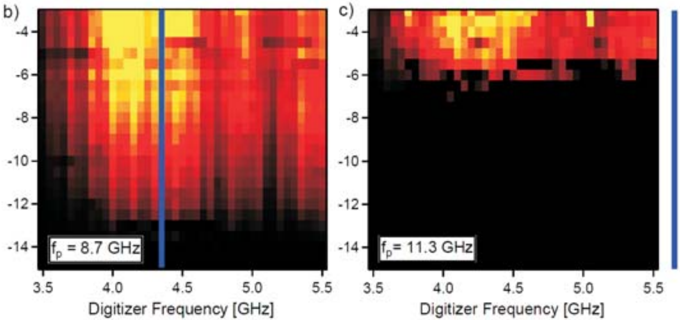
- in this system, the effective velocities may be extremely high, leading to high photon creation rates

  ($\sim 10^5/s$ around 90Ghz)

Experimental proposals

Coplanar waveguide experiment

- **JR Johansson** *et al*, 2010: theoretical details;
- **CM Wilson** *et al*, 2010: preliminary observations;
- **CM Wilson** *et al* "*Observation of the Dynamical Casimir Effect in a Superconducting Circuit*", arXiv: 1105.4714v1.

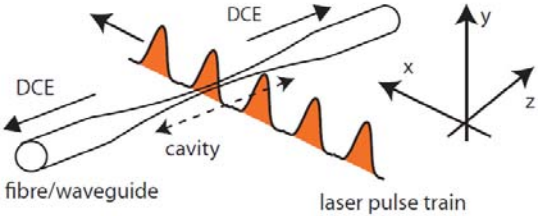


Note the asymmetry of the spectral distribution.

Experimental proposals

DCE in optically modulated cavities (**Faccio-Casurotto**-2011)

- An appropriate train of laser pulses is applied perpendicularly to a cavity made of a non-linear optical fiber.



- Efficient modulation of the effective optical length of a cavity mode in the near IR region.

$$n_{eff}(t) = n_0 + \delta n(t) = n_0 + n_2 I_p(t)$$

Robin boundary conditions

Main features

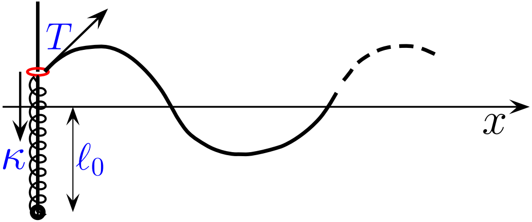
- For a scalar field ϕ in 3+1, **Robin** BC is defined by

$$\left(\phi - \beta\frac{\partial\phi}{\partial n}\right)\bigg|_{\text{boundary}} = 0$$

  where β is a constant parameter with dimension of length.
- They interpolate continuously **Dirichlet** ($\beta \to 0$) and **Neumann** ($\beta \to \infty$) boundary conditions
- For $\omega \ll \omega_P$, β plays the role of the plasma wavelength
  Phenomenological model for penetrable surfaces
  (Mostepanenko and Trunov - 1985)
- They appear in Mechanics, electromagnetism, quantum mechanics and QFT (Casimir effect,..), among others.

Robin boundary conditions

- **Classical mechanics:** it appears in vibrating strings coupled to harmonic oscillators at its edges (Chen and Zhou - 1992)



For $|\frac{\partial \phi}{\partial x}| \ll 1$ Newton's law applied to the massless ring leads to

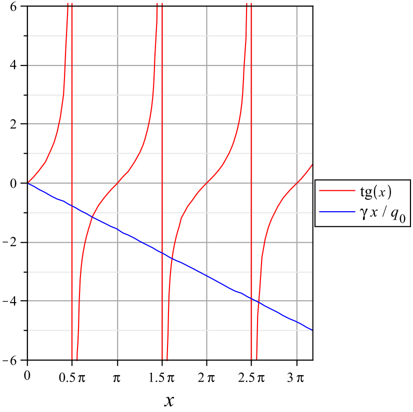
$$\phi(x,t)\Big|_{x=0} = \frac{T}{\kappa}\frac{\partial \phi(x,t)}{\partial x}\Big|_{x=0},$$

where T/κ plays the role of the parameter β.

Robin BC in the static Casimir effect

- Eigenfrequencies are roots of a *transcendental Eq*. For a 1+1 cavity with Robin-Dirichlet BC,

$$\left[\mathrm{sen}(kx) + \beta k \cos(kx)\right]_{x=a} = 0 \quad \Longrightarrow \quad \mathrm{tg}(ka) = -\frac{\beta}{a}(ka)$$



Robin BC in the static Casimir effect

For the more general case of **Robin**(β_1) - **Robin**(β_2) BC, the eigenfrequencies are the roots of $g(z) = 0$, with

$$g(z) = (\beta_1 + \beta_2) z \cos(za) \; + \; (1 \; - \; z^2 \beta_1 \beta_2) \, \text{sen}(za) \, .$$

The regularized zero point energy,

$$E_0^{(reg)}(a, \sigma) = \sum_{n=1}^{\infty} \frac{1}{2} \omega_n e^{-\sigma \omega_n} = \sum_{n=1}^{\infty} f(\omega_n) \, ,$$

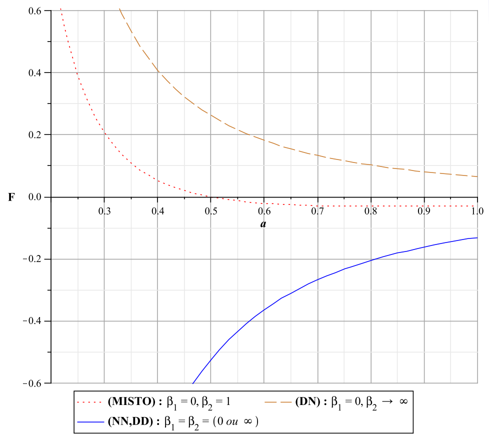
can be computed with the aid of the **Argument Theorem**,

$$\sum_n r_n \, f(z_n) - \sum_n s_n \, f(p_n) = \frac{1}{2\pi i} \oint_{\mathcal{C}} dz \, f(z) \, \frac{d}{dz} \log g(z) \; ,$$

After subtracting unphysical terms, the Casimir force reads

$$F_{Cas}(a, \beta_1, \beta 2) = -\frac{1}{\pi} \int_0^{\infty} dy \; y \left[\frac{(1 + \beta_1 y)(1 + \beta_2 y)}{(1 - \beta_1 y)(1 - \beta_2 y)} e^{2ay} - 1 \right]^{-1} \, .$$

Casimir force $\times$ a for $\neq$ values of β_1 e β_2 (arbitrary units)



D-D or N-N BC (blue solid line), N-D BC (brown dashed line)
Restoring Casimir forces (red dotted line) (**Romeo/Saharian-2002**)

- A detailed discussion of BC can be found in many recent papers by Asorey, García Álvarez, Clemente-Gallardo and Muñoz-Castañeda.

Robin BC in the dynamical Casimir effect

Force on the moving mirror:

- massless scalar field ϕ in $1+1$ and one mirror in a **prescribed** and **non-relativistic** motion with **small amplitudes**,

$$|\delta\dot{q}(t)| << c \quad \text{and} \quad |\delta q(t)| << c/\omega_0 \, ,$$

where ω_0 corresponds to the mechanical frequency.

- Solving $\partial^2\phi(t,x) = 0$, submitted to **Robin BC**

$$\left[\frac{\partial}{\partial x} + \delta\dot{q}(t)\frac{\partial}{\partial t}\right]\phi(t,x)|_{x=\delta q(t)} = \frac{1}{\beta}\phi(t,x)|_{x=\delta q(t)} + \mathcal{O}(\delta\dot{q}^2/c^2)\, ,$$

in the Ford-Vilenkin **perturbative** approach ($\phi = \phi_0 + \delta\phi$), one can show that the susceptibility acquires a real part

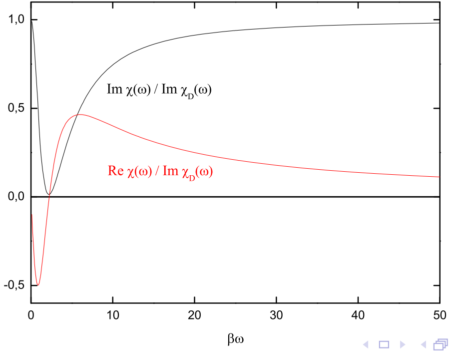
$$\delta\mathcal{F}(\omega) = \chi(\omega)\delta Q(\omega), \quad \text{with} \quad \chi(\omega) = \mathcal{R}e\chi(\omega) + i\mathcal{I}m\chi(\omega)$$

Robin BC in the dynamical Casimir effect

- **Total work** on the moving plate: only $\mathcal{I}m\chi(\omega)$ contributes,

$$\int_{-\infty}^{+\infty} F(t)\delta\dot{q}(t)\,dt = -\frac{1}{\pi}\int_0^{\infty} d\omega\,\omega\,\mathcal{I}m\,\chi(\omega)|\delta Q(\omega)|^2\ .$$

- $\mathcal{R}e\chi(\omega)$ and $\mathcal{I}m\chi(\omega)$, normalized by $\mathcal{I}m\chi_D(\omega) = \omega^3/(6\pi)$ as functions of $\beta\omega$ (possible suppression of dissipative effects):



Robin BC in the dynamical Casimir effect

Particle creation:

- The spectral density is given by (**Mintz** *et al* - **2006**)

$$\frac{dN(\omega)}{d\omega} = \frac{4\omega}{1+\beta^2\omega^2}\int_0^\infty \frac{d\omega'}{2\pi}\frac{[\delta Q(\omega-\omega')]^2}{1+\beta^2\omega'^2}\,\omega'\left[1-\beta^2\omega\omega'\right]^2$$

- For a **typical oscillatory motion**, given by

$$\delta q(t) = \delta q_0\, e^{-|t|/T}\cos(\omega_0 t), \quad \text{with} \quad \omega_0 T \gg 1$$

$\delta Q(\omega)$ is a very narrow function around $\pm\omega_0$, so that

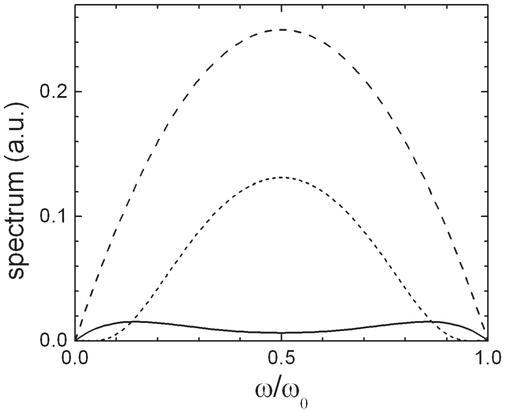
$$\frac{dN}{d\omega}(\omega) = (\delta q_0)^2 T\omega(\omega_0-\omega)\frac{[1-\beta^2\omega(\omega_0-\omega)]^2\Theta(\omega_0-\omega)}{(1+\beta^2\omega^2)(1+\beta^2(\omega_0-\omega)^2)}$$

For $\beta = 0$ or $\beta \to \infty$ we get (**Lambrecht** *et al*-**1996**)

$$\frac{dN}{d\omega}(\omega) = (\delta q_0)^2 T\omega(\omega_0-\omega)\Theta(\omega_0-\omega)\,.$$

Robin BC in the dynamical Casimir effect

Spectral distribution $dN/d\omega$ as a function of ω/ω_0 for $\neq$ values of β



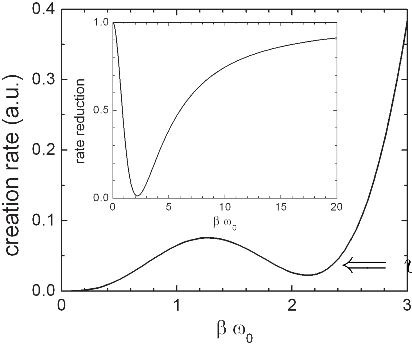
Dashed line $\longleftrightarrow$ Dirichlet BC; **solid line** $\longleftrightarrow$ $\beta\omega_0 = 1, 7$

Robin BC in the dynamical Casimir effect

- Total number of created particles and creation rate:

$$N = \int_0^{\omega_0} \frac{dN}{d\omega}(\omega)\, d\omega = \left[\frac{\delta q_0^2\, T}{12\pi}\, \omega_0^3 \right] 6F(\beta\omega_0)\ ; \qquad R = \frac{N}{T}\,,$$

$$F(\xi) = \frac{\xi[4\xi + \xi^3 + 12\arctan(\xi)] - 6(2+\xi^2)\ln(1+\xi^2)}{6\xi^2(4+\xi^2)}$$



$\Longleftarrow$ $with\ fixed\ \beta$

Modeling a moving boundary by a static one

- **Purpose:** to make a simple theoretical model that describes static surfaces which simulate moving mirrors;
- **motivation:** recent experimental proposals;
- **theoretical model:** since the Robin parameter is related to the penetration depth, a time-dependent Robin parameter may simulate a moving mirror;
- **we consider** a massless scalar field in $1+1$ submitted to a Robin BC with time-dependent parameter at $x = 0$:

$$\phi(0, t) \;=\; \gamma(t) \frac{\partial \phi}{\partial x}\bigg|_{x=0}$$

- to apply the Ford/Vilenkin perturbative approach, we assume

$$\gamma(t) = \gamma_0 + \delta\gamma(t) \; ; \quad \mathit{with} \quad \max |\delta\gamma(t)| \ll \gamma_0 \,.$$

Modeling a moving boundary by a static one

- By assumption, $\delta\gamma$ is a **prescribed** function of t that vanishes in the remote past and distant future;
- after a straightforward calculation, we obtain (Bogoliubov transformation)

$$
\begin{aligned}
a_{\text{out}}(\omega) &= a_{\text{in}}(\omega) - 2i\sqrt{\frac{\omega}{1+\gamma_0^2\omega^2}} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \sqrt{\frac{\omega'}{1+\gamma_0^2\omega'^2}} \times \\
&\times \left[\Theta(\omega')a_{\text{in}}(\omega') - \Theta(-\omega')a_{\text{in}}^{\dagger}(-\omega')\right] \delta\Gamma(\omega-\omega') ,
\end{aligned}
$$

where $\delta\Gamma(\omega)$ is the Fourier transformation of $\delta\gamma(t)$.

- Note the presence of $a_{\text{in}}^{\dagger}(-\omega')$ in the expression for $a_{\text{out}}(\omega)$.

Modeling a moving boundary by a static one

- Using the previous Bogoliubov transformation and

$$\frac{dN(\omega)}{d\omega} = \frac{1}{2\pi} \langle 0_{\text{in}} | \, a^{\dagger}_{\text{out}}(\omega) a_{\text{out}}(\omega) \, | 0_{\text{in}} \rangle,$$

we obtain for the spectral distribution

$$\frac{dN(\omega)}{d\omega} = \frac{2}{\pi} \left(\frac{\omega}{1 + \gamma_0^2 \omega^2} \right) \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\omega'}{1 + \gamma_0^2 \omega'^2} \left| \delta\Gamma(\omega - \omega') \right|^2 \Theta(\omega').$$

- In order to compare the present results with previous ones, we choose for the time-dependence of the Robin parameter an oscillatory behaviour, namely,

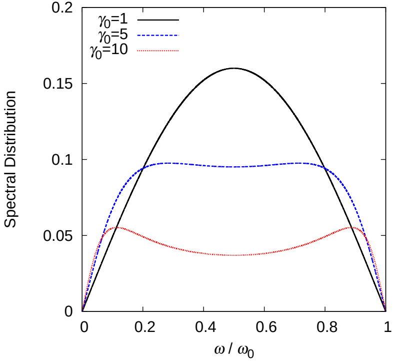
$$\delta\gamma(t) = \epsilon_0 \cos(\omega_0 t) \, e^{-|\,t\,|/T},$$

which substituted into the previous equation leads to the following spectral distribution

Modeling a moving boundary by a static one

$$\frac{dN(\omega)}{d\omega} = \left(\frac{\epsilon_0^2 T}{2\pi}\right) \frac{\omega\,(\omega_0 - \omega)}{(1 + \gamma_0^2\omega^2)\left[1 + \gamma_0^2(\omega_0 - \omega)^2\right]} \Theta(\omega_0 - \omega),$$

which is completely analogous to that found for a moving mirror and a time-independent Robin parameter,



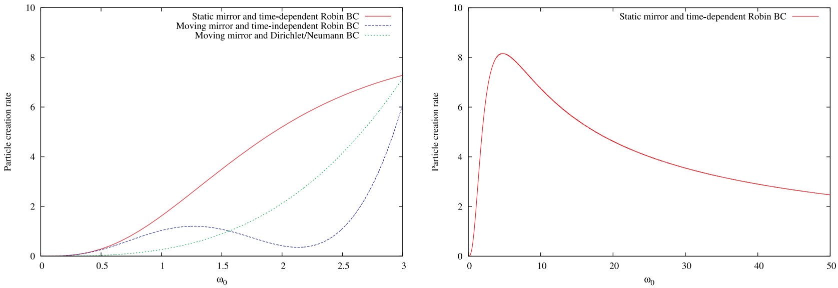
Modeling a moving boundary by a static one

- However, things are different for the particle creation rate

$$R = \frac{N}{T} = \frac{1}{T}\int_0^\infty \frac{dN(\omega)}{d\omega}\,d\omega = \left(\frac{\epsilon_0^2\omega_0^3}{2\pi}\right) G(\omega_0\gamma_0)$$

where

$$G(\xi) = \frac{(2+\xi^2)\ln(1+\xi^2) - 2\xi\arctan(\xi)}{\xi^4(4+\xi^2)}$$



Modified Robin BC

- We shall now consider a slightly different boundary condition:

$$\phi(t,x)\,|_{x=0} \;=\; \gamma(t)\left[\partial_x\phi(t,x) - \alpha_0\partial_t^2\phi(t,x)\right]_{x=0}\,.$$

Motivation: this BC appeared in the discussion of the SQUID experiment (there the α_0-term can be neglected).

- As before, $\gamma(t) = \gamma_0 + \delta\gamma(t)$ and we adopt Ford/Vilenkin perturbative approach, $\phi(t,x) = \phi_0(t,x) + \delta\phi(t,x)$.

- The boundary condition for ϕ_0 is given by

$$\phi_0(t,x)\,|_{x=0} = \gamma_0\left[\partial_x\phi_0(t,x) - \alpha_0\partial_t^2\phi_0(t,x)\right]_{x=0}\;,$$

while that for $\delta\phi$ can be written as

$$\left[1-\gamma_0\left(\partial_x - \alpha_0\partial_t^2\right)\right]\delta\phi(t,x)\Big|_{x=0} =$$

$$= \;\; \delta\gamma(t)\left(\partial_x - \alpha_0\partial_t^2\right)\phi_0(t,x)\Big|_{x=0}.$$

Modified Robin BC

- Let us take the same time-dependence for $\gamma(t) = \gamma_0 + \delta\gamma(t)$,

$$\delta\gamma\left(t\right) = \epsilon_0 \cos\left(\omega_0 t\right) e^{-t/\tau}, \quad \omega_0 \tau \gg 1\,.$$

- Introducing Fourier transf. $\Phi(\omega, x)$, $\Gamma(\omega, x)$, ..., and relating Φ_{in} and Φ_{out} we get the Bogoliubov transf. which lead to

$$\begin{aligned}
\frac{dN(\omega)}{d\omega} &= \frac{1}{2\pi}\langle a^\dagger_{\text{out}}(\omega) a_{\text{out}}(\omega)\rangle \\
&= \frac{\epsilon_0^2 \tau/(2\pi)}{(1+\alpha_0\gamma_0\omega^2)^4} \frac{\omega\left[1 + 2\alpha_0\gamma_0\left(\omega_0 - \omega\right)^2\right]^2}{(1+\alpha_0\gamma_0\omega^2)^2 + \gamma_0^2\omega^2} \times \\
&\times \quad \frac{(\omega_0 - \omega)\Theta(\omega_0 - \omega)}{[1+\alpha_0\gamma_0(\omega_0-\omega)^2]^2 + \gamma_0^2(\omega_0-\omega)^2}\,,
\end{aligned}$$

Modified Robin BC

- Spectral distribution for a fixed γ_0 and increasing values of α_0

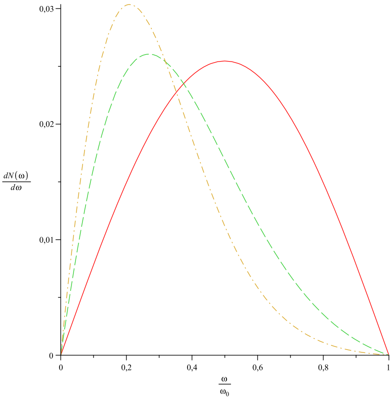


Figura: $\alpha_0 = 0$ (solid), $\alpha_0 = 1/2$ (dashed), $\alpha_0 = 1$ (dotted-dashed).

The introduction of the α_0-term leads to an **assymmetric** spectral distribution.

Modified Robin BC

- Total number of created particles as a function of the **effective oscillating frequency** ω_0:

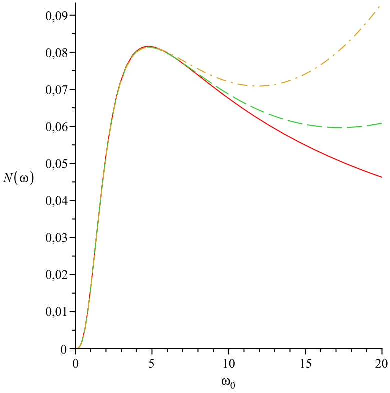


Figura: The total number as a function of ω_0 for $\alpha_0 = 0$ (solid line), $\alpha_0 = 1 \times 10^{-3}$ (dashed line) and $\alpha_0 = 2 \times 10^{-3}$ (dot-dashed line).

Final remarks and perspectives

- **Dynamical Casimir effect:** after $\sim 40\ years$ finally observed (though in the context of Circuit QED);
- There are a few other promising experimental proposals;
- Robin BC with cte parameter at a moving mirror and time-dependent parameter at a static mirror have intriguing properties in DCE (for cte γ, see the poster of **ACL Rego**);
- Different situations can be simulated by choosing appropriately the time-dependence of the Robin parameter;
- cte γ and $\gamma(t)$: quite different behaviours for large $\omega_0 \to \infty$.
- **Numerical estimatives** for comparison with experiments;
- Understand the asymmetric observed spectral distribution
- **thermal effects**, generalization to **3+1 dimensions**;
- **cavity in 1+1** with time-dependent Robin parameters;