Entropy of Quasiblack holes and entropy of black holes in membrane approach

José P. S. Lemos
Centro Multidisciplinar de Astrofísica, CENTRA,
Lisbon, Portugal

Oleg B. Zaslavskii
Department of Physics and Technology,
Kharkov V.N. Karazin National University,
Kharkov, Ukraine

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Entropy of black hole versus entropy without horizon: membrane paradigme

K. Thorne, Damour, etc.

Lemos and O.Z., PRD 2011

Streched horizon, vacuum inside, shell

Timelike boundary vs. lighlike. How S approaches BH A/4 ?
Schwarzschild (Parikh, theses 1999)

$$ds^2 = -N^2 dt^2 + h_{ij} dx^i dx^j,$$

$$Z = \exp(-I)$$

$$I = \beta E - S.$$

$$I_g = I_R + I_b,$$

$$I_R = -\frac{1}{16\pi}\int d^4x\sqrt{-g}R,$$

$$I_b = \frac{1}{8\pi}\int d^3x\sqrt{\gamma}(K - K_0).$$

$$I_{\text{matter}} = \beta_0 \int d^3x \sqrt{-g}\rho - S_{\text{matter}}$$

$$\beta_0 \equiv T_0^{-1},$$

$$T = \frac{T_0}{N}$$

00 Einstein equation, 3+1 splitting

No black hole case

$$I_{\text{withoutbh}} = \int_{\text{eb}} d\sigma \beta \varepsilon - S_{\text{matter}}$$

$$\varepsilon = \frac{k - K_0}{8\pi}.$$

Black hole case

$$S_{\text{tot}} = S_{\text{matter}} + \frac{A}{4},$$

York 1986 (spherical symmetry)

O. Z. 1991 (static, without symmetry)

Black hole entropy in the membrane approach

Internal boundary 1) $r_{\mathrm{mb}} + \delta$ or 2) $r_{\mathrm{mb}} - \delta,$

Black hole case 2)

Physical noundary on inner side of shell

$$I = \int ds\, b\, e - S \qquad S = S_m + S_{mb} \qquad S_{\mathrm{mb}} = \frac{\beta_0}{8\pi} \int_{\mathrm{mb}} d\sigma \left(\frac{\partial N}{\partial n}\right)_+,$$

$(\frac{\partial N}{\partial n})_+ \longrightarrow \kappa$ Surface gravity $T_{\mathrm{H}} = \frac{\kappa}{2\pi}$ $T_0 = T_{\mathrm{H}},$

$$\lim_{l \to 0} S_{\mathrm{mb}} = \frac{A}{4}.$$

Nonextremal case

Black hole: S=A/4 or S=A/4 + S(matter)

Without horizon: S(matter)

Let $R = r_+(1+d)$ $d \rightarrow 0$

What happens to entropy? Why S(matter) approaches A/4 ?

A/4 due to horizon, light-like surface. Now space-like

Whether and how continuity achieved?

Quasiblack hole $e \rightarrow 0$

$$N = e\, f(x^i)$$

Extremal case, puzzles. Matter system without horizon

Membrane: vacuum inside
Shell near gravitational radius

Without horizon: S(matter)

Quasiblack hole
Physical (nonvacuum) system on threshold of collapse

QBH: $N = e f(x^i)$ Everywhere inside $e \to 0$

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Black hole: S=A/4 or S=A/4 + S(matter)

Let $R = r_+ (1 + d)$

Nonextremal case

$d \to 0$

Extremal case, puzzles. Matter system without horizon

A/4 due to horizon, light-like surface. Now space-like

Whether and how continuity achieved?

Answer expected to be model-independent

Quasiblack hole
System on threshold of horizon formation
But horizon does not form

True BH: $N = f(r),\ f(r_0) = 0$ N Lapse function

QBH: $N = e\, f(x^i)$ $e \to 0$

but $f > 0$

QBH: definition and general properties

Gravitational radius is approached by sequence of static configurations

Horizon almost forms but does not form

1) Methodical tool: mass and entropy of black hole from system without horizon

2) Real physical systems (extremal case)

Remote obsrver at infinity – no difference from true BH

Vicinity of quasihorizon – crucial difference

Usual situation: size approaches gravitational radius, system collapses

Special cases when gravitational radius is approached by sequence of static configurations

Majumdar –Papapetrou systems $p = 0,$

Compact objects: Bonnor stars $r = r_e$

Sphere of neutral hydrogen lost 10^{-18} of its electrons

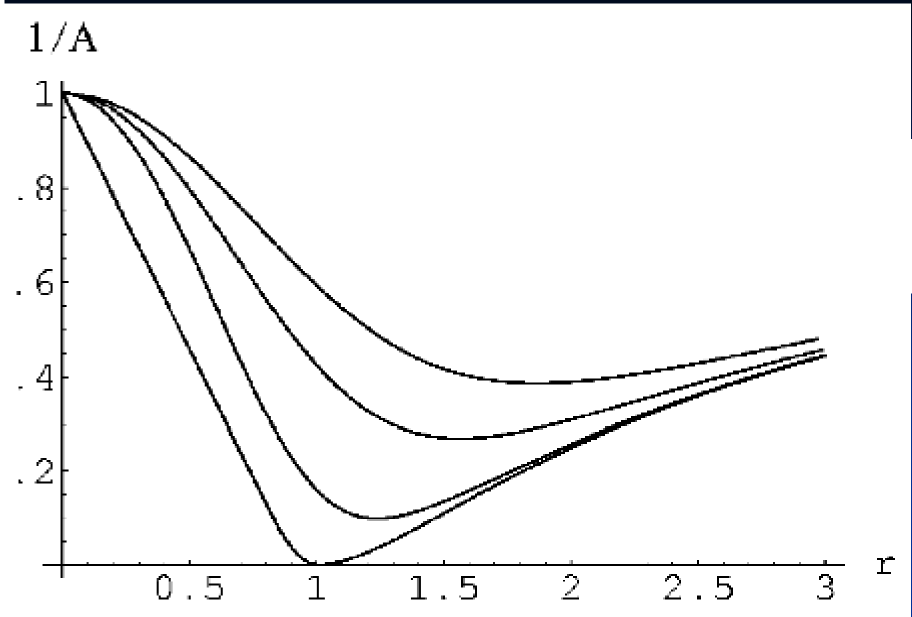
Self-gravitating magnetic monopole

Threshold of formation of event horizon. Quasihorizon

Massive charged extremal shells

Different physical systems share common features: geometry of spacetime

behavior of tidal forces



Lemos and E. Weinberg 2004

FIG. 1. A plot of $1/A$ as a function of r for $q=1$ and, reading from the top down, $c=0.5$, 0.3, 0.1, 0.001. The emergence of the quasihorizon is quite evident in the $c=0.001$ curve.

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(dq^2 + \sin^2 q\, df^2)$$

Lue and E. Weinberg 2000

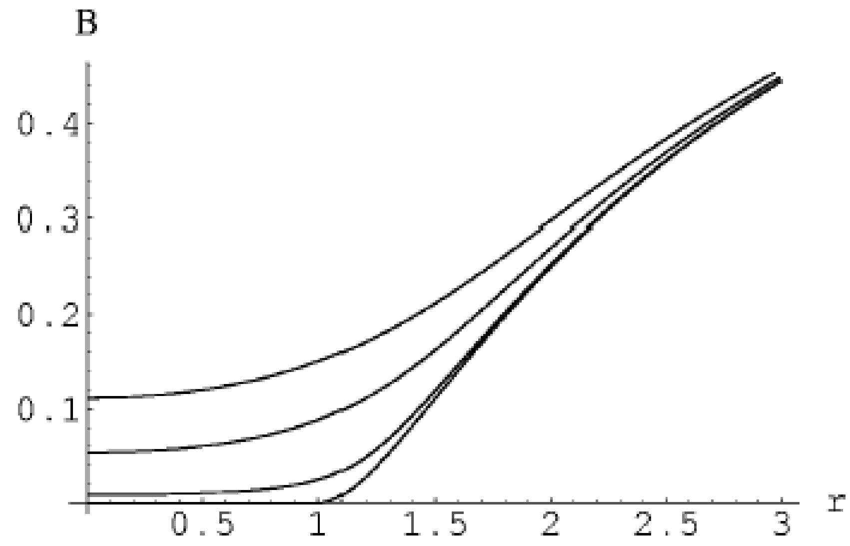
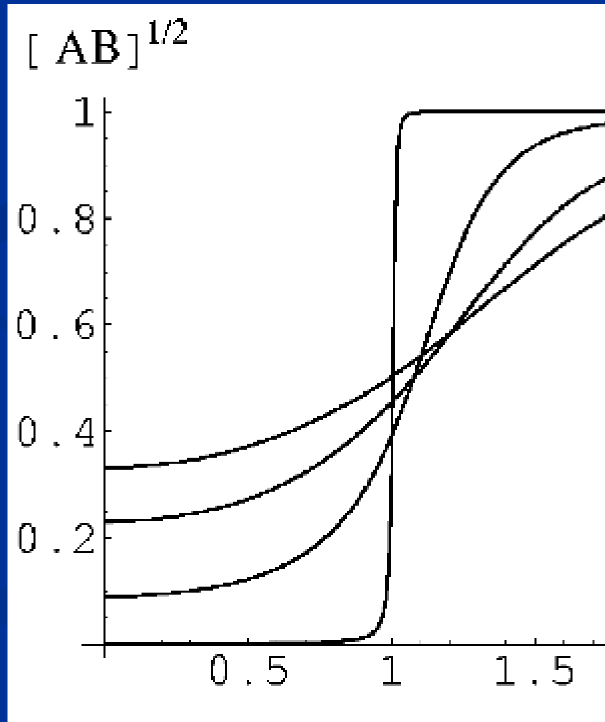


FIG. 2. A plot of $B(r)$ for $q=1$ and, reading from the top down, $c=0.5$, 0.3, 0.1, 0.001.



General approach

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(dq^2 + \sin^2 q\, df^2) \qquad A = \frac{1}{V}$$

(i) V(r) attains minimum at $r^* \neq 0$ $\quad V(r^*) = e = 1$

(ii) $e \neq 0$ regular configuration

(iii) In limit $e \to 0$ $V(r^*) \to 0$ $B(r) \to 0$ for all $r \leq r^*$

Consequences:

(a) infinite redshift

(b) infinite tidal forces for free-falling observer

Limit $e \to 0$ Singular (degenerate) or regular?

Properties of spacetimes -?

Extremal RN outside – Minkowski inside (shell)

Classical model of electron
A. V. Vilenkin, P. I. Fomin 1978

Outer metric

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \left(1-\frac{m}{r}\right)^{-2} dr^2 + r^2 d\Omega^2 \qquad r > r_0$$

Inner metric

$$ds^2 = -\left(1-\frac{m}{r_0}\right)^2 dt^2 + dr^2 + r^2 d\Omega^2 \qquad r < r_0$$

Inside. Two alternatives

1) Using time *t as* "good" coordinate. Then $g_{00} \to 0$

in the entire region $r \le r_0$ Degenerate behavior

But Riemann tensor =0 there!

Surface $r = r_0$ becomes light-like in limit $r_0 \to m + 0$

Redshift

$$w(\infty) = w\sqrt{A(r)} \qquad A \to 0 \qquad \text{in the whole inner region}$$

infinite redshift in quasi-horizon limit

2) Let us introduce inside the coordinate *T* :

$$t = \frac{T r_0}{r_0 - m} \qquad ds^2 = -dT^2 + dr^2 + r^2 d\Omega^2$$

Finite intervals of *T* – infinite intervals of *t*

$$-dT^2 = -\frac{(r_0 - m)^2}{r^2{}_0} dt^2$$

Finite intervals of *t* – vanishing intervals of *T*

Let *T* be legitimate coordinate

$r_0 \to m$ Time-like surface. No matching between inside and outside

Complementarity and mutual impenetrability

No penetration from outside to inside because of infinite tidal forces (naked behavior)

No penetration from inside to outside because of infinite redshift

Role of surface stresses: mass formula

$$M = \frac{\kappa A_h}{4\pi} + \varphi_h Q + M_{out}^{matter}$$

J. Lemos and O. Z. PRD 2008

Contribution of inner region à 0

$$M_{surf} = \int_{surface} (-T_0^0 + T_k^k) N \sqrt{g_3} d^3x.$$

$$M_{surf} = \frac{1}{8\pi} \int \alpha N d\sigma,$$

$$\alpha = 8\pi(S_a^a - S_0^0),$$

$$\alpha = \frac{2}{N}\left[\left(\frac{\partial N}{\partial l}\right)_+ - \left(\frac{\partial N}{\partial l}\right)_-\right]$$

Inside: $N = e\, f(x^i) \qquad e \to 0$

Surface stresses: nonextremal versus extremal

1) Nonextremal. Main contribution from the term

$$\alpha = \frac{2}{N}\left[\left(\frac{\partial N}{\partial l}\right)_+ - \left(\frac{\partial N}{\partial l}\right)_-\right]$$

$$a \approx \frac{k}{N} \to \infty$$

k surface gravity

$$N = e\, f(x^i) \qquad e \to 0$$

a diverges but $a\, N \approx k$ gives finite contribution to mass

$$\frac{k\, A_h}{4p}$$

2) Extremal $N \approx const\, \exp(-A l)$

a finite $a\, N$ gives zero contribution to mass

Byproduct: Abraham-Lorentz classical "electron" in GR

Original idea: there are no extrenal forces, no non-electromagnetic stresses, self-consistent solution

Electromagnetic forces + gravitation (GR)

In QBH approach requirement is weakened:

In extremal case non-electromagnetic stresses are present
but they do not contribute to the mass!

II. ENTROPY AND FIRST LAW OF THERMODYNAMICS FOR QUASIBLACK HOLES

$$ds^2 = -N^2dt^2 + dl^2 + g_{ab}dx^a dx^b,$$

$$Td(s\sqrt{g}) = d(\sqrt{g}\epsilon) + \frac{\Theta^{ab}}{2}\sqrt{g}dg_{ab} + \varphi d(\sqrt{g}\rho_e).$$

$$T = \frac{T_0}{N}$$

$$\Theta_{ab} = \Theta^g_{ab} - \Theta^0_{ab},$$

$$8\pi\Theta^g_{ab} = K_{ab} + \left(\frac{N'}{N} - K\right)g_{ab},$$

K_{ab} extrinsic curvature

Dominant contribution: $\frac{1}{N}\left(\frac{\partial N}{\partial l}\right)_+$

Role of surface stresses

$$8\pi\Theta^{g}_{ab} = K_{ab} + \left(\frac{N'}{N} - K\right)g_{ab},$$

In the outer region $\left(\frac{\partial N}{\partial l}\right)_+ \longrightarrow \kappa$

Integrate along subset of configurations near would-be horizon

$$d(s\sqrt{g}) = \frac{\kappa}{16\pi T_0}\sqrt{g}g^{ab}dg_{ab},$$

$$T_0 = T_{\mathrm{H}} = \frac{\kappa}{2\pi}$$

$$d(s\sqrt{g}) = \tfrac{1}{4}d\sqrt{g}$$

$$S = \tfrac{1}{4}A,$$

Predecessors:
F. Pretorius, D. Vollick, and W. Israel,
Phys. Rev. D 1998

An operational approach to black hole entropy

Spherical shell near the gravitational radius,
Euler relation between pressure,
entropy density and energy density.
Model of three surfaces

Now: model-independent approach, spherical symmetry is not required,
Euler relation is in general invalid
Concept of QBH

Spherically symmetric configurations: Entropy issues

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right)e^{2\psi(r)}dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

$$T_0 dS = \exp\psi(R)(dm + 4\pi p_r R^2 dR)$$

$T_0 \rightarrow T_H$ when $d \rightarrow 0$

Otherwise backreaction is unbounded on horizon

How to integrate

Let $R = r_+(1 + d)$ $d << 1$

In space of parameters we move along this line

By substitution into 1st law, we obtain after integration

S=A/4
Constant of integration =0

If the system shrinks, S vanishes.

Simplified example: shell in vacuum

Minkowski inside, Schwarzschild outside

$TdS = dE + pdA$ A area $A = 4\pi R^2$

E quasilocal Brown and York energy $E = R(1-\sqrt{V})$ $V = g^{rr} = 1 - \frac{r_+}{R}$

p gravitational pressure $$p = \frac{(1-\sqrt{V})^2}{16\pi R\sqrt{V}}$$

We consider vicinity of horizon only

Let $R = r_+(1+d)$ $d << 1$

$dE \approx dR \approx dr_+$

$p \approx \frac{1}{16\pi R\sqrt{V}}$

$p \sim \frac{1}{\sqrt{V}} \to \infty$

Term with p dominates 1st law $TdS = dE + pdA$

After integration the same BH entropy S=A/4 reobtained

Entropy in nonextremal case. General features

(i) Systems without horizons
Bridge between thermal matter configurations and black hole entropy

(ii) Entropy comes from surface layer. Details of interior are irrelevant
QBH deletes information

(iii) Role of huge surface stresses. A/4 in model-independent way

(iv) Role of surface layer– quantum states on quasihorizon

(v) Interplay between matter and space-time geometry

Extremal case

Eucludean action approach

$$I = bE - S_{tot}$$

Free energy and Euclidean action

$$S_{tot} = S_m + S_{bh}$$

$$S_{bh} = \frac{A}{4}\frac{T_H}{T_0}$$

1) Nonextremal case: $T_0 = T_H$ $\quad S_{bh} = \frac{A}{4}$

2) Extremal case $T_H = 0$ $\quad S_{bh} = 0$

Temperature arbitrary (Hawking, Horowitz, Ross 1995)

Geometrical explanation: proper distance infinite

Classically, with quantum corrections neglected,
picture self-consistent

Quantum corrections change picture drastically (P. Anderson et al)

Quantum stress-energy tensor:

$$T_m^{\ n} = (\ldots)\,\frac{T^4{}_H - T^4{}_0}{g^2{}_{00}} + finite$$

Divergences destroy horizon

Contradiction

Now $T_H = 0$

$T_0 \neq 0$

Another attempt:

$$S = -\frac{\partial F}{\partial T}$$

But T is fixed =0

Direct definition faces difficulties. QBH approach helps

Lemos and OZ PLB 2011

$$ds^2 = -V \exp(2\psi)\, dt^2 + \frac{dr^2}{V} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right) ,$$

Extremal RN outside

$$R \geq r_+$$

No horizon, “ordinary: system at each stage

1st law

$$T dS = dE + \lambda\, dA - \varphi\, de$$

$$T = \frac{T_0}{1 - \frac{r_+}{r}}$$

$R \geq r_+$

$$TdS = dE + \lambda\, dA - \varphi\, de$$

$$T = \frac{T_0}{1 - \frac{r_+}{r}}$$

$$E = R[1 - \sqrt{V(R)}]$$

$$E = r_+ = e$$

$p_{r(matter)} \to 0$ when $R \to r_+$ (property of QBH with matter)

$$p_r{}^{\mathrm{matter}}(r) = \frac{b(r_+, R)}{4\pi R^2}\left(1 - \frac{r_+}{R}\right)$$

$$\lambda = \frac{1}{8\pi}\frac{b(r_+, R)}{R}$$

Simplest example: extremal charged shell in vacuum

$TdS = dr_+.$

Integrability condition: $T = T(r_+),$ T is local temperature

To compare with uncharged case $T_0 dS = dm$ T_0 temperature at infinity

$$T_0 = T(r_+)\left(1 - \frac{r_+}{R}\right)$$

$$S = S(r_+) = \int_0^{r_+} d\bar{r}_+ \frac{1}{T(\bar{r}_+)}$$

Now, $T_0 \to 0$ but T_{loc} Is finite

This resolves the problem with quantum backreaction:

$$T_m^{\ n} = (\ldots)\, T^4_{loc} + \mathit{finite}$$

More general case

$$S = S(r_+) = \int_0^{r_+} d\bar{r}_+ \frac{D(\bar{r}_+)}{T(\bar{r}_+)}$$

$$D(r_+) = 1 + b_+ - \varphi_+$$

Extremal case. Features of entropy.

S is model-dependent – two functions T, D

Memory: dependence on prehistory

encoded in the law how temperature at
Infinity approaches zero:

$$T_0 = T(r_+)\left(1 - \frac{r_+}{R}\right)$$

Unusual thermodynamics

Agrees with Israel et al PRD 1996
for particular state

General remarks

Considering systems which almost collapse but do not form BH (membrane, QBHs)
lets us gain insight into properties of true black holes

Thank you!