Quantum Field Theory under the Influence of External Conditions
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Aharonov-Bohm effect in scattering of quasiclassical partiles

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Outline

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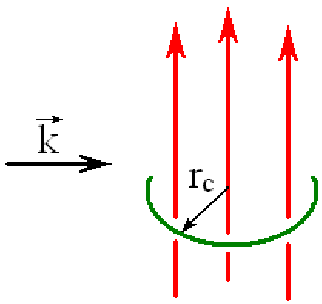
2. Scattering by an impermeable tube: Fraunhofer diffraction

3. Scattering by an impermeable vortex:

    - quasiclassical limit
    - Fraunhofer diffraction
    - scattering amplitude and differential cross section
    - total cross section and optical theorem

4. Conclusion



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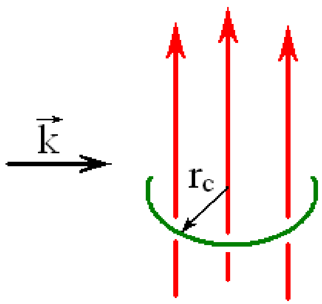
The Aharonov-Bohm effect (1959) is the purely quantum effect

Scattering

$$\frac{d\sigma}{d\varphi} = \frac{1}{2\pi k} \frac{\sin^2(\Phi \Phi_0^{-1} \pi)}{\sin^2(\varphi/2)}, \qquad k r_c \ll 1$$

$\Phi_0 = h c e^{-1}$ is the London flux quantum



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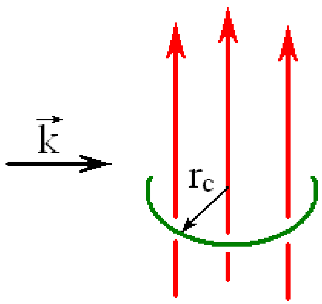

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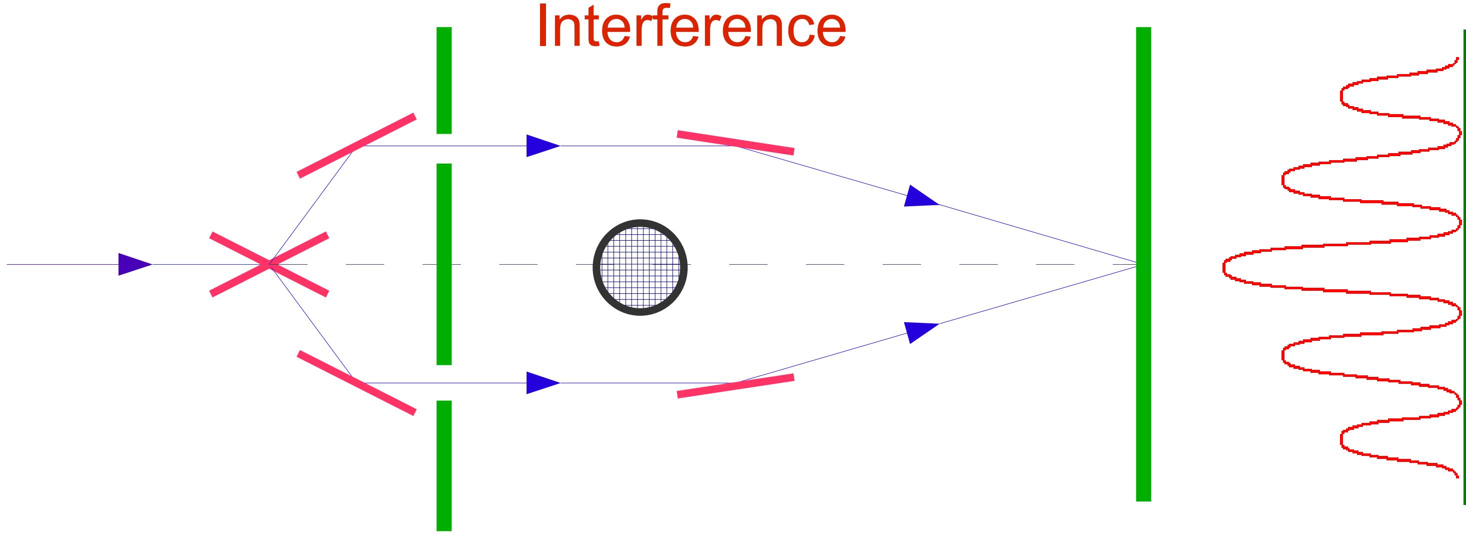
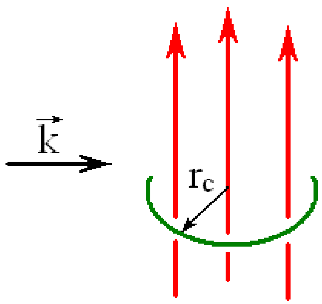


$$\psi_{\pm} = \psi_{\pm}^{(0)} e^{i\chi_{\pm}}, \quad \chi_{\pm} = \frac{e}{\hbar c} \int_{C\pm} \vec{A} \cdot d\vec{x}, \quad \chi_{+} - \chi_{-} = \frac{e}{\hbar c} \oint \vec{A} \cdot d\vec{x} = 2\pi \frac{\Phi}{\Phi_0}, \quad \Phi = \oint \vec{A} \cdot d\vec{x}$$



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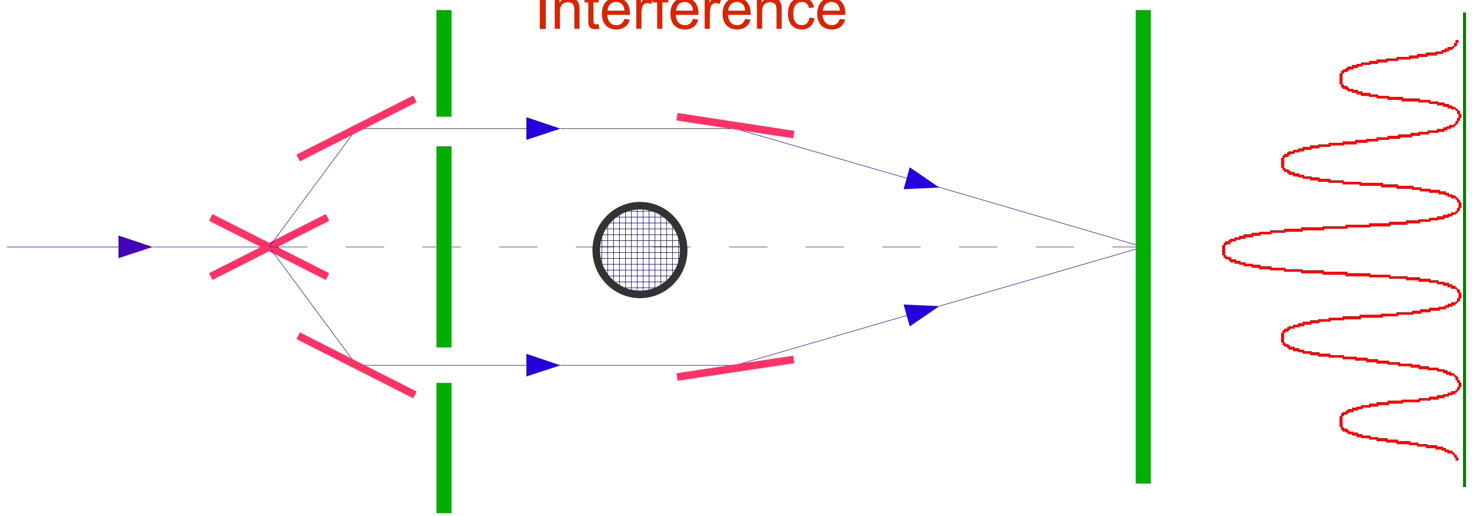

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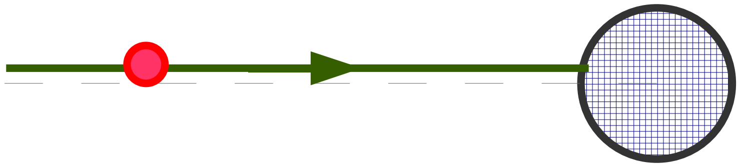
high-energy (short-wavelength) limit: $k = \frac{p}{\hbar} \to \infty$?

Scattering on the impermeable magnetic vortex



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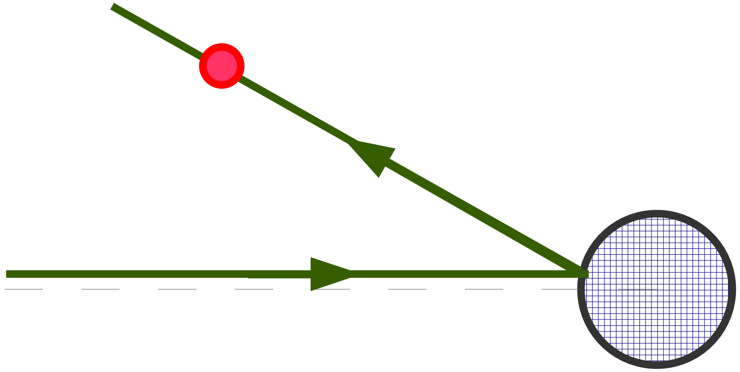

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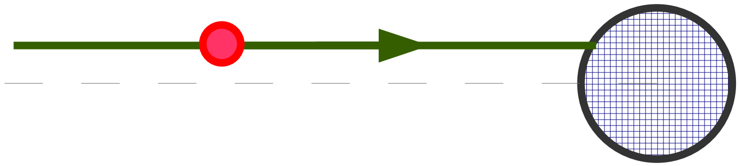

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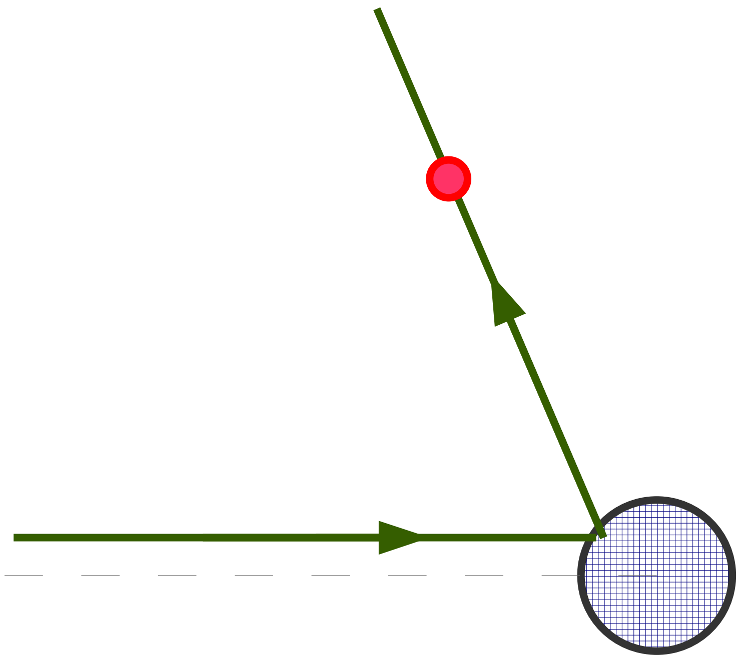

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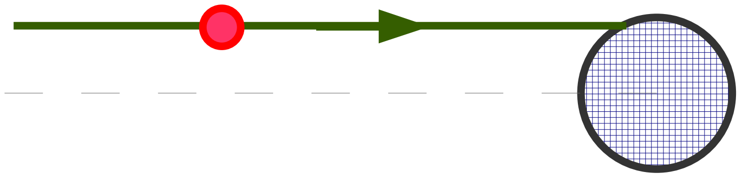

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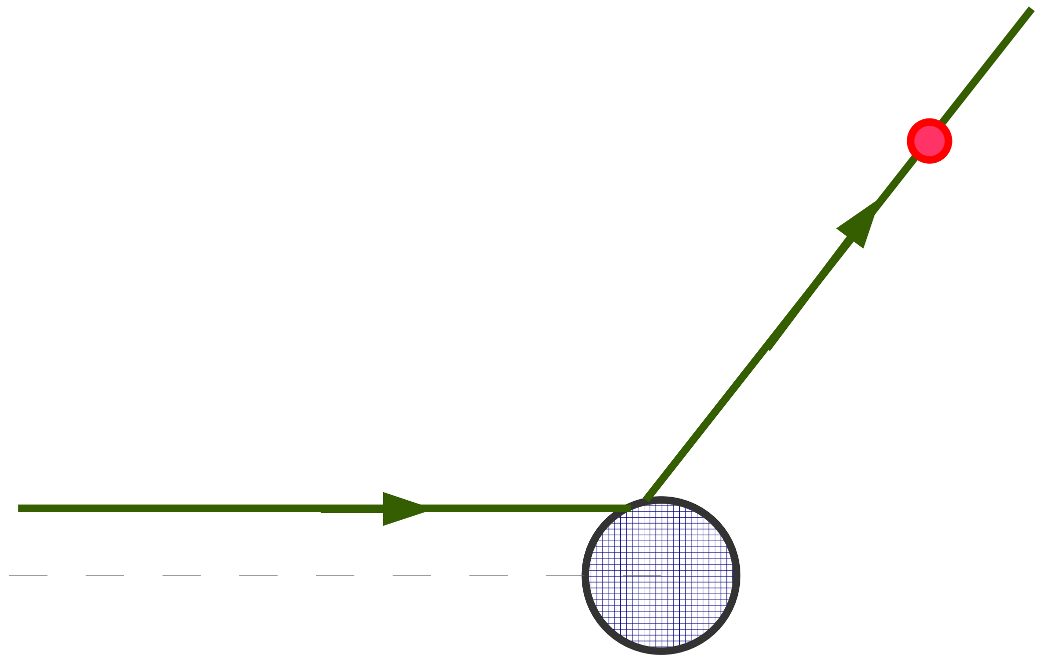

quantum-classical correspondence

quantum physics

classical physics

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quasiclassical physics

classical physics

Since the Aharonov-Bohm effect is the purely quantum effect which has no analogues in classical physics, it becomes evidently more manifest in the limit of long wavelengths of a scattered particle, when the wave aspects of the matter are exposed to the maximal extent. As the particle wavelength decreases, the wave aspects of matter are suppressed in favour of the corpuscular ones, and therefore the persistence of the Aharonov-Bohm effect in the limit of short wavelengths (or high energies) seems to be rather questionable.

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- high-energy (quasiclassical) limit $k \longrightarrow \infty$?
- dependence on the boundary condition ?

Plane wave

$$\psi_{\boldsymbol{k}}^{(0)}(\boldsymbol{r})=e^{ikr\cos\varphi}=\sum_{n\in\mathbb{Z}} e^{in\varphi}\, e^{\frac{i}{2}\lceil n\rceil\pi} J_{\lceil n\rceil}(kr)=\frac{1}{2}\sum_{n\in\mathbb{Z}} e^{in\varphi}\, e^{\frac{i}{2}\lceil n\rceil\pi}\left[H_{\lceil n\rceil}^{(1)}(kr)+H_{\lceil n\rceil}^{(2)}(kr)\right]$$

where **r** and **k** are the two-dimensional vectors, φ is the angle between them, $J_{\alpha}(u)$, $H_{\alpha}^{(1)}(u)$, $H_{\alpha}^{(2)}(u)$ are the Bessel, first- and second-kind Hankel functions of order α, $\mathbb{Z}$ is the set of integer numbers

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At large distances:

$$\psi_{\boldsymbol{k}}^{(0)}(\boldsymbol{r})\underset{r\to\infty}{=}\frac{1}{\sqrt{2\pi kr}}\sum_{n\in\mathbb{Z}} e^{in\varphi} e^{\frac{i}{2}\lceil n\rceil\pi}\left[e^{i(kr-\frac{1}{2}\lceil n\rceil\pi-\frac{1}{4}\pi)}+e^{-i(kr-\frac{1}{2}\lceil n\rceil\pi-\frac{1}{4}\pi)}\right]=$$

$$=\sqrt{2\frac{\pi}{kr}}e^{i(kr-\pi/4)}\Delta(\varphi)+\sqrt{2\frac{\pi}{kr}}e^{-i(kr-\pi/4)}\Delta(\varphi-\pi)$$

where $\Delta(\varphi)=\frac{1}{2\pi}\sum_{n\in\mathbb{Z}} e^{in\varphi}$ is the delta-function for the azimuthal angle, $\Delta(\varphi+2\pi)=\Delta(\varphi)$

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The plane wave passing through the origin (r=0) can be naturally interpreted at large distances from the origin as a superposition of two cylindrical waves: the diverging one, e^{ikr}, in the forward, $\varphi=0$, direction and the converging one, e^{-ikr}, from the backward, $\varphi=\pi$, direction.

Scattering by an impermeable tube

$$\psi_{\boldsymbol{k}}(\boldsymbol{r})=\sum_{n\in\mathbb{Z}} e^{in\varphi} e^{\frac{i}{2}|n|\pi}\left[J_{|n|}(kr)-\frac{J_{|n|}(kr_c)}{H^{(1)}_{|n|}(kr_c)}H^{(1)}_{|n|}(kr)\right]$$

$$\psi_{\boldsymbol{k}}(\boldsymbol{r})_{r=r_c}=0\,,\qquad \lim_{r\to\infty} e^{ikr}\psi_{\boldsymbol{k}}(\boldsymbol{r})_{\varphi=\pm\pi}=1$$

At large distances:

$$\psi_{\boldsymbol{k}}(\boldsymbol{r})\underset{kr\gg1}{=}\frac{-1}{\sqrt{2\pi k r}}e^{i(kr-\pi/4)}\sum_{n\in\mathbb{Z}} e^{in\varphi}\frac{H^{(2)}_{|n|}(kr_c)}{H^{(1)}_{|n|}(kr_c)}+\sqrt{\frac{2\pi}{kr}}e^{-i(kr-\pi/4)}\Delta(\varphi-\pi)$$

at low energies ($k\to0$):

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where $\Delta_x(\varphi)=\frac{1}{2\pi}\sum_{|n|\le x} e^{in\varphi}$ is the regularized (smoothed) angular delta-function,

$$\lim_{x\to\infty}\Delta_x(\varphi)=\Delta(\varphi), \qquad \Delta_x(0)=\frac{x}{\pi}$$

Quantitative description

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \psi_{\boldsymbol{k}}^{(0)}(\boldsymbol{r}) + f(k,\ \varphi)\frac{e^{i(kr+\pi/4)}}{\sqrt{r}} + O(r^{-3/2})$$

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$$f(k,\varphi) = -\sqrt{\frac{\pi}{2\mathrm{k}}}|\ln(kr_c)|^{-1}\left[1+\left(\gamma - i\frac{\pi}{2}\right)|\ln(kr_c)|^{-1}\right] + k^{-1/2}O\left[|\ln(kr_c)|^{-3}\right], \qquad kr_c \ll 1$$

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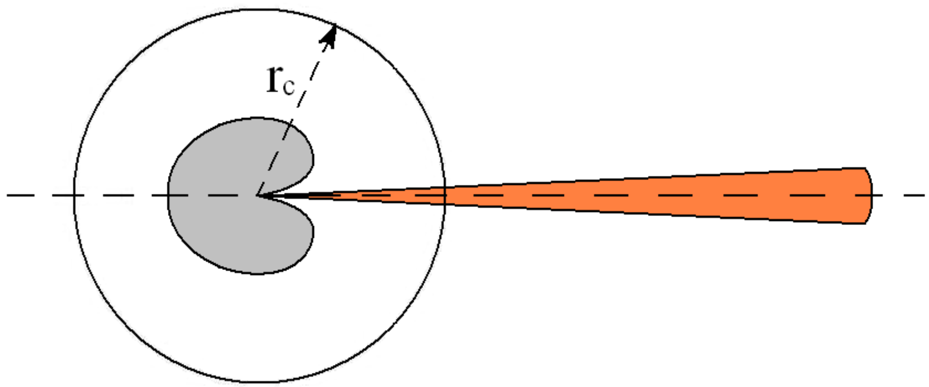
Fraunhofer diffraction — classical reflection

Differential cross section in the high-energy limit

$$\frac{d\sigma}{d\varphi}=|f_c(k,\varphi)|^2=2r_c\tilde{\Delta}_{kr_c}(\varphi)+\frac{r_c}{2}|\sin(\varphi/2)|+r_c O[(kr_c)^{-1/2}]$$

where $$\tilde{\Delta}_x(\varphi)=\frac{1}{4\pi x}\frac{\sin^2(x\varphi)}{\sin^2(\varphi/2)} \qquad (x\gg 1,\ -\pi<\varphi<\pi)$$

$$\lim_{x\to\infty}\tilde{\Delta}_x(\varphi)=\Delta(\varphi), \qquad \tilde{\Delta}_x(0)=\frac{x}{\pi}$$



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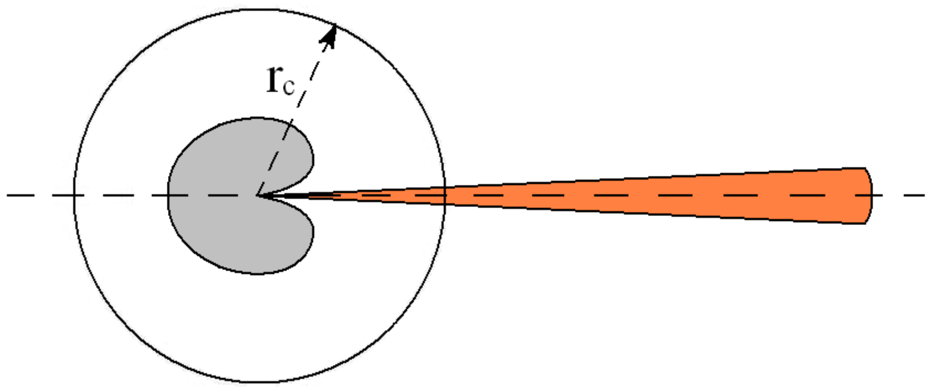

Total cross section

$$\sigma=\int_{-\pi}^{\pi}d\varphi\ \frac{d\sigma}{d\varphi}=\sigma^{(peak)}+\sigma^{(class)}=4r_c$$

$$\sigma^{(peak)}=\sigma^{(class)}=2r_c$$

S-matrix:

$$S(k,\varphi;k',\varphi')=\frac{1}{k}\delta(k-k')\Delta(\varphi-\varphi')+\delta(k-k')\frac{i}{\sqrt{2\pi k}}f(k,\varphi-\varphi')$$

Unitarity of S-matrix $S\,S^{+}=S^{+}S=I$

$$\frac{1}{i}\sqrt{\frac{k}{2\pi}}\left[f(k,\varphi'-\varphi'')-f^{*}(k,\varphi''-\varphi')\right]=\frac{k}{2\pi}\int_{-\pi}^{\pi}d\varphi\;\; f^{*}(k,\varphi-\varphi')\,f(k,\varphi-\varphi'')$$

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Optical theorem:

$$2\sqrt{\frac{2\pi}{k}}\,Im\,f(k,0)=\sigma$$

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at $k \rightarrow 0 \quad \frac{\pi^{2}}{k\;\ln^{2}(kr_{c})} = \frac{\pi^{2}}{k\;\ln^{2}(kr_{c})}$

at $k \rightarrow \infty \quad 4r_{c} = 4r_{c}$

Scattering by an impermeable magneti vortex

Schrödinger equation out of the vortex

$$\frac{-\hbar^2}{2\mathrm{m}}\left[\partial_r^2+\frac{1}{r}\partial_r+\frac{1}{r^2}\left(\partial_\varphi-i\frac{\Phi}{\Phi_0}\right)^2\right]\psi_{\boldsymbol{k}}(\boldsymbol{r})=\frac{\hbar^2k^2}{2\mathrm{m}}\psi_{\boldsymbol{k}}(\boldsymbol{r})$$

where Φ is the total flux of the vortex and $\Phi_0=2\pi\hbar c e^{-1}$ is the London flux quantum

1º condition $\lim_{r\to\infty} e^{ikr}\psi_{\boldsymbol{k}}(\boldsymbol{r})_{\varphi=\pm\pi}=1$

2º condition (Robin) $\left[\cos(\rho\pi)+r_c\sin(\rho\pi)\frac{\partial}{\partial r}\right]\psi_{\boldsymbol{k}}(\boldsymbol{r})\Big|_{r=r_c}=0$

$\rho=0$: Dirichlet (perfect conductivity of the boundary)

$\rho=1/2$: Neumann (absolute rigidity of the boundary)

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$$\psi_{\boldsymbol{k}}(\boldsymbol{r})=\sum_{n\in\mathbb{Z}} e^{in\varphi}e^{i\left(|n|-\frac{1}{2}|n-\mu|\right)\pi}\left[J_{|n-\mu|}(kr)-Y^{(\rho)}_{|n-\mu|}(kr_c)H^{(1)}_{|n-\mu|}(kr)\right]$$

where $\mu=\Phi\,\Phi_0^{-1}$ and

$$Y^{(\rho)}_{\alpha}(u)=\frac{\cos(\rho\pi)J_\alpha(u)+\sin(\rho\pi)u\frac{d}{du}J_\alpha(u)}{\cos(\rho\pi)H^{(1)}_\alpha(u)+\sin(\rho\pi)u\frac{d}{du}H^{(1)}_\alpha(u)}$$

Asymptotics at large distances

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \psi_{\boldsymbol{k}}^{(0)}(\boldsymbol{r}) e^{i\mu[\varphi - sgn(\varphi)\pi]} + f(k,\varphi)\frac{e^{i(kr+\pi/4)}}{\sqrt{r}} + O(r^{-3/2})$$

where

$$f(k,\varphi) = f_0(k,\varphi) + f_c(k,\varphi)$$

$$f_0(k,\varphi) = \frac{\sin(\mu\pi)}{\sqrt{2\pi k}} \sum_{n\in\mathbb{Z}} sgn(n-\mu) e^{in\varphi}$$

$$f_c(k,\varphi) = i\sqrt{\frac{2}{k\pi}} \sum_{n\in\mathbb{Z}} e^{in\varphi} e^{i(|n| - |n-\mu|\pi)} \Upsilon_{|n-\mu|}^{(\rho)}(kr_c)$$

Asymptotics at large distances

$$\psi_{\boldsymbol{k}}(\boldsymbol{r})=\psi_{\boldsymbol{k}}^{(0)}(\boldsymbol{r})e^{i\mu[\varphi-sgn(\varphi)\pi]}+f(k,\varphi)\frac{e^{i(kr+\pi/4)}}{\sqrt{r}}+O(r^{-3/2})$$

where

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S-matrix

$$S(k,\varphi;k',\varphi')=\cos(\mu\pi)\frac{1}{k}\delta(k-k')\Delta(\varphi-\varphi')+\delta(k-k')\frac{i}{\sqrt{2\pi k}}f(k,\varphi-\varphi')$$

Low-energy (ultraquantum) limit $k \to 0$: $f \to f_0$

$$f_0(k,\varphi) = i\,\frac{e^{i(\nu+1/2)\varphi}}{\sqrt{2\pi k}}\,\frac{\sin(\mu\pi)}{\sin(\varphi/2)},$$ ν is the integer part of μ

Y.Aharonov, D.Bohm. Phys.Rev. 115, 485 (1959)

$$\frac{d\sigma_0}{d\varphi} = \frac{1}{2\pi k}\,\frac{\sin^2(\mu\pi)}{\sin^2(\varphi/2)}$$

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S-matrix unitarity condition

$$\sin^2(\mu\pi)\,\Delta(\varphi'-\varphi'') = \frac{k}{2\pi}\int_{-\pi}^{\pi} d\varphi\; f_0^*(k,\varphi-\varphi')\,f_0(k,\varphi-\varphi'')$$

Optical theorem: $\Delta(0) = \Delta(0)$

$$\sigma_0 = \infty$$

High-energy (quasiclassical) limit $k \to \infty : f \to \lim_{kr_c \gg 1} f_c$

S-matrix unitarity condition

$$\frac{1}{i}\sqrt{\frac{k}{2\pi}}\cos(\mu\pi)[f_c(k,\varphi'-\varphi'')-f_c^*(k,\varphi''-\varphi')]+2\sin^2(\mu\pi)\Delta_{kr_c}^{(\nu)}(\varphi'-\varphi'')+O(\sqrt{kr_c})=$$

$$=\frac{k}{2\pi}\int_{-\pi}^{\pi} d\varphi\ f_c^*(k,\varphi-\varphi')f_c(k,\varphi-\varphi'')$$

where

$$\Delta_x^{(\nu)}(\varphi)=\frac{1}{2\pi}\sum_{|n-\mu|\le x} e^{in\varphi}, \qquad \Gamma_x^{(\nu)}(\varphi)=\frac{1}{2\pi i}\sum_{|n-\mu|\le x} sgn(n-\mu)\ e^{in\varphi}$$

Optical theorem

$$2\sqrt{\frac{2\pi}{k}}\cos(\mu\pi)Im f_c(k,0)+4r_c\sin^2(\mu\pi)+O(k^{-1})=\sigma, \quad \sigma=\int_{-\pi}^{\pi} d\varphi|f_c(k,\varphi)|^2, \quad kr_c\gg 1$$

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Scattering amplitude in the $kr_c \gg 1$ case:

$$f_c(k,\varphi)=i\sqrt{\frac{2\pi}{k}}\left[\cos(\mu\pi)\Delta_{kr_c}^{(\nu)}(\varphi)-\sin(\mu\pi)\Gamma_{kr_c}^{(\nu)}(\varphi)\right]-\sqrt{\frac{r_c}{2}|\sin(\varphi/2)|}\times$$
$$\times\exp\left\{-2ikr_c|\sin(\varphi/2)|+i\mu[\varphi-sgn(\varphi)\pi]\right\}\exp\left\{-i[2\chi^{(\rho)}(kr_c,\varphi)+\pi/4]\right\}+\sqrt{r_c}O[(kr_c)^{-1/6}], \quad kr_c\gg 1,$$

where $\chi^{(\rho)}(kr_c,\varphi)=\arctan\left[\frac{2kr_c|\sin^3(\varphi/2)|}{2\cot(\rho\pi)\sin^2(\varphi/2)-1}\right]$

Yu.A.S.,N.D.Vlasii, J.Phys.A 44, 315301 (2011)

Differential cross section in the high-energy limit

$$\frac{d\sigma}{d\varphi}=|f_c(k,\varphi)|^2=$$

$$=2r_c\left\{\cos(2\mu\pi)\tilde{\Delta}_{kr_c}(\varphi)+[1-\cos(2\mu\pi)+\sin(2\mu\pi)\sin(kr_c\varphi)]\tilde{\Delta}_{\frac{1}{2}kr_c}(\varphi)\right\}+$$

$$+\frac{r_c}{2}|\sin(\varphi/2)|+r_c O[(kr_c)^{-1/2}],$$

where $\tilde{\Delta}_x(\varphi)=\frac{1}{4\pi x}\frac{\sin^2(x\varphi)}{\sin^2(\varphi/2)} \qquad (x\gg 1,\ -\pi<\varphi<\pi)$

$$\lim_{x\to\infty}\tilde{\Delta}_x(\varphi)=\Delta(\varphi), \qquad \tilde{\Delta}_x(0)=\frac{x}{\pi}$$

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classical reflection

Fraunhofer diffraction

where $\tilde{\Delta}_x(\varphi) = \dfrac{1}{4\pi x}\dfrac{\sin^2(x\varphi)}{\sin^2(\varphi/2)} \qquad (x \gg 1,\ -\pi < \varphi < \pi)$

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Total cross section

$$\sigma = \int_{-\pi}^{\pi} d\varphi\ \frac{d\sigma}{d\varphi} = \sigma^{(peak)} + \sigma^{(class)} = 4r_c$$

$$\sigma^{(peak)} = \sigma^{(class)} = 2r_c$$

Yu.A.S., N.D.Vlasii, Ann.Phys. 326, 1441 (2011)
EPL 92, 60001 (2010)

In the strictly forward direction

$$\frac{d\sigma^{(peak)}}{d\varphi}=\frac{2}{\pi}k\,r_C^2\cos^2(\Phi\Phi_0^{-1}\pi),\qquad \varphi=0$$

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$$\Phi = n\Phi_0 / 2$$

$$\left.\frac{d\sigma}{d\varphi}\right|_{\varphi=0} = \left\{ \begin{array}{lr} 2kr_C^2\pi^{-1}, & even \quad n \\ 0 \quad , & odd \quad n \end{array} \right\}$$

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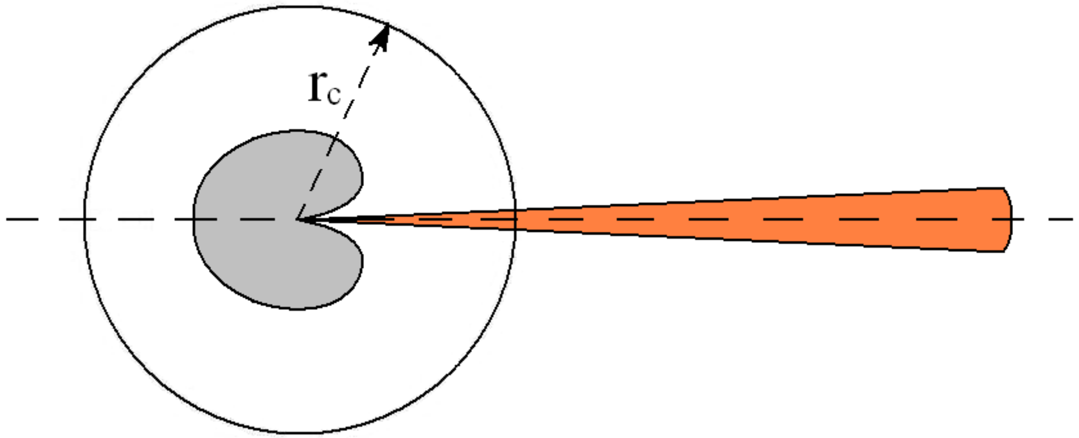

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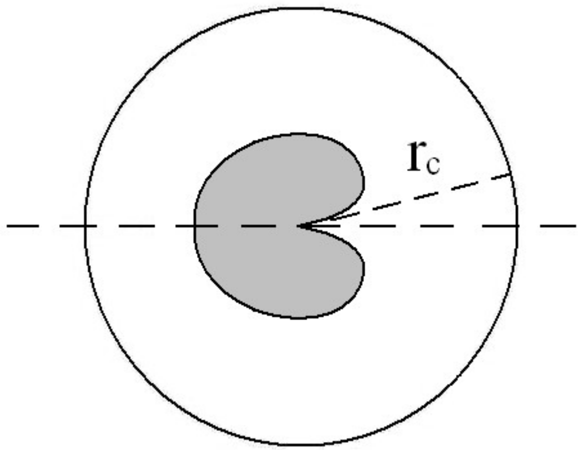

Conclusion

- Although the Aharonov-Bohm effect is the purely quantum effect that is alien to classical physics, it persists in the quasiclassical limit owing to the diffraction persisting in the short-wavelength limit in the forward direction.

- Hence, the enclosed magnetic flux serves as a gate for the propagation of high-energy, almost classical, particles.

- A direct scattering experiment with the use of quasiclassical (fast-moving) particles is quite feasible

FINAL REMARKS

The existence of the forward peak of the Fraunhofer diffraction in the hard-core scattering in the short-wavelength limit was known theoretically long before the theoretical discovery of the Aharonov-Bohm effect. Whereas the classical reflection is surely observed, the forward peak of the Fraunhofer diffraction is elusive to experimental measurements: as is noted in the monographs of P. M. Morse and H. Feshbach [Methods of Theoretical Physics II (McGraw-Hill, New York, 1953) Chapter 11, section 11.2.], it seems more likely that the measurable quantity is the classical cross section, although the details of this phenomenon depend on the method of measurement.

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We would like to draw attention to this long-standing experimental problem by pointing at the circumstances when the detection of the forward diffraction peak will be the detection of the Aharonov-Bohm effect persisting in the quasiclassical limit.

Thank you!