Massive, massless and ghost modes of gravitational waves from higher-order gravity

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Summary

- Field equations in higer-order gravity
- Polarization states of gravitational waves (GWs)
- Massive, massles and ghost modes
- Response of GW detectors
- The sthocastic background of GWs
- Conclusions

Field equations in higer-order gravity

Let us consider the action with curvature invariants

$$S = \int d^4x \sqrt{-g}\, f(R, P, Q)$$

$$P \equiv R_{ab}R^{ab}$$
$$Q \equiv R_{abcd}R^{abcd}$$

Varying with respect to the metric, one gets the field equations

$$\begin{aligned} F G_{\mu\nu} = {} & \frac{1}{2} g_{\mu\nu}\left(f - R\, F\right) - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F \\ & -2\left(f_P R^a_\mu R_{a\nu} + f_Q\, R_{abc\mu}R^{abc}{}_\nu\right) \\ & -g_{\mu\nu}\nabla_a\nabla_b(f_P R^{ab}) - \Box(f_P R_{\mu\nu}) \\ & +2\nabla_a\nabla_b\left(f_P\, R^a{}_{(\mu}\delta^b{}_{\nu)} + 2 f_Q\, R^a{}_{(\mu\nu)}{}^b\right) \end{aligned}$$

Field equations in higer-order gravity

Taking the trace

$$\Box\left(F+\frac{f_P}{3}R\right)=\frac{1}{3}\left(2f-RF-2\nabla_a\nabla_b((f_P+2f_Q)R^{ab})-2(f_PP+f_QQ)\right)$$

If we define

$$\Phi\equiv F+\frac{2}{3}(f_P+f_Q)R$$

and

$$\frac{dV}{d\Phi}\equiv\mathrm{RHS}$$

We get a Klein-Gordon equation for the scalar field

$$\Box\Phi=\frac{dV}{d\Phi}$$

To find the various GW modes, we need to linearize gravity about a Minkowski background:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$
$$\Phi=\Phi_0+\delta\Phi$$

Field equations in higer-order gravity

We get

$$\delta\Phi = \delta F + \frac{2}{3}(\delta f_P + \delta f_Q)R_0 + \frac{2}{3}(f_{P0} + f_{Q0})\delta R$$

$$R_0 \equiv R(\eta_{\mu\nu}) = 0$$

$$f_{P0} = \frac{\partial f}{\partial P}|_{\eta_{\mu\nu}}$$

denote the first order perturbation on the Ricci scalar that, after perturbing the Riemann and Ricci tensors, are given by

$$\delta R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\sigma\partial_\nu h_{\mu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma}\right)$$
$$\delta R_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma\partial_\nu h^\sigma{}_\mu + \partial_\sigma\partial_\mu h^\sigma{}_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu}\right)$$
$$\delta R = \partial_\mu\partial_\nu h^{\mu\nu} - \Box h$$

$$h = \eta^{\mu\nu} h_{\mu\nu}$$

The first term is

$$\delta F = \frac{\partial F}{\partial R}|_0\ \delta R + \frac{\partial F}{\partial P}|_0\ \delta P + \frac{\partial F}{\partial Q}|_0\ \delta Q$$

$$\delta F \simeq F_{,R0}\ \delta R$$

are second order

Gravitational Waves

And...

$$\delta\Phi = \left(F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0}) \right) \delta R$$

The Klein-Gordon equation for the scalar perturbation

$$\Box\delta\Phi = \frac{1}{3}\frac{F_0}{F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0})}\delta\Phi - \frac{2}{3}\delta R^{ab}\partial_a\partial_b(f_{P0} + 2f_{Q0}) - \frac{1}{3}\delta R\Box(f_{P0} + 2f_{Q0}) = m_s^2\delta\Phi$$

=0 since $f_{P0},\ f_{Q0}$ are consts.

We define the scalar mass of gravitational modes as

$$m_s^2 \equiv \frac{1}{3}\frac{F_0}{F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0})}.$$

Gravitational Waves

Perturbing the field equations, we get:

$$F_0(\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta R) = -(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)(\delta\Phi - \frac{2}{3}(f_{P0} + f_{Q0})\delta R)$$
$$-\eta_{\mu\nu}\partial_a\partial_b(f_{P0}\delta R^{ab}) - \Box(f_{P0}\delta R_{\mu\nu}) + 2\partial_a\partial_b(f_{P0}\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)} + 2f_{Q0}\ \delta R^a{}_{(\mu\nu)}{}^b)$$

It is convenient to work in Fourier space so that

$$\partial_\gamma h_{\mu\nu} \to ik_\gamma h_{\mu\nu}$$ and $$\Box h_{\mu\nu} \to -k^2 h_{\mu\nu}$$

Then the above equations become

$$F_0(\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta R) = (\eta_{\mu\nu}k^2 - k_\mu k_\nu)(\delta\Phi - \frac{2}{3}(f_{P0} + f_{Q0})\delta R)$$
$$+\eta_{\mu\nu}k_a k_b(f_{P0}\delta R^{ab}) + k^2(f_{P0}\delta R_{\mu\nu}) - 2k_a k_b(f_{P0}\ \delta R^a{}_{(\mu}\delta^b{}_{\nu)}) - 4k_a k_b(f_{Q0}\ \delta R^a{}_{(\mu\nu)}{}^b)$$

Gravitational Waves

We can rewrite the metric perturbation as

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\,\eta_{\mu\nu} + \eta_{\mu\nu} h_f$$

and use the gauge:

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \quad \text{and} \quad \bar{h} = 0$$

The first of these conditions implies that

$$k_\mu \bar{h}^{\mu\nu} = 0$$

while the second gives

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \eta_{\mu\nu} h_f$$

$$h = 4h_f$$

and we have

$$\delta R_{\mu\nu} = \frac{1}{2}\left(2k_\mu k_\nu h_f + k^2 \eta_{\mu\nu} h_f + k^2 \bar{h}_{\mu\nu}\right)$$

$$\delta R = 3k^2 h_f$$

$$k_\alpha k_\beta\ \delta R^{\alpha}{}_{(\mu\nu)}{}^{\beta} = -\frac{1}{2}\left((k^4 \eta_{\mu\nu} - k^2 k_\mu k_\nu) h_f + k^4 \bar{h}_{\mu\nu}\right)$$

$$k_a k_b\ \delta R^{a}{}_{(\mu} \delta^{b}{}_{\nu)} = \frac{3}{2} k^2 k_\mu k_\nu h_f$$

Gravitational Waves

...after some algebra, we get

$$\frac{1}{2}\left(k^2 - k^4\frac{f_{P0}+4f_{Q0}}{F_0}\right)\bar{h}_{\mu\nu} = (\eta_{\mu\nu}k^2 - k_\mu k_\nu)\frac{\delta\Phi}{F_0} + (\eta_{\mu\nu}k^2 - k_\mu k_\nu)h_f$$

$$h_f \equiv -\frac{\delta\Phi}{F_0}$$

the perturbation equation is

$$\left(k^2 + \frac{k^4}{m^2_{spin2}}\right)\bar{h}_{\mu\nu} = 0$$

$$m^2_{spin2} \equiv -\frac{F_0}{f_{P0}+4f_{Q0}}$$

we have a modified dispersion relation which corresponds to a massless spin-2 field ($k^2=0$) and a massive 2-spin ghost mode

$$k^2 = \frac{F_0}{\frac{1}{2}f_{P0}+2f_{Q0}} \equiv -m^2_{spin2}$$

Gravitational Waves

note that the propagator for $\bar{h}_{\mu\nu}$ can be rewritten as

$$G(k) \propto \frac{1}{k^2} - \frac{1}{k^2 + m^2_{spin2}}$$

the second term has the opposite sign, which indicates the presence of a ghost

Also, we can see that for the Gauss-Bonnet term

$$\mathcal{L}_{GB} = Q - 4P + R^2$$

$$f_{Q0} = 1 \qquad f_{P0} = -4$$

We have

Finally we obtain

$$k^2 \bar{h}_{\mu\nu} = 0$$

in this case we have no ghosts as expected

Gravitational Waves

$$\left(k^2 + \frac{k^4}{m^2_{spin2}}\right)\bar{h}_{\mu\nu} = 0$$

$$\Box h_f = m_s^2 h_f$$

GR

Solutions are plane waves

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p})\cdot exp(ik^\alpha x_\alpha) + cc$$

$$h_f = a(\overrightarrow{p})\cdot exp(iq^\alpha x_\alpha) + cc$$

Massive mode

Dispertion relations

$$k^\alpha \equiv (\omega_{m_{spin2}}, \overrightarrow{p}) \qquad \omega_{m_{spin2}} = \sqrt{m^2_{spin2} + p^2}$$

$$q^\alpha \equiv (\omega_{m_s}, \overrightarrow{p}) \qquad \omega_{m_s} = \sqrt{m_s^2 + p^2}.$$

Note:

The velocity of any ordinary" mode $\bar{h}_{\mu\nu}$ is the light speed c

But.... the dispersion law for the modes of h_f is that of a massive field which can be considered like a wave-packet

The h_f wave-packet group-speed, centered in p, is

$$\vec{v}_G = \frac{\vec{p}}{\omega},$$

$$v_G = \frac{\sqrt{\omega^2 - m^2}}{\omega}.$$

$$m = \sqrt{1 - v_G^2}\,\omega.$$

$$0\ \text{eV} \leqslant m \leqslant 10^{-33}\ \text{eV}$$

Polarization states of GWs

The above two conditions depend on the value of k^2

For $k^2=0$ mode → a massless spin-2 field with two independent polarizations plus a scalar mode

For $k^2 \neq 0$ mode → a massive spin-2 ghost mode and there are five independent polarization tensors plus a scalar mode

First case: massless spin-2 modes

In the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different from zero can be chosen. The condition $h = 0$ gives $A_{11} = -A_{22}$.

In this frame we may take the polarization bases defined in this way

$$e^{(+)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^{(\times)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^{(s)}_{\mu\nu} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

...the characteristic amplitude

$$h_{\mu\nu}(t,z) = A^{+}(t-z)e^{(+)}_{\mu\nu} + A^{\times}(t-z)e^{(\times)}_{\mu\nu} + h_s(t - v_G z)e^{s}_{\mu\nu}$$

two standard polarizations of GW arise from GR

the massive field arising from the generic higher-order theory

S.Capozziello, C. Corda, M. De Laurentis: PLB 669 (2008) 255

Second case: massive spin-2 modes

...we have six polarizations defined by

$$e_{\mu\nu}^{(+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(\times)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(B)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad e_{\mu\nu}^{(C)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e_{\mu\nu}^{(D)} = \frac{\sqrt{2}}{3}\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad e_{\mu\nu}^{(s)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

....and the amplitude in terms of the 6 polarization states as

$$h_{\mu\nu}(t,z) = A^{+}(t - v_{G_{s2}}z)e_{\mu\nu}^{(+)} + A^{\times}(t - v_{G_{s2}}z)e_{\mu\nu}^{(\times)} + B^{B}(t - v_{G_{s2}}z)e_{\mu\nu}^{(B)}$$
$$+ C^{C}(t - v_{G_{s2}}z)e_{\mu\nu}^{(C)} + D^{D}(t - v_{G_{s2}}z)e_{\mu\nu}^{(D)} + h_{s}(t - v_{G}z)e_{\mu\nu}^{s}.$$

$v_{G_{s2}}$ is the group velocity of the massive spin-2 field and it is given by

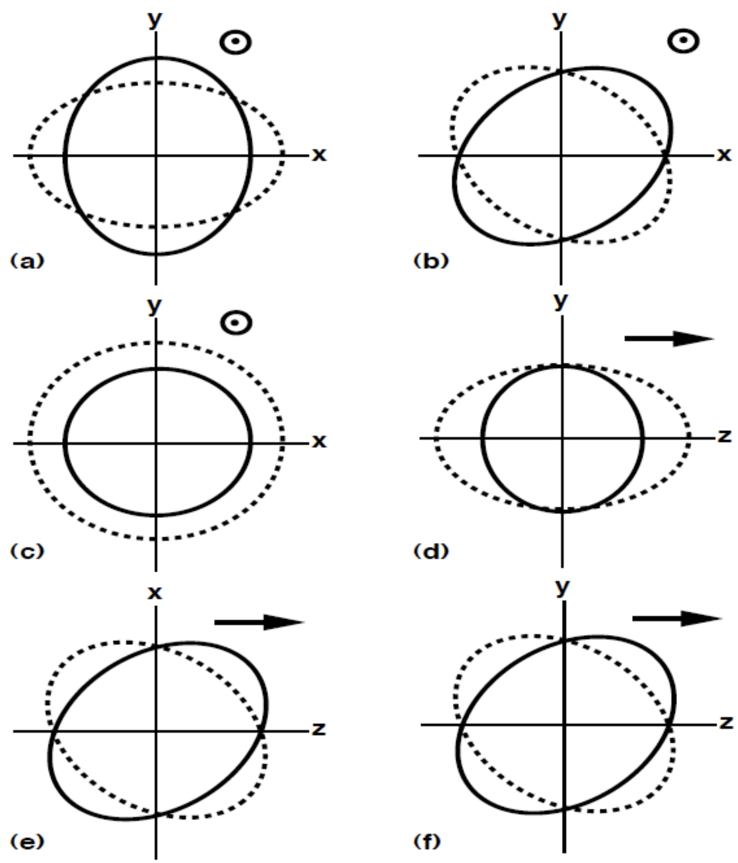
$$v_{G_{s2}} = \frac{\sqrt{\omega^2 - m_{s2}^2}}{\omega}$$

Polarization modes of GWs

Displacement induced by each mode on a sphere of test particles.

The wave propagates out of the plane in (a), (b), (c), and it propagates in the plane in (d), (e) and (f).

In (a) and (b) we have respectively the plus mode and cross mode, in (c) the scalar mode, in (d), (e) and (f) the D, B and C mode.



C. Bogdanos, S.Capozziello, M. De Laurentis, S. Nesseris, Astrpart. Phys. 34 (2010) 236

Ghost modes

The presence of ghost modes may seem a pathology
from a quantum-mechanical viewpoint

The ghost mode can be viewed as either a particle state of

positive energy and negative probability density!

or

positive probability density state with a negative energy

Like gaps and electrons in semiconductivity!!

Ghost modes

- The presence of such quasi-particle induces unitarity violation.

- The negative energy states lead to an unbounded theory where there is no minimal energy and the system thus becomes unstable.

- The vacuum can decay into pairs of ordinary and ghost gravitons leading to a catastrophic instability

A way out

• Imposing a very weak coupling of the ghost with the rest of the particles in the theory, such that the decay rate of the vacuum becomes comparable to the inverse of the Hubble scale coming out of the horizon.
• The today observed vacuum state becomes sufficiently stable.

This is not a viable option in our theory, since the ghost states come out in the gravitational sector, which couples to all kinds of matter. This solution seems physically and mathematically unlikely since ghost gravitons should couple differently than the standard massless gravitons (exotic forms of matter???).

Another way out

- Assuming that this picture does not hold up at arbitrarily high energies and that at some cutoff scale M_{cutoff} the theory gets modified appropriately as to ensure a ghost-free behavior and a stable ground state

This can happen for example if we assume that Lorentz invariance is violated at M_{cutoff}, thereby restricting any potentially harmful decay rates (R. Emparan and J. Garriga, JHEP 0603 (2006) 028)

Forthcoming answers at LHC (CERN) where gravitational massive states are expected at $M_{cutoff} \approx 1 \div 10$ TeV
S. Capozziello, G. Basini, M. De Laurentis EPJC 71 (2011) 1679

Ghost modes

The presence of massive ghost gravitons would induce on interferometers the same effects as an ordinary massive graviton transmitting the perturbation, but with the opposite sign in the displacement.

Tidal stretching from a polarized wave on the polarization plane will be turned into shrinking and viceversa.

This signal will, at the end, be a superposition of the displacements coming from the ordinary massless spin-2 gravitons and the massive ghosts.

Since these induce two competing effects, this fact will lead to a less pronounced signal than the one we would expect if the ghost mode was absent, setting in this way less severe constraints on the theory

The presence of the new modes will also affect the total energy density carried by the gravitational waves and this may also appear as a candidate signal in stochastic backgrounds, as we will see in the following

Detector response

Now one can study the detector response to each GW polarization without specifying, a priori, the theoretical model

...the angular pattern function of a detector to GWs is

$$F_A(\hat{\Omega}) = \mathbf{D} : \mathbf{e}_A(\hat{\Omega}) \,,$$

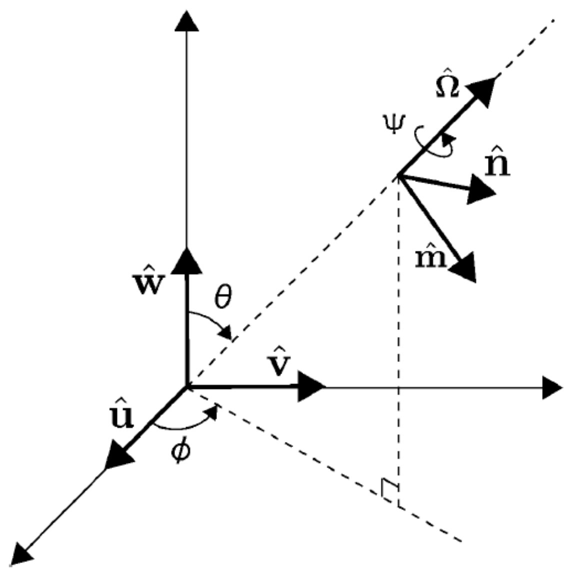
$$\mathbf{D} = \frac{1}{2}\left[\hat{\mathbf{u}} \otimes \hat{\mathbf{u}} - \hat{\mathbf{v}} \otimes \hat{\mathbf{v}}\right]$$

holds only when the arm length of the detector is smaller and smaller than the GW wavelength that we are taking into account

$$A = +, \times, B, C, D, s$$

detector tensor = response of a laser-interferometric detector

maps the metric perturbation in a signal on the detector



This is relevant for dealing with ground-based laser interferometers but this condition could not be valid when dealing with space interferometers like LISA

Detector response

the coordinate system for the GW, rotated by angles (θ ,ϕ), is

$$\begin{cases} \hat{\mathbf{u}}' = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\ \hat{\mathbf{v}}' = (-\sin\phi, \cos\phi, 0) \\ \hat{\mathbf{w}}' = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \end{cases}$$

The rotation with respect to the angle ψ , around the GW-propagating axis, gives the most general choice for the coordinate system

$$\begin{cases} \hat{\mathbf{m}} = \hat{\mathbf{u}}'\cos\psi + \hat{\mathbf{v}}'\sin\psi \\ \hat{\mathbf{n}} = -\hat{\mathbf{v}}'\sin\psi + \hat{\mathbf{u}}'\cos\psi \\ \hat{\mathbf{\Omega}} = \hat{\mathbf{w}}' \end{cases}$$

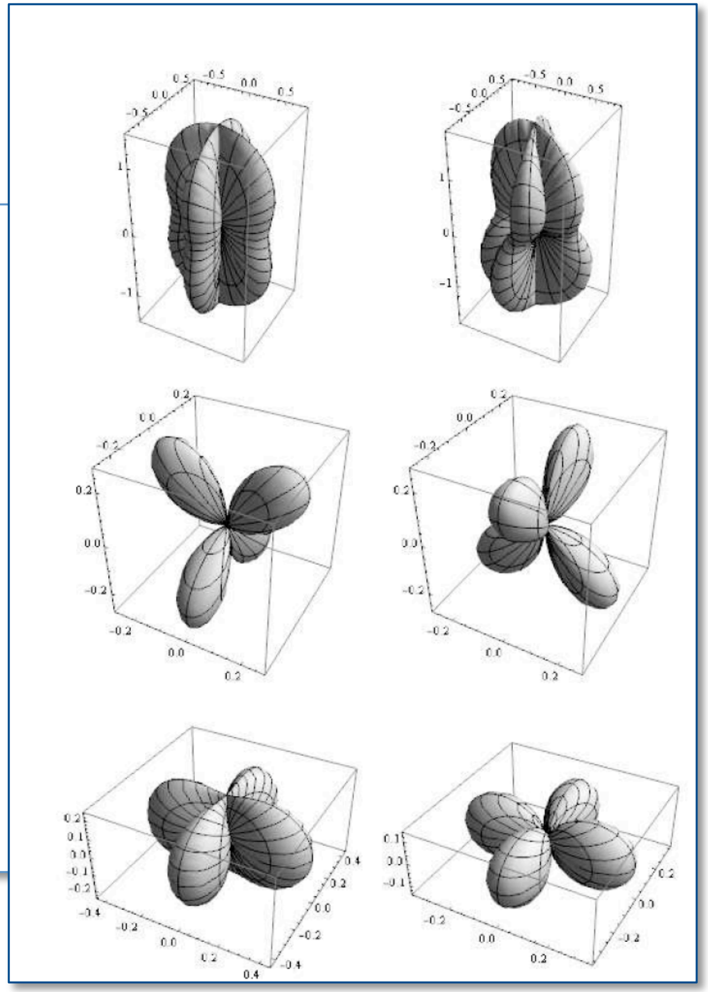
the polarization tensors are

$$\begin{aligned} \mathbf{e}_{+} &= \frac{1}{\sqrt{2}}(\hat{\mathbf{m}}\otimes\hat{\mathbf{m}} - \hat{\mathbf{n}}\otimes\hat{\mathbf{n}}) \ , \\ \mathbf{e}_{\times} &= \frac{1}{\sqrt{2}}(\hat{\mathbf{m}}\otimes\hat{\mathbf{n}} + \hat{\mathbf{n}}\otimes\hat{\mathbf{m}}) \ , \\ \mathbf{e}_{B} &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{m}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{m}}\right) \ , \\ \mathbf{e}_{C} &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{n}}\otimes\hat{\mathbf{\Omega}} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{n}}\right) \ . \\ \mathbf{e}_{D} &= \frac{\sqrt{3}}{2}\left(\frac{\hat{\mathbf{m}}}{2}\otimes\frac{\hat{\mathbf{m}}}{2} + \frac{\hat{\mathbf{n}}}{2}\otimes\frac{\hat{\mathbf{n}}}{2} + \hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \\ \mathbf{e}_{s} &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{\Omega}}\otimes\hat{\mathbf{\Omega}}\right) \ , \end{aligned}$$

Detector response

the angular patterns for each polarization

$$
\begin{aligned}
F_+(\theta,\phi,\psi) &= \frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi \cos 2\psi \\
&\quad - \cos\theta \sin 2\phi \sin 2\psi \ , \\
F_\times(\theta,\phi,\psi) &= -\frac{1}{\sqrt{2}}(1+\cos^2\theta)\cos 2\phi \sin 2\psi \\
&\quad - \cos\theta \sin 2\phi \cos 2\psi \ , \\
F_B(\theta,\phi,\psi) &= \sin\theta\,(\cos\theta\cos 2\phi\cos\psi - \sin 2\phi \sin\psi) \ , \\
F_C(\theta,\phi,\psi) &= \sin\theta\,(\cos\theta\cos 2\phi\sin\psi + \sin 2\phi \cos\psi) \ , \\
F_D(\theta,\phi) &= \frac{\sqrt{3}}{32}\cos 2\phi\left(6\sin^2\theta + (\cos 2\theta + 3)\cos 2\psi\right) \\
F_s(\theta,\phi) &= \frac{1}{\sqrt{2}}\sin^2\theta\cos 2\phi \ .
\end{aligned}
$$



The stochastic background of GWs

The contributions to the gravitational radiation coming from higher order gravity could be efficiently selected if it would be possible to investigate gravitational sources in extremely strong field regimes.

The further polarizations coming from the higher order contributions could be, in principle, investigated by the response of a single GW detector

The only realistic approach to investigate these further contribution seems the cosmological background

stochastic background of GWs

The stochastic background of GWs

GW background can be roughly divided into two classes of phenomena:

• the background generated by the incoherent superposition of gravitational radiation emitted by large populations of astrophysical sources, and

• the primordial GW background generated by processes in the early cosmological eras

it can be described and characterized by a dimensionless spectrum

$$\Omega_{sgw}(f) = \frac{1}{\rho_c} \frac{d\rho_{sgw}}{d\ln f},$$

→ energy density of part of the gravitational radiation contained in the frequency range f to $f + df$.

$$\rho_c \equiv \frac{3H_0^2}{8\pi G}$$

→ today observed Hubble expansion rate

↓ today critical energy density of the Universe

The stochastic background of GWs

The GW stochastic background energy density of all modes can be written as

$$\Omega^{A}_{\rm gw} \equiv \Omega^{+}_{\rm gw} + \Omega^{\times}_{\rm gw} + \Omega^{B}_{\rm gw} + \Omega^{C}_{\rm gw} + \Omega^{D}_{\rm gw} + \Omega^{s}_{\rm gw}$$

GR

$$\Omega^{GR}_{\rm gw} = \Omega^{+}_{\rm gw} + \Omega^{\times}_{\rm gw}$$

$$\Omega^{+}_{\rm gw} = \Omega^{\times}_{\rm gw}$$

Higher-order

$$\Omega^{HOG}_{\rm gw} = \Omega^{B}_{\rm gw} + \Omega^{C}_{\rm gw} + \Omega^{D}_{\rm gw}$$

$$\Omega^{B}_{\rm gw} = \Omega^{C}_{\rm gw} = \Omega^{D}_{\rm gw}$$

Scalar term

The stochastic background of GWs

The equation for the characteristic amplitude adapted to one of the components of the GWs can be used

$$h_A(f) \simeq 8.93 \times 10^{-19} \left(\frac{1Hz}{f}\right) \sqrt{h_{100}^2 \Omega_{gw}(f)}$$

and then we obtain for the GR modes

$$h_{GR}(100Hz) < 1.3 \times 10^{-23}$$

while for the higher-order modes

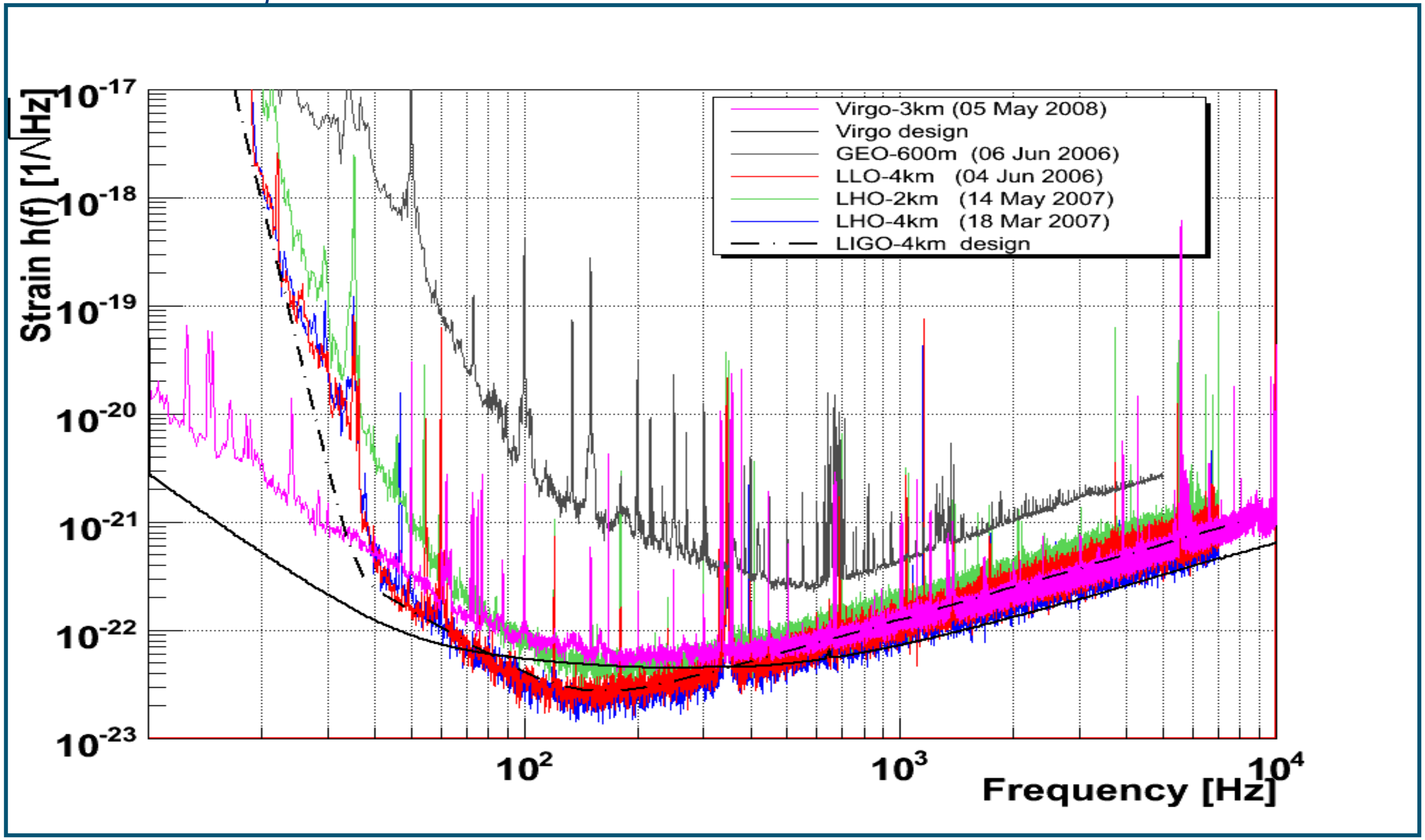
$$h_{HOG}(100Hz) < 7.3 \times 10^{-25}$$

and for scalar modes

$$h_s(100Hz) < 2 \times 1.410^{-26}$$

The stochastic background of GWs

Then, since we expect a sensitivity of the order of 10^{-22} for the above interferometers at ≈ 100Hz, we need to gain at least three orders of magnitude.



The stochastic background of GWs

Let us analyze the situation also at smaller frequencies.

The sensitivity of the VIRGO interferometer is of the order of 10^{-21} at ≈ 10Hz and in that case it is for the GR modes

$$h_{GR}(100Hz) < 1.3 \times 10^{-22}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-24}$$

and for scalar modes

$$h_{s}(100Hz) < 1.4 \times 10^{-25}$$

Still, these effects are below the sensitivity threshold to be observed today but new generation interferometers could be suitable (e.g. Advanced VIRGO-LIGO)

The stochastic background of GWs

The sensitivity of the LISA interferometer will be of the order of 10^{-22} at $\approx 10^{-3}Hz$ and in that case it is

$$h_{GR}(100Hz) < 1.3 \times 10^{-18}$$

while for the higher-order modes

$$h_{HOG}(100Hz) < 7.3 \times 10^{-20}$$

and for scalar modes

$$h_s(100Hz) < 1.4 \times 10^{-21}$$

This means that a stochastic background of relic GWs could be, in principle, detected by the LISA interferometer, including the additional modes.

Conclusions and remarks

- Our analysis covers the most extended gravity models with a generic class of Lagrangian density with higher-order terms as $f(R, P, Q)$, where $P \equiv R_{ab}R^{ab}$ and $Q \equiv R_{abcd}R^{abcd}$

- We have linearized the field equations for this class of theories around a Minkowski background and found that, besides a massless spin-2 field (the graviton), the theory contains also spin-0 and spin-2 massive modes with the latter being, in general, ghosts (detectable at LHC??!!).

- we have investigated the detectability of additional polarization modes of a stochastic GW with ground-based laser-interferometric detectors and space-interferometers. Such polarization modes, in general, appear in the ETG and can be used to constrain theories beyond GR.

Conclusions and remarks

....a point has to be discussed in detail !!!

if the source is coherent → The interferometer is directionally sensitive and we also know the orientation of the source

the massive mode coming from the simplest extension, f(R)-gravity, would induce longitudinal displacements along the direction of propagation which should be detectable and only the amplitude due to the scalar mode would be the true, detectable, "new" signal

we could have a second scalar mode inducing a similar effect, coming from the massive ghost, although with a minus sign.

one has deviations from the prediction of f(R)-gravity, even if only the massive modes are considered as new signal.

Conclusions and remarks

....another point !!!

if the source is not coherent (case of the stochastic background) and no directional detection of the gravitational radiation ⇒ the background has to be isotropic, the signal should be the same regardless of the orientation of the interferometer, no matter how the plane is rotated, it would always record the characteristic amplitude h_c.

There is no way to disentangle the modes in the background, being hc related to the total energy density of the gravitational radiation, which depends on the number of modes available

Every mode contributes in the same way, at least in the limit where the mass of massive and ghost modes are very small

It should be the number of available modes that makes the difference, not their origin.

Conclusions and remarks

- The massive modes are certainly of interest for direct detection by the LISA experiment

- Massive GW modes could be produced in more significant quantities in cosmological or early astrophysical processes in extended theories of gravity, being this possibility still unexplored
- They could constitute the new frontier of physics, if the Higgs boson is not discovered at LHC.