Covariant Lagrange multiplier constrained higher derivative gravity with scalar projectors

Based on the works with S.D. Odintsov,

Phys.Rev.D81:043001,2010, arXiv:0905.4213 [hep-th];

Phys.Lett.B691:60-64,2010, arXiv:1004.3613 [hep-th];

Phys.Rev.D83:023001,2011, arXiv:1007.4856 [hep-th]

and J. Kluson,

Phys.Lett.B701:117-126,2011, arXiv:1104.4286 [hep-th] .

Shin'ichi Nojiri (Nagoya Univ.)

Introduction

Petr Hořava, "Quantum Gravity at a Lifshitz Point"

Phys.Rev.D79:084008,2009, arXiv:0901.3775 [hep-th]

- A candidate of quantum field theory of gravity which is power-counting renormalizable.
- Anisotropy between space and time (explicit breaking of covariance). At long distances, the Lorentz symmetry could be recovered.

Problem: the existence of extra scalar mode violating the Newton law.

Today's talk: A Proposal of covariant and power-counting renormalizable model of gravity, where only massless graviton propagates.

Brief review on Hořava gravity

Einstein's general relativity:

non-renormalizable as a quantum field theory

In the flat background: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ (κ: gravitational coupling)

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R \\
&= \int d^4x\Big[-\frac{1}{2}\partial h\partial h + \kappa h\partial h\partial h + \kappa^2 h^2\partial h\partial h + \cdots \\
&\qquad +\kappa^n h^n \partial h\partial h + \cdots\Big]
\end{aligned}
$$

The dimension of κ : $[L]$ (L: length) $\Rightarrow$ non-renormalizable

If the propagator $1/p^2 \ \to 1/p^4,\ 1/p^6,\ \cdots$ (p_μ: four momentum)

$\Rightarrow$ UV behavior could be improved. $\Rightarrow$ higher derivative theory

unitarity could be broken in general, due the higher derivative with respect to t

Hořava's idea:

anisotropic treatment between space and time

higher derivative theory with respect to only spacial coordinates ($\boldsymbol{p}$: spacial momentum).

$1/p^2 \to 1/\boldsymbol{p}^4, 1/\boldsymbol{p}^6, \cdots, (|\boldsymbol{p}| \to \infty)$

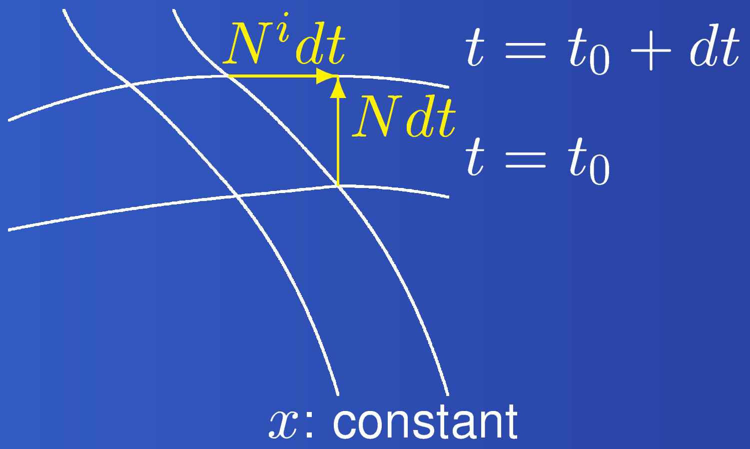
Anisotropy: Under the scale transformation with a constant b,

$$\boldsymbol{x} \to b\boldsymbol{x}, t \to b^z t, (z = 2, 3, \cdots)$$

ADM decomposition

Metric

$$ds^2 = -N^2 dt^2 + g^{(3)}_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)\ , \quad (i = 1, 2, 3)$$



N : lapse variable

N^i : shift variable

$$\int d^4x\sqrt{-g}R \Rightarrow \int d^3x dt N\sqrt{g^{(3)}}\left(K^{ij}K_{ij} - K^2 + R^{(3)}\right)$$

Extrinsic curvature : $K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)\ , \quad K = K^i_{\ i}$

Action of Hořava gravity: kinetic term $+$ "potential"

Kinetic term (including the derivatives with respect to time $\sim (\partial_t)^2$)

$$S_K = \frac{2}{\kappa^2} \int dt\, d^3\boldsymbol{x}\, \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 \right) , \quad \lambda \ : \ \text{parameter}$$

Symmetry: spacial diffeomorphism $\otimes$ temporal diffeomorphism

$$\delta x^i = \zeta^i(t, \boldsymbol{x}), \qquad \delta t = f(t)$$

Dimension of time t : $[L^z]$ (L: length)

$$ds^2 = -N^2 dt^2 + \cdots \ \Rightarrow \ [N] = [L^{1-z}]$$

In perturbation theory, gauge fixing : $N = N_0$

Effective coupling constant: $1/\kappa_{\mathrm{eff}}^2 = N_0/\kappa^2$, $[\kappa_{\mathrm{eff}}^2] = [L^{3-z}]$

When $z = 3$, κ_{eff}: dimensionless $\Rightarrow$ power counting renormalizable

“Potential” (not including the derivatives with respect to time)

Generalized De Witt “metric on the space of metrics”

$$\mathcal{G}^{ijk\ell} = \frac{1}{2}\left(g^{ik}g^{j\ell} + g^{i\ell}g^{jk}\right) - \lambda g^{ij}g^{k\ell}$$

Potential with “detailed balance”

$$S_V = \frac{\kappa^2}{8}\int dt\, d^3\boldsymbol{x}\,\sqrt{g}N\, E^{ij}\mathcal{G}_{ijk\ell}E^{k\ell} \qquad \sqrt{g}E^{ij} = \frac{\delta W[g_{k\ell}]}{\delta g_{ij}}$$

$z = 2$ model $\cdots\ (\partial_t)^2 \leftrightarrow \left(\partial_x^2\right)^2$

$$W = \frac{1}{\kappa_W^2}\int d^3\boldsymbol{x}\,\sqrt{g}(R - 2\Lambda_W)$$

$$\Rightarrow\ S_V = \frac{\kappa^2}{8\kappa_W^4}\int dt\, d^3\boldsymbol{x}\,\sqrt{g}N\left(R^{ij} - \frac{1}{2}Rg^{ij} + \Lambda_W g^{ij}\right)\mathcal{G}_{ijk\ell}\left(R^{k\ell} - \frac{1}{2}Rg^{k\ell} + \Lambda_W g^{k\ell}\right)$$

$z = 3$ model $\cdots$ $(\partial_t)^2 \leftrightarrow (\partial_x^3)^2$

$$W = \frac{1}{w^2}\int_\Sigma \omega_3(\Gamma) \quad w^2 \ : \text{ dimensionless coupling}$$

$\omega_3(\Gamma)$: gravitational Chern-Simons term

$$\omega_3(\Gamma) = \mathrm{Tr}\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) \equiv \varepsilon^{ijk}\left(\Gamma^m_{i\ell}\partial_j \Gamma^\ell_{km} + \frac{2}{3}\Gamma^n_{i\ell}\Gamma^\ell_{jm}\Gamma^m_{kn}\right) d^3\boldsymbol{x}$$

$\Rightarrow$

$$S = \int dt\, d^3\boldsymbol{x}\, \sqrt{g}\, N \left\{ \frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} \right\}$$

$$= \int dt\, d^3\boldsymbol{x}\, \sqrt{g}\, N \left\{ \frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2w^4}\left(\nabla_i R_{jk} \nabla^i R^{jk} - \nabla_i R_{jk}\nabla^j R^{ik} - \frac{1}{8}\nabla_i R \nabla^i R\right)\right\}$$

Cotton tensor : $C^{ij} = \varepsilon^{ik\ell}\nabla_k\left(R^j_\ell - \frac{1}{4}R\delta^j_\ell\right)$

Problems on Hořava gravity

Symmetry: spacial diffeomorphism $\otimes$ temporal diffeomorphism

$$\delta x^i = \zeta^i(t, \boldsymbol{x}), \qquad \delta t = f(t)$$

Projectability condition: Assume $N = N(t)$ from the beginning

Degrees of freedom of Hořava gravity $\neq$ Degrees of freedom of Einstein gravity

$\Rightarrow$ does not reproduce general relativity even in the low energy region

C. Charmousis, G. Niz, A. Padilla, P.M. Saffin,

"Strong coupling in Horava gravity" arXiv:0905.2579

Miao Li, Yi Pang, "A Trouble with Hořava-Lifshitz Gravity", arXiv:0905.2751

D. Blas, O. Pujolas, S. Sibiryakov,

"Consistent Extension of Horava Gravity" arXiv:0909.3525

etc.

Covariant Model

Correct degrees of freedom

$\Leftrightarrow$ Covariance (full diffeomorphism invariance)

$\Rightarrow$ Proposal of covariant and renormalizable models:

S.N., S.D. Odintsov, + J. Kluson

arXiv:0905.4213 [hep-th]; arXiv:1004.3613 [hep-th]; arXiv:1007.4856 [hep-th];

arXiv:1104.4286 [hep-th]

Idea:

Spontaneous breakdown of Lorentz symmetry

$$\partial_\mu \phi \neq 0$$

The breakdown $\Leftarrow$ constraint

$\sim$ Stückelberg formulation of massive vector field

Models (two kinds of models):

$$S_{2n+2} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \alpha\left\{(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho)^n\right.\right.$$
$$\left.\times P_\alpha^{\ \mu}P_\beta^{\ \nu}\left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\}$$
$$\times\left\{(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho)^n P^{\alpha\mu}P^{\beta\nu}\left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\}$$
$$\left.-\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right],$$

for $z = 2n+2$ model $(n = 0, 1, 2, \cdots)$

λ: Lagrange multiplier field

U_0: constant

$P_\mu^{\ \nu}$: projection operator $P_\mu^{\ \nu} \equiv \delta_\mu^{\ \nu} + \frac{\partial_\mu\phi\partial^\nu\phi}{2U_0}$

$$
S_{2n+3} = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \alpha\left\{(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho)^n\right.\right.
$$

$$
\left.\times P_\alpha{}^\mu P_\beta{}^\nu\left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\}
$$

$$
\times\left\{(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho)^{n+1}\,P^{\alpha\mu}P^{\beta\nu}\left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\}
$$

$$
\left.-\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right]\,.
$$

for $z = 2n + 3$ model $(n = 0, 1, 2, \cdots)$

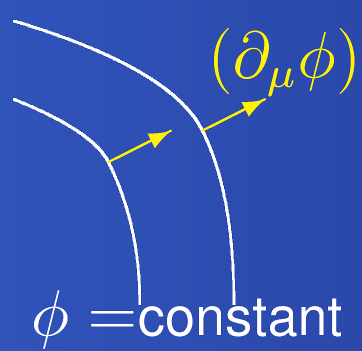
λ: Lagrange multiplier field

$$\Rightarrow \text{ constraint } : \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0 = 0 \Rightarrow (\partial_\mu\phi) \ : \text{ time-like}$$

Locally, one can choose the direction of time to be parallel to $(\partial_\mu\phi)$

$$\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 = U_0$$

Spacial region : ϕ =constant hypersurface (equipotential surface)



Spontaneous breakdown of Lorentz symmetry

The actions admit a flat space vacuum solution.

Field equations:

$$0 = \frac{1}{2\kappa^2}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + G^{\text{higher}}_{\mu\nu} - \frac{\lambda}{2}\partial_\mu\phi\partial_\nu\phi + \frac{\lambda}{2}g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + U_0\right) .$$

Assuming the flat vacuum solution,

$$0 = \lambda\partial_\mu\phi\partial_\nu\phi .$$

$\Rightarrow \lambda = 0$.

$\Rightarrow$ Flat space vacuum solution with $\lambda = 0$.

Perturbation from the flat background

Diffeomorphism invariance with respect to time coordinate

$$\Rightarrow \quad \phi = \sqrt{2U_0}t \quad \text{(unitary) gauge condition .}$$

Perturbation from flat background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$S_{2n+2} \to \int d^4x \left[-\frac{1}{8\kappa^2} \left\{ -2h_{tt} \left(\delta^{ij}\partial_k\partial^k - \partial^i\partial^j \right) h_{ij} \right.\right.$$
$$+2h_{ti} \left(\delta^{ij}\partial_k\partial^k - \partial^i\partial^j \right) h_{tj} + h_{ti} \left(2\delta^{jk}\partial^i - \delta^{ik}\partial^j - \delta^{ij}\partial^k \right) \partial_t h_{jk}$$
$$+h_{ij} \left(\left(\delta^{ij}\delta^{kl} - \frac{1}{2}\delta^{ik}\delta^{jl} - \frac{1}{2}\delta^{il}\delta^{jk} \right) \left(-\partial_t^2 + \partial_k\partial^k \right) \right.$$
$$\left. \left. -\delta^{ij}\partial^k\partial^l - \delta^{kl}\partial^i\partial^j + \frac{1}{2} \left(\delta^{ik}\partial^j\partial^l + \delta^{il}\partial^j\partial^k + \delta^{jk}\partial^i\partial^l + \delta^{jl}\partial^i\partial^k \right) \right) h_{kl} \right\}$$
$$-2^{2n-2}\alpha U_0^{2n} \left\{ \left(\partial_k\partial^k \right)^n \left(h_{ki,j}^{\ \ \ \ k} + h_{kj,i}^{\ \ \ \ k} - h_{ij,k}^{\ \ \ \ k} - \partial_i\partial_j \left(h_\mu^{\ \mu} \right) \right) \right\}$$
$$\left. \times \left\{ \left(\partial_k\partial^k \right)^n \left(h_{k\ \ ,}^{\ i\ \ jk} + h_{k\ \ ,}^{\ j\ \ ik} - h^{ij}_{\ \ ,k}{}^{k} - \partial^i\partial^j \left(h_\mu^{\ \mu} \right) \right) \right\} + U_0\lambda h_{tt} \right] ,$$

$$
\begin{aligned}
S_{2n+3} \to \int d^4x \Bigg[& -\frac{1}{8\kappa^2} \Big\{ -2h_{tt} \left(\delta^{ij}\partial_k\partial^k - \partial^i\partial^j\right) h_{ij} \\
& +2h_{ti}\left(\delta^{ij}\partial_k\partial^k - \partial^i\partial^j\right) h_{tj} + h_{ti}\left(2\delta^{jk}\partial^i - \delta^{ik}\partial^j - \delta^{ij}\partial^k\right)\partial_t h_{jk} \\
& +h_{ij}\left(\left(\delta^{ij}\delta^{kl} - \frac{1}{2}\delta^{ik}\delta^{jl} - \frac{1}{2}\delta^{il}\delta^{jk}\right)\left(-\partial_t^2 + \partial_k\partial^k\right)\right. \\
& \left. -\delta^{ij}\partial^k\partial^l - \delta^{kl}\partial^i\partial^j + \frac{1}{2}\left(\delta^{ik}\partial^j\partial^l + \delta^{il}\partial^j\partial^k + \delta^{jk}\partial^i\partial^l + \delta^{jl}\partial^i\partial^k\right)\right) h_{kl} \Big\} \\
& -2^{2n-1}\alpha U_0^{2n+1} \left\{ \left(\partial_k\partial^k\right)^n \left(h_{ki,j}{}^{k} + h_{kj,i}{}^{k} - h_{ij,k}{}^{k} - \partial_i\partial_j\left(h_\mu{}^\mu\right)\right)\right\} \\
& \times \left\{ \left(\partial_k\partial^k\right)^{n+1} \left(h_k{}^{i}{}_{,}{}^{jk} + h_k{}^{j}{}_{,}{}^{ik} - h^{ij}{}_{,k}{}^{k} - \partial^i\partial^j\left(h_\mu{}^\mu\right)\right)\right\} + U_0\lambda h_{tt} \Bigg] .
\end{aligned}
$$

We show that the **only propagating mode is higher derivative graviton** while scalar and vector modes do not propagate.

$\delta\lambda \;\Rightarrow\; h_{tt} = 0.$

$\delta h_{tt} \;\Rightarrow$

$$\begin{aligned} \lambda &= -\frac{1}{4\kappa^2 U_0}\left(\delta^{ij}\partial_k\partial^k - \partial^i\partial^j\right)h_{ij} \\ &+2^{2n-1}\alpha U_0^{2n-1}\left(\partial_k\partial^k\right)^{2n}\partial^i\partial^j\left(h_{ki,j}{}^{k} + h_{kj,i}{}^{k} - h_{ij,k}{}^{k} - \partial_i\partial_j\left(h_\mu^{\ \mu}\right)\right)\,, \end{aligned}$$

or

$$\begin{aligned} \lambda &= -\frac{1}{4\kappa^2 U_0}\left(\delta^{ij}\partial_k\partial^k - \partial^i\partial^j\right)h_{ij} \\ &+2^{2n}\alpha U_0^{2n}\left(\partial_k\partial^k\right)^{2n+1}\partial^i\partial^j\left(h_{ki,j}{}^{k} + h_{kj,i}{}^{k} - h_{ij,k}{}^{k} - \partial_i\partial_j\left(h_\mu^{\ \mu}\right)\right)\,. \end{aligned}$$

$\delta\phi \;\Rightarrow$

$$\begin{aligned} 0 &= \partial_t\left\{\lambda + 2^{2n-1}\alpha U_0^{2n-1}\left(\partial_k\partial^k\right)^{2n}\partial^i\partial^j\left(h_{ki,j}{}^{k} + h_{kj,i}{}^{k} - h_{ij,k}{}^{k} - \partial_i\partial_j\left(h_\mu^{\ \mu}\right)\right)\right\}\,, \\ 0 &= \partial_t\left\{\lambda + 2^{2n}\alpha U_0^{2n}\left(\partial_k\partial^k\right)^{2n+1}\partial^i\partial^j\left(h_{ki,j}{}^{k} + h_{kj,i}{}^{k} - h_{ij,k}{}^{k} - \partial_i\partial_j\left(h_\mu^{\ \mu}\right)\right)\right\}\,. \end{aligned}$$

Decomposition of h_{ti} (shift function N_i)

$$h_{ti} = \partial_i s + v_i \,, \quad \partial^i v_i = 0 \,, \quad s\text{: spatial scalar}$$

Linearized diffeomorphism invariance transformations with respect to the spatial coordinates:

$$\delta x^i = \partial^i u + w^i \,, \quad \partial_i w^i = 0 \;\Rightarrow\; \delta s = \partial_t u \,, \quad \delta v_i = \partial_t w_i \,,$$

Gauge fixing condition $s = v^i = 0 \;\Rightarrow\; h_{ti} = 0$

$$\delta h_{ti} \;\Rightarrow\; \partial_t \left(-2\delta^{jk}\partial^i + \delta^{ik}\partial^j + \delta^{ij}\partial^k \right) h_{jk} = 0 \,,$$

identical with that in the Einstein gravity
(not including higher derivative terms)

Decomposition of h_{ij}

$$h_{ij} = \delta_{ij}A + \partial_j B_i + \partial_i B_j + C_{ij} + \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\partial_k\partial^k\right)E\,,$$

$$\partial^i B_i = 0\,, \quad \partial^i C_{ij} = \partial^j C_{ij} = 0\,, \quad C_i{}^i = 0\,,$$

$$\delta h_{ti} \;\Rightarrow\; 0 = \partial_t\left(-4\partial_i A + 2\partial_k\partial^k B_i + \frac{4}{3}\partial_i\partial_k\partial^k E\right) \cdots (*)$$

$$\partial^i\times \;\Rightarrow\; \partial_t\partial_i\partial^i\left(-4A + \frac{4}{3}\partial_k\partial^k E\right) = 0 \;\Rightarrow\; A = \frac{1}{3}\partial_k\partial^k E \cdots (**)$$

by assuming A, $E \to 0$ at spatial infinity.

$$(*), (**) \;\Rightarrow\; \partial_t\partial_j\partial^j B_i = 0 \;\Rightarrow\; B_i = 0$$

by assuming $B_i \to 0$ at spatial infinity.

δh_{tt} and $\delta\phi$ equations:

$$\begin{aligned} \lambda &= \frac{1}{2\kappa^2 U_0}\partial_k\partial^k\left(-A+\frac{1}{3}\partial_j\partial^j E\right) - 2^{2n}\alpha U_0^{2n-1}\left(\partial_k\partial^k\right)^{2n+2}\left(-A+\frac{1}{3}\partial_j\partial^j E\right), \\ 0 &= \partial_t\left\{\lambda + 2^{2n}\alpha U_0^{2n-1}\left(\partial_k\partial^k\right)^{2n+2}\left(-A+\frac{1}{3}\partial_j\partial^j E\right)\right\}, \end{aligned}$$

or

$$\begin{aligned} \lambda &= \frac{1}{2\kappa^2 U_0}\partial_k\partial^k\left(-A+\frac{1}{3}\partial_j\partial^j E\right) - 2^{2n+1}\alpha U_0^{2n}\left(\partial_k\partial^k\right)^{2n+3}\left(-A+\frac{1}{3}\partial_j\partial^j E\right), \\ 0 &= \partial_t\left\{\lambda + 2^{2n+1}\alpha U_0^{2n}\left(\partial_k\partial^k\right)^{2n+3}\left(-A+\frac{1}{3}\partial_j\partial^j E\right)\right\}. \end{aligned}$$

$\Rightarrow \lambda = 0$ since $A = \frac{1}{3}\partial_k\partial^k E$

Scalar modes λ and the vector mode B_i do not propagate.

$\delta A \Rightarrow$

$$
\begin{aligned}
0 &= +\frac{1}{8\kappa^2}\left\{-12\left(-\partial_t^2+\partial_k\partial^k\right)A+8\partial_k\partial^k A+\frac{4}{3}\left(\partial_k\partial^k\right)^2 E\right\} \\
&\quad -2^{2n-1}\alpha U_0^{2n}\left(-\partial_i\partial_j-\delta_{ij}\partial_k\partial^k\right)\left\{\left(\partial_k\partial^k\right)^{2n}\right. \\
&\quad \times\left.\left(-\partial^i\partial^j A-\delta^{ij}\partial_k\partial^k A+\frac{1}{3}\partial^i\partial^j\partial_k\partial^k E+\frac{1}{3}\delta^{ij}\left(\partial_k\partial^k\right)^2 E\right)\right\},
\end{aligned}
$$

or

$$
\begin{aligned}
0 &= +\frac{1}{8\kappa^2}\left\{-12\left(-\partial_t^2+\partial_k\partial^k\right)A+8\partial_k\partial^k A+\frac{4}{3}\left(\partial_k\partial^k\right)^2 E\right\} \\
&\quad -2^{2n}\alpha U_0^{2n+1}\left(-\partial_i\partial_j-\delta_{ij}\partial_k\partial^k\right)\left\{\left(\partial_k\partial^k\right)^{2n+1}\right. \\
&\quad \times\left.\left(-\partial^i\partial^j A-\delta^{ij}\partial_k\partial^k A+\frac{1}{3}\partial^i\partial^j\partial_k\partial^k E+\frac{1}{3}\delta^{ij}\left(\partial_k\partial^k\right)^2 E\right)\right\},
\end{aligned}
$$

$\delta E \Rightarrow$

$$0=\partial_k\partial^k\left[\frac{1}{8\kappa^2}\left\{\frac{4}{3}\left(-\partial_t^2+\partial_k\partial^k\right)\partial_k\partial^k E+\frac{4}{3}\partial_k\partial^k A+\frac{16}{9}\left(\partial_k\partial^k\right)^2 E\right\}\right.$$
$$+\frac{2^{2n-1}}{3}\alpha U_0^{2n}\left(-\partial_i\partial_j-\delta_{ij}\partial_k\partial^k\right)\left\{\left(\partial_k\partial^k\right)^{2n}\right.$$
$$\left.\left.\times\left(-\partial^i\partial^j A-\delta^{ij}\partial_k\partial^k A+\frac{1}{3}\partial^i\partial^j\partial_k\partial^k E+\frac{1}{3}\delta^{ij}\left(\partial_k\partial^k\right)^2 E\right)\right\}\right],$$

or

$$0=\partial_k\partial^k\left[\frac{1}{8\kappa^2}\left\{\frac{4}{3}\left(-\partial_t^2+\partial_k\partial^k\right)\partial_k\partial^k E+\frac{4}{3}\partial_k\partial^k A+\frac{16}{9}\left(\partial_k\partial^k\right)^2 E\right\}\right.$$
$$+\frac{2^{2n}}{3}\alpha U_0^{2n+1}\left(-\partial_i\partial_j-\delta_{ij}\partial_k\partial^k\right)\left\{\left(\partial_k\partial^k\right)^{2n+1}\right.$$
$$\left.\left.\times\left(-\partial^i\partial^j A-\delta^{ij}\partial_k\partial^k A+\frac{1}{3}\partial^i\partial^j\partial_k\partial^k E+\frac{1}{3}\delta^{ij}\left(\partial_k\partial^k\right)^2 E\right)\right\}\right],$$

$\Rightarrow\ 0=\partial_t^2 A$ (using $A=\frac{1}{3}\partial_k\partial^k E$) $\Rightarrow A=E=0.$

All the scalar modes ϕ, λ, h_{tt}, s, A, and E

and all the vector modes v_i and B_i do not propagate.

The only propagating mode is massless graviton C_{ij}.

($\Leftrightarrow$ Hořava quantum gravity)

$$\begin{aligned} S_{2n+2} &= \int d^4x \left[\frac{1}{8\kappa^2} \left\{ C_{ij} \left(-\partial_t^2 + \partial_k \partial^k \right) C^{ij} \right\} \right. \\ &\left. -2^{2n-2} \alpha U_0^{2n} \left\{ \left(\partial_k \partial^k \right)^{n+1} C_{ij} \right\} \left\{ \left(\partial_k \partial^k \right)^{n+1} C^{ij} \right\} \right], \\ S_{2n+3} &= \int d^4x \left[\frac{1}{8\kappa^2} \left\{ C_{ij} \left(-\partial_t^2 + \partial_k \partial^k \right) C^{ij} \right\} \right. \\ &\left. -2^{2n-1} \alpha U_0^{2n+1} \left\{ \left(\partial_k \partial^k \right)^{n+1} C_{ij} \right\} \left\{ \left(\partial_k \partial^k \right)^{n+2} C^{ij} \right\} \right]. \end{aligned}$$

Propagator:

$$\langle h_{ij}(p)h_{kl}(-p)\rangle = \langle C_{ij}(p)C_{kl}(-p)\rangle$$
$$= \frac{1}{2}\left\{\left(\delta_{ij} - \frac{p_ip_j}{\boldsymbol{p}^2}\right)\left(\delta_{kl} - \frac{p_kp_l}{\boldsymbol{p}^2}\right) - \left(\delta_{ik} - \frac{p_ip_k}{\boldsymbol{p}^2}\right)\left(\delta_{jl} - \frac{p_jp_l}{\boldsymbol{p}^2}\right)\right.$$
$$\left. - \left(\delta_{il} - \frac{p_ip_l}{\boldsymbol{p}^2}\right)\left(\delta_{jk} - \frac{p_jp_k}{\boldsymbol{p}^2}\right)\right\}$$
$$\times \begin{cases} \left(p^2 - 2^{2n}\alpha\kappa^2 U_0^{2n}\boldsymbol{p}^{4(n+1)}\right)^{-1}, & z = 2n+2 \text{ case} \\ \left(p^2 - 2^{2n-1}\alpha\kappa^2 U_0^{2n+1}\boldsymbol{p}^{2(2n+3)}\right)^{-1}, & z = 2n+3 \text{ case} \end{cases} .$$

$\boldsymbol{p}^2 = \sum_{i=1}^3 \left(p^i\right)^2$, $p^2 = -\left(p^0\right)^2 + \boldsymbol{p}^2$.

Assume $\alpha < 0$ in order to avoid tachyon pole.

In the ultraviolet region, ($\boldsymbol{k}$), Propagator behaves

$\sim 1/\left|\boldsymbol{p}\right|^4$ for $z = 2$ ($n = 0$) case

$\sim 1/\left|\boldsymbol{p}\right|^6$ for $z = 3$ ($n = 0$) case: power-counting renormalizable.

$z = 2n+2$ ($n \geq 1$) or $z = 2n+3$ ($n \geq 1$) case: super-renormalizable.

FRW cosmology

Start with a little bit general action:

$$
\begin{aligned}
S \quad = \quad & \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \sum_{n=0}^{n_{\max}} \alpha_n \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho\right)^n P_\alpha^{\ \mu} P_\beta^{\ \nu} \right.\right. \\
& \left. \times \left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\} \\
& \times \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho\right)^n P^{\alpha\mu} P^{\beta\nu} \left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\} \\
& - \sum_{m=0}^{m_{\max}} \tilde{\alpha} \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho\right)^m P_\alpha^{\ \mu} P_\beta^{\ \nu} \right. \\
& \left. \times \left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\} \\
& \times \left\{ \left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu - \partial_\mu\phi\partial^\mu\phi\nabla^\rho\nabla_\rho\right)^{m+1} P^{\alpha\mu} P^{\beta\nu} \left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right\} \\
& \left. -\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right] .
\end{aligned}
$$

In low energy, Einstein-Hilbert term $+\, n = 0$ term could dominate:

$$\begin{aligned} S \quad \sim \quad & \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \alpha_0 P_\alpha{}^\mu P_\beta{}^\nu \left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right)\right. \\ & \times P^{\alpha\mu}P^{\beta\nu}\left(R_{\mu\nu} - \frac{1}{2U_0}\partial_\rho\phi\nabla^\rho\nabla_\mu\nabla_\nu\phi\right) \\ & \left. -\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right] . \end{aligned}$$

FRW metric:

$$ds^2 = -\mathrm{e}^{2b(t)}dt^2 + a(t)^2 \sum_{i=1,2,3} \left(dx^i\right)^2 .$$

FRW equation:

$$\frac{3}{\kappa^2}H^2 + 81\alpha_0 H^4 = \rho_{\rm matter}\,.$$

$\rho_{\rm matter}$: energy-density of the matter

1st order differential equation with respect to the scale factor $a(t)$

If $\alpha_0 < 0$ and $\rho_{\rm matter} = 0$, de Sitter solution

$$H^2 = -\frac{1}{27\alpha_0\kappa^2}\,.$$

Inflation in the early universe?

Summary

- We formulated covariant higher derivative gravity with Lagrange multiplier constraint and scalar projectors.
  - The theory admits flat space solution.
  - Its gauge-fixing formulation is fully developed.
  - The only propagating mode is (higher derivative) graviton, while scalar and vector modes do not propagate.
  - The theory could be power-counting renormalizable.

- The preliminary study of FRW cosmology indicates to the possibility of inflationary universe solution.
  - 1st FRW equation in the theory turns out to be the first order differential equation which is quite unusual for higher derivative gravity which normally leads to third order differential equation with respect to scale factor.

Backup

Stückelberg formulation of massive vector field

Lagrangian

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) \\ &\quad -\frac{1}{2}m^2\left(\partial_\mu\varphi - A_\mu\right)\left(\partial^\mu\varphi - A^\mu\right)\end{aligned}$$

Gauge symmetry

$$A_\mu \to A_\mu + \partial_\mu\epsilon\,, \quad \varphi \to \varphi + \epsilon$$

Gauge fixing condition: $\varphi = 0 \Rightarrow$ massive vector field

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \frac{1}{2}m^2 A_\mu A^\mu$$

the variation of $\varphi \Rightarrow$ physical state condition: $\partial_\mu A^\mu = 0$

By using constrained complex scalar field η $(\eta^*\eta = 1)$

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \frac{1}{2}m^2\left\{\left(\partial_\mu - iA_\mu\right)\eta^*\right\}\left\{\left(\partial^\mu + iA^\mu\right)\eta\right\} \quad \eta = \mathrm{e}^{i\varphi}$$

which is obtained from the double-well potential for complex scalar field ζ:

$$V = \frac{1}{4}M^2\left(\zeta^*\zeta - 1\right)^2 \quad \zeta = \rho\mathrm{e}^{i\varphi}$$

In the limit of $M \to \infty$ ($\rho \to 1, \zeta \to \eta$)

By replacing η : non-trivial representation of $U(1)$

$\longrightarrow$ constrained vector field $\partial_\mu\phi$ or A_μ

$\Rightarrow$ spontaneous breakdown of Lorentz symmetry

Perturbation from the flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$P_i^{\ \mu} P_j^{\ \nu} R_{\mu\nu} = \frac{1}{2}\left(h^\rho_{\ i,j\rho} + h^\rho_{\ j,i\rho} - \partial_\rho \partial^\rho h_{ij,} - \partial_i \partial_j \left(h_\rho^{\ \rho} \right) \right) ,$$

$$\frac{1}{2U_0} P_i^{\ \mu} P_j^{\ \nu} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi = -\frac{1}{2}\left(h_{ti,jt} + h_{tj,it} - h_{ij,tt} \right) ,$$

$$\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu + 2U_0 \nabla^\rho \nabla_\rho = 2U_0 \partial_k \partial^k ,$$

$$P_i^{\ \mu} P_j^{\ \nu} \left(R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right)$$

$$= \frac{1}{2}\left(h_{ki,j}^{\ \ \ \ k} + h_{kj,i}^{\ \ \ \ k} - h_{ij,k}^{\ \ \ \ k} - \partial_i \partial_j \left(h_\mu^{\ \mu} \right) \right) .$$

In the present model, there is no propagating vector or scalar mode at least on the tree level. The change of the tensor structure of the propagator means that the vector or scalar mode could appear, that is, the vector or scalar mode must correspond to a composite state, which usually does not appear at any perturbative level. Therefore, it is expected the tensor structure should not be changed by the quantum corrections.

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