



SIMULACIÓN Y PREVENCIÓN DE INUNDACIONES

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Fronteras de la Computación

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th CENTRO DE CIENCIAS
DE BENASQUE
PEDRO PASCUAL

Grupo de **HIDRÁULICA COMPUTACIONAL**



Área de Mecánica de Fluidos
Escuela de Ingeniería y Arquitectura
Universidad de Zaragoza
ghc.unizar.es

GHC: Líneas de Investigación

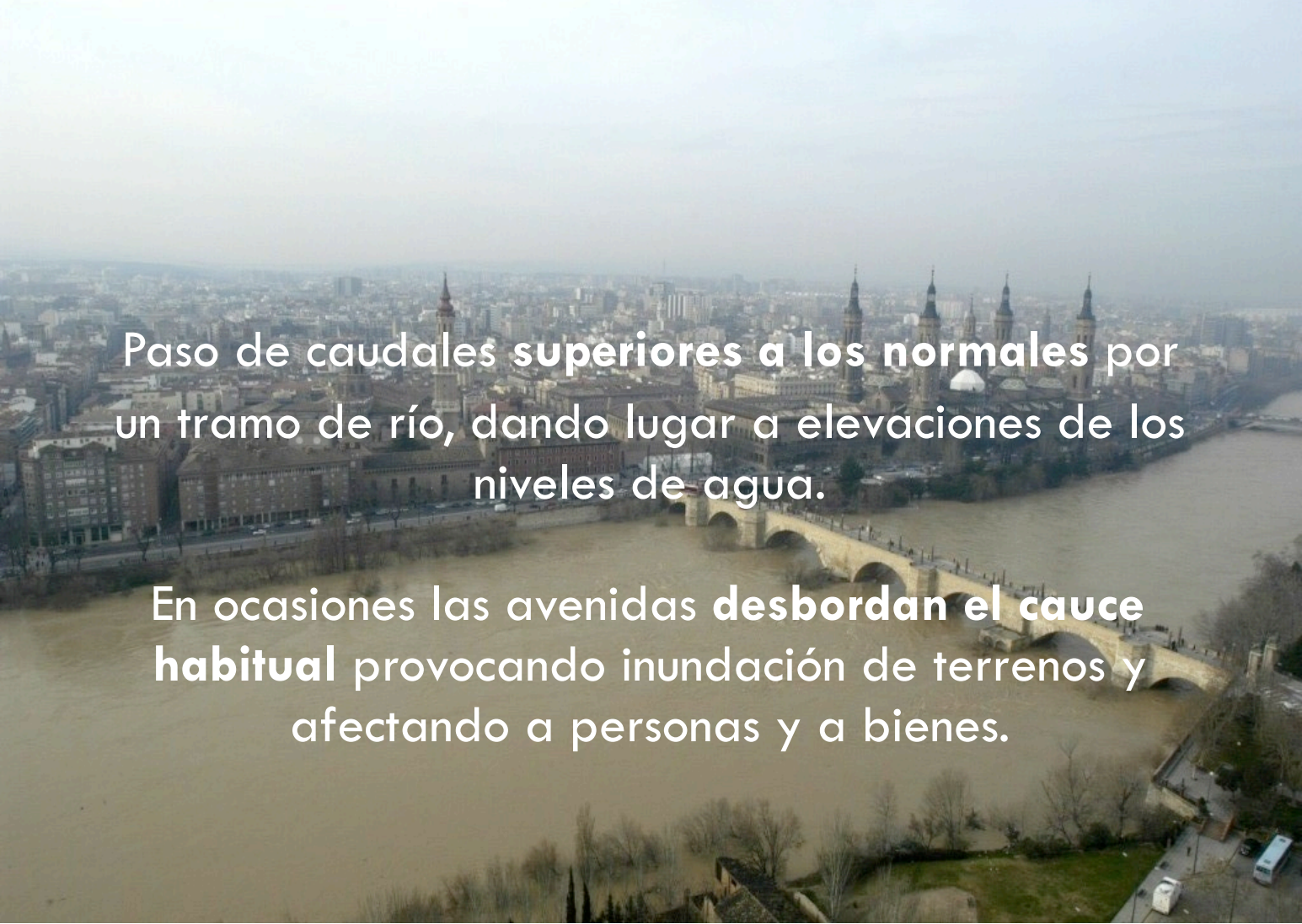
Desarrollo de:

- Técnicas numéricas para las ecuaciones de aguas poco profundas 1D
- Modelos numéricos para las ecuaciones de aguas poco profundas 2D
- Esquemas de alta resolución en 1D/2D
- Técnicas de adaptación de mallas
- Modelos de simulación de transporte con convección-difusión
- Modelos de simulación de flujo sobre lecho deformable

Aplicación a:

- Modelos de optimización del riego por inundación
- Simulación de flujo en ríos: ondas de crecida y de inundación. Capacidad erosiva de un río.
- Transporte, difusión y reacción de nutrientes en un flujo de agua.
- Transporte, difusión y términos de forzado de la temperatura del agua.
- Simulación en Hidrología.

Definición de avenida, crecida, riada

An aerial photograph of a city, likely Salamanca, Spain, showing a wide river (the Tago) with a large stone bridge. The water is significantly higher than its normal level, indicating a flood. The city buildings and the bridge are visible in the background.

Paso de caudales **superiores** a los normales por un tramo de río, dando lugar a elevaciones de los niveles de agua.

En ocasiones las avenidas **desbordan el cauce habitual** provocando inundación de terrenos y afectando a personas y a bienes.

Barrancos usualmente secos...



Huerva en Zaragoza, Mayo 2003



Ebro en Zaragoza, Febrero 2003



Algunas cifras

Daños a nivel mundial*:

- 25000 víctimas/año
- 520 millones de afectados/año (10%)
- 60000 millones \$/año

Daños a nivel de Europa (1998–2004):

- Más de 100 inundaciones graves
- 700 víctimas mortales
- 25000 M€

* Información extraída del 1^{er} Informe sobre el Desarrollo de los Recursos Hídricos en el Mundo de las Naciones Unidas “Agua para Todos, Agua para la Vida”

Estimación daños materiales en España por riesgos naturales (1986 y 2016):

- **Inundaciones: 570 M€/año**
- Incendios: 300 M€/año
- Erosión: 175 M€/año
- Deslizamientos: 150 M€/año
- Terremotos: 13 M€/año

* Información extraída de Ayala et al. 1987

Tipos de inundaciones en medios fluviales

- Originadas por **precipitaciones y/o fusión nival**:
 - Inundaciones en la misma zona donde tiene lugar la precipitación
 - Inundaciones aguas abajo del área de precipitación por desbordamiento de ríos
- Originadas por **rotura u operación incorrecta** de obras de infraestructura hidráulica

Factores que condicionan la magnitud de una avenida

- Características de la precipitación: **Intensidad, duración y distribución espacial**
- La **temperatura** (condiciona la fusión nival)
- Las **características físicas** de la cuenca (área, relieve, geología, forma, cubierta vegetal, densidad del drenaje, morfología del cauce, grado de urbanización...)
- Estado de **humedad del suelo**

Factores a tener en cuenta para evaluar los efectos de una avenida (Severidad)

- La **magnitud de la avenida** ($Q_{\text{máx}}$, velocidad, calado, carga sólida, duración)
- La **capacidad de transporte del cauce** (erosión-sedimentación, existencia de obstáculos naturales o artificiales, vegetación, existencia de estructuras transversales...)
- Capacidad **de laminación**
 - Natural: llanuras de inundación
 - Artificial: embalses

Tipos de medidas a adoptar

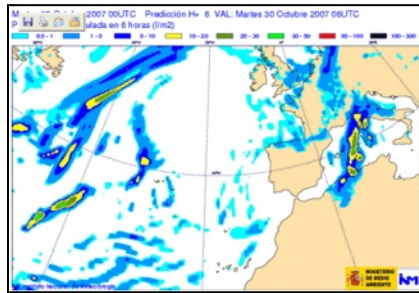
✓ Medidas preventivas

- Gestión del territorio
- Estructurales

✓ Medidas correctivas

- Gestión de las infraestructuras y del dominio público hidráulico
- Planes de emergencia
- Sistemas de vigilancia y alerta temprana

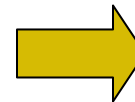
SAIH-SAD



Previsiones meteorológicas
del modelo HIRLAM

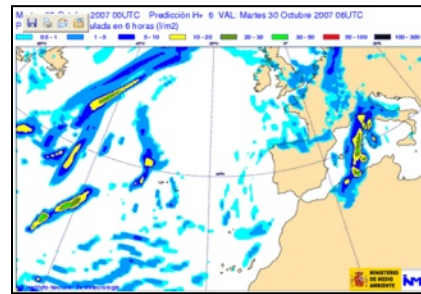


Funcionamiento: Entradas y Salidas al SAD



Caudales previstos en la
Cuenca del Ebro
(www.chebro.es)

SAIH-SAD



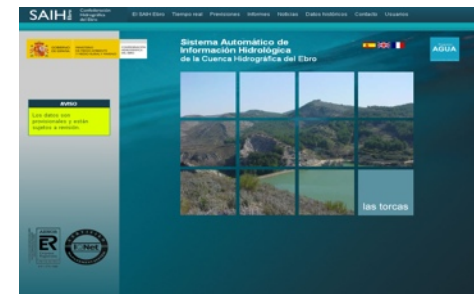
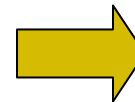
Previsiones meteorológicas del modelo HIRLAM



Datos hidrológicos en tiempo real

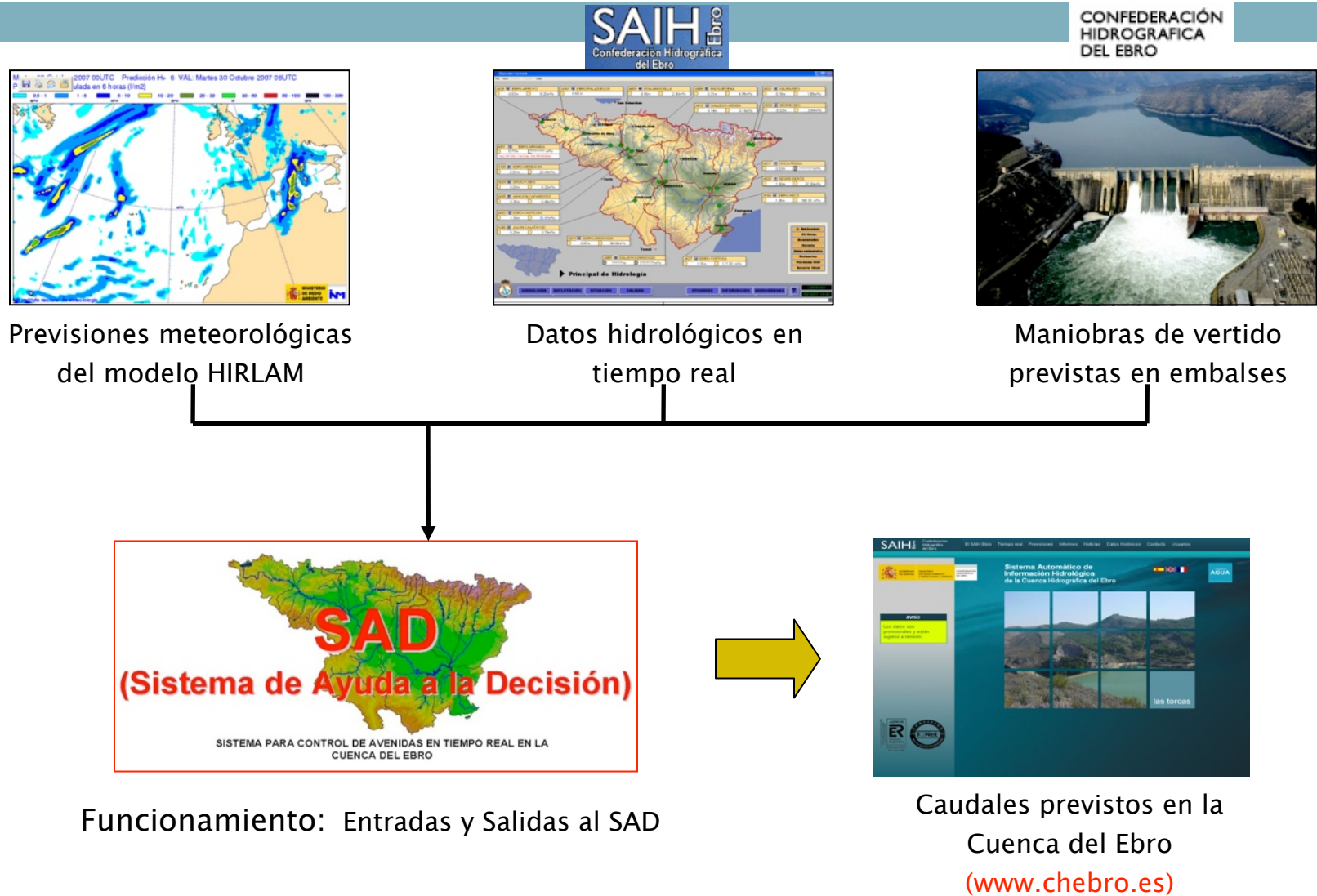


Funcionamiento: Entradas y Salidas al SAD



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www.chebro.es

SAIH-SAD



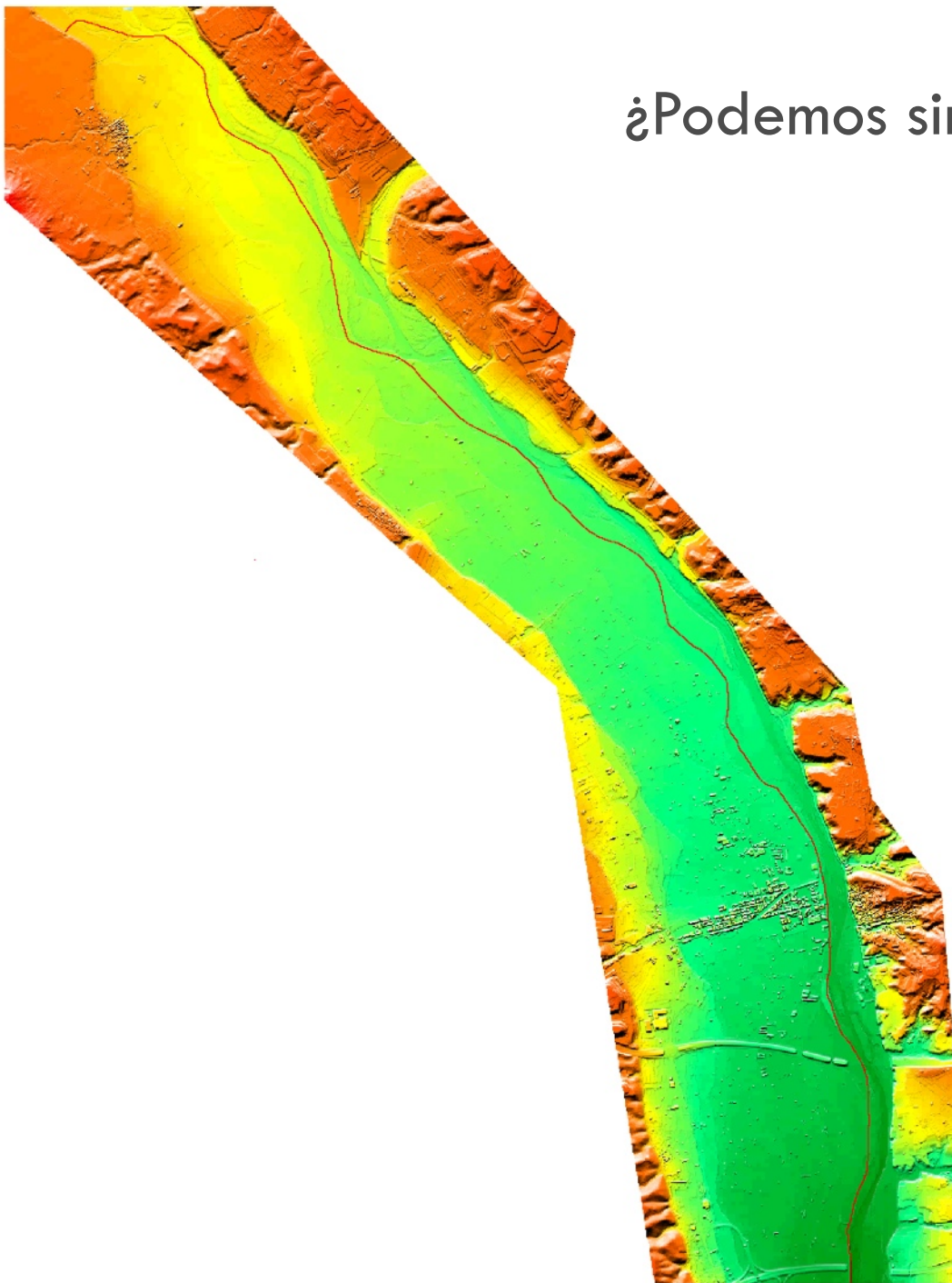
¿Podemos simularlo en el ordenador?

Cortesía INCLAM (www.inclam.com)

- **Longitud modelizada**
15km del río Cinca.
- **Numero de celdas del terreno modelizado:**
1.108.376.
- **Condiciones de contorno de entrada:**
Hidrogramas de entrada de 500 años de periodo de retorno.
- **Condiciones de contorno de salida:**
Curva de desagüe de la última sección

¿Podemos simularlo en el ordenador?

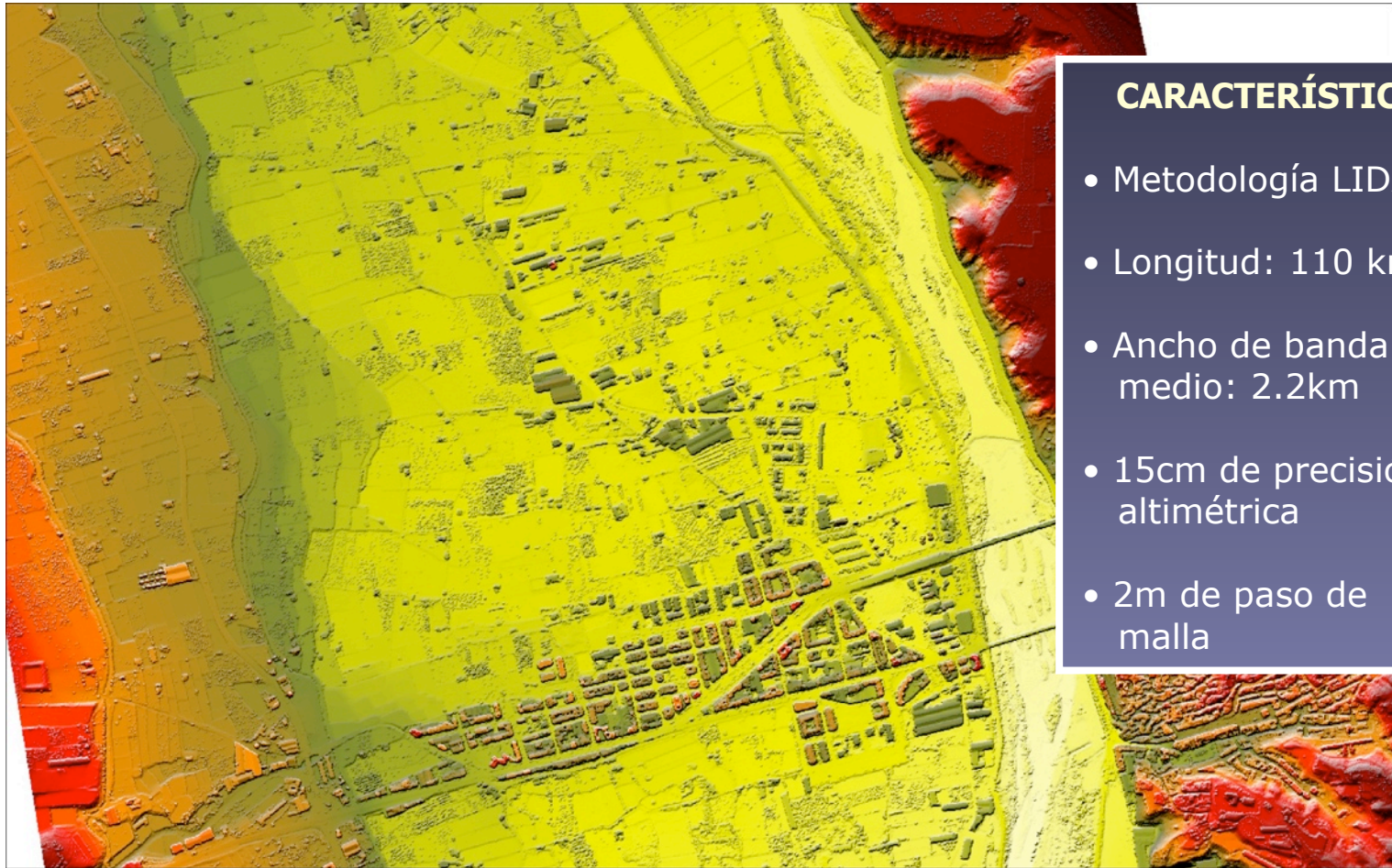
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Trabajos previos: Topografía/Cartografía

MDT

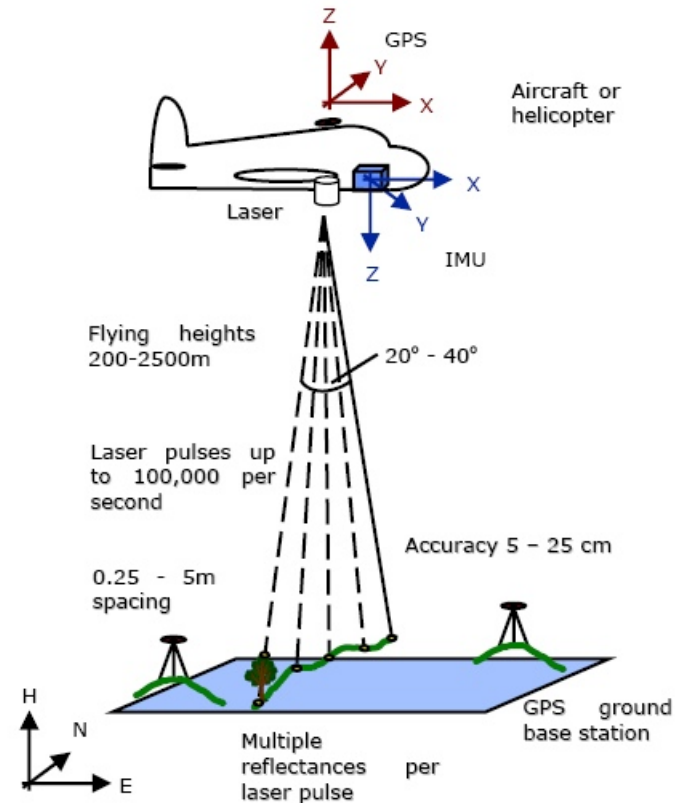
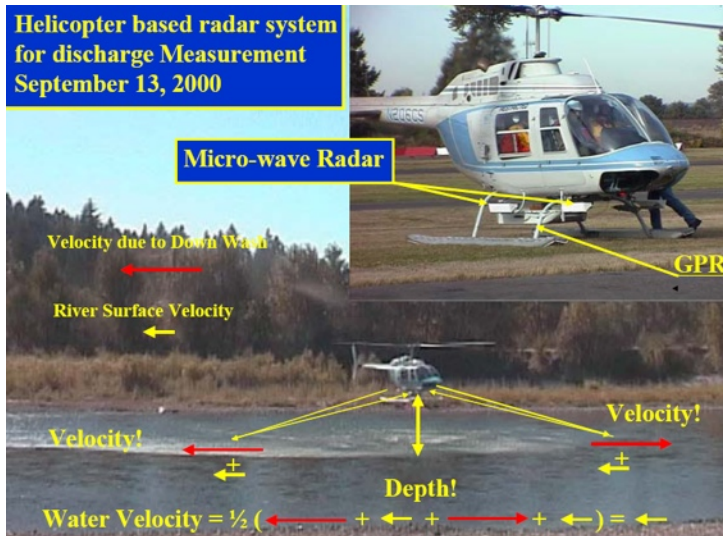


CARACTERÍSTICAS

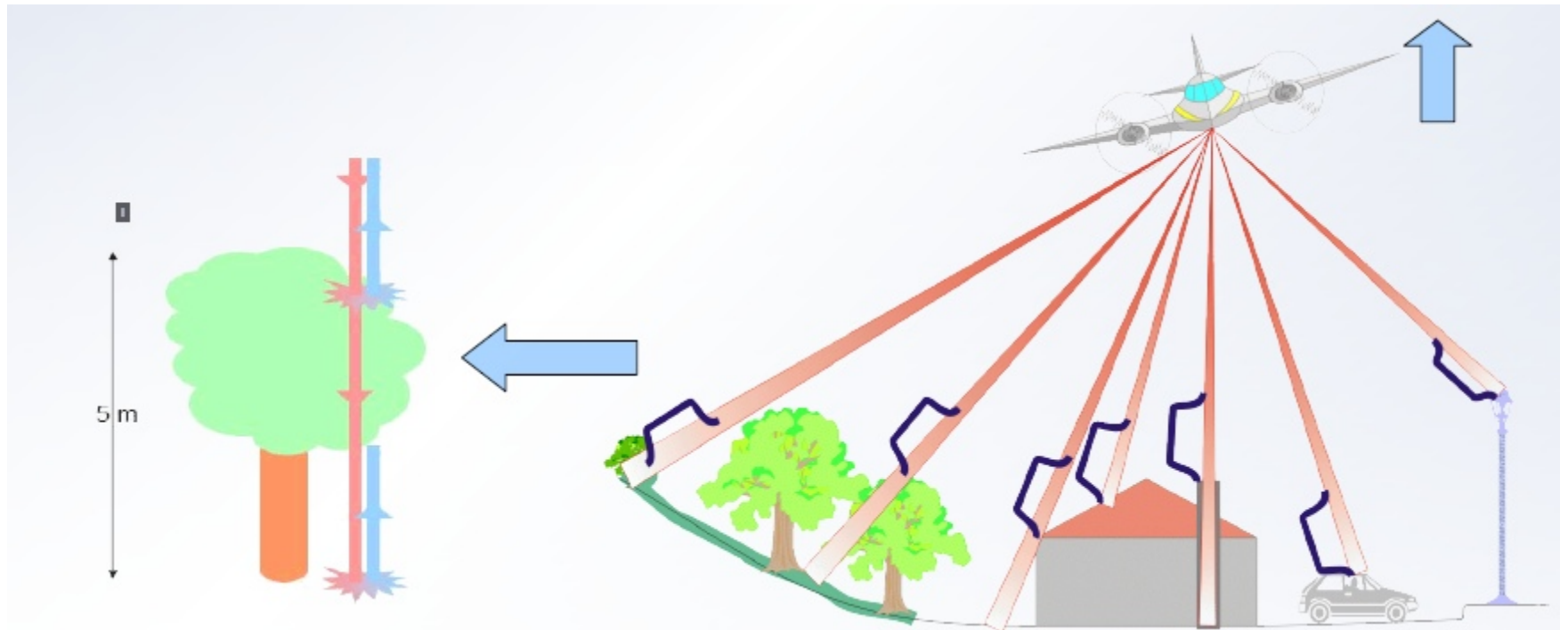
- Metodología LIDAR
- Longitud: 110 km.
- Ancho de banda medio: 2.2km
- 15cm de precisión altimétrica
- 2m de paso de malla

Fuentes de datos:

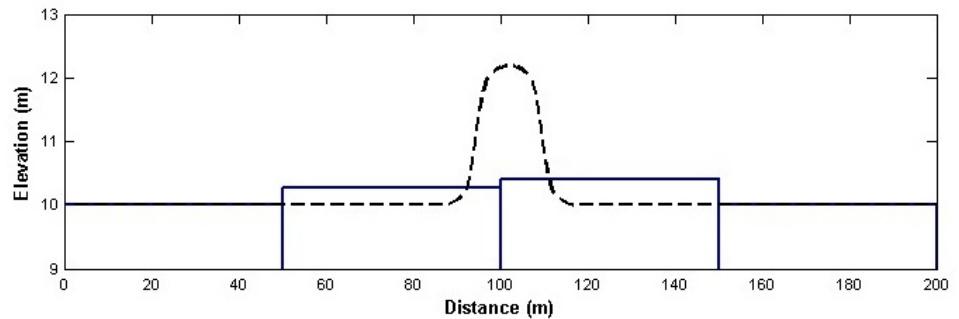
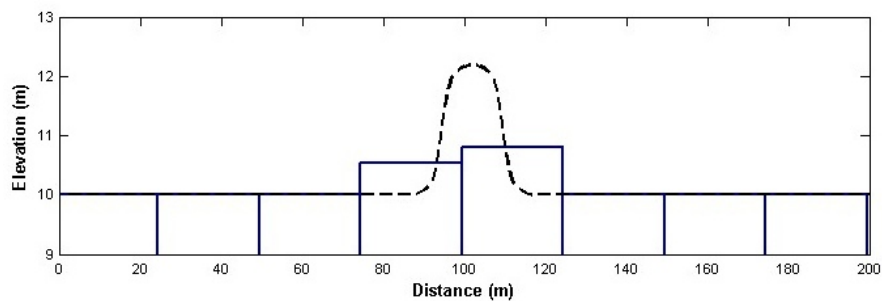
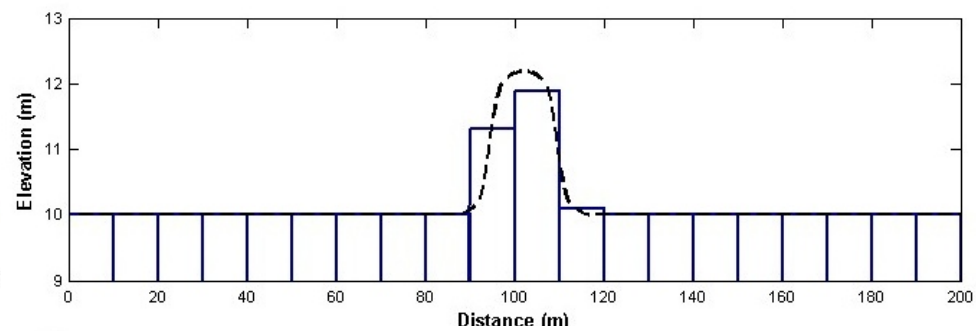
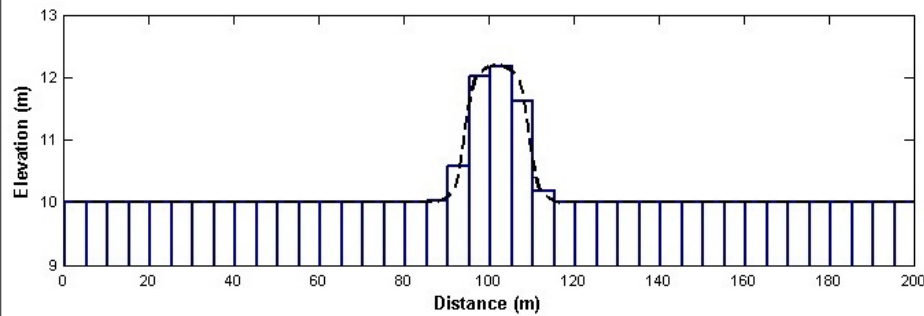
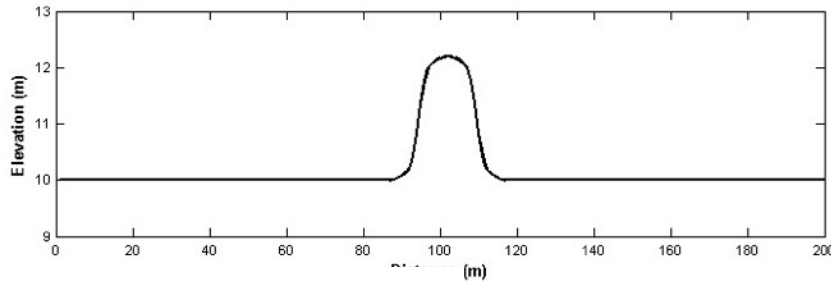
Helicopter based radar system for discharge Measurement
September 13, 2000



Técnicas LIDAR

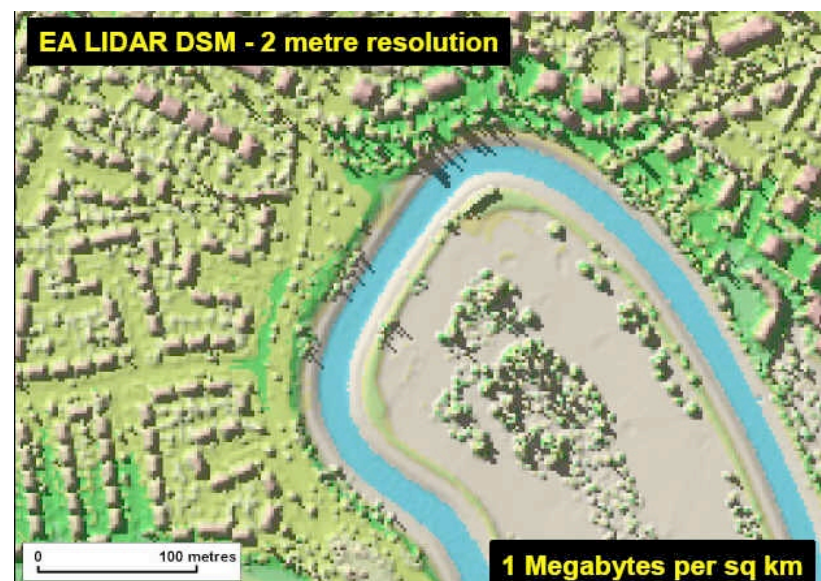
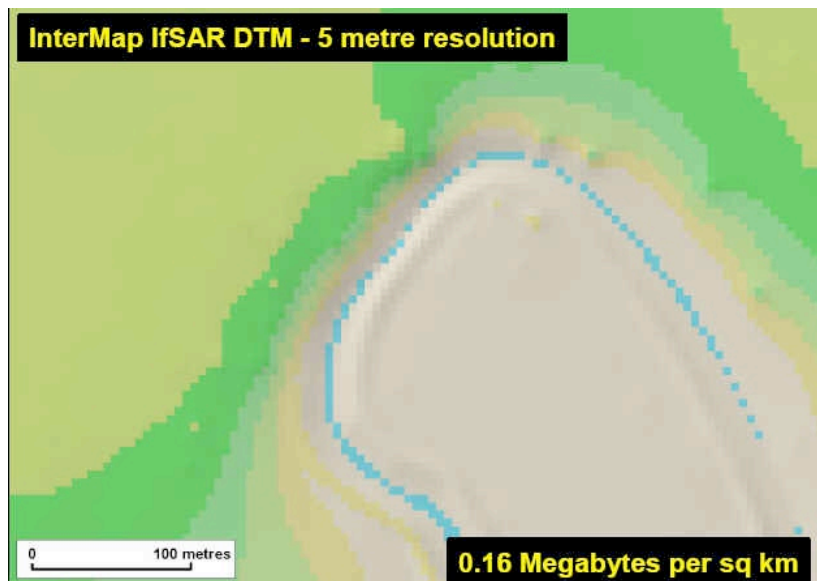
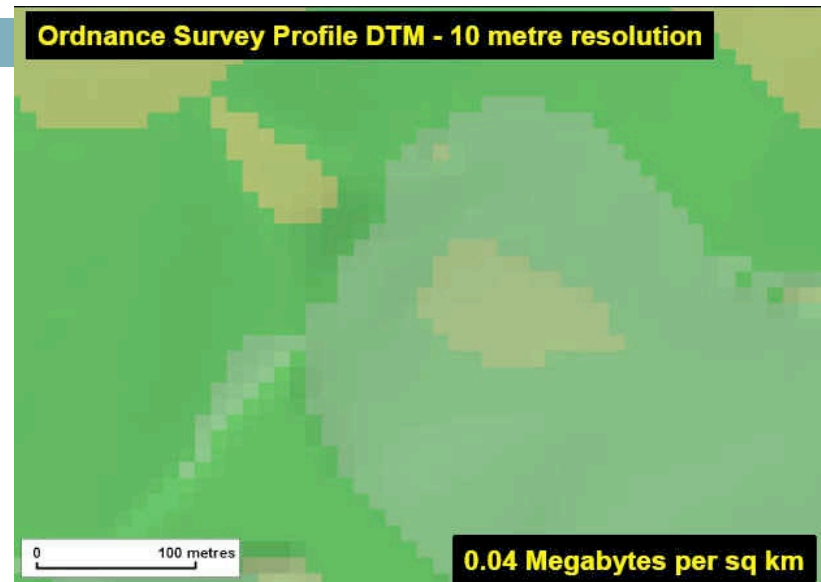
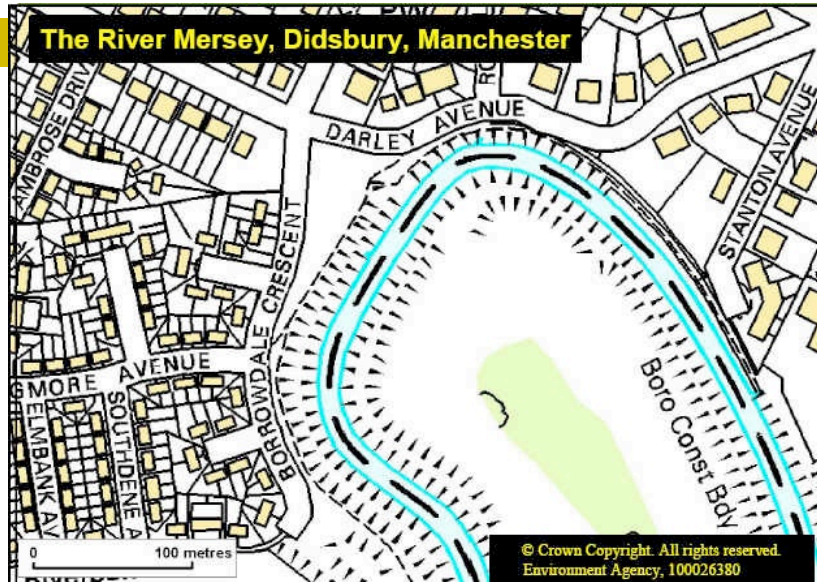


La resolución de la malla...



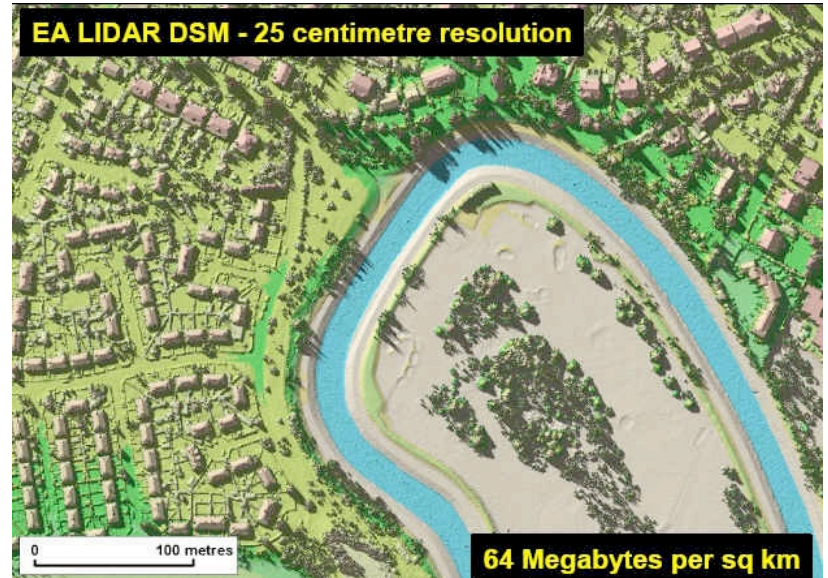
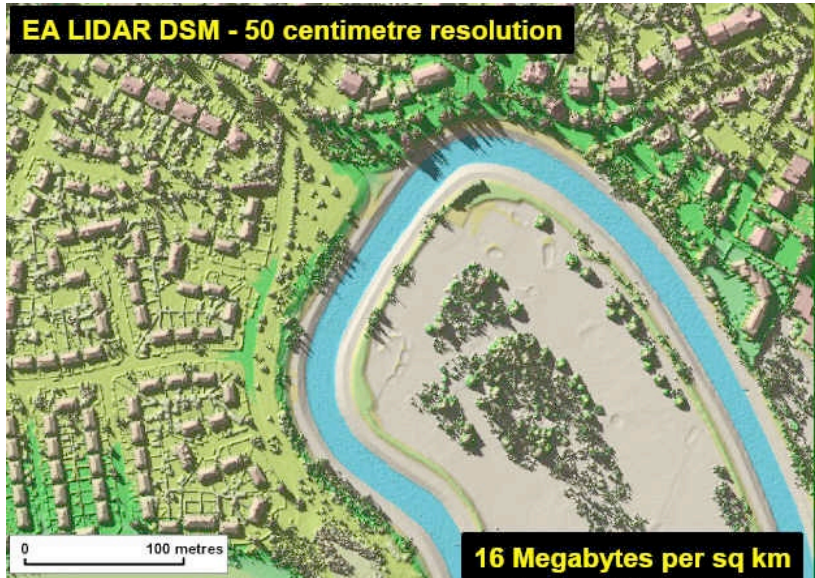
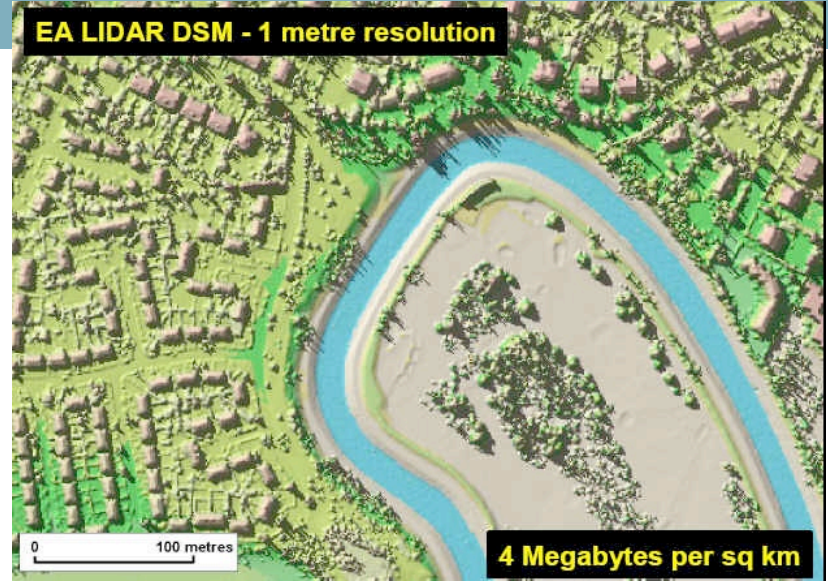
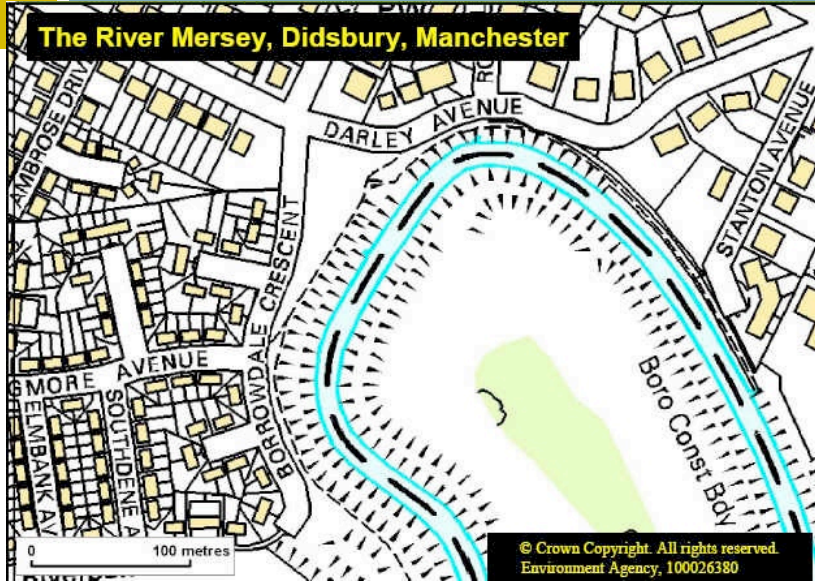
Resolución espacial de los datos del terreno.

Cortesía de Environment Agency, UK, 2005



Resolución espacial de los datos del terreno.

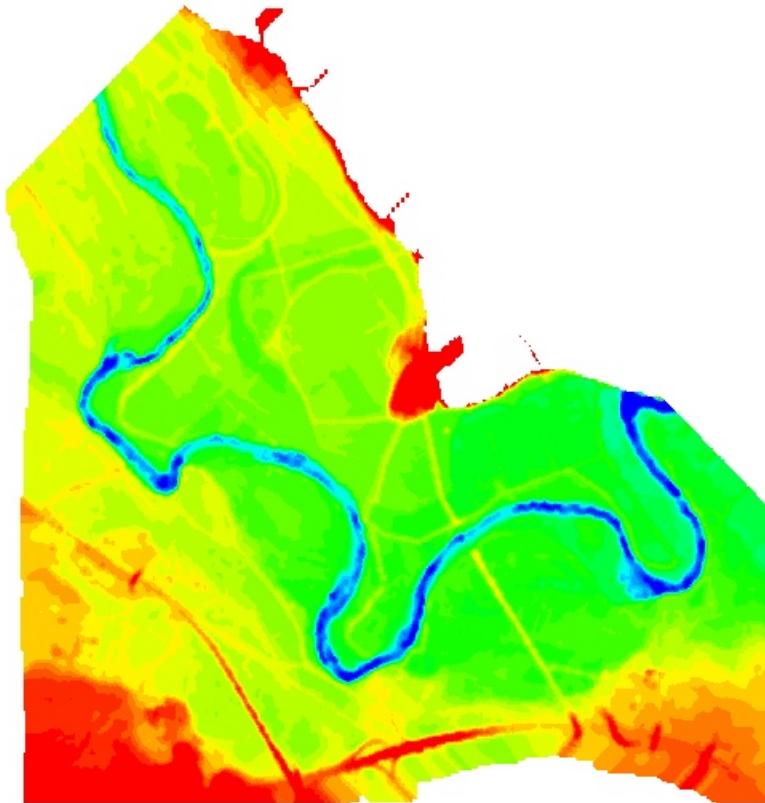
Cortesía de Environment Agency, UK, 2005



Motivación:

El avance tecnológico en la adquisición y gestión de datos del terreno es un reto para la simulación numérica y para el desarrollo de herramientas informáticas más sofisticadas en

- la predicción de inundaciones



¿Modelos para el movimiento del agua?



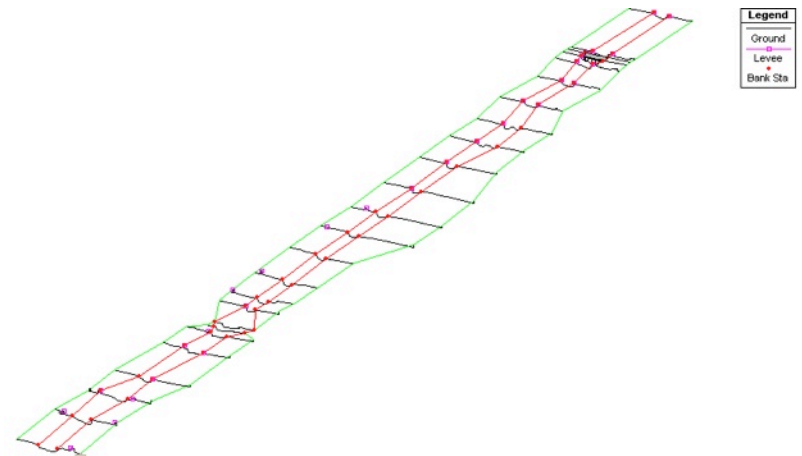
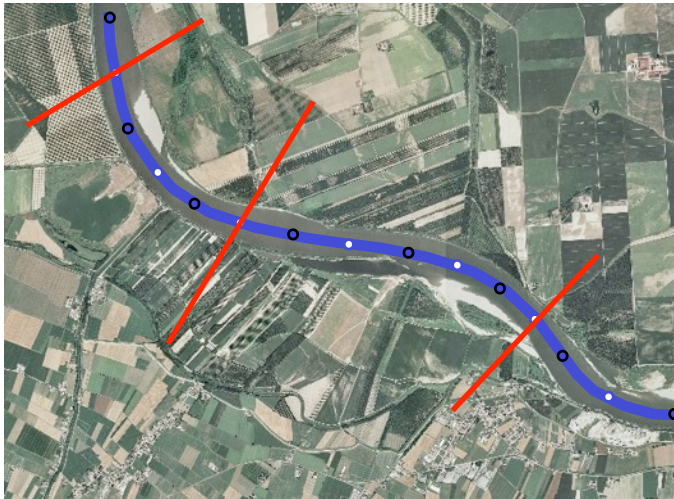
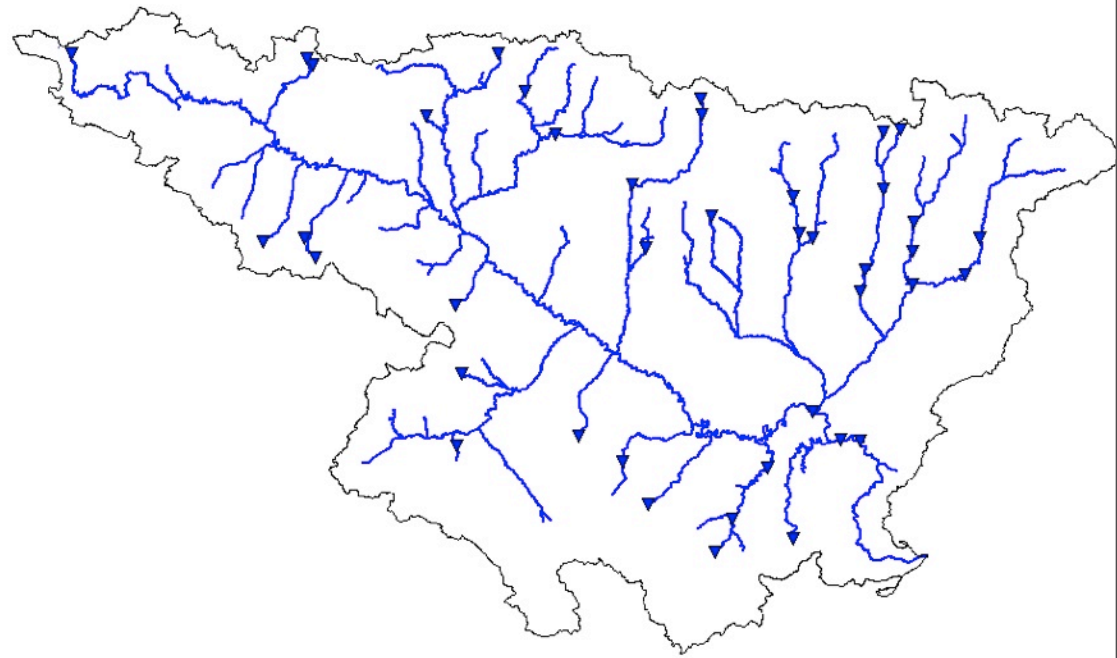
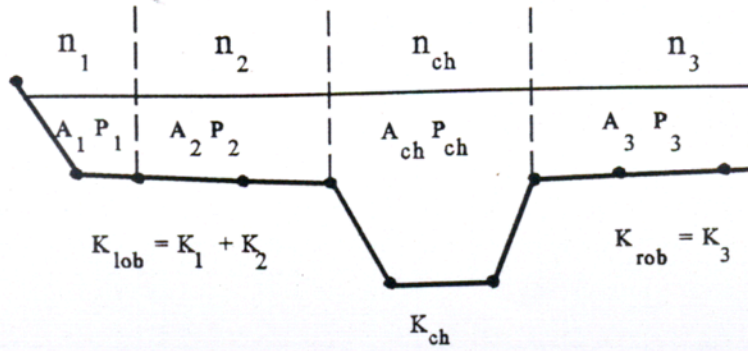
Modelo de simulación

- Finalidad:
 - Académica: comprender, investigar
 - Profesional
- Requiere:
 - Representación del fenómeno físico por ecuaciones diferenciales/integrales
 - Representación de la realidad por información discreta.
 - **MÉTODO NUMÉRICO**
 - Aproximación de las ecuaciones a expresiones algebraicas que hay que resolver.
 - **PROGRAMACIÓN**
- Proporciona:
 - Conjuntos de números que hay que interpretar/representar

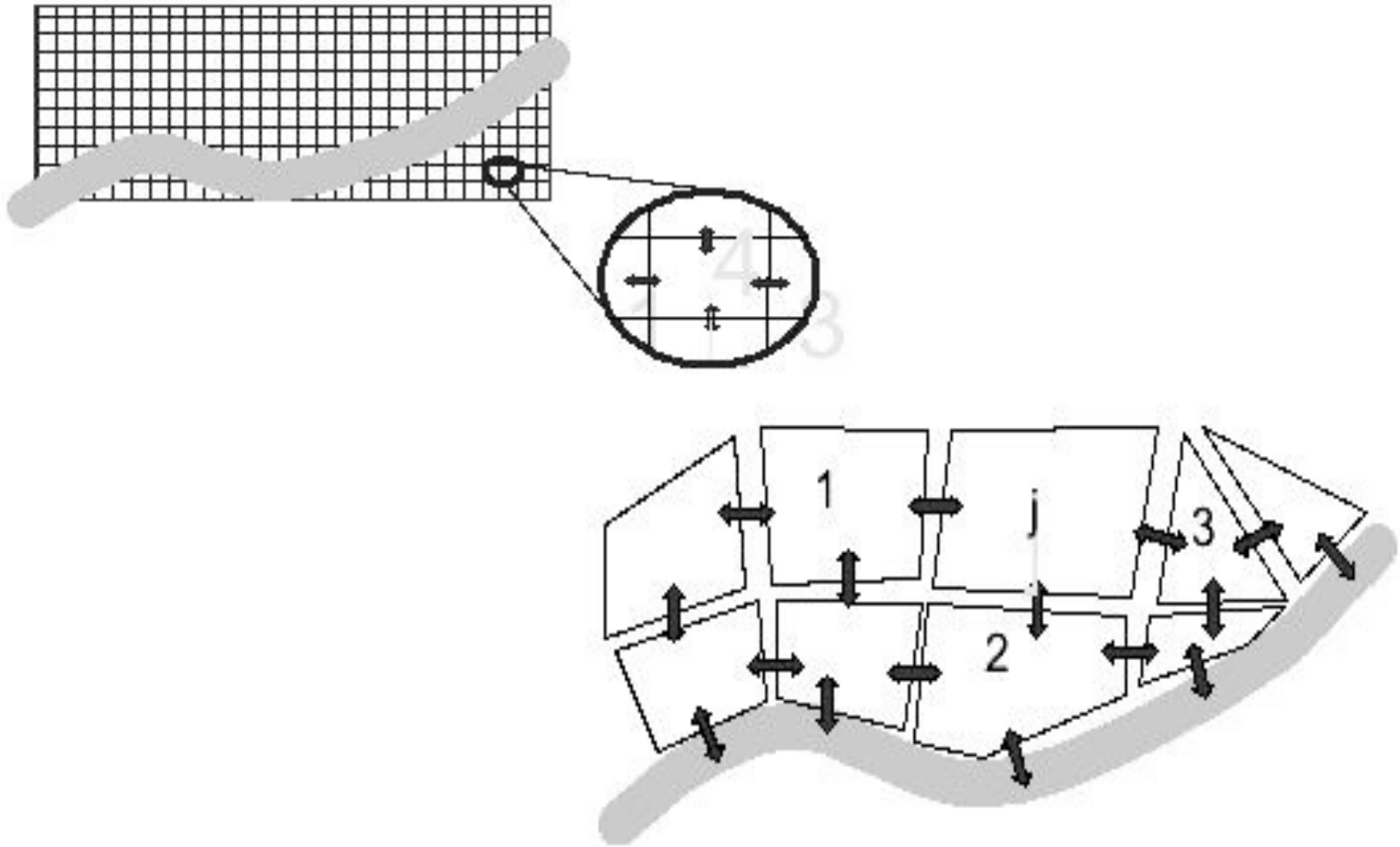
¿Hidráulica 1D o 2D?



Hidráulica 1D



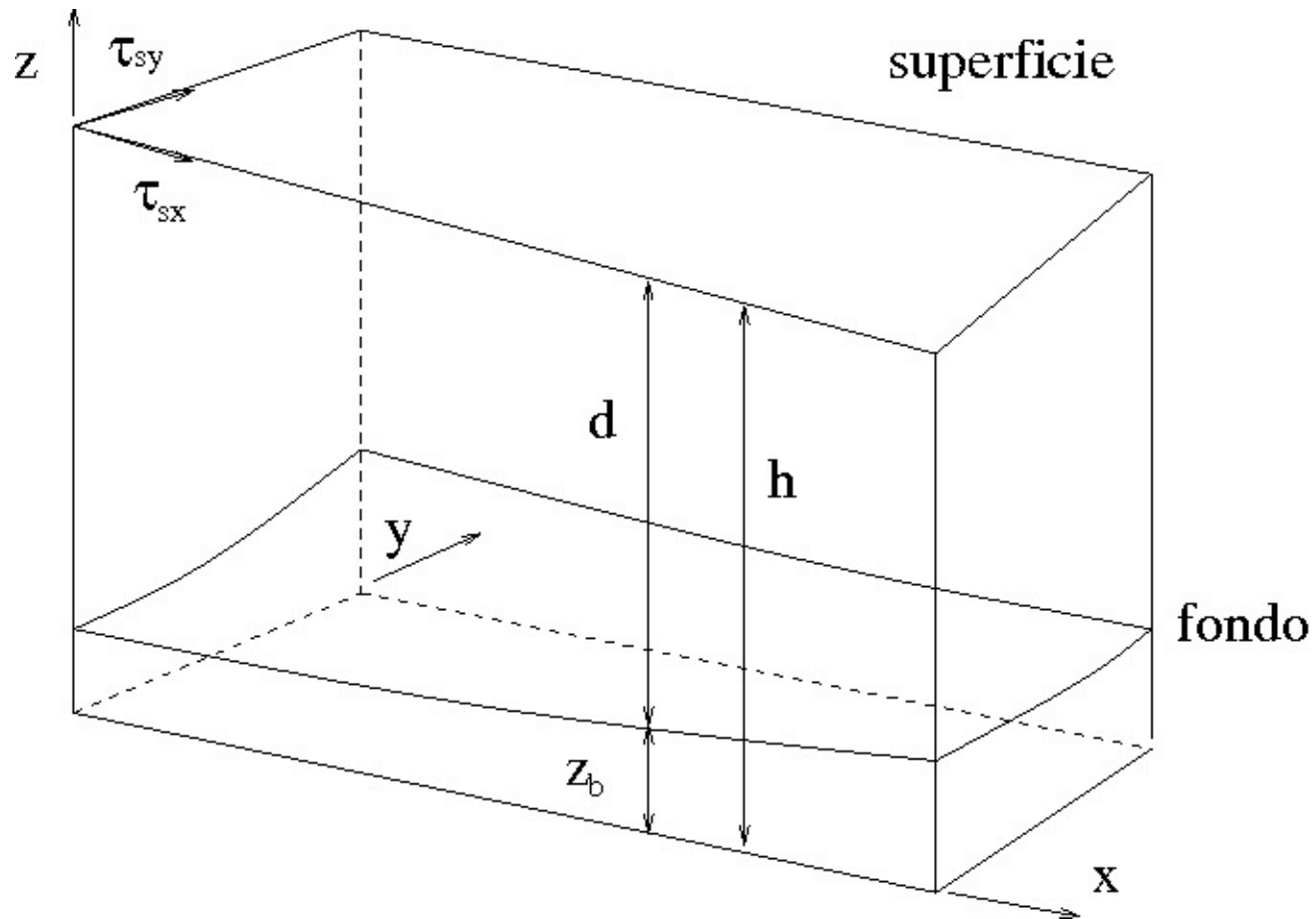
¿Cómo plantear un modelo 2D?





LAS ECUACIONES

Dominio de cálculo 3D y sistema de referencia



Ecuaciones de Navier-Stokes

La conservación de la **masa** se expresa

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

La conservación de la **cantidad de movimiento** se expresa

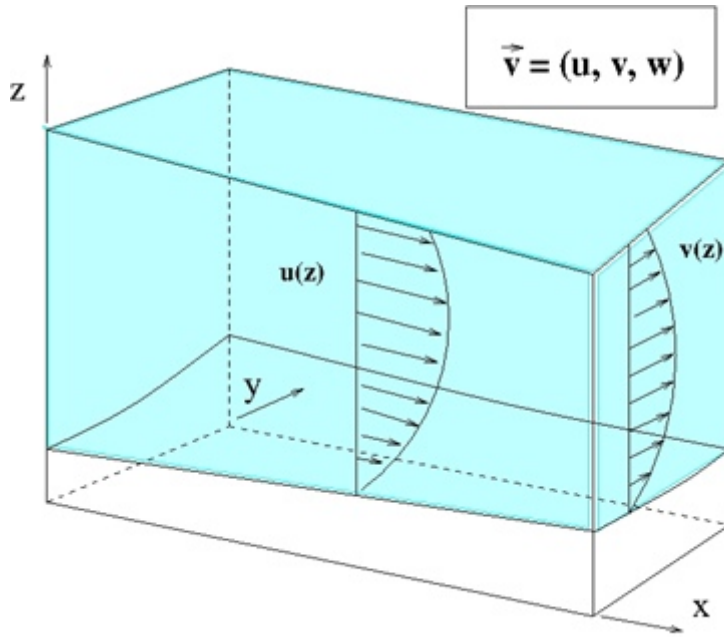
$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho uv}{\partial y} + \frac{\partial \rho uw}{\partial z} - \rho f v + \frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho uv}{\partial x} + \frac{\partial \rho v^2}{\partial y} + \frac{\partial \rho vw}{\partial z} + \rho f u + \frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} - \frac{\partial \tau_{yz}}{\partial z} = 0$$

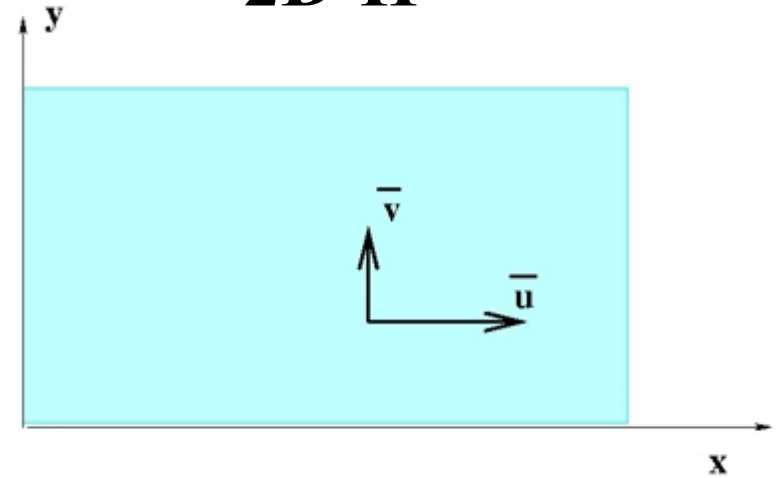
$$\frac{\partial \rho w}{\partial t} + \frac{\partial \rho uw}{\partial x} + \frac{\partial \rho vw}{\partial y} + \frac{\partial \rho w^2}{\partial z} + \rho g + \frac{\partial p}{\partial z} - \frac{\partial \tau_{xz}}{\partial x} - \frac{\partial \tau_{yz}}{\partial y} - \frac{\partial \tau_{zz}}{\partial z} = 0$$

Modelos simplificados

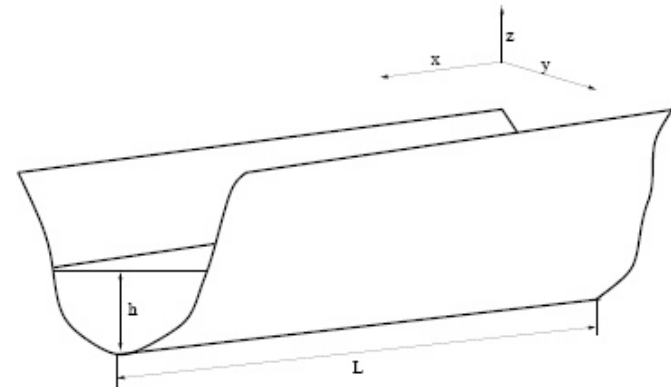
3D



2D-H



1D



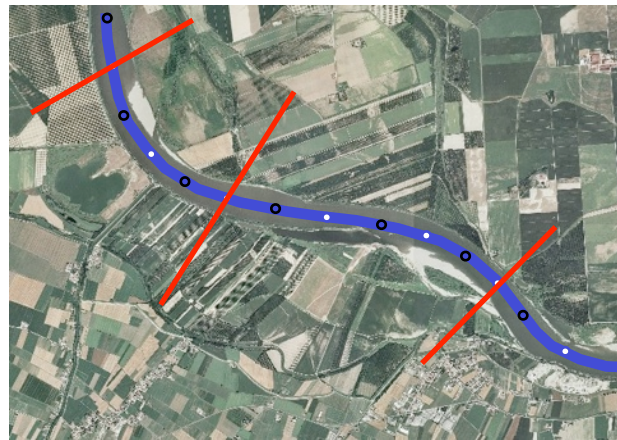
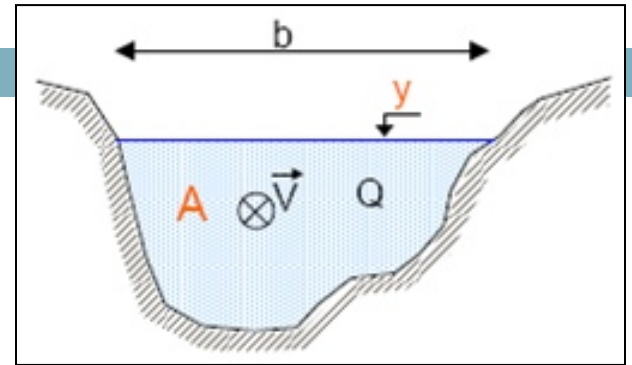
Modelos 'shallow water'

- Aptos para flujos en los que la escala vertical es mucho menor que la horizontal
- Las ecuaciones del flujo se integran en la vertical sobre la columna líquida eliminando
 - ▣ La coordenada vertical de la formulación
 - ▣ La superficie libre como contorno
- Con esto cambian:
 - ▣ La naturaleza de las ecuaciones
 - ▣ Los tipos de solución y las metodologías

En aproximación 1D

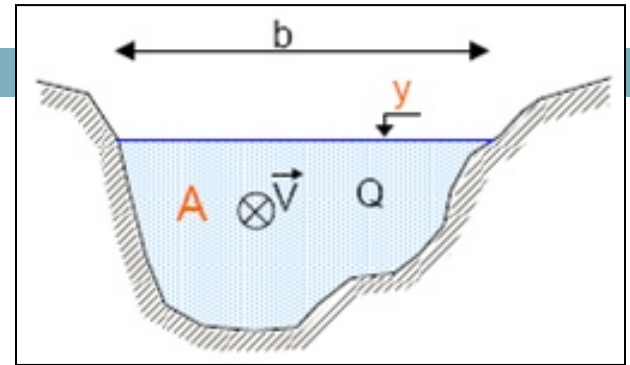
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = i$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + gI_1 \right) = gA(S_0 - S_f) + gI_2$$



En aproximación 1D

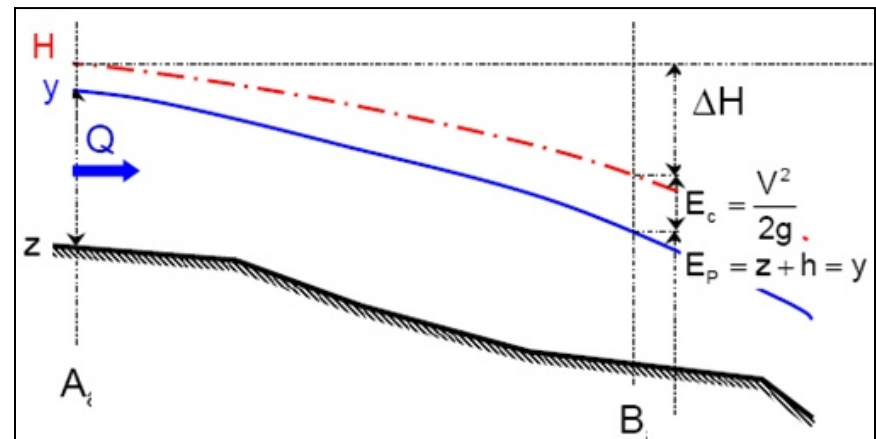
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = i$$



$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + gI_1 \right) = gA(S_0 - S_f) + gI_2$$

$$S_0 = -\frac{\partial z}{\partial x} = \tan \alpha \approx \text{sen} \alpha$$

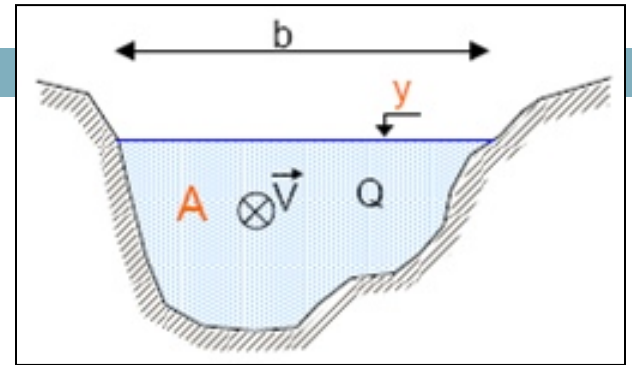
$$S_f = \frac{Q|Q|n^2}{A^2 R^{4/3}}$$



En aproximación 1D

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = i$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + gI_1 \right) = gA(S_0 - S_f) + gI_2$$



Fuerzas de presión hidrostática

$$I_1 = \int_0^h (h - \eta) \sigma(x, \eta) d\eta$$

$$I_2 = \int_0^h (h(x) - \eta) \frac{\partial \sigma(x, \eta)}{\partial x} d\eta$$

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{x}, \mathbf{U})}{\partial x} = \mathbf{H}(\mathbf{x}, \mathbf{U}) \begin{cases} \mathbf{U} &= (A, Q)^T \\ \mathbf{F} &= (Q, \frac{Q^2}{A} + gI_1)^T \\ \mathbf{H} &= (0, gI_2 + gA(S_0 - S_f))^T \end{cases}$$

La matriz Jacobiana del sistema es

$$\mathbf{J} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 1 \\ g\frac{A}{b} - \frac{Q^2}{A^2} & 2\frac{Q}{A} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ c^2 - u^2 & 2u \end{pmatrix},$$

Valores propios: $\lambda^{1,2} = u \pm c$

Vectores propios: $\mathbf{e}^{1,2} = (1, u \pm c)^T.$

■ Formulación desacoplada

En los casos en que $\mathbf{F} = \mathbf{F}(\mathbf{U})$

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{J} \frac{\partial \mathbf{U}}{\partial x} = \mathbf{H} \Rightarrow \frac{\partial \mathbf{W}}{\partial t} + \mathbf{\Lambda} \frac{\partial \mathbf{W}}{\partial x} = \mathbf{P}^{-1} \mathbf{H}$$

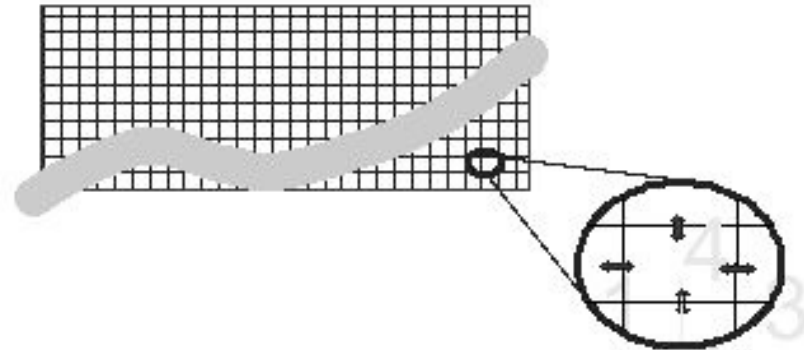
Es decir:

$$\mathbf{J} = \mathbf{P} \mathbf{\Lambda} \mathbf{P}^{-1} \quad , \quad \mathbf{\Lambda} = \mathbf{P}^{-1} \mathbf{J} \mathbf{P}$$

$$\delta \mathbf{W} = \mathbf{P}^{-1} \delta \mathbf{U} \quad , \quad \delta \mathbf{U} = \mathbf{P} \delta \mathbf{W} = \sum_{k=1}^m \alpha_k \mathbf{e}_k$$

Esto permite desacoplar el sistema y explotar las técnicas desarrolladas para ecuaciones escalares

En aproximación 2D



- Se amplían los contornos a todo el perímetro. MALLA.
- Se expresan las variables promediadas en la vertical

Aproximación 2D



$$\frac{\partial}{\partial t} \mathbf{U} + \nabla(\mathbf{F}, \mathbf{G}) = \mathbf{H}^1 + \mathbf{H}^2$$

$$\mathbf{U} = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + g \frac{h^2}{2} \\ huv \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + g \frac{h^2}{2} \end{pmatrix}$$

$$\mathbf{H}^1 = \begin{pmatrix} 0 \\ ghS_{0x} \\ ghS_{0y} \end{pmatrix} \quad S_{0x} = -\frac{\partial z_b}{\partial x} \quad S_{0y} = -\frac{\partial z_b}{\partial y} \quad \mathbf{H}^2 = \begin{pmatrix} 0 \\ -ghS_{fx} \\ -ghS_{fy} \end{pmatrix}$$

$$S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}}$$

$$S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}}$$

Forma quasi-conservativa

$$\frac{\partial U}{\partial t} + (\mathbf{A}, \mathbf{B}) \nabla U = R$$

Las matrices jacobianas son:

$$\mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 1 & 0 \\ c^2 - u^2 & 2u & 0 \\ -uv & v & u \end{pmatrix}, \quad \mathbf{B} = \frac{\partial \mathbf{G}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 0 & 1 \\ -uv & v & u \\ c^2 - v^2 & 0 & 2v \end{pmatrix}$$

NOTA: No es posible la diagonalización simultánea de **A** y **B**

... redefinición del problema

□ Matriz jacobiana del flujo normal

$$\mathbf{J}_n = \frac{\partial(\mathbf{E} \times \mathbf{n})}{\partial \mathbf{U}} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} n_x + \frac{\partial \mathbf{G}}{\partial \mathbf{U}} n_y$$

$$\mathbf{J}_n = \begin{pmatrix} 0 & n_x & n_y & \vdots \\ \left(gh - \frac{q_x^2}{h^2} \right) n_x - \frac{q_x q_y}{h^2} n_y & \frac{q_y}{h} n_y + \frac{2q_x}{h} n_x & \frac{q_x}{h} n_y & \vdots \\ \left(gh - \frac{q_y^2}{h^2} \right) n_y - \frac{q_x q_y}{h^2} n_x & \frac{q_y}{h} n_x & \frac{q_x}{h} n_x + \frac{2q_y}{h} n_y & \vdots \end{pmatrix}$$

- Valores propios

$$\lambda^1 = un_x + vn_y + c$$

$$\lambda^2 = un_x + vn_y$$

$$\lambda^3 = un_x + vn_y - c$$

- Vectores propios

$$e^1 = \begin{pmatrix} 1 \\ u + cn_x \\ v + cn_y \end{pmatrix}, \quad e^2 = \begin{pmatrix} 0 \\ -cn_y \\ cn_x \end{pmatrix}, \quad e^3 = \begin{pmatrix} 1 \\ u - cn_x \\ v - cn_y \end{pmatrix},$$

Usando los vectores propios se pueden construir matrices P y P^{-1} que diagonalicen al jacobiano J_n

$$J_n = P\Lambda P^{-1}$$

Ecuaciones en forma integral

$$\frac{\partial}{\partial t} \mathbf{U} + \nabla(\mathbf{F}, \mathbf{G}) = \mathbf{H}$$

Usando el flujo tensorial $\mathbf{E} = (\mathbf{F}, \mathbf{G})$

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{U} d\Omega + \int_{\Omega} \nabla \times \mathbf{E} d\Omega = \int_{\Omega} \mathbf{H} d\Omega$$

Y aplicando el teorema de Gauss a la segunda integral:

$$\frac{\partial}{\partial t} \int_{\Omega} \mathbf{U} d\Omega + \oint_C \mathbf{E} \times \mathbf{n} dC = \int_{\Omega} \mathbf{H} d\Omega$$

Donde C es el contorno del volumen Ω

Propiedades del modelo

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{E} = \mathbf{H}$$

- Sistema hiperbólico

$$\mathbf{J}_n = \frac{\partial(\mathbf{E} \times \mathbf{n})}{\partial U} = \frac{\partial(\mathbf{F})}{\partial U} n_x + \frac{\partial(\mathbf{G})}{\partial U} n_y$$

- Variables:

Conservativas/primitivas/características

- Términos convectivos/términos fuente
- Estados estacionarios/transitorios
- Formación de ondas de choque
- Fronteras seco/mojado
- **NECESIDAD DE SOLUCIONES NUMERICAS**



RESOLUCIÓN NUMÉRICA

Opciones

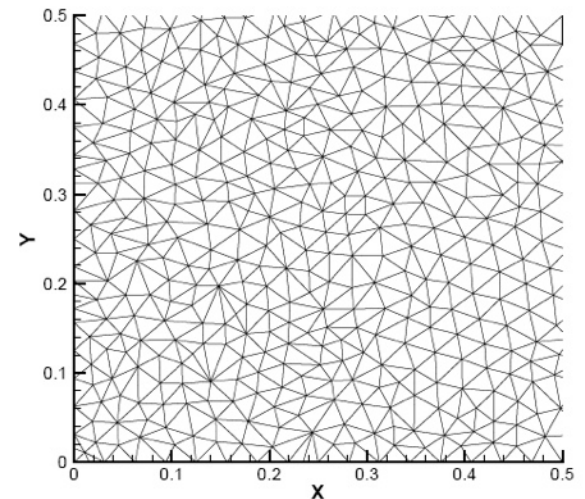
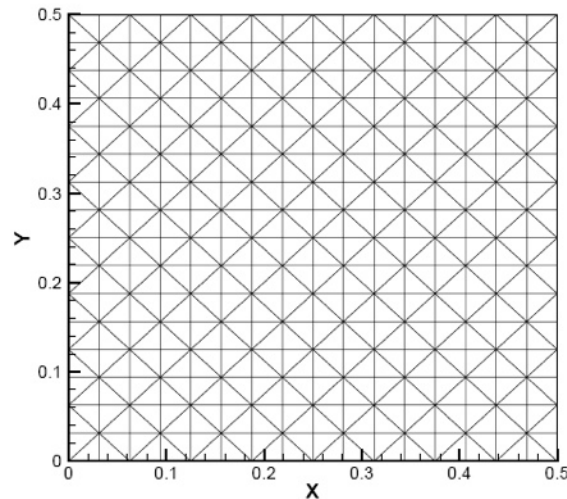
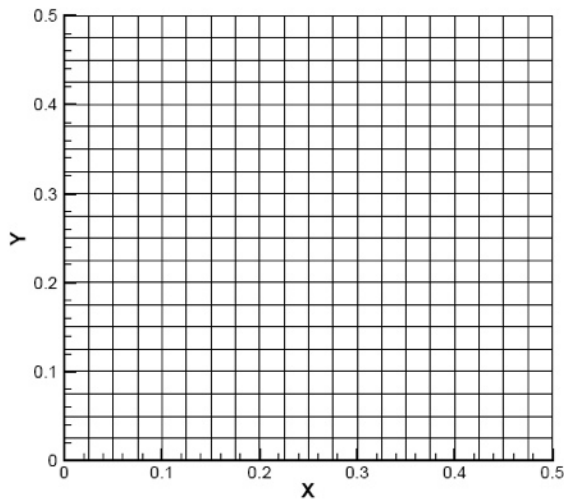
- Discretización
 - Diferencias finitas
 - **Volúmenes finitos**
 - Elementos finitos

Opciones

- Discretización
- Tipo de malla
 - ▣ Estructurada/No Estructurada
 - ▣ Fija/Adaptativa

Opciones

- Discretización
- Tipo de malla
 - Estructurada/No Estructurada
 - Fija/Adaptativa



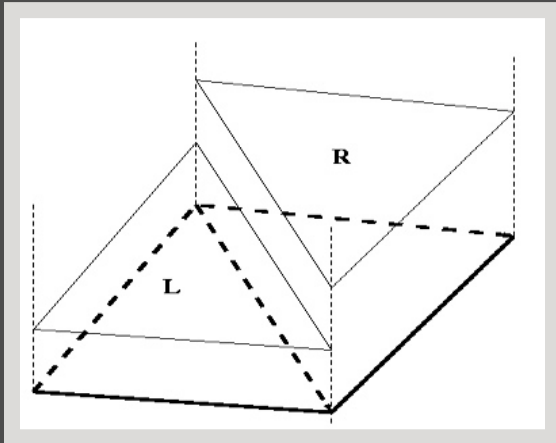
Opciones

- Discretización
- Tipo de malla
- Esquema numérico
 - ▣ **Euleriano**/lagrangiano
 - ▣ Implícito/**Explícito**
 - ▣ Centrado/**Upwind**

Cosas a tener en cuenta

- Discretización
- Tipo de malla
- Esquema numérico
- Términos fuente
- Condiciones de contorno
- NECESARIA CALIBRACION Y VALIDACION

Ecuaciones 2D SW-homogéneas



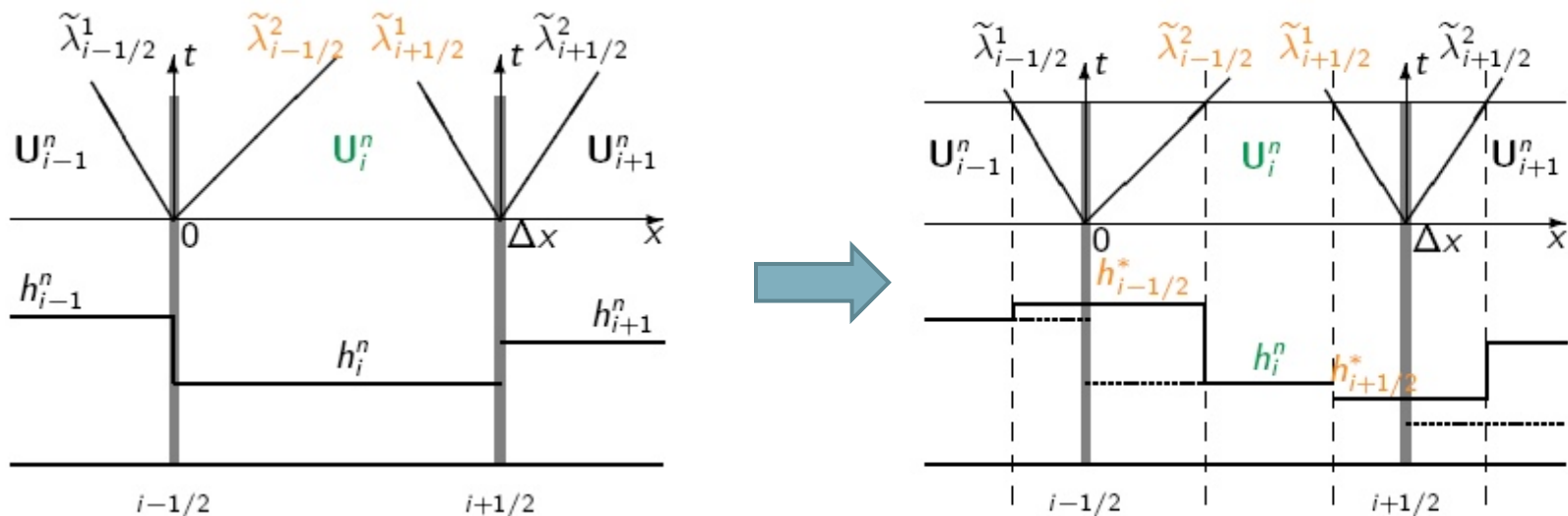
$$\frac{\partial}{\partial t} \mathbf{U} + \nabla(\mathbf{F}, \mathbf{G}) = 0$$

- ✓ Sólo válidas en casos de fondo plano y sin rozamiento
- ✓ No lineales y sin solución exacta
- ✓ Requieren simulación numérica y validación

Esquema conservativo 1D

Método de Godunov

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$



$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - (\alpha \tilde{\mathbf{e}} \tilde{\lambda})_{i-1/2}^2 \frac{\Delta t}{\Delta x} - (\alpha \tilde{\mathbf{e}} \tilde{\lambda})_{i+1/2}^1 \frac{\Delta t}{\Delta x}$$

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \left((\delta \mathbf{F})_{i-1/2}^+ + (\delta \mathbf{F})_{i+1/2}^- \right) \frac{\Delta t}{\Delta x}$$

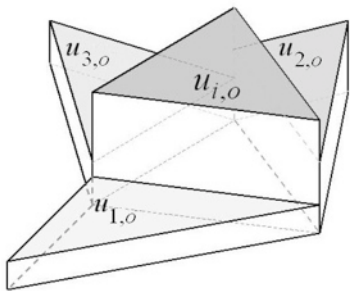
Método upwind- 1^{er} orden

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \mathbf{E} = 0$$

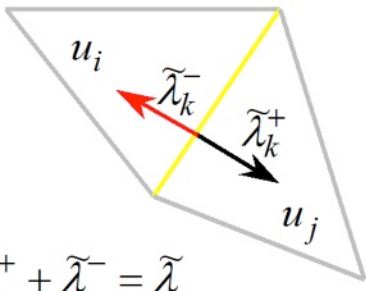
$$\mathbf{J}_n = \frac{\partial(\mathbf{E} \times \mathbf{n})}{\partial U} = \frac{\partial(\mathbf{F})}{\partial U} n_x + \frac{\partial(\mathbf{G})}{\partial U} n_y$$

$$\tilde{\mathbf{J}}_{n,k} = (\tilde{\mathbf{P}} \tilde{\mathbf{E}} \tilde{\mathbf{P}}^{-1})_k$$

$$\tilde{\mathbf{E}}^\pm = (\tilde{\mathbf{E}} \pm |\tilde{\mathbf{E}}|)/2$$



$$\delta(\mathbf{E} \times \mathbf{n})_k = \underbrace{\tilde{\mathbf{P}}_k \tilde{\mathbf{E}}^- \tilde{\mathbf{P}}_k^{-1} \delta \mathbf{U}_k}_{\text{ondas entrantes}} + \underbrace{\tilde{\mathbf{P}}_k \tilde{\mathbf{E}}^+ \tilde{\mathbf{P}}_k^{-1} \delta \mathbf{U}_k}_{\text{ondas salientes}}$$



$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} (\tilde{\lambda}^- \alpha \tilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$

Restricción en el paso temporal

Condición Courant-Friedrichs-Lewy :

$$\Delta t = CFL \Delta t_{\max}$$

$$\Delta t_{\max} = \min \left\{ \Delta t_{\max,i} \right\}_{i=1, NCELL}$$

Para sistemas homogéneos:

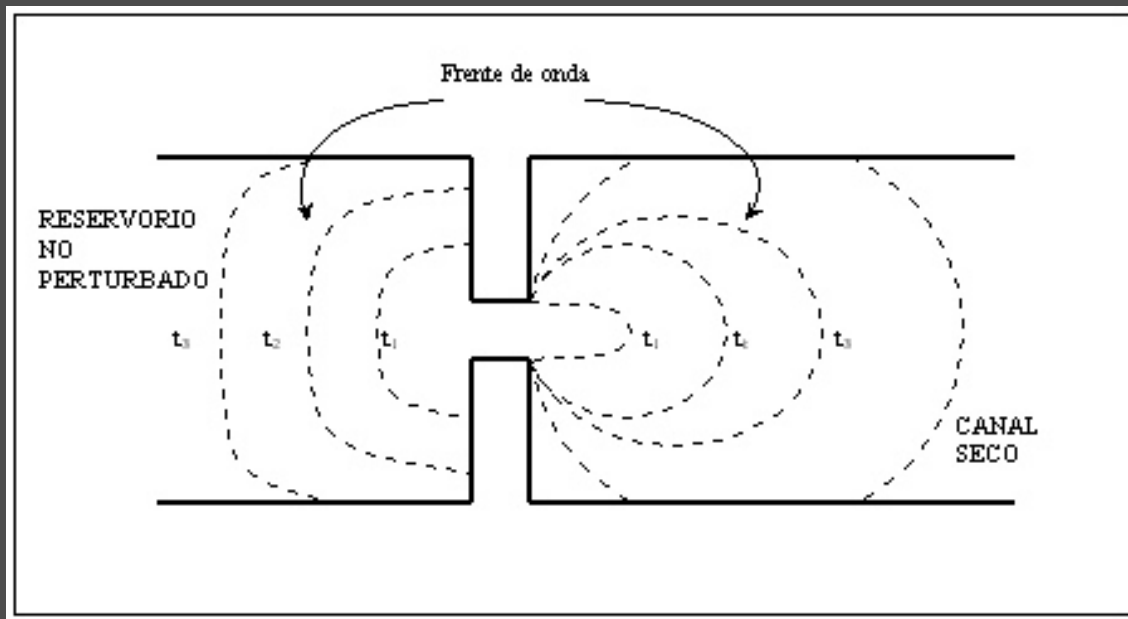
$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} (\tilde{\lambda}^{-\alpha} \tilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$

$$\Delta t_{\max,i} = \min \left\{ \min \left(\frac{A_{\min,k}}{|\tilde{\lambda}_k^m| l_k} \right)_{m=1, \dots, N\lambda} \right\}_{k=1, NE}$$

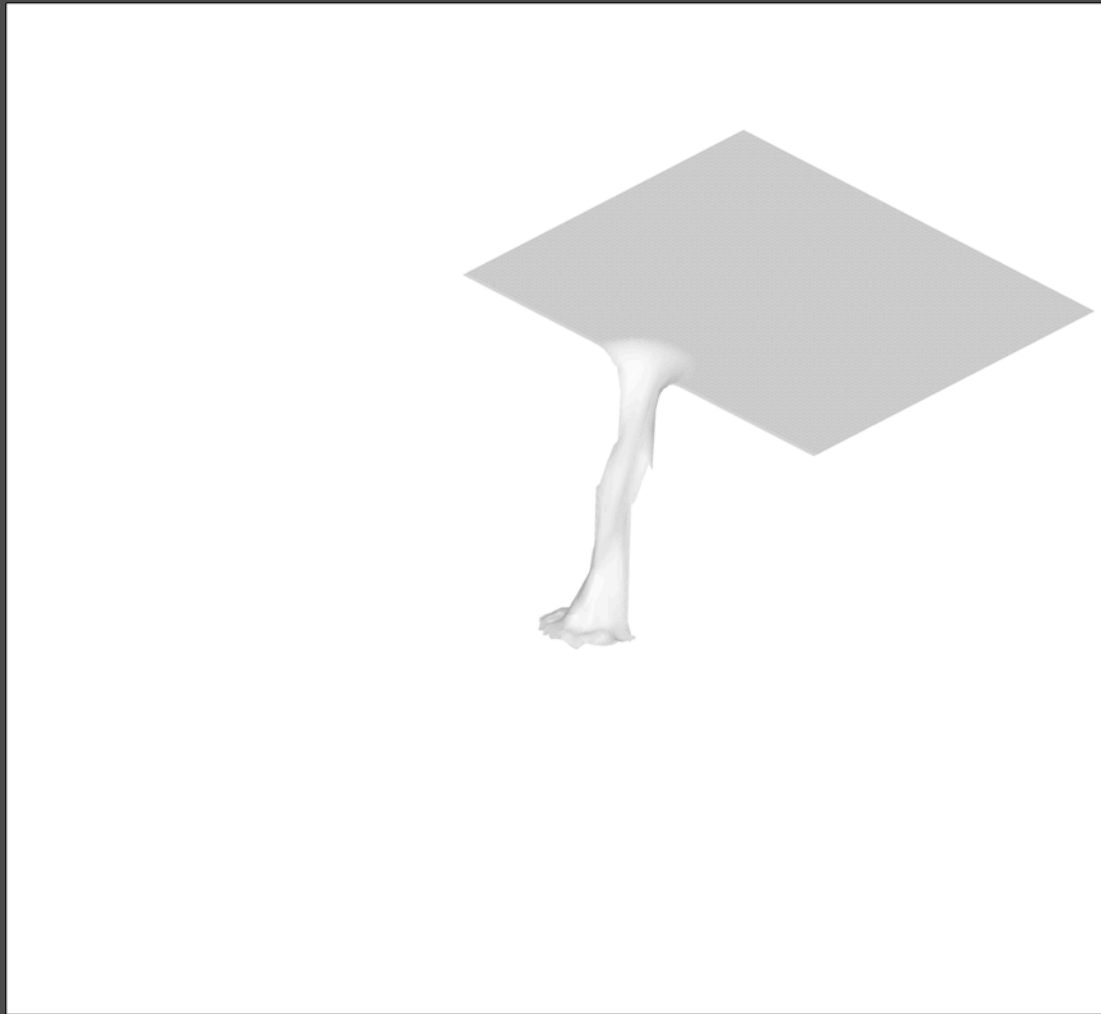
Flujo en una rotura de presa

Descripción del laboratorio

(UNIVERSITY OF PAVIA)



T = 0.08 s



$T = 0.24 \text{ s}$



$T = 0.32 \text{ s}$

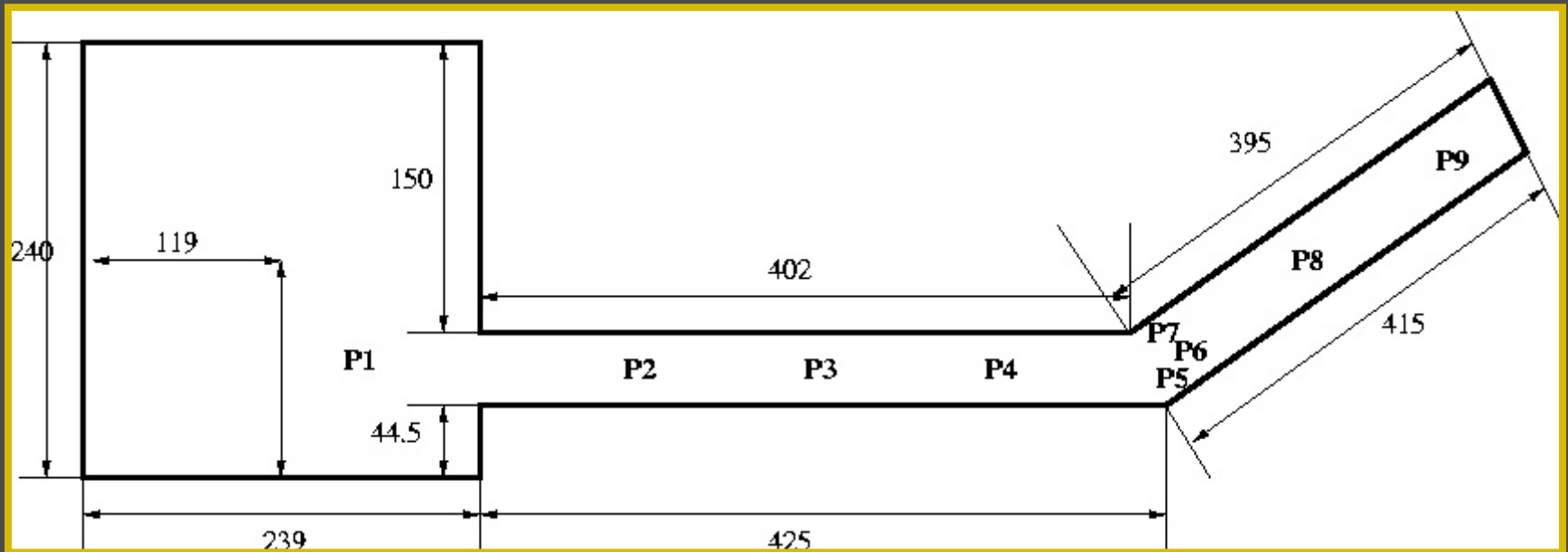


$T = 0.40 \text{ s}$

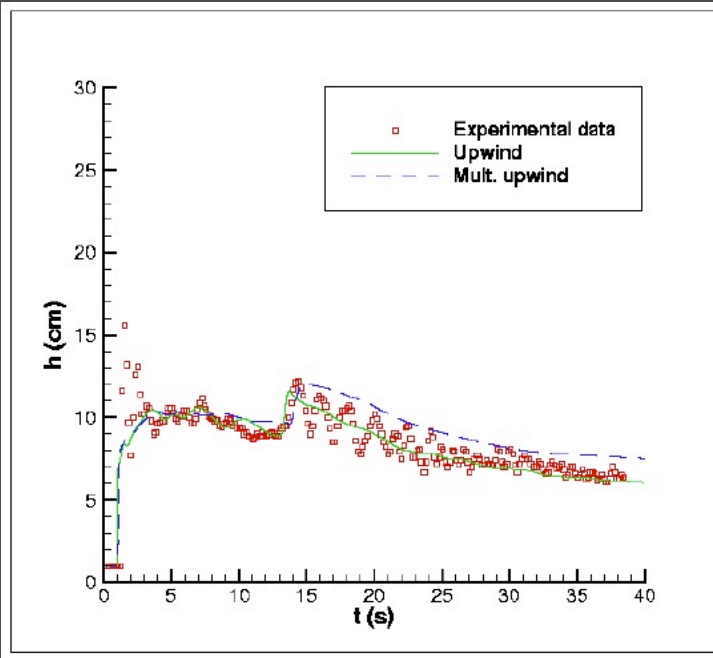


Rotura de presa: Validación experimental

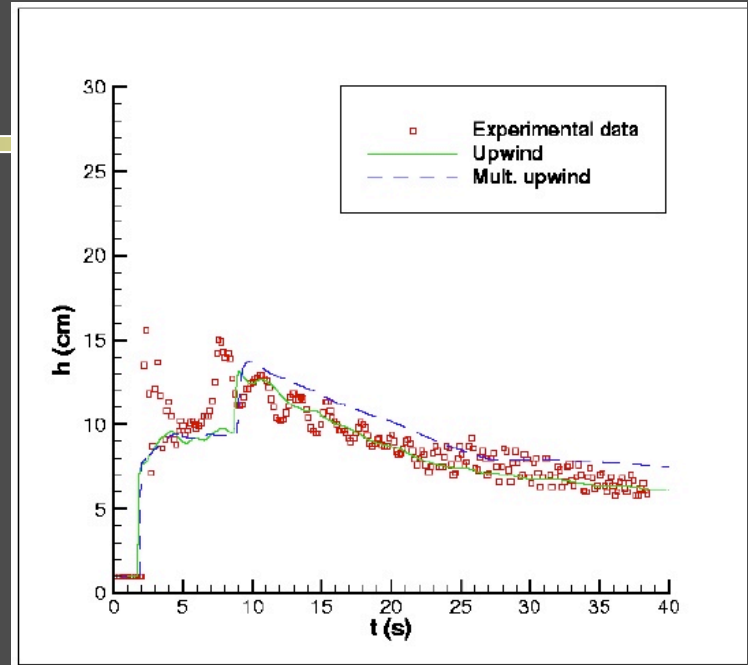
(Univ. Cath. Louvain, Belgium. CADAM. 1998)



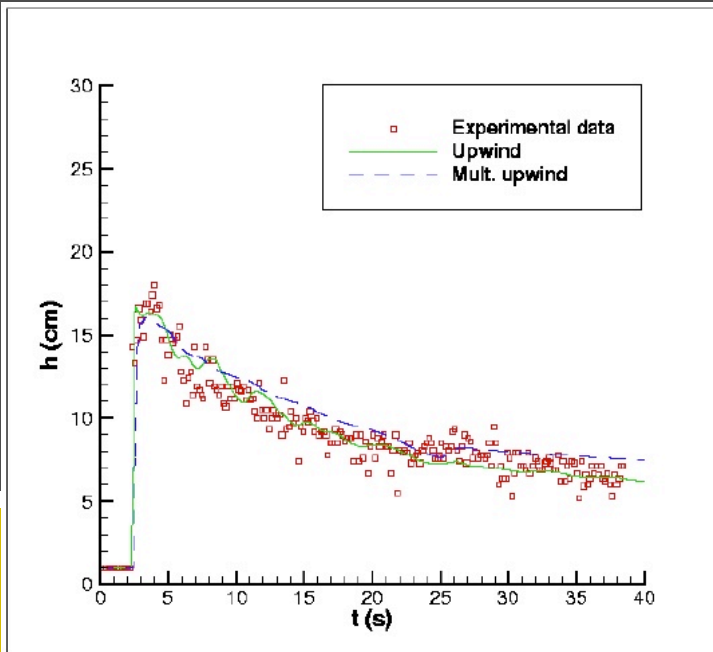
P3



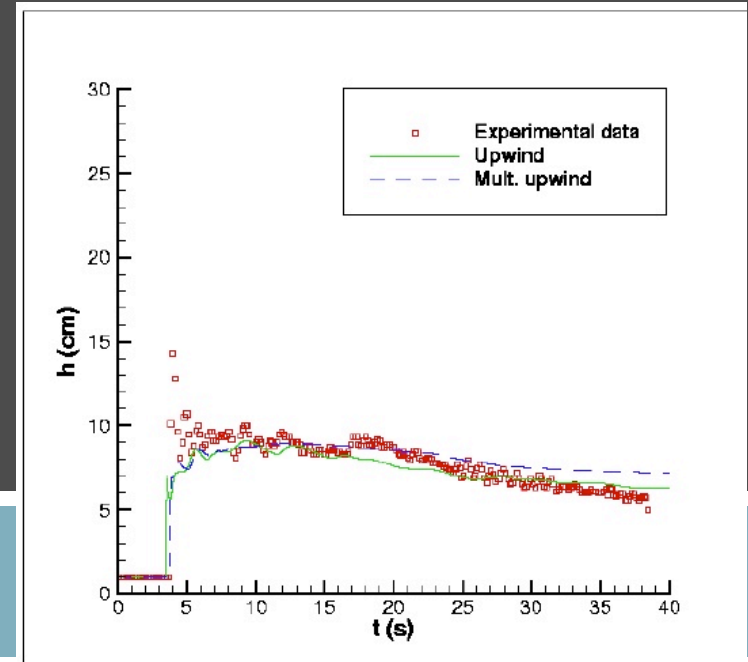
P4



P5



P9



Esquema upwind: 1^{er} orden

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \mathbf{E} = \mathbf{H}$$

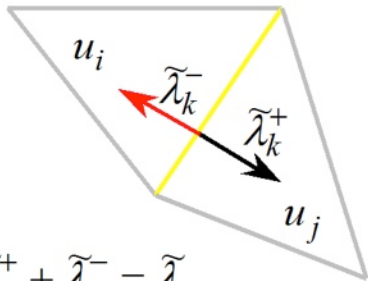
$$\mathbf{J}_n = \frac{\partial(\mathbf{E} \times \mathbf{n})}{\partial \mathbf{U}} = \frac{\partial(\mathbf{F})}{\partial \mathbf{U}} n_x + \frac{\partial(\mathbf{G})}{\partial \mathbf{U}} n_y$$

$$\tilde{\mathbf{J}}_{n,k} = (\tilde{\mathbf{P}} \tilde{\mathbf{E}} \tilde{\mathbf{P}}^{-1})_k$$

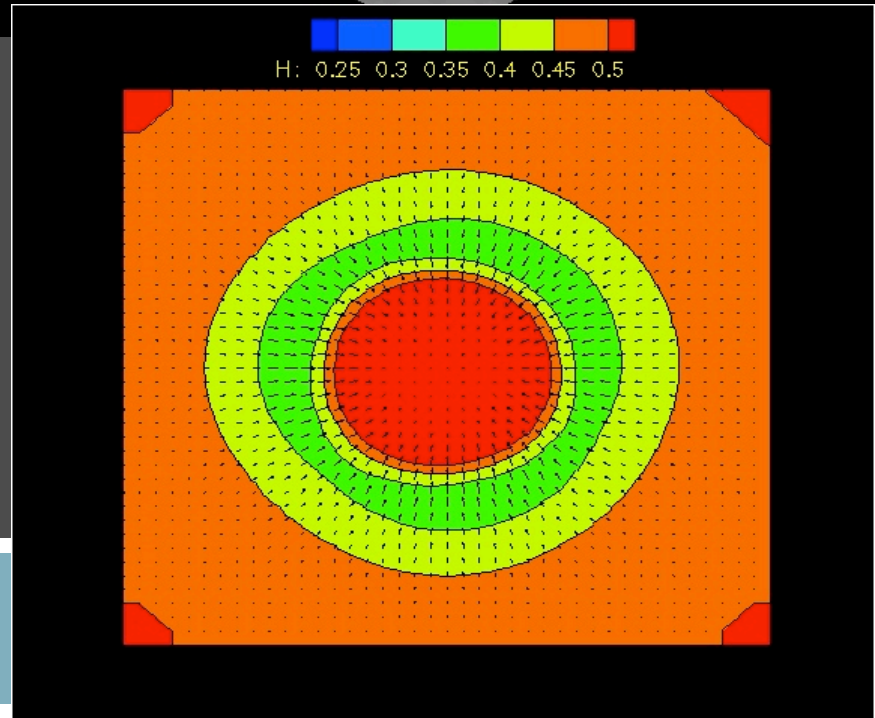
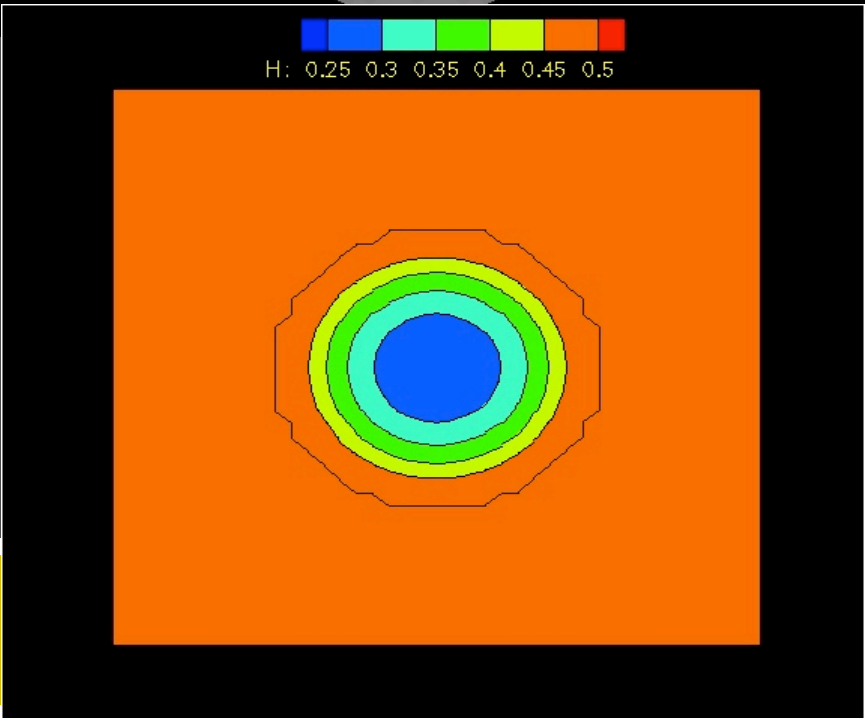
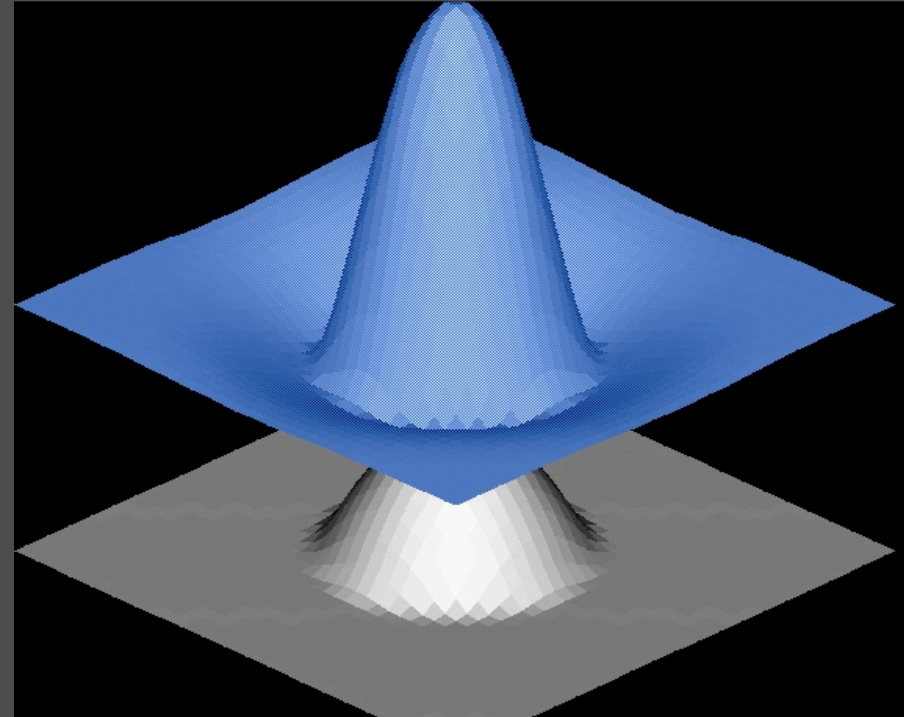
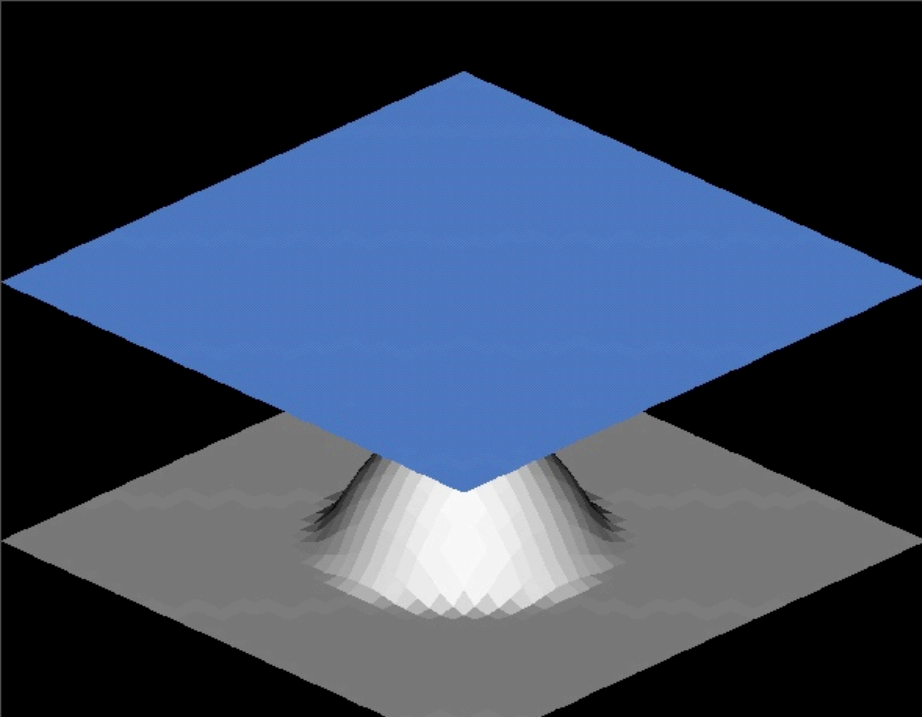
$$\tilde{\mathbf{E}}^\pm = (\tilde{\mathbf{E}} \pm |\tilde{\mathbf{E}}|)/2$$

$$\delta(\mathbf{E} \times \mathbf{n})_k = \underbrace{(\mathbf{P} \tilde{\mathbf{E}}^- \mathbf{P}^{-1} \delta \mathbf{U})_k}_{\text{ondas entrantes}} + \underbrace{(\mathbf{P} \tilde{\mathbf{E}}^+ \mathbf{P}^{-1} \delta \mathbf{U})_k}_{\text{ondas salientes}}$$

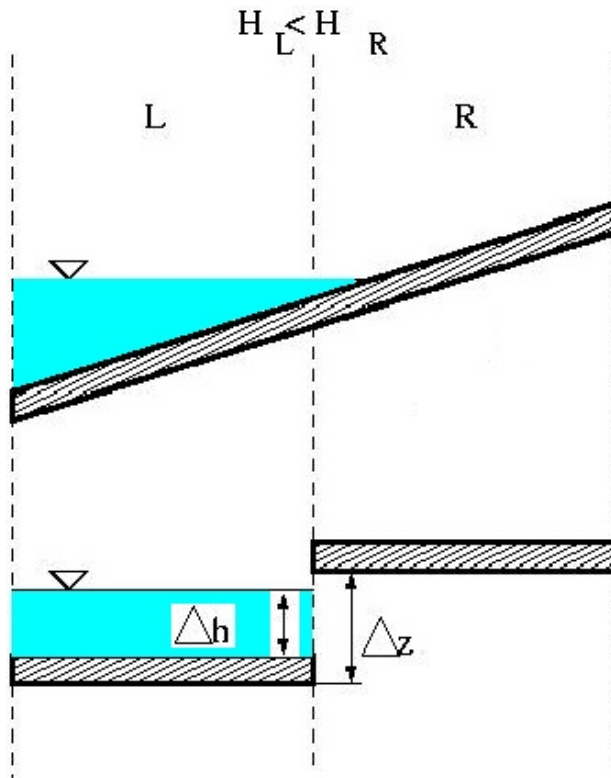
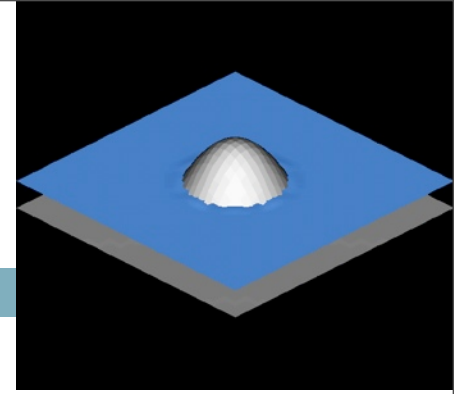
$$\delta(\mathbf{H} \times \mathbf{n})_k = \underbrace{(\delta \mathbf{H} \times \mathbf{n})_k^-}_{\text{ondas entrantes}} + \underbrace{(\delta \mathbf{H} \times \mathbf{n})_k^+}_{\text{ondas salientes}}$$



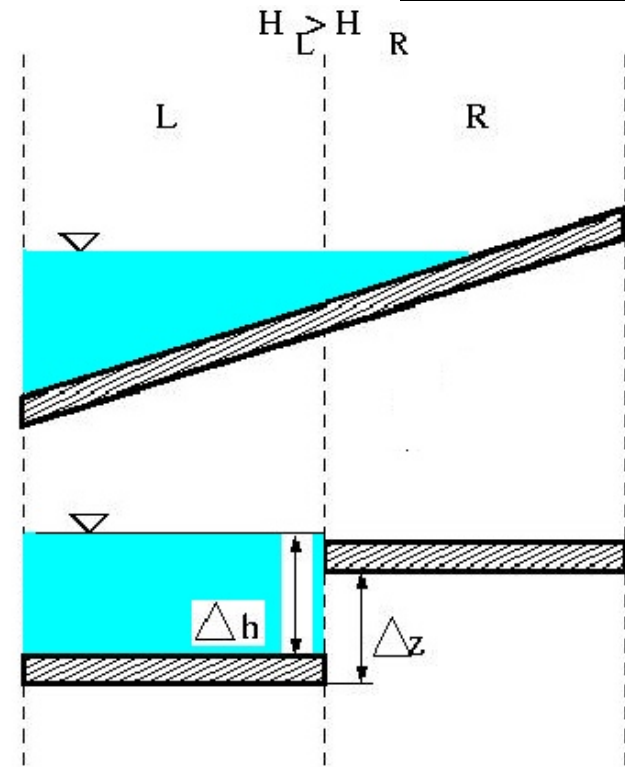
$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} ((\tilde{\lambda}^- \alpha - \beta^-) \tilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$



Fronteras seco/mojado



$$h_L + z_L < h_R + z_R$$



$$h_L + z_L > h_R + z_R$$

Fronteras seco/mojado

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} ((\tilde{\lambda}^{-\alpha} - \beta^{-}) \tilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$

Definición:

$$\lambda_k^{*m} = (\tilde{\lambda} - \beta / \alpha)_k^{-m}$$

Control basado en:

$$Fr_k = \left| \frac{\tilde{\mathbf{u}} \cdot \mathbf{n}}{\tilde{c}} \right|_k < 1$$

De forma que:

$$\lambda_k^{*m} < 0$$

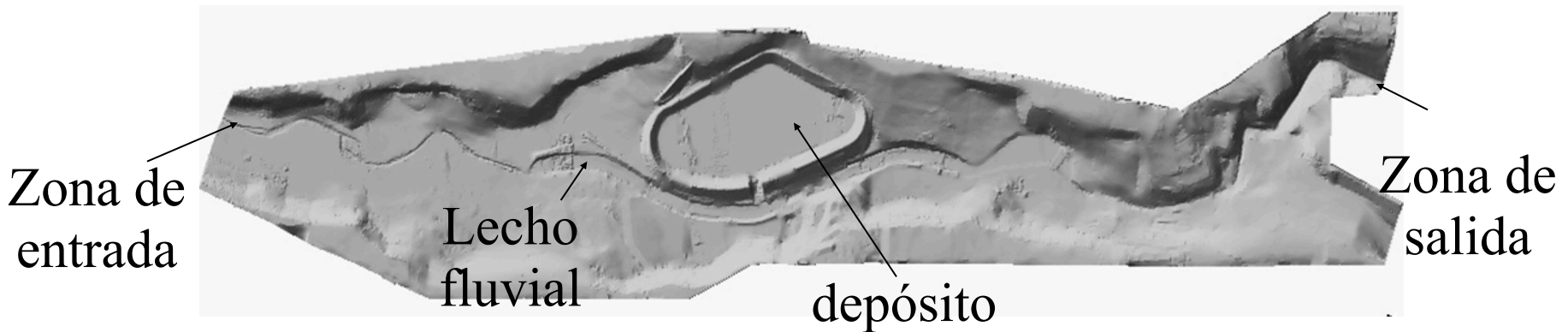
Hay flujo cruzando el lado k

$$\lambda_k^{*m} > 0$$

No hay flujo cruzando lado k

Caso test: Río Toce

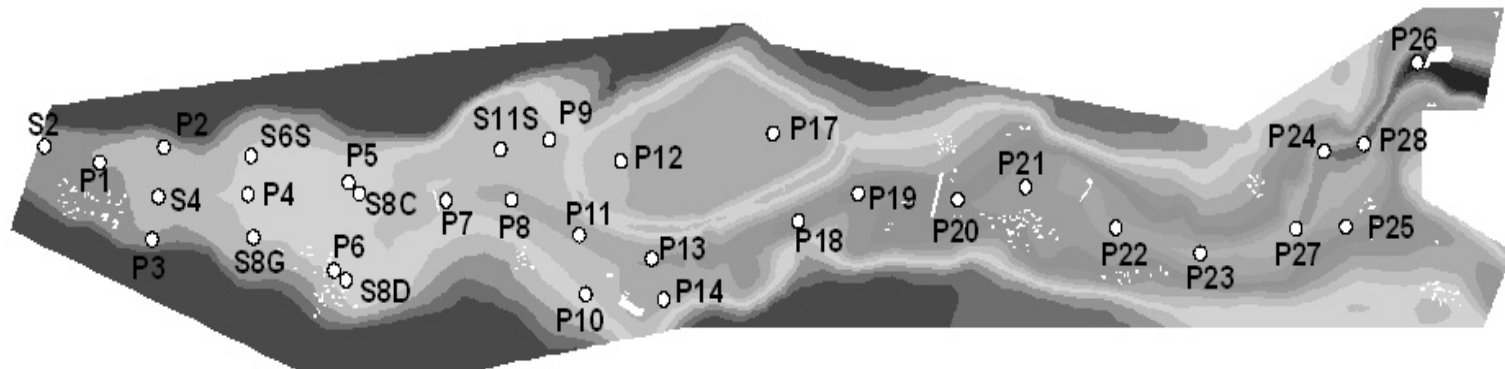
- Caso experimental realizado en un modelo a escala del río Toce (Italia) se usa para probar la calidad de las soluciones



- Se introduce un volumen de agua a la entrada sobre un valle seco

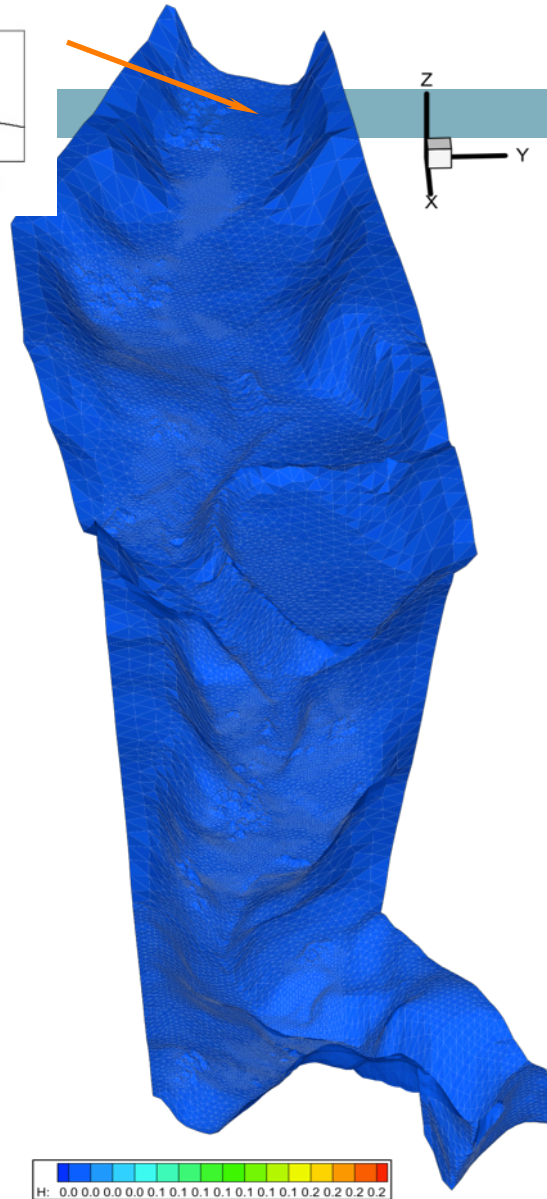
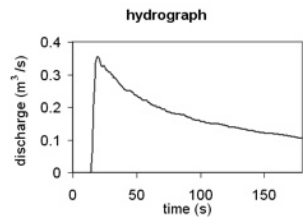
Caso test: Rio Toce

- Varias sondas miden la evolución temporal del nivel de agua

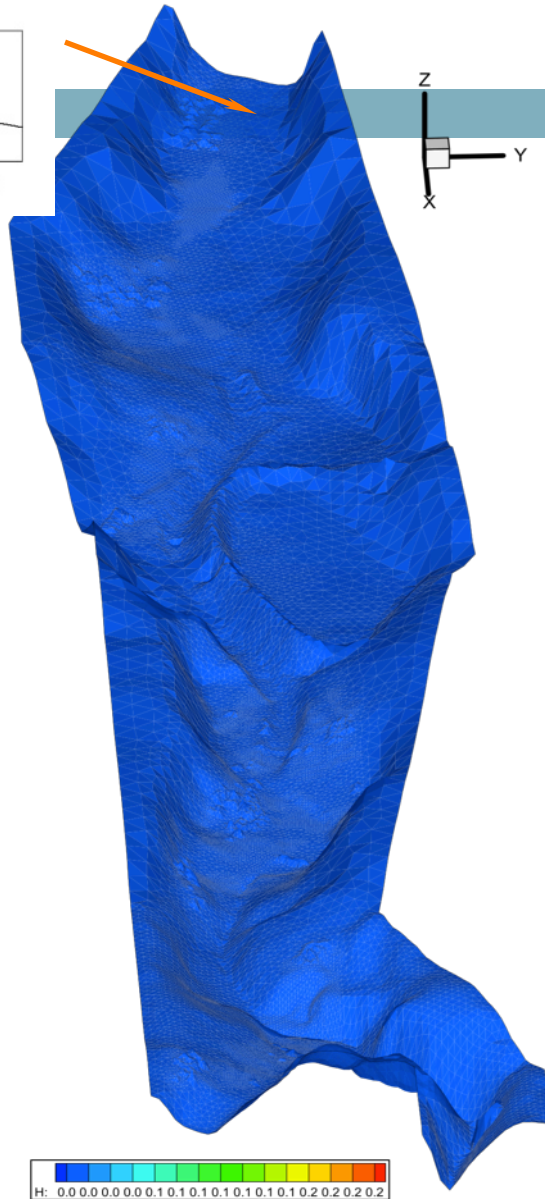
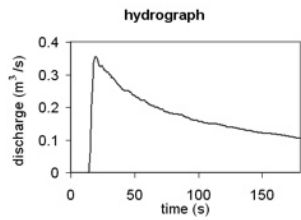


- Nos permiten validar los resultados numéricos

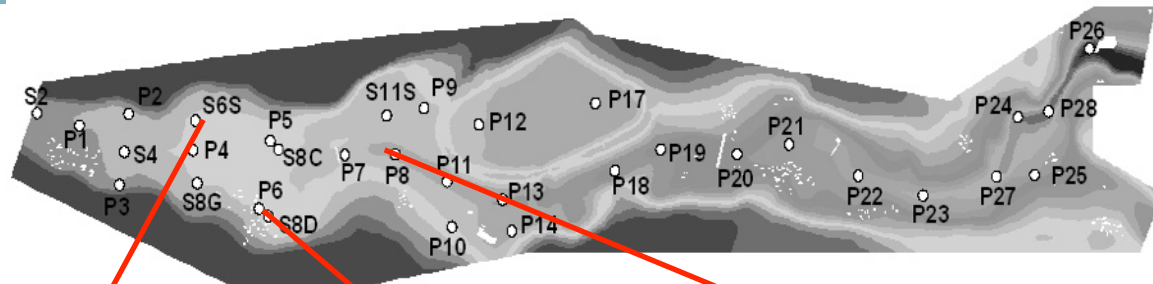
Evolución temporal del nivel superficial



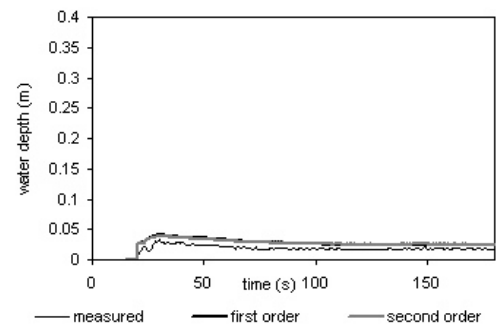
Evolución temporal del nivel superficial



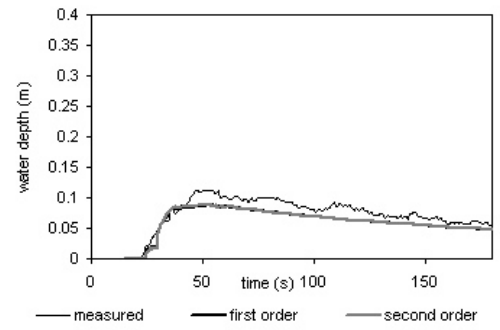
Medido frente a Calculado



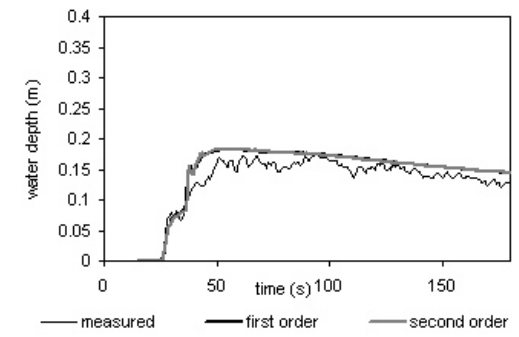
probe S6S



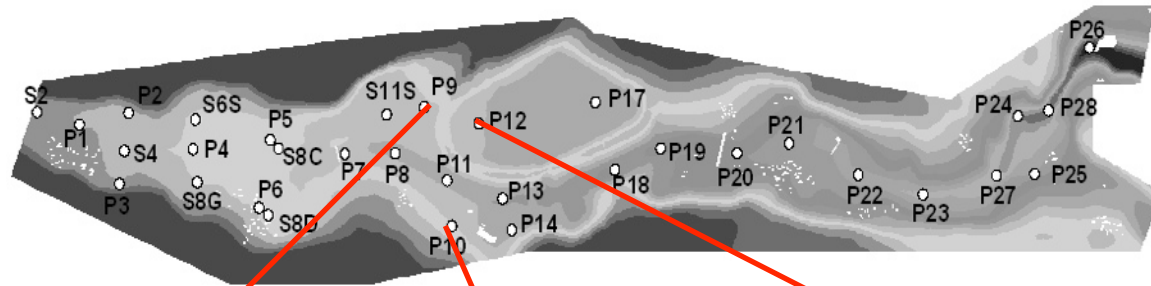
probe S8D



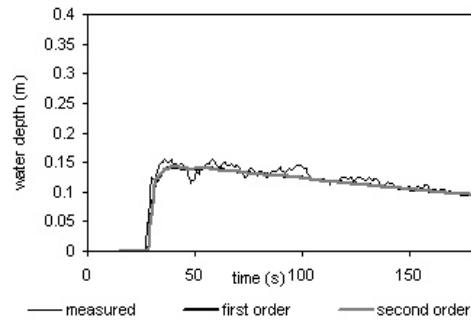
probe P8



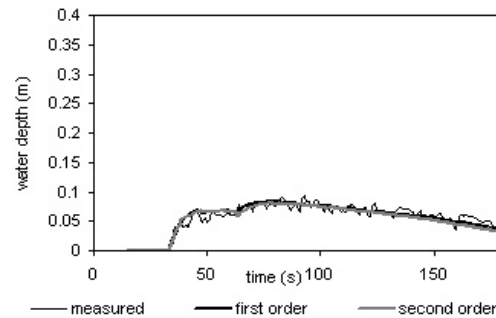
Medido frente a Calculado



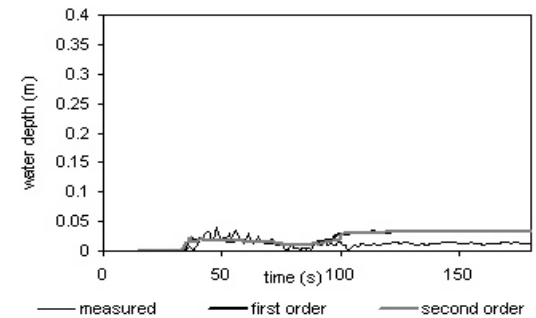
probe P9



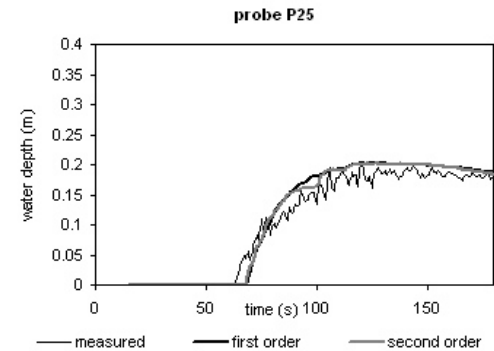
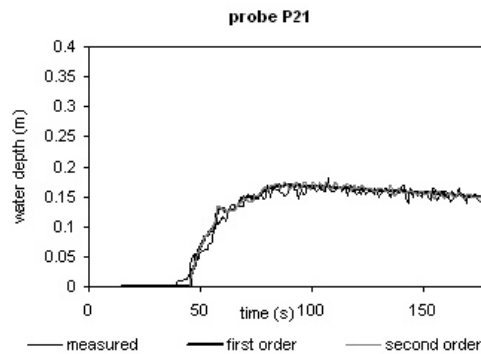
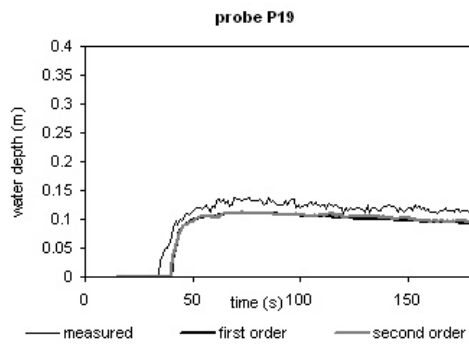
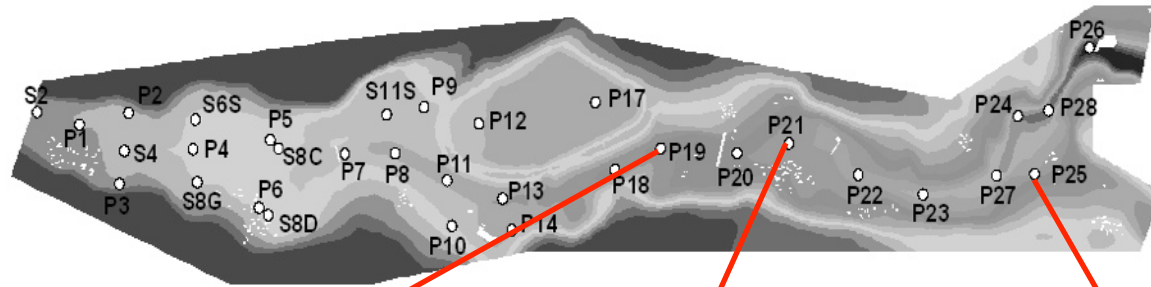
probe P10



probe P12



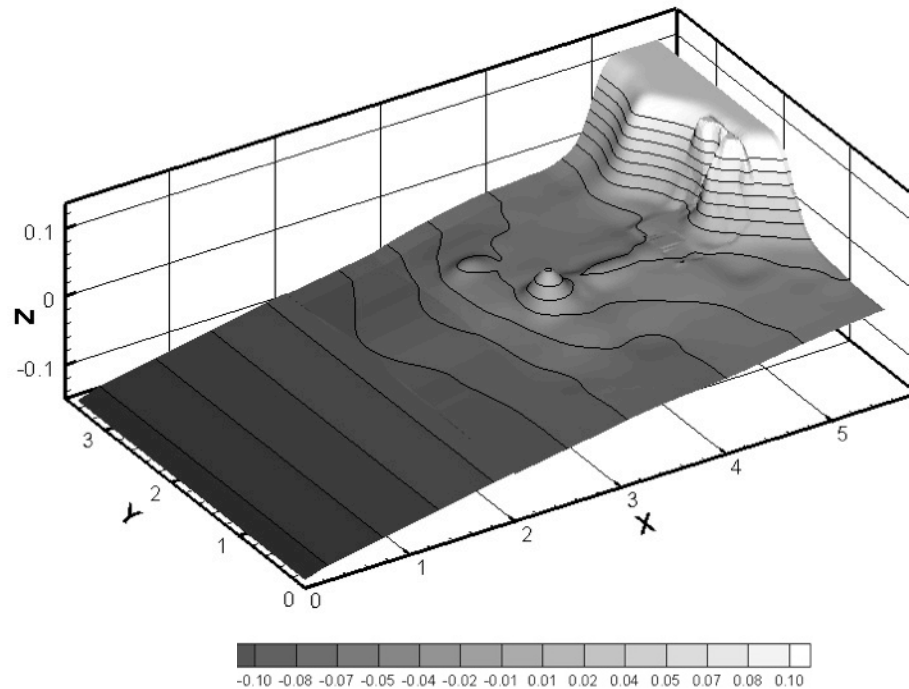
Medido frente a Calculado



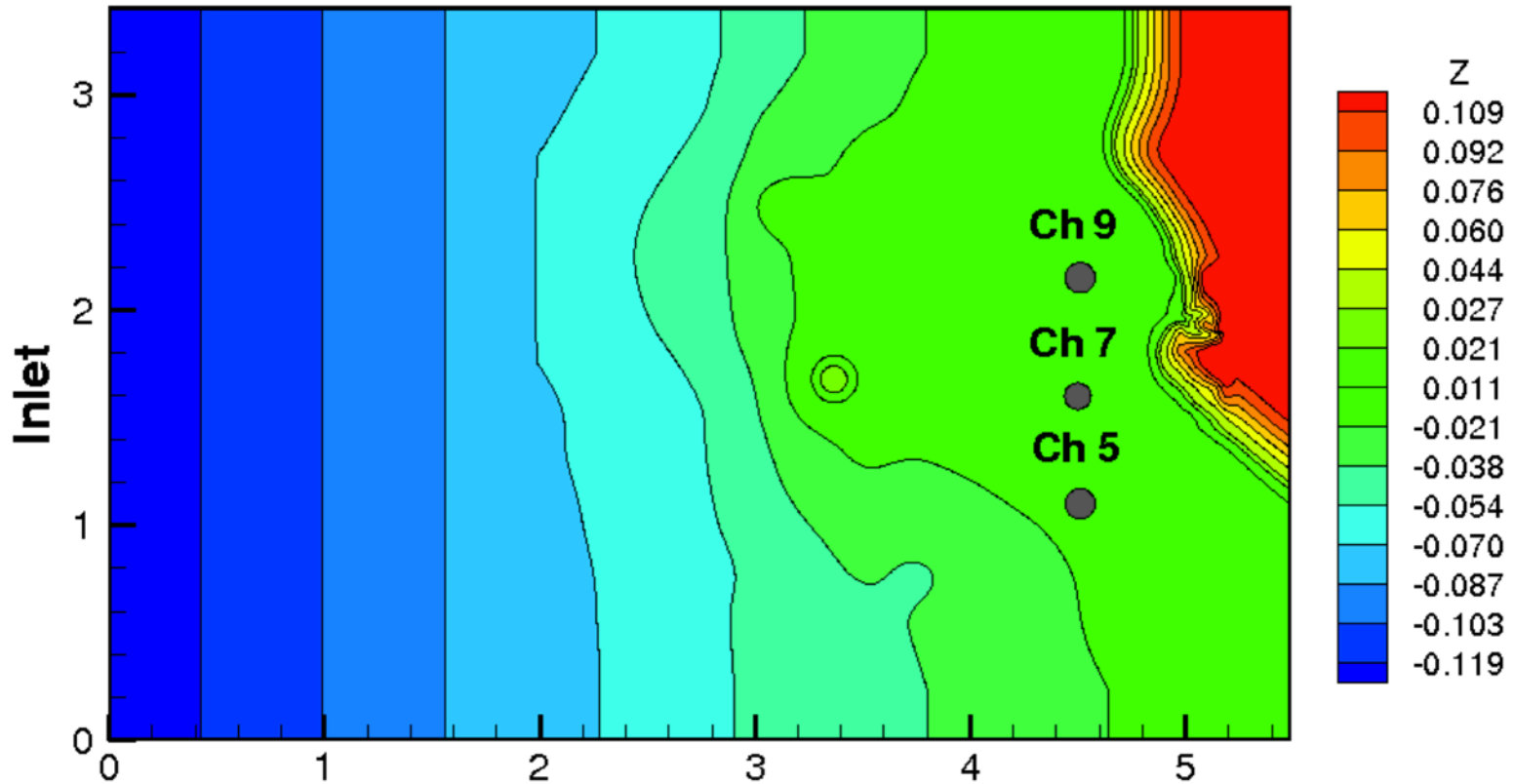
MODELO A ESCALA: Onda de Tsunami experimental

Experimento de laboratorio a escala 1/400 de la llegada a la isla de Monai de una onda de Tsunami.

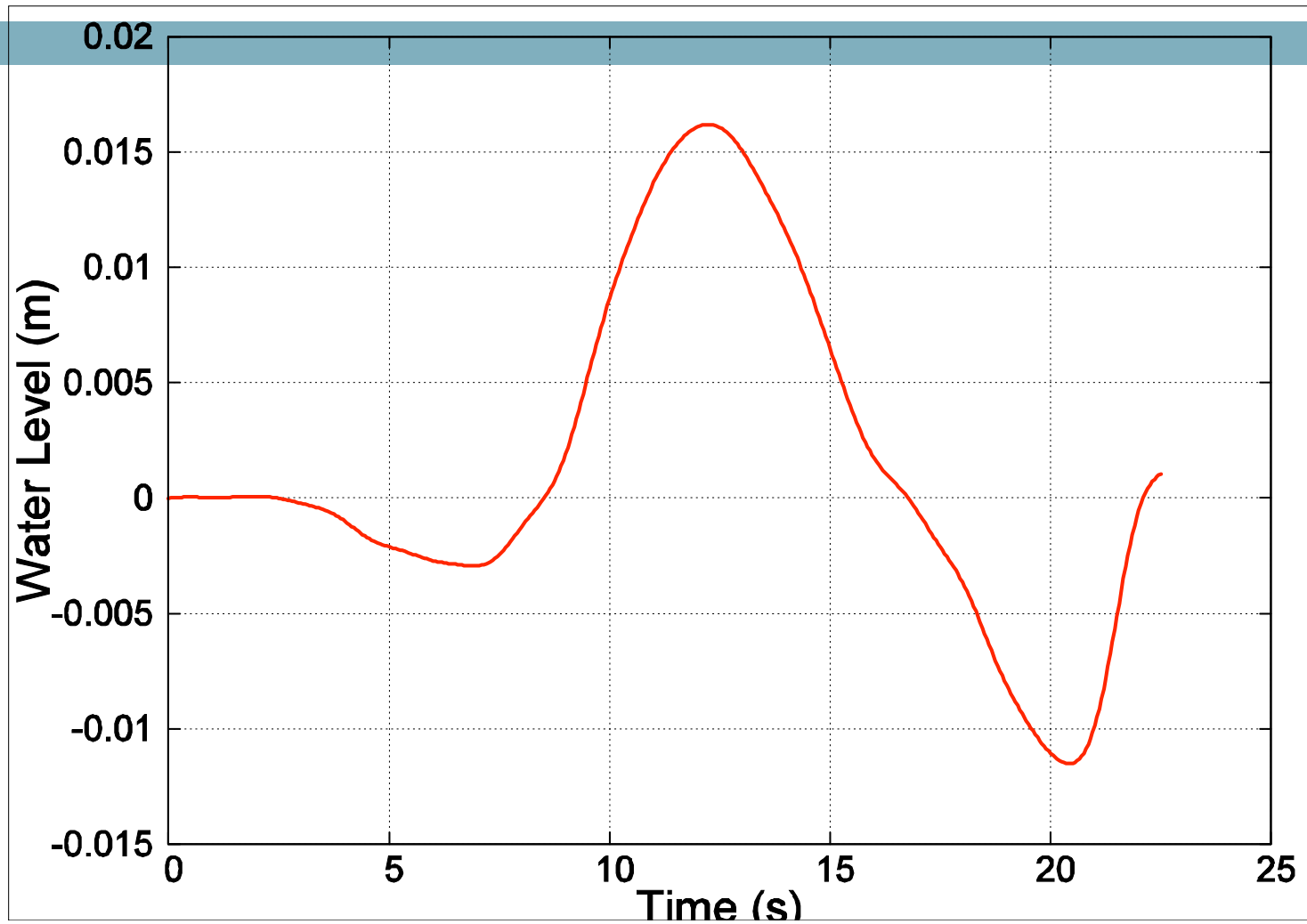
Tanque de 205 m de largo, 6 m de profundo, 3.4 m de ancho) at Central Research Institute for Electric Power Industry (CRIEPI) in Abiko, Japan.

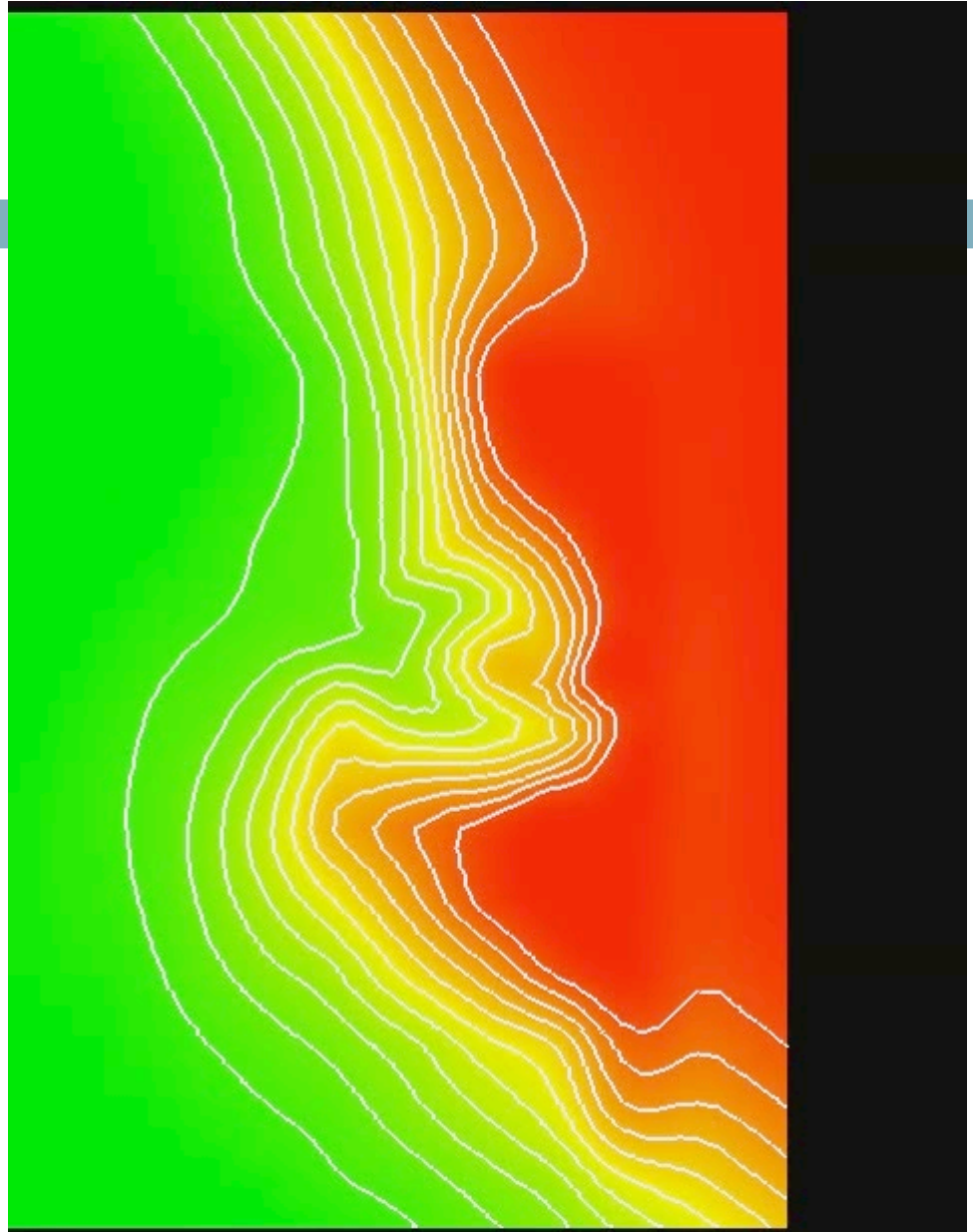
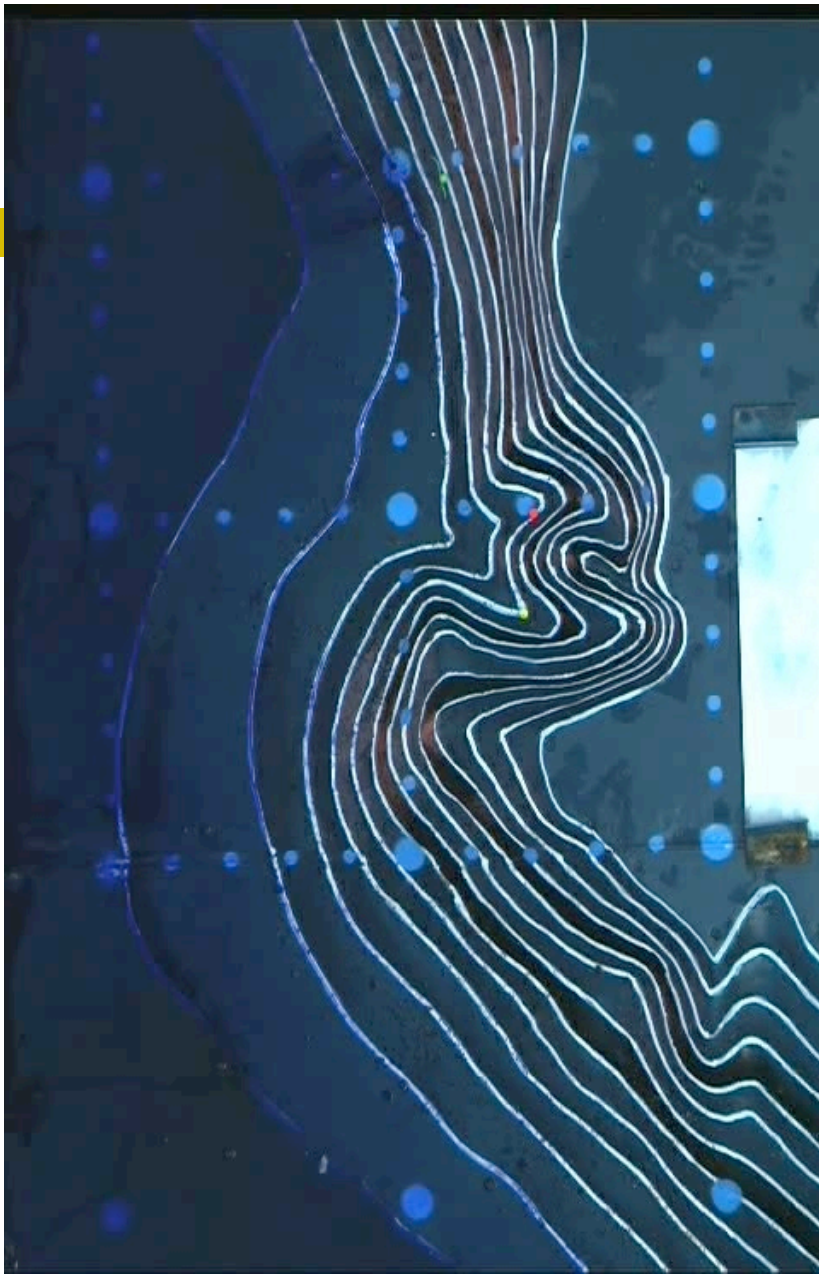


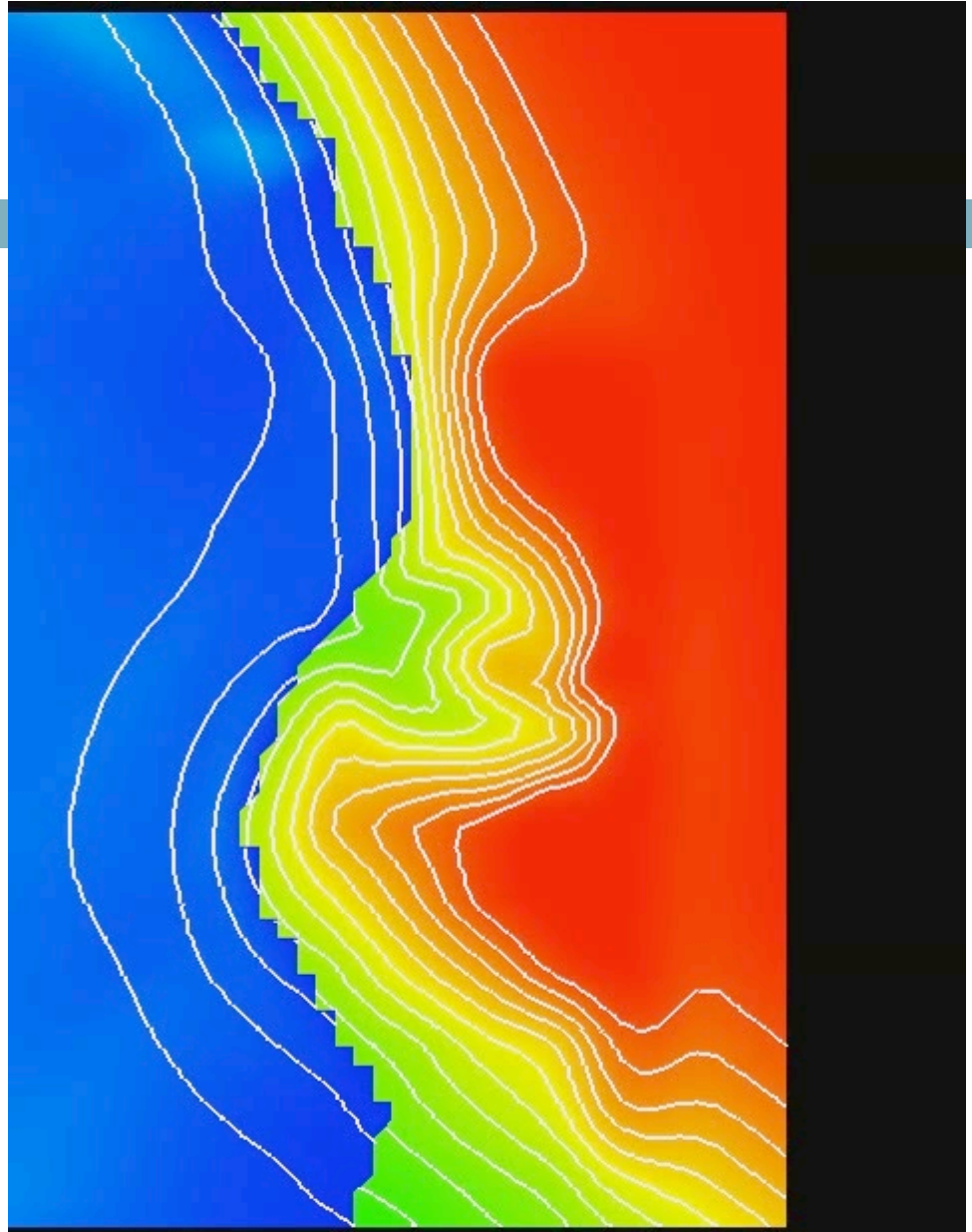
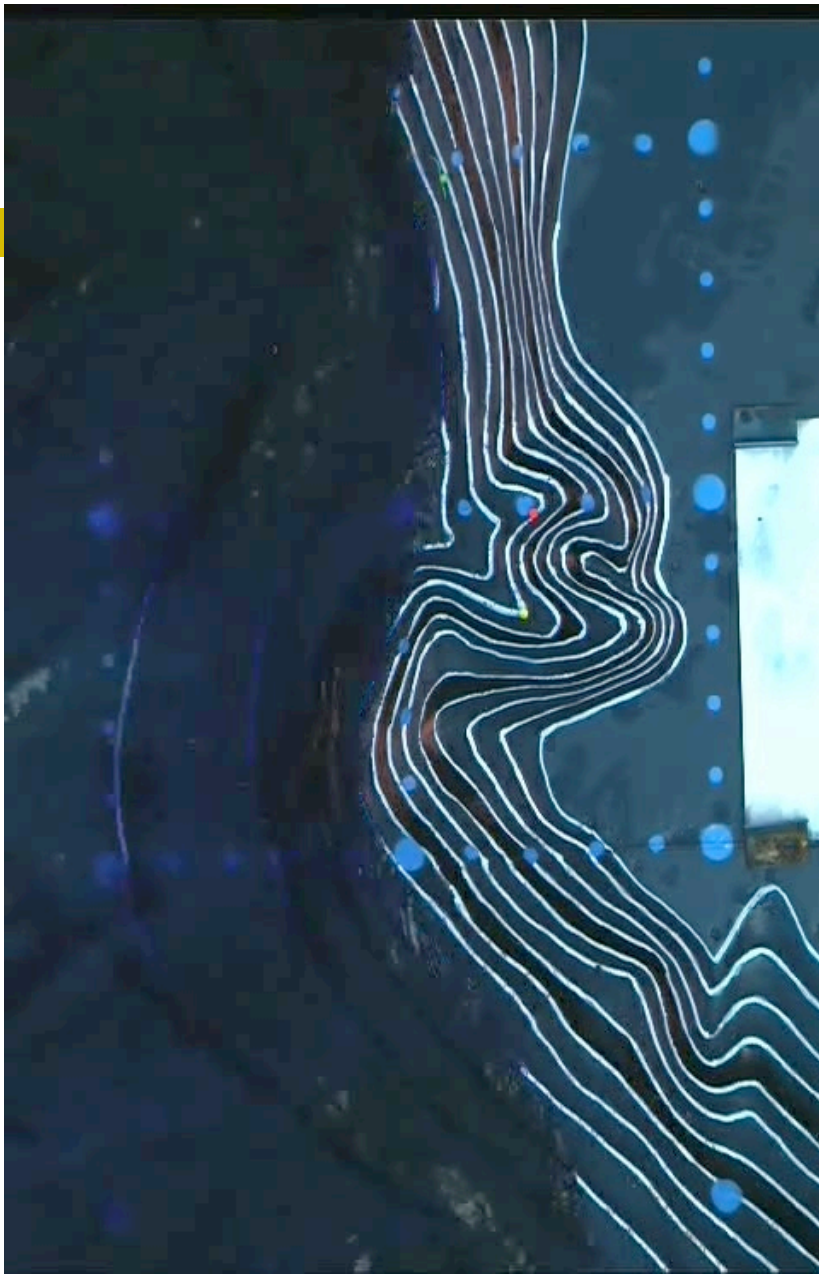
Batimetría en el modelo a escala

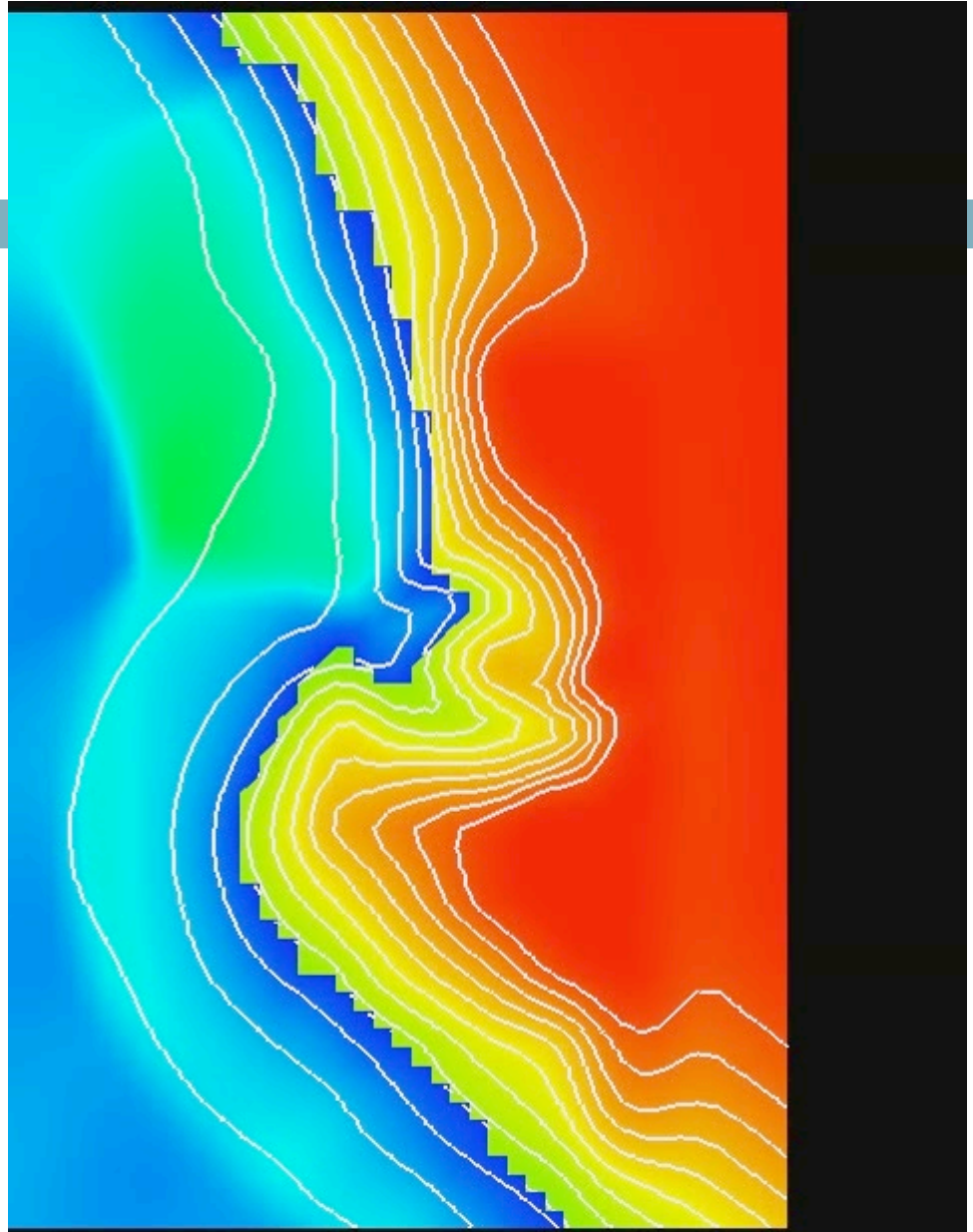


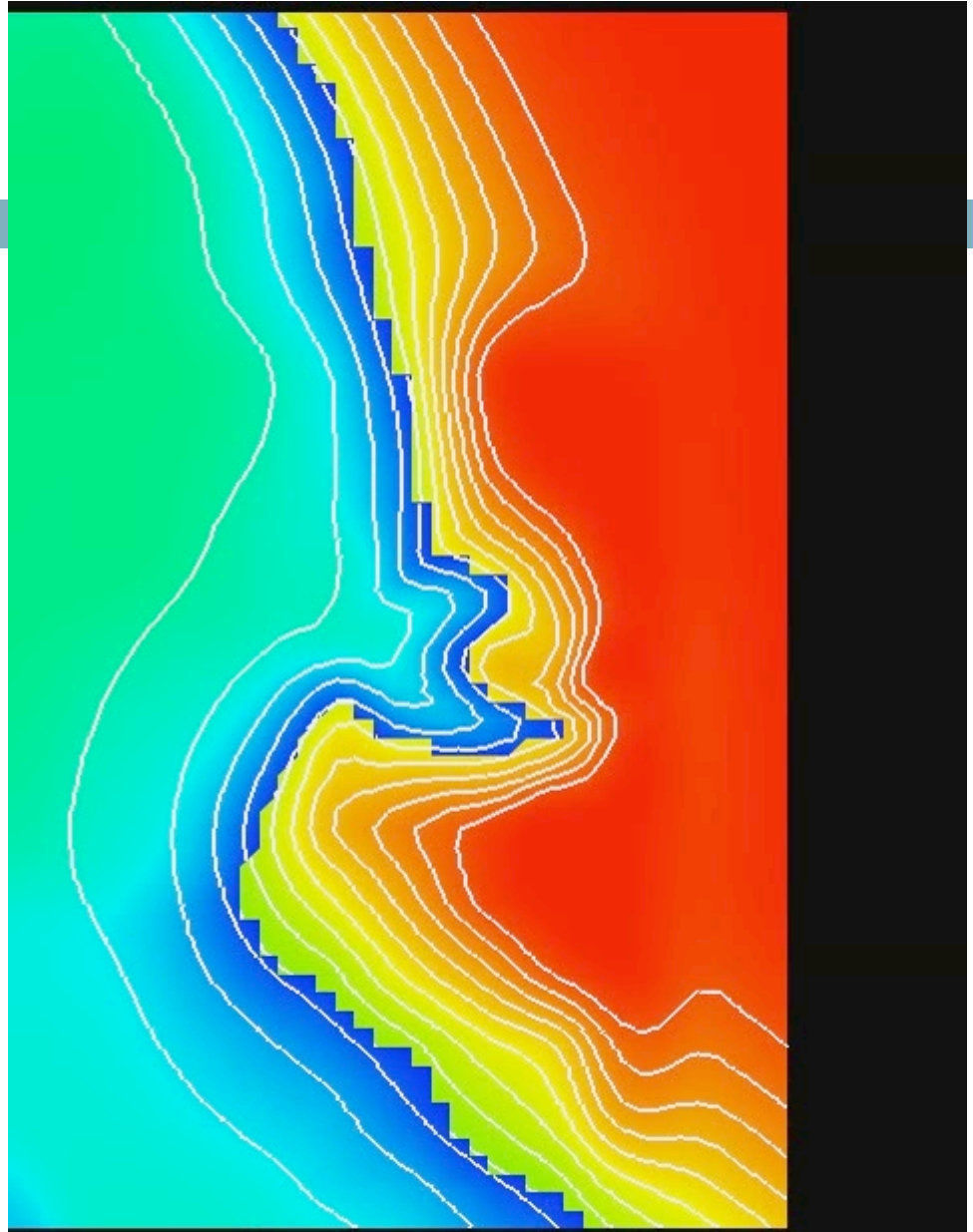
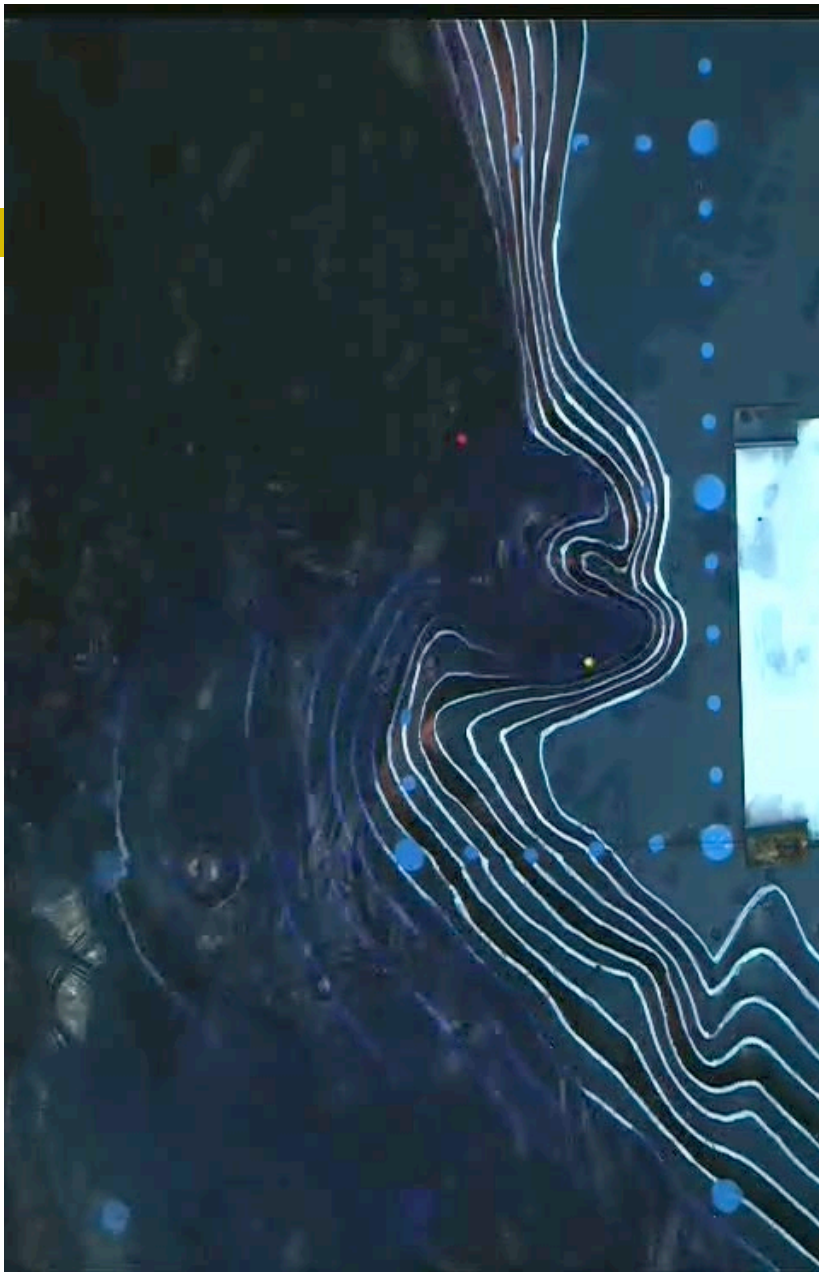
Condición de contorno de entrada

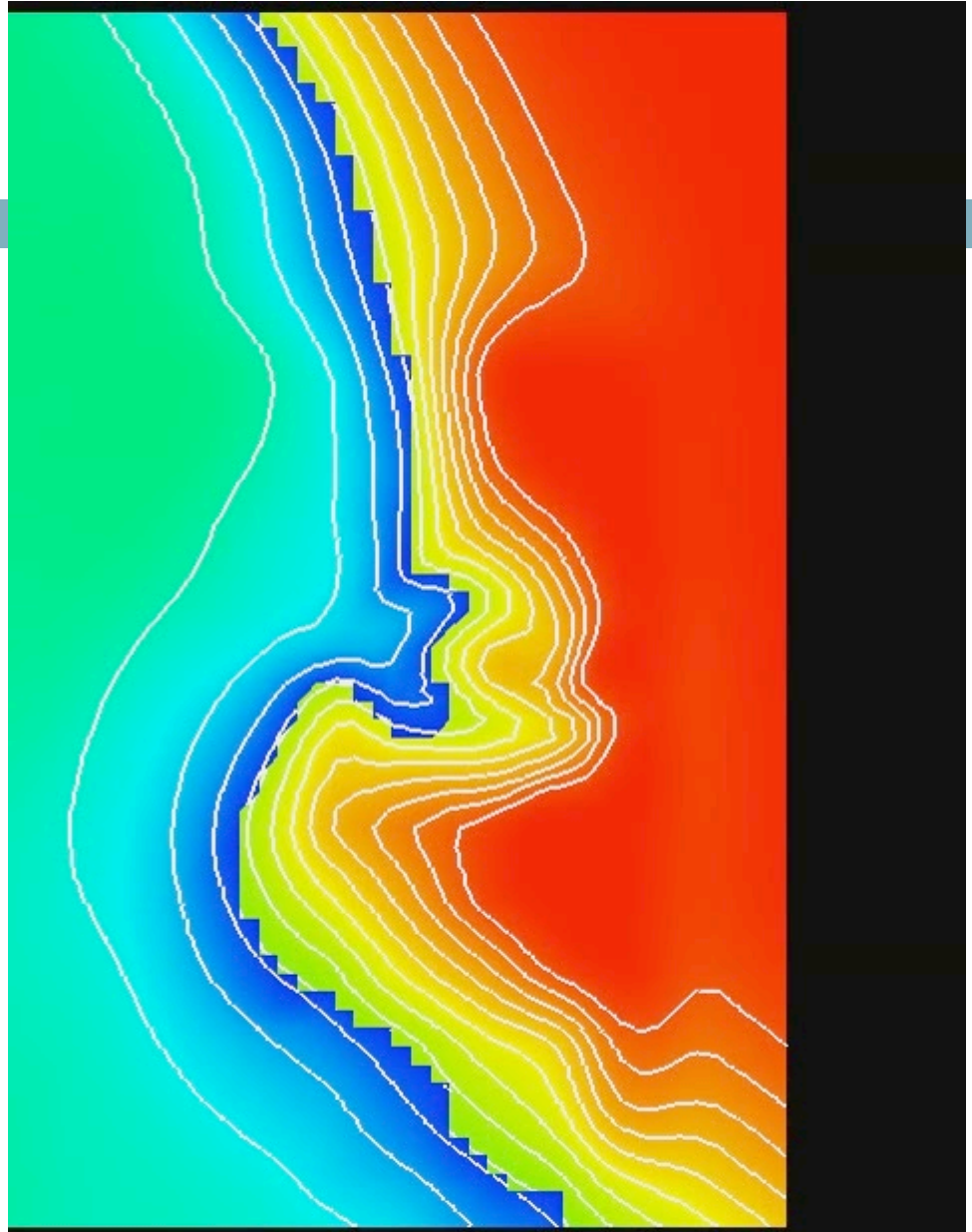


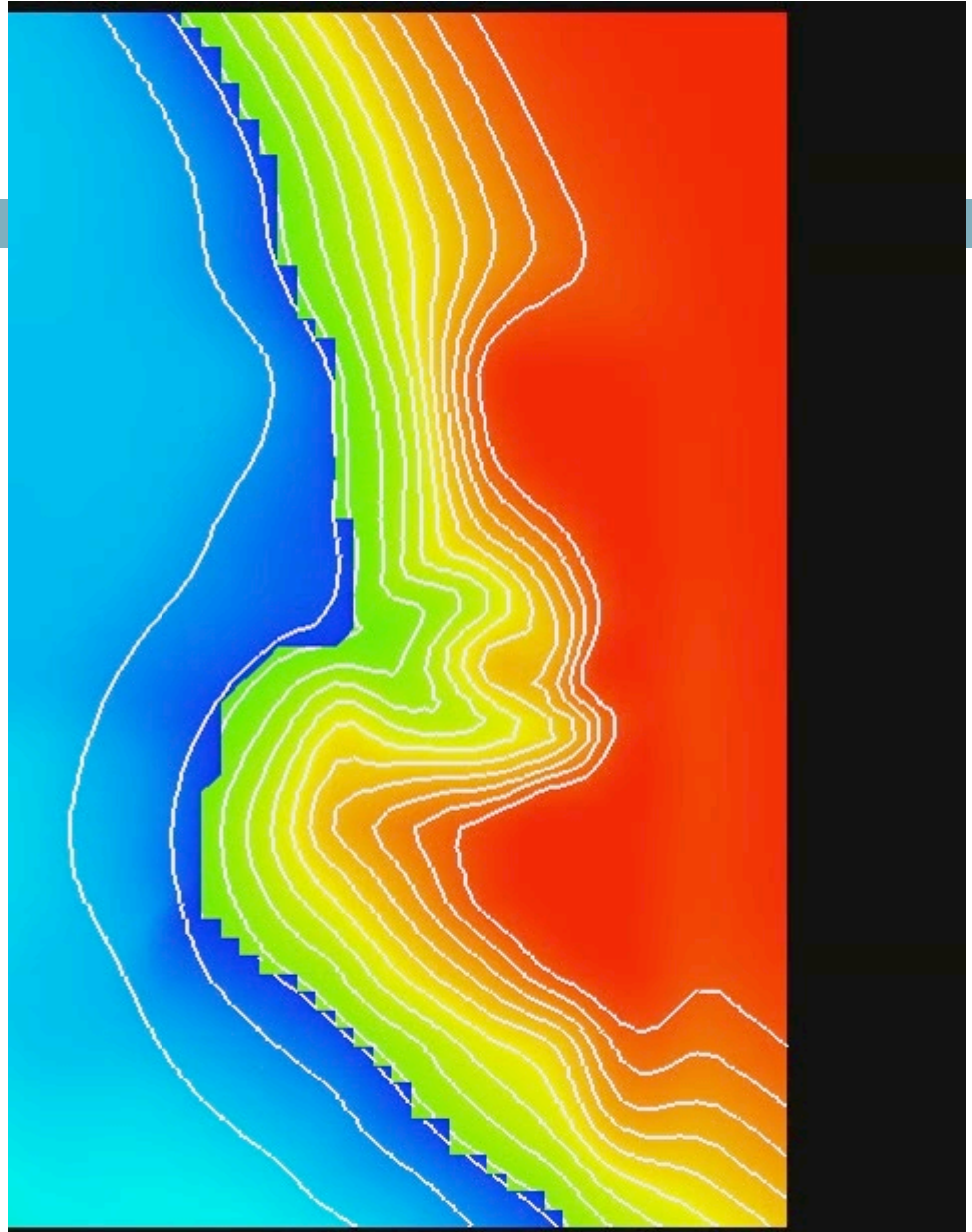
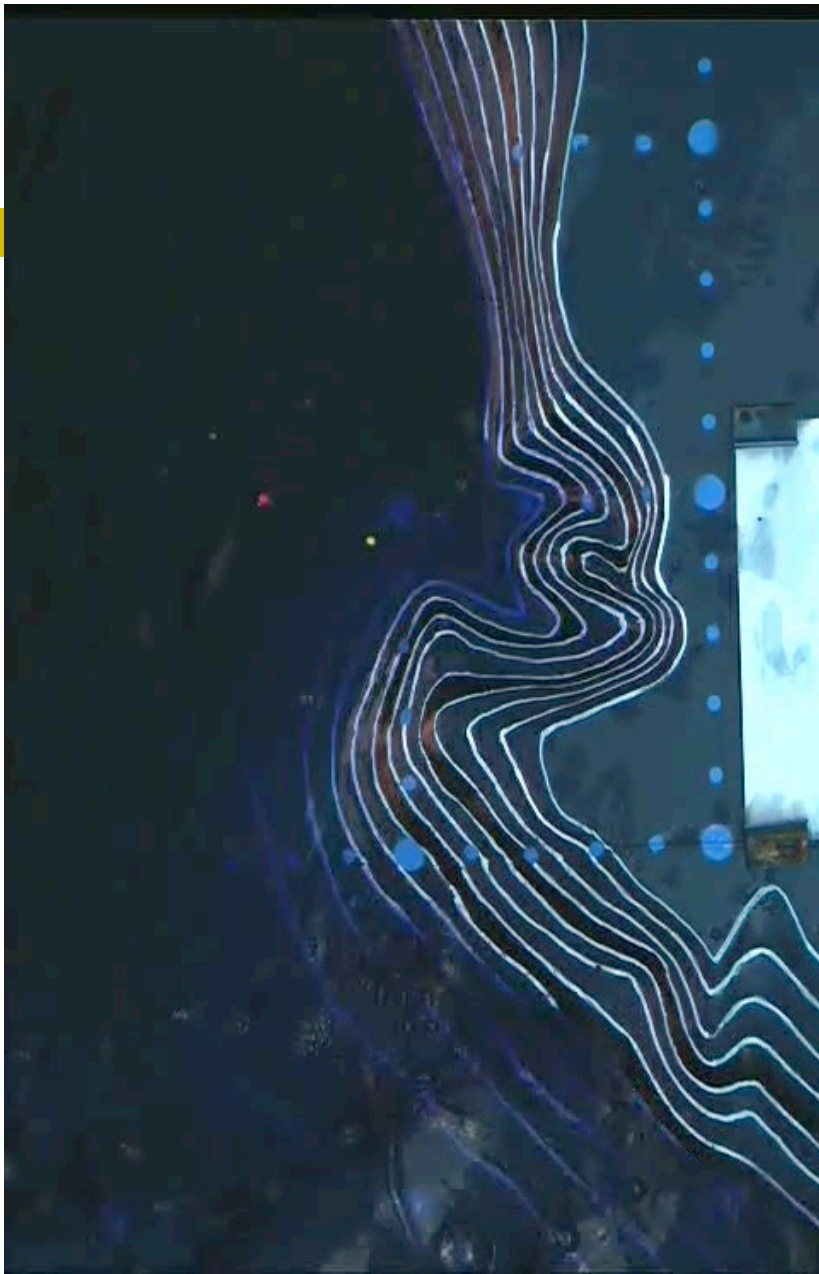


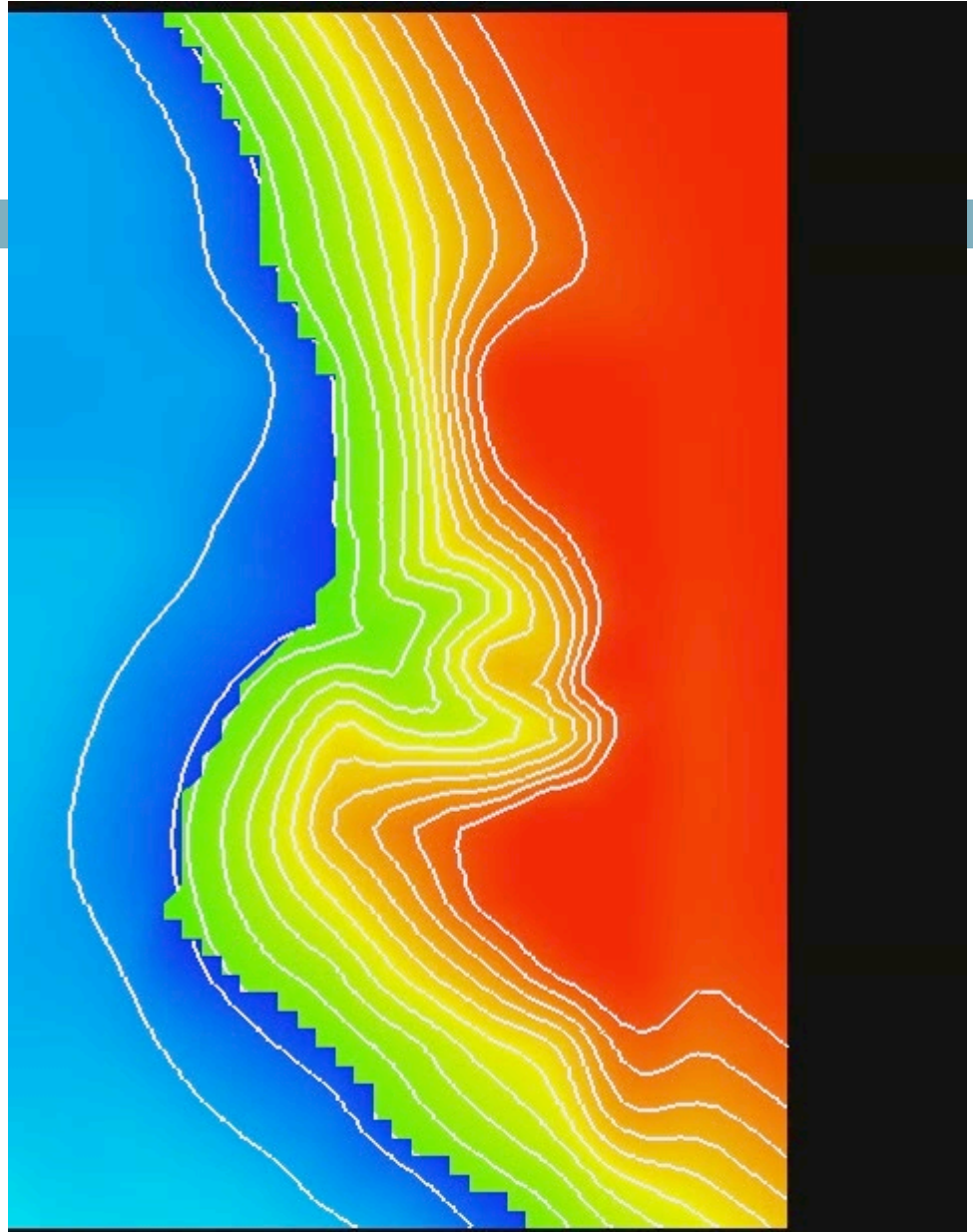


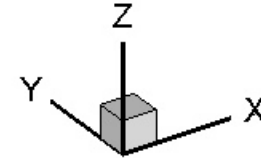
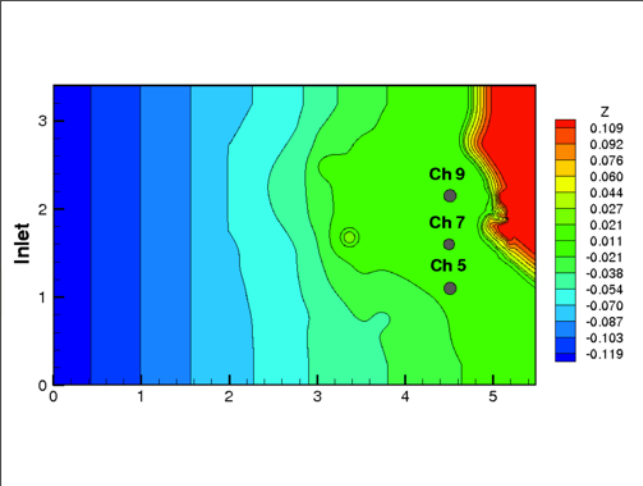




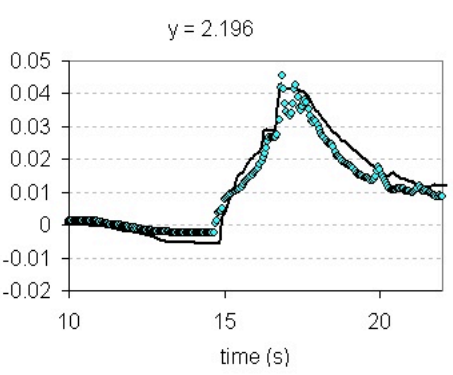
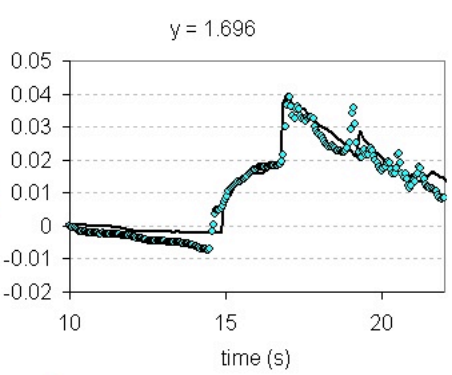
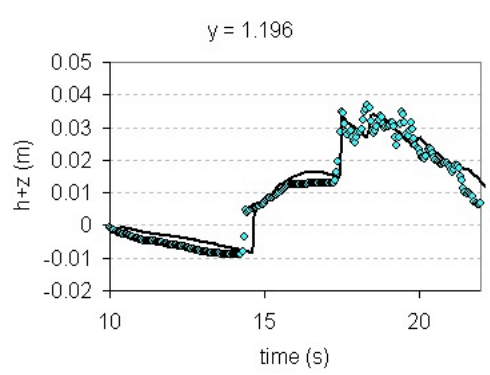
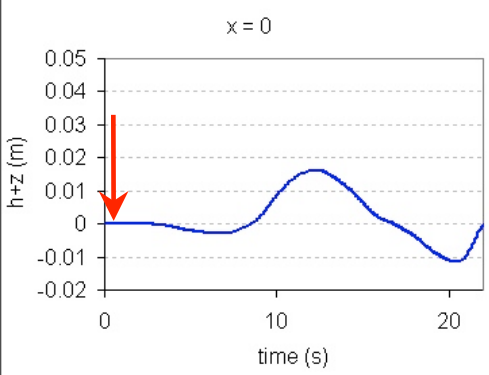
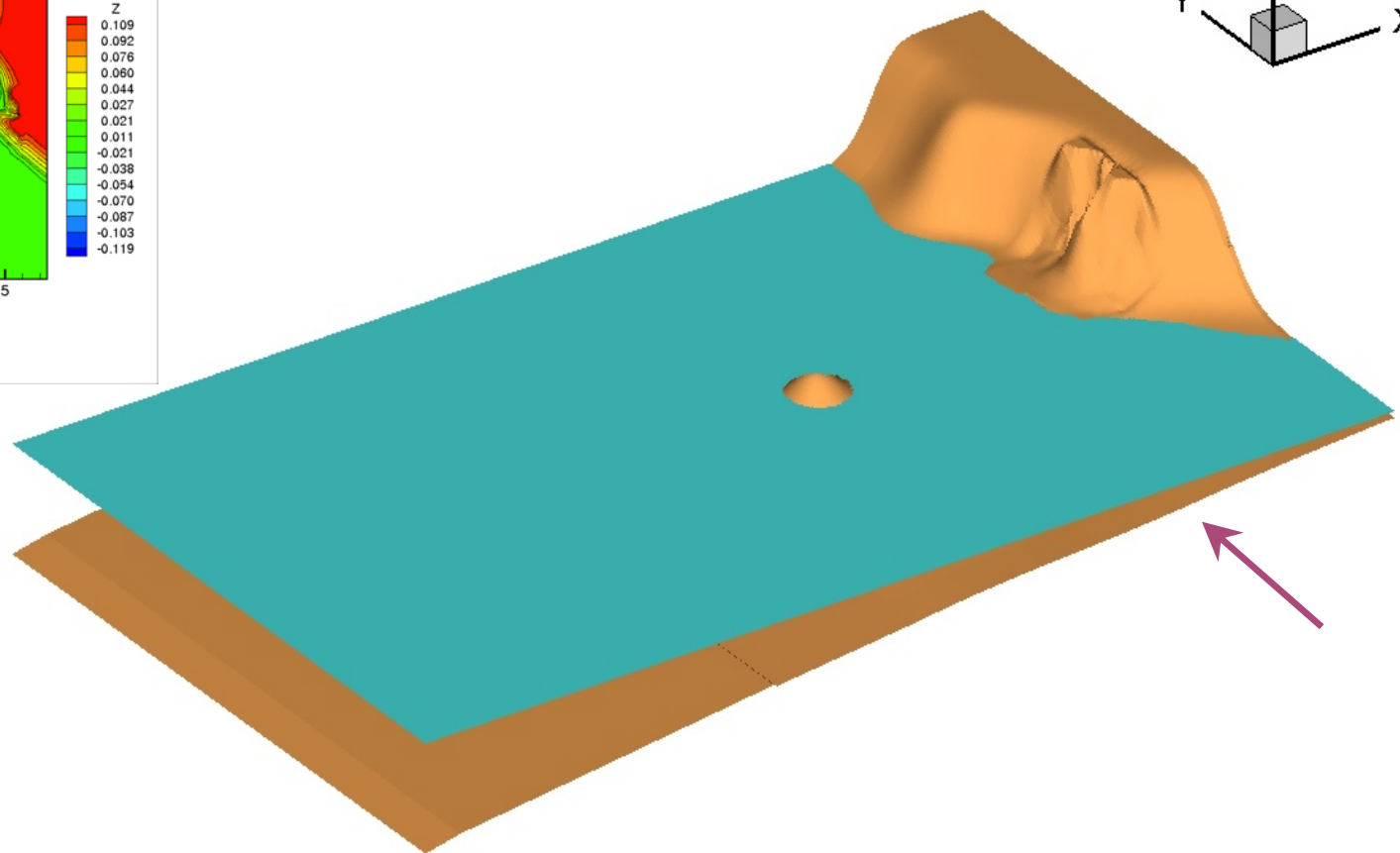




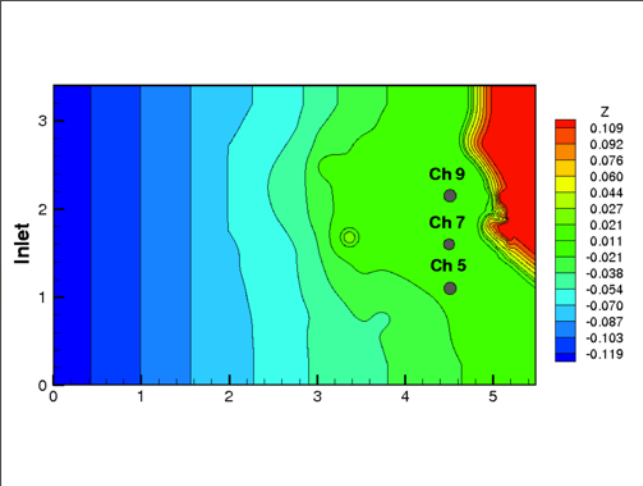




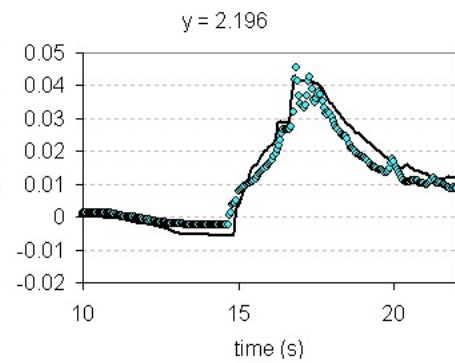
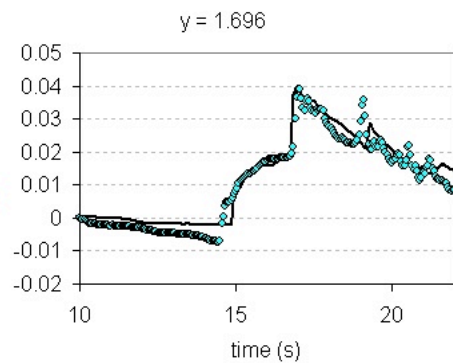
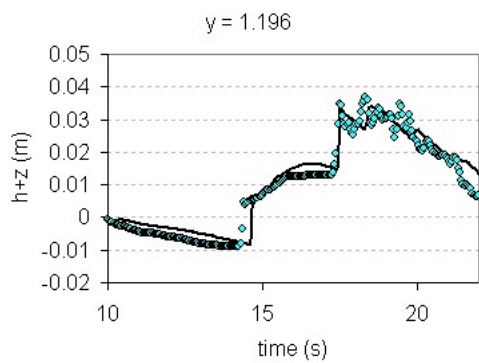
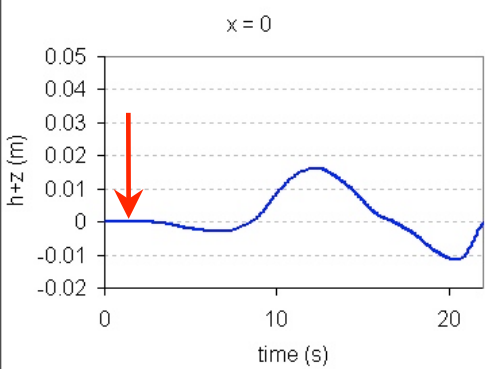
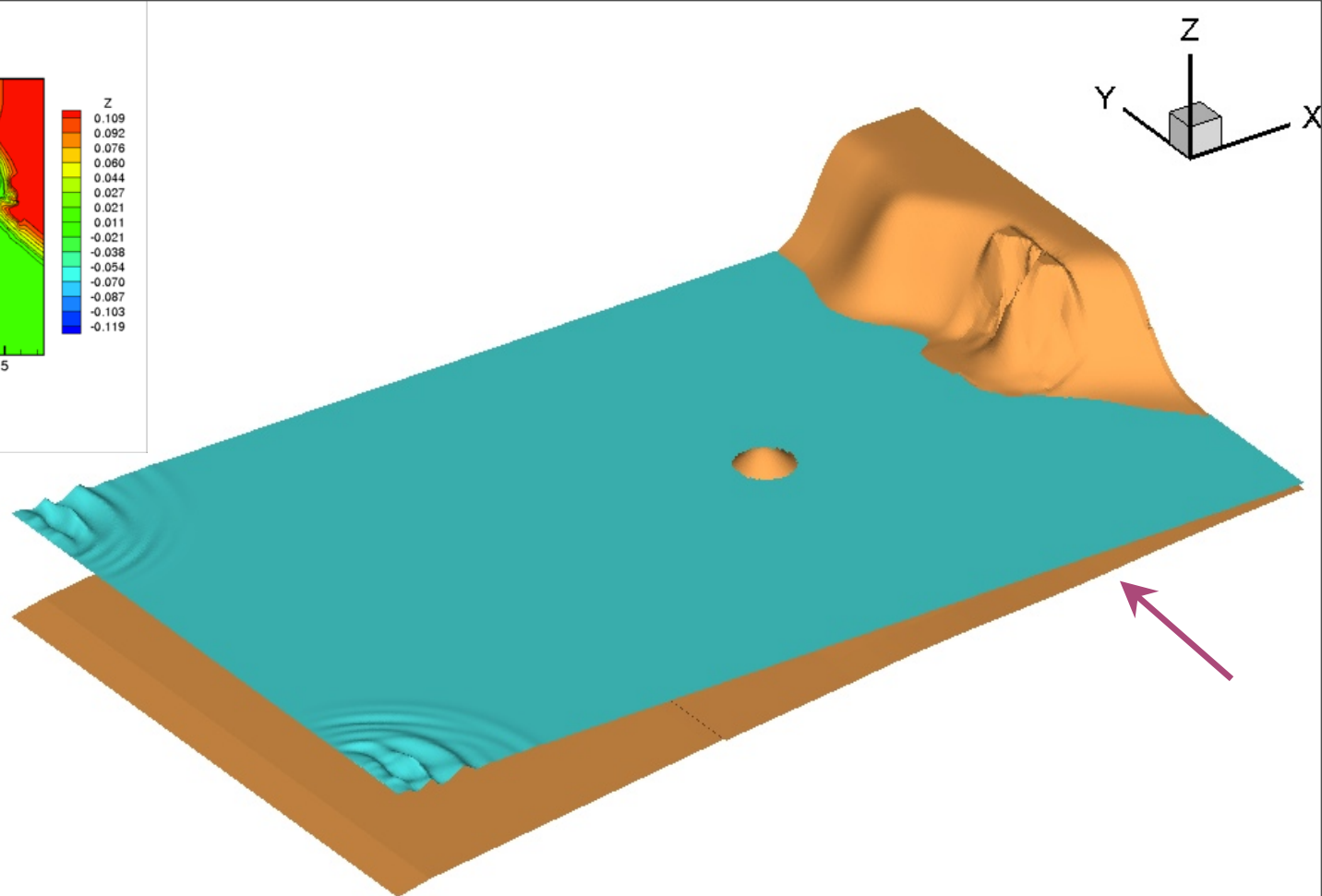
$t = 1 \text{ s}$



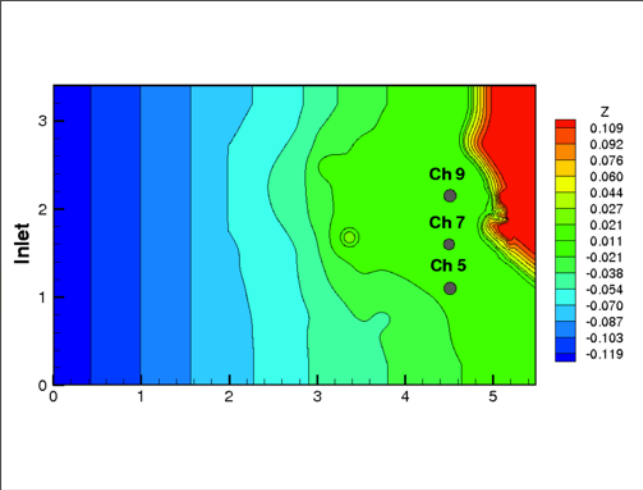
◆ measured — computed



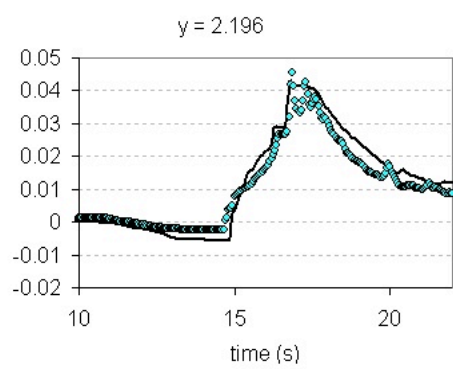
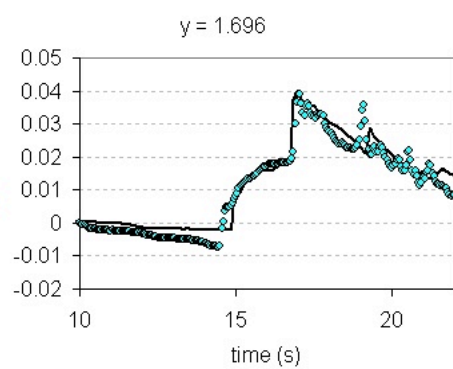
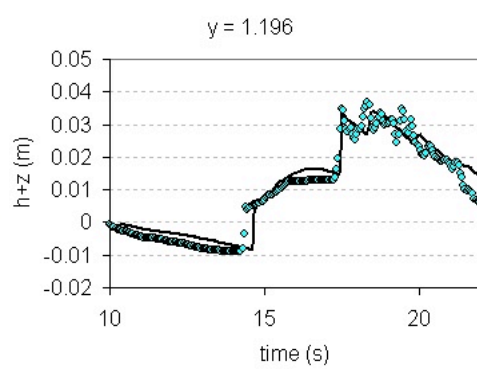
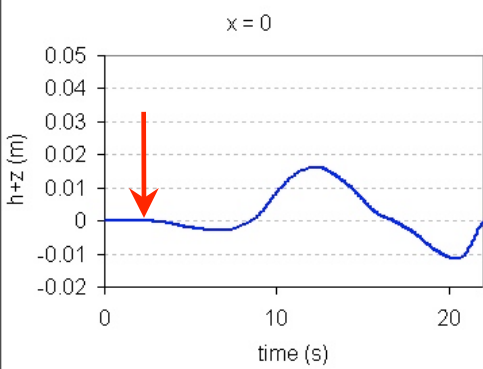
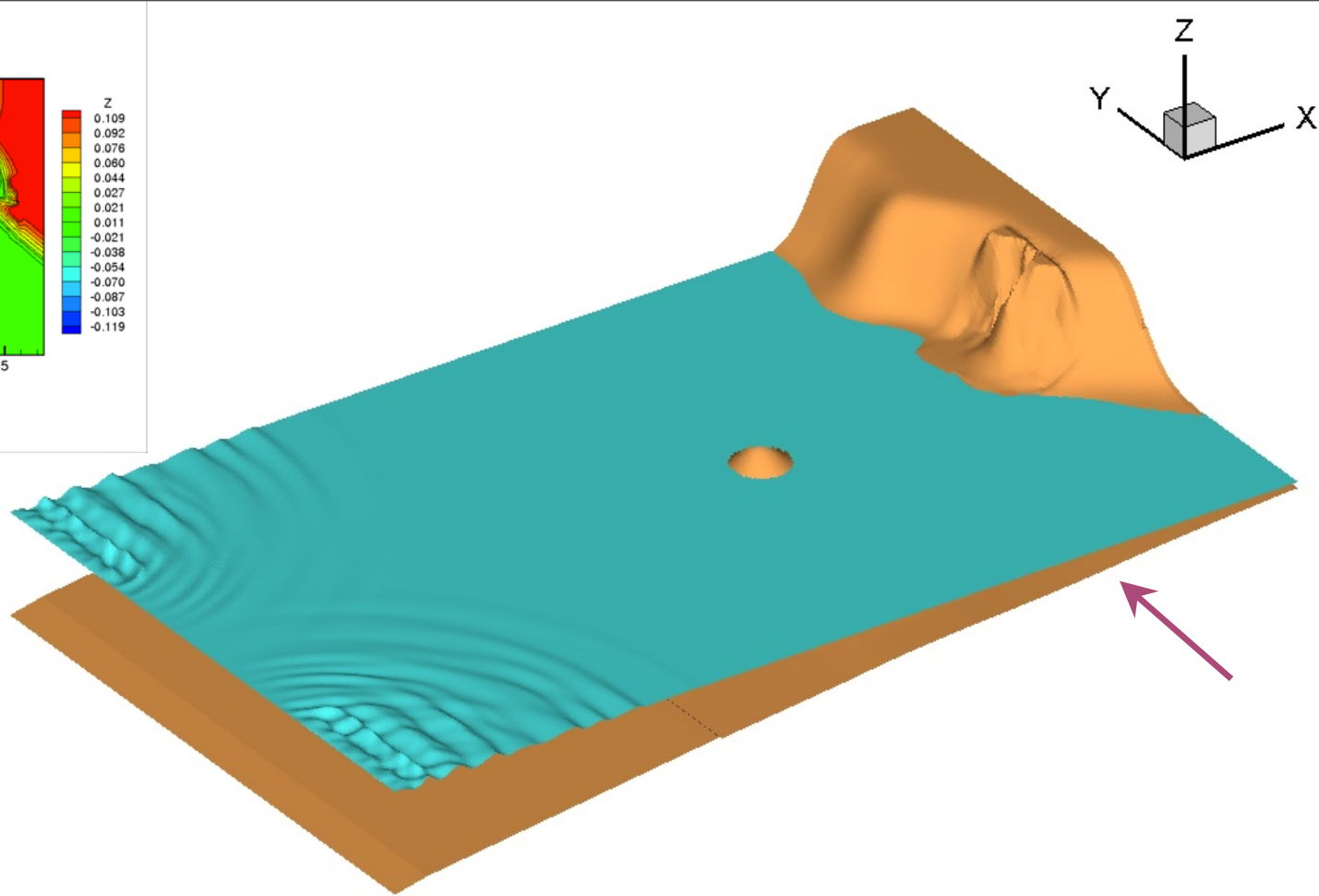
$t = 2 \text{ s}$



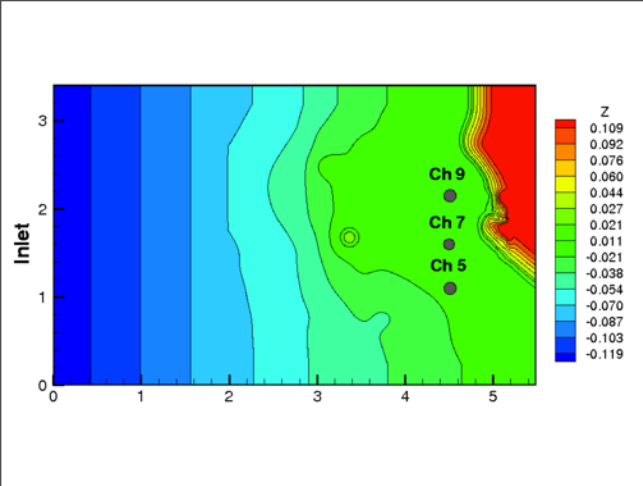
◆ measured — computed



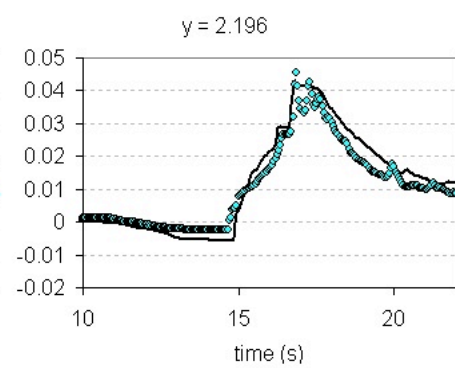
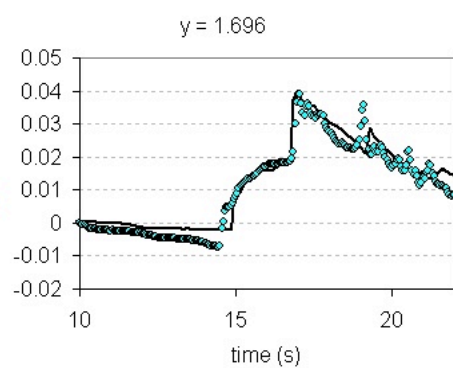
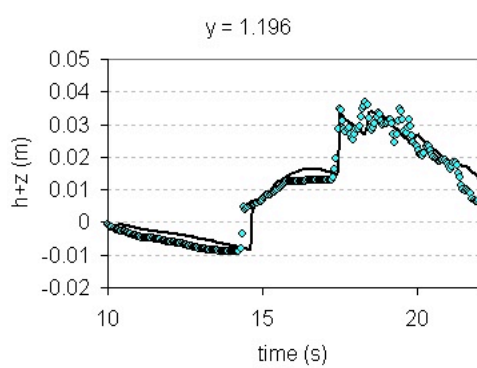
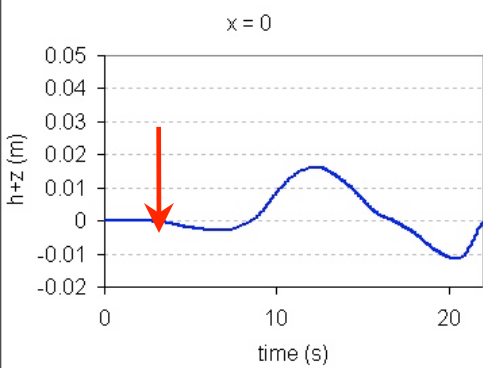
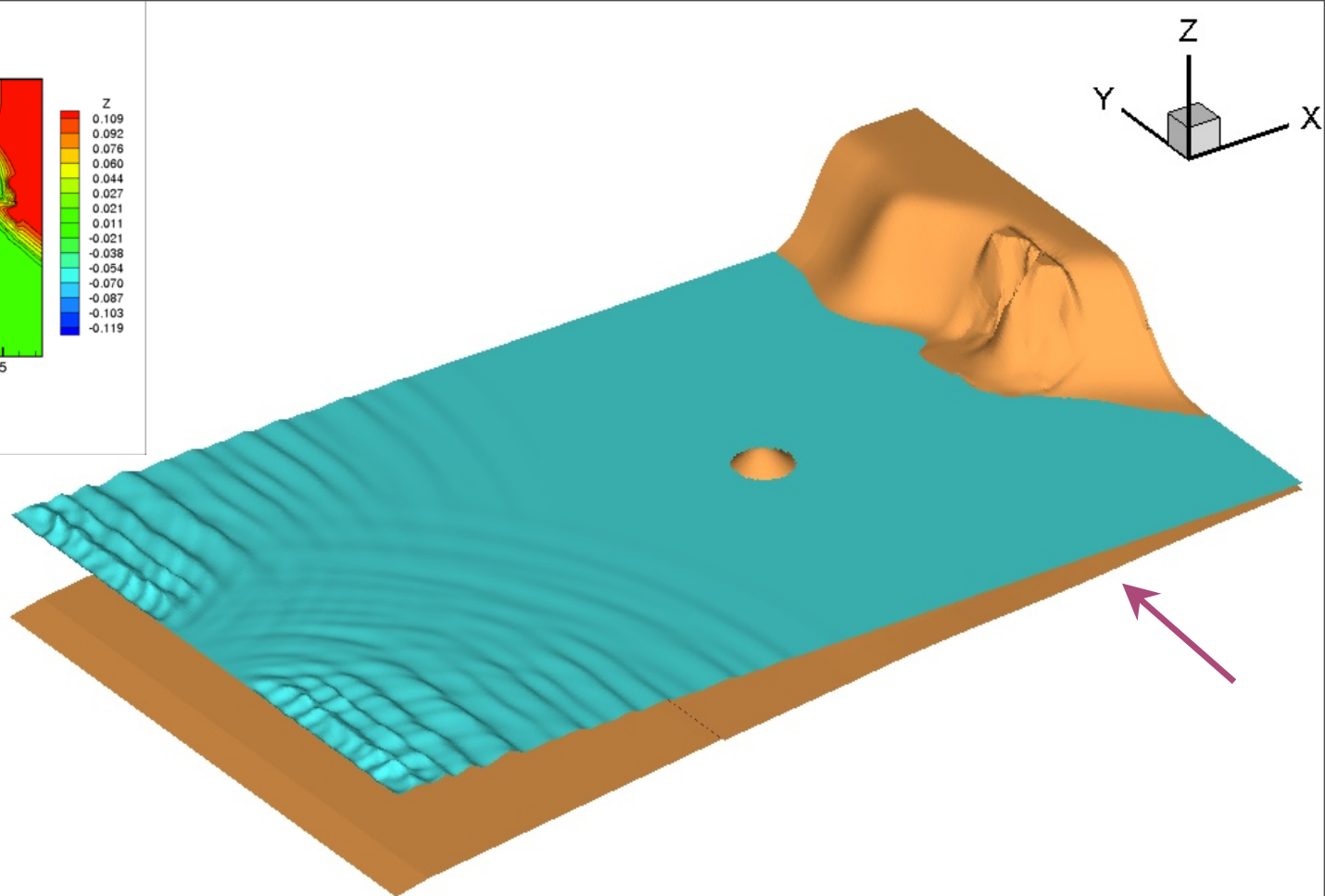
$t = 3 \text{ s}$



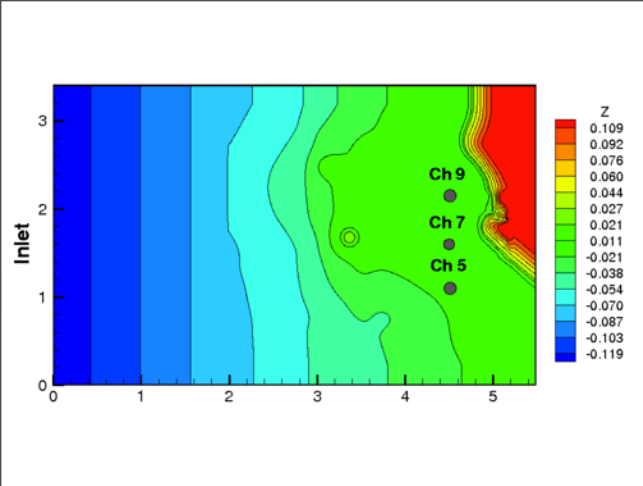
◆ measured — computed



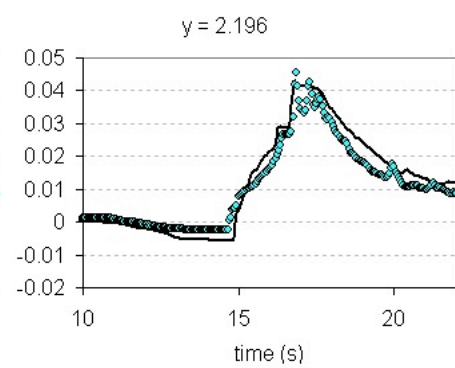
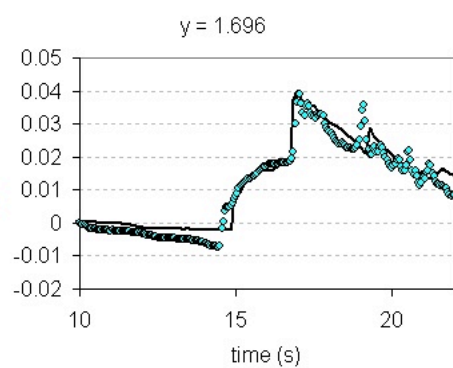
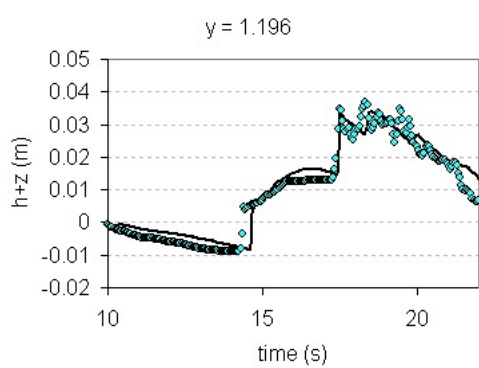
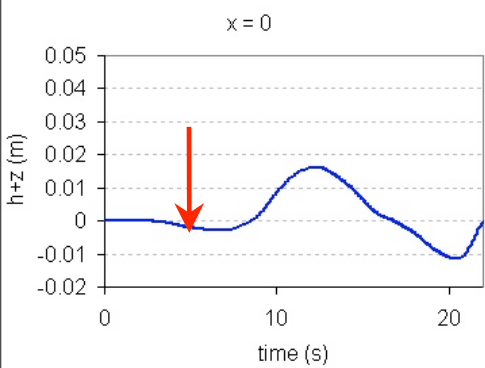
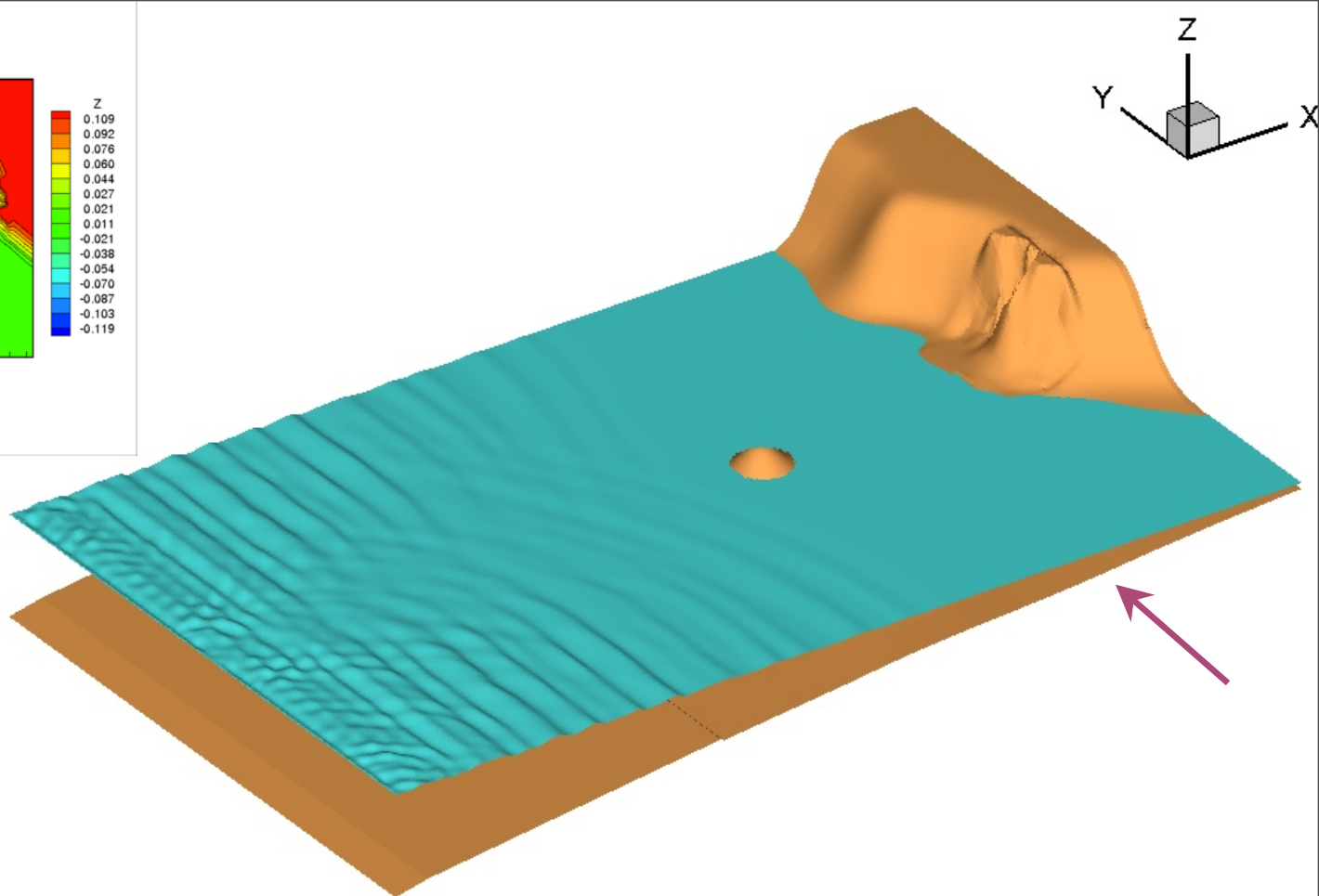
$t = 4 \text{ s}$



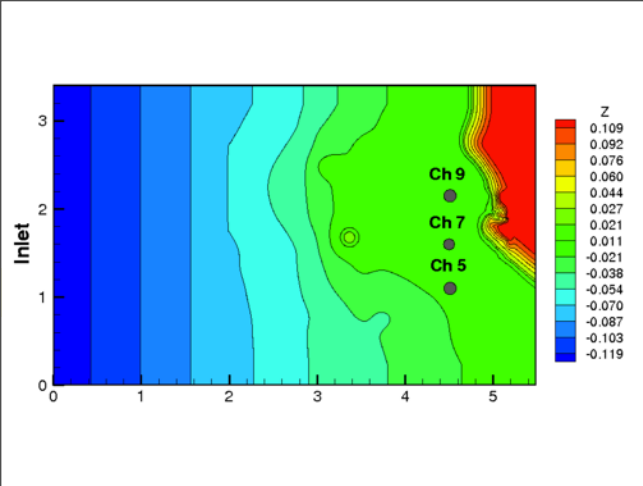
◆ measured — computed



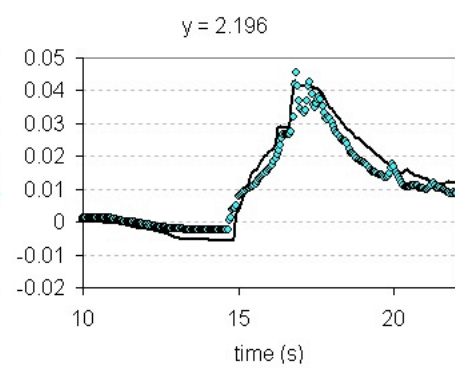
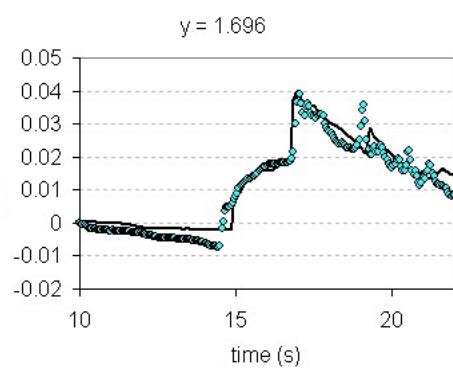
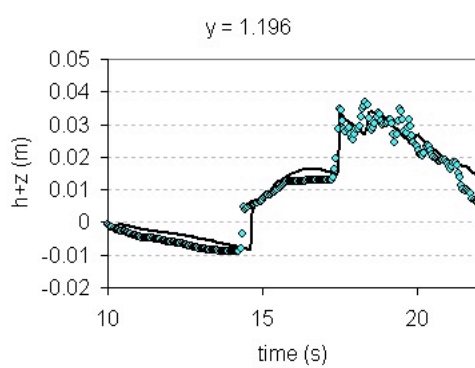
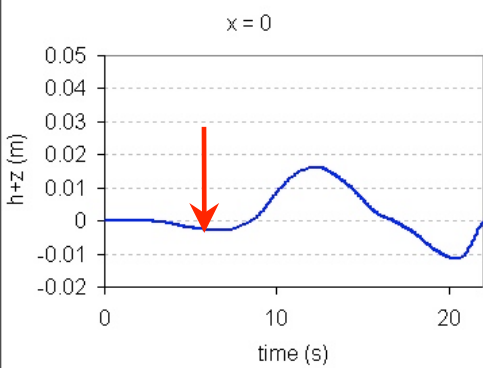
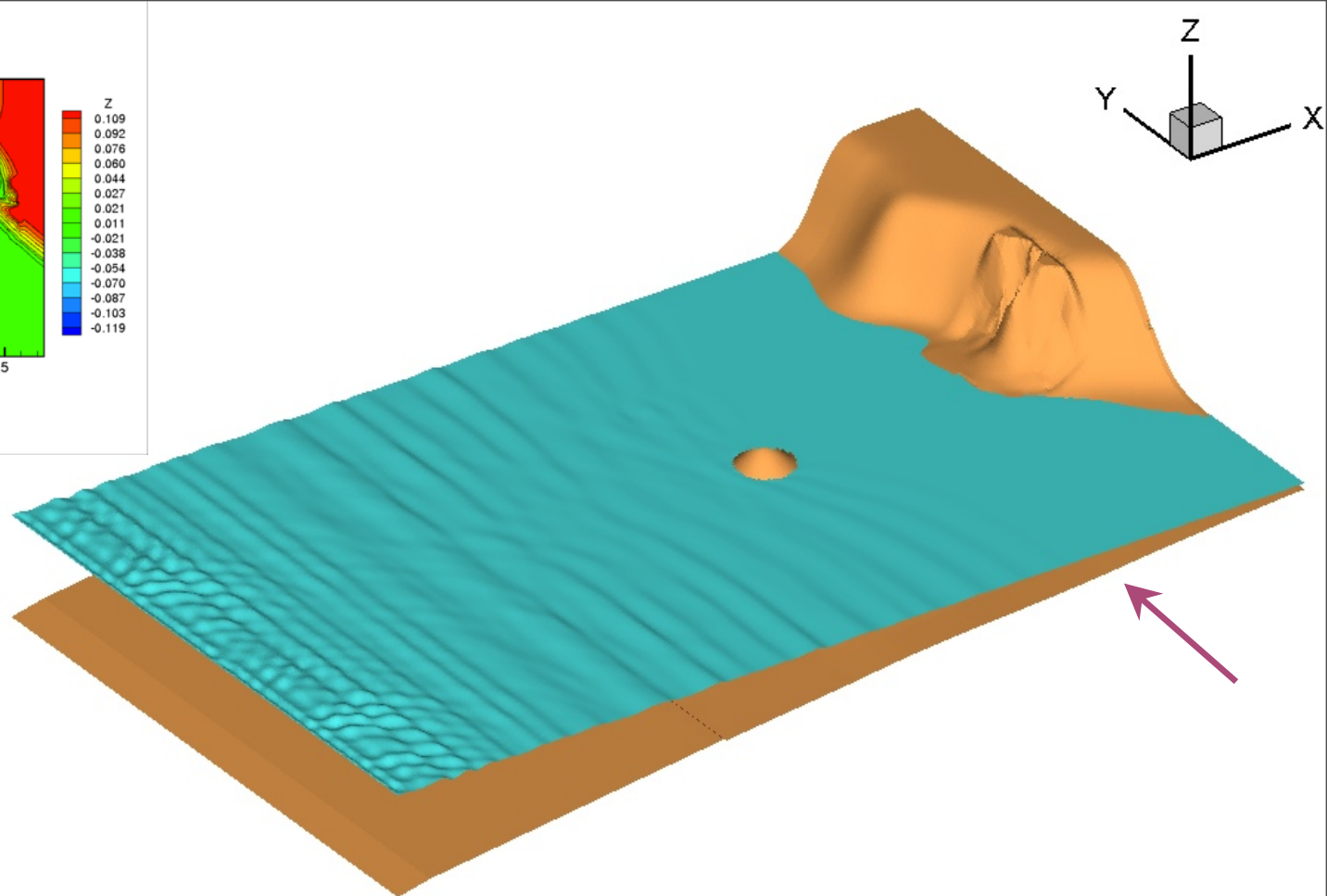
$t = 5 \text{ s}$



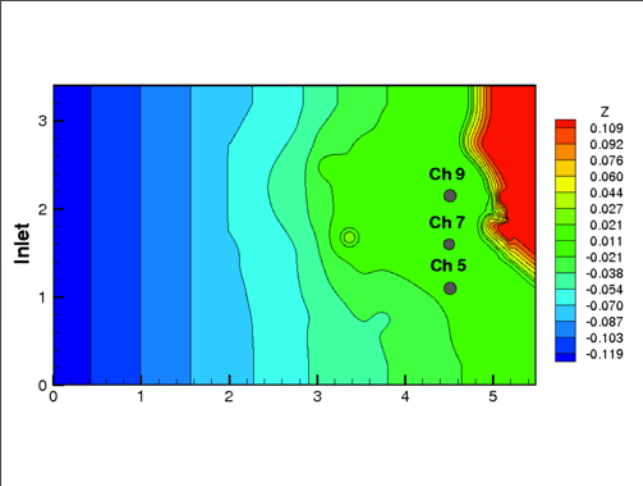
◆ measured — computed



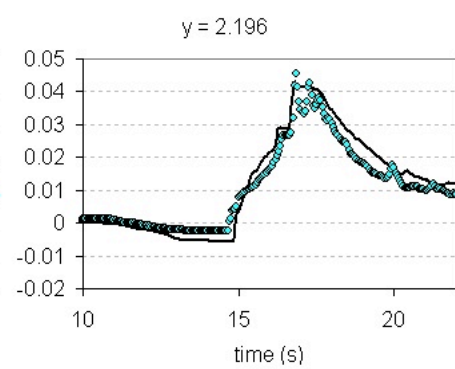
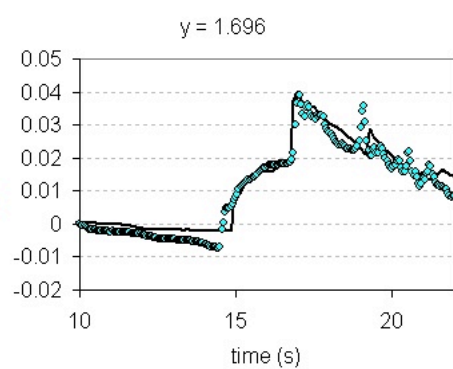
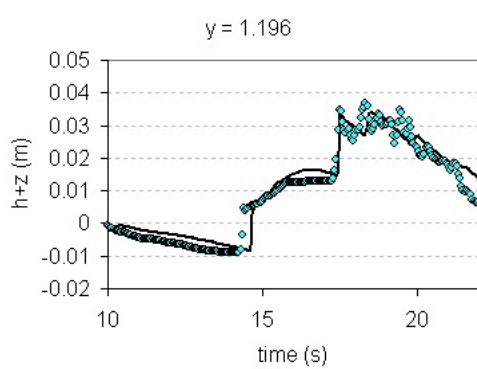
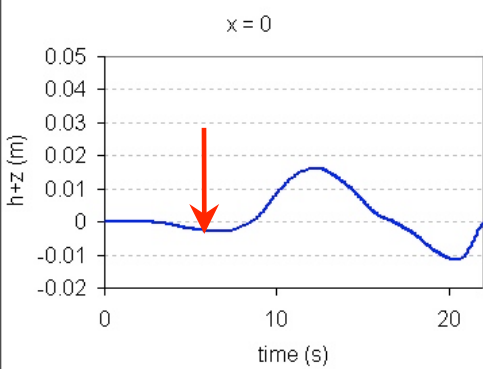
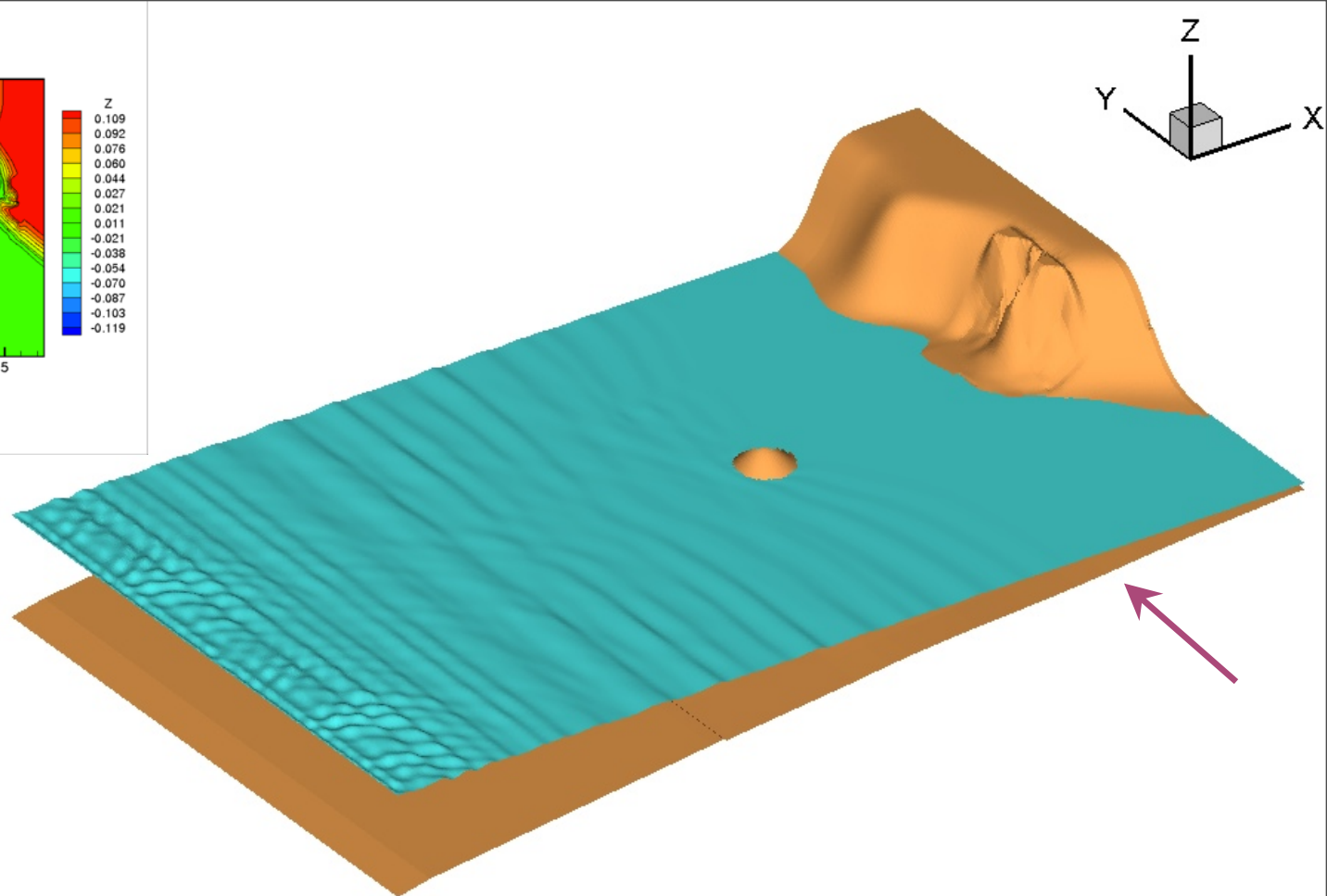
$t = 6 \text{ s}$



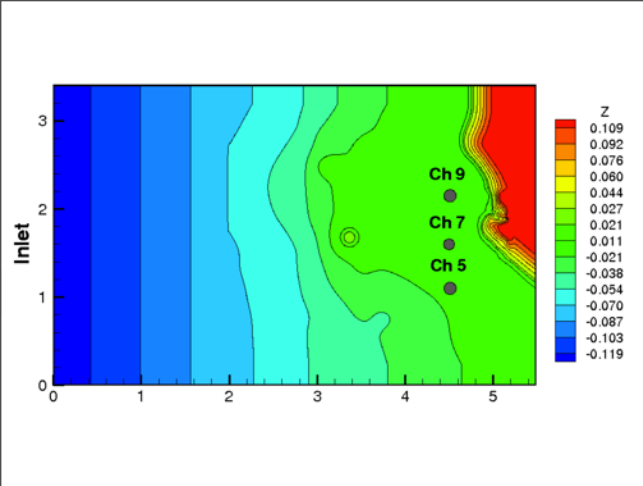
◆ measured — computed



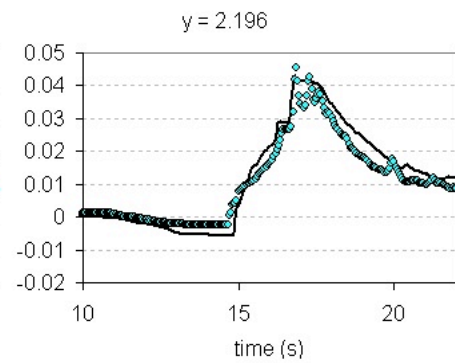
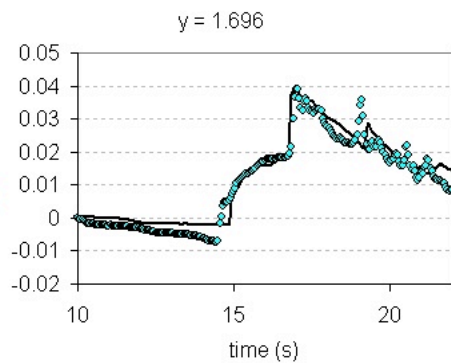
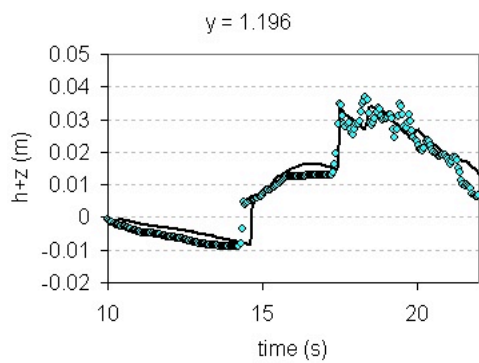
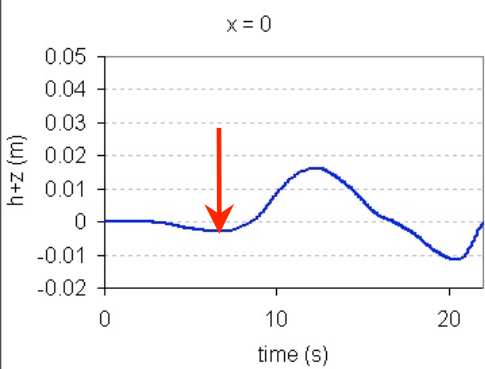
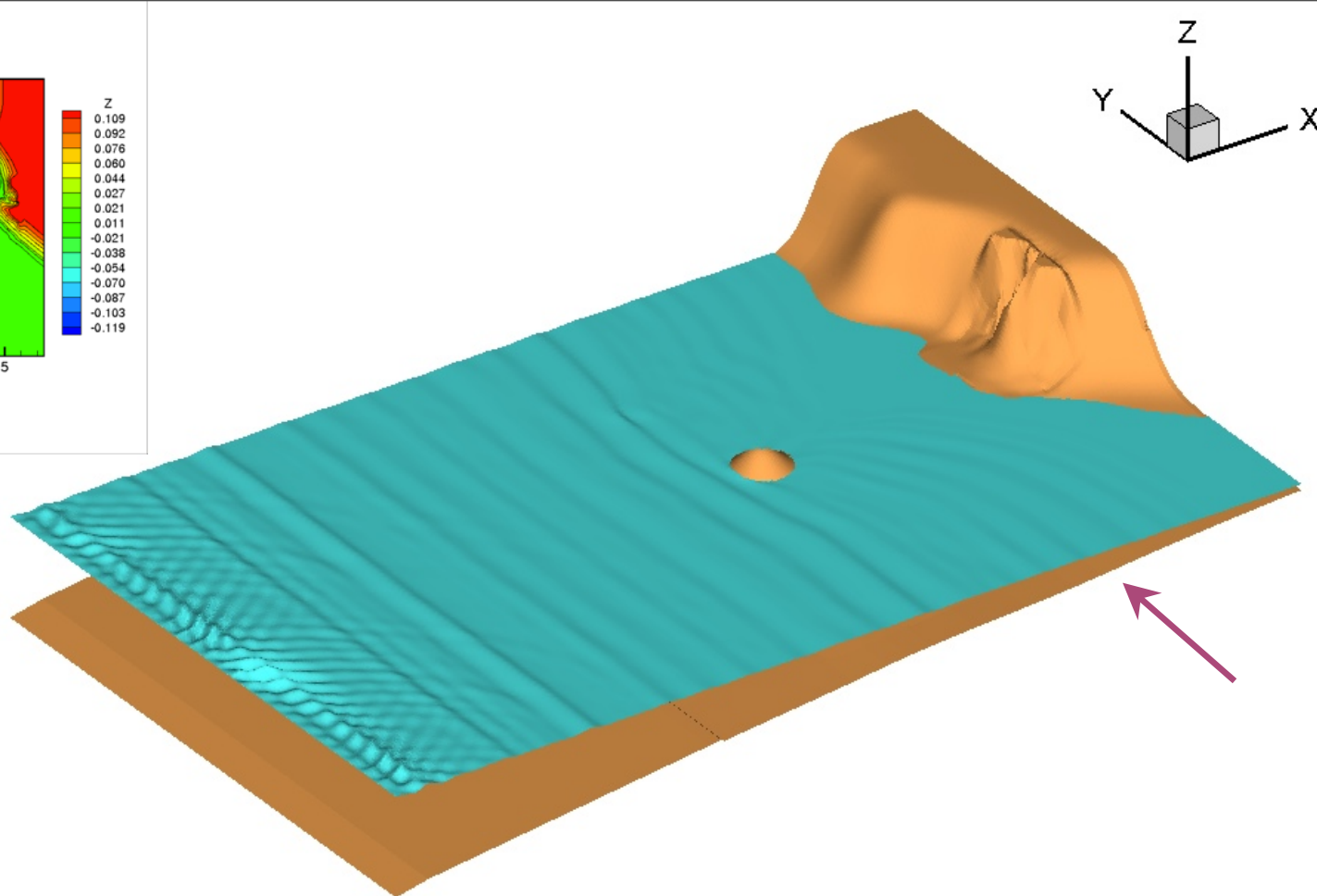
$t = 6 \text{ s}$



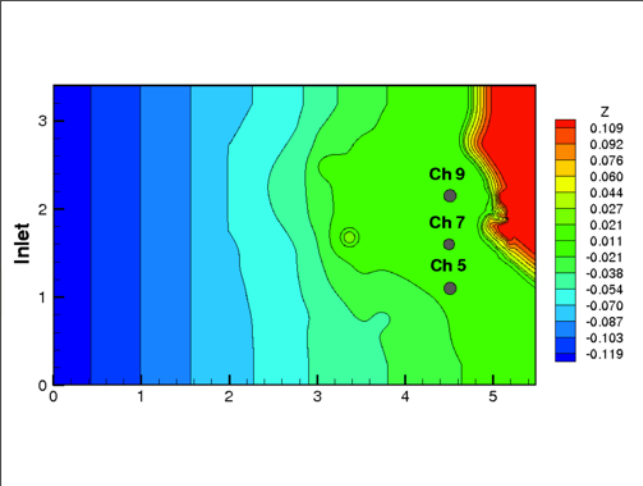
◆ measured — computed



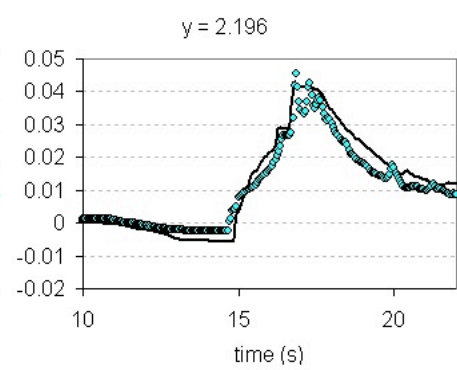
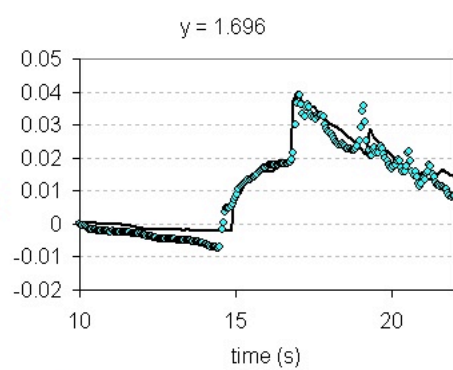
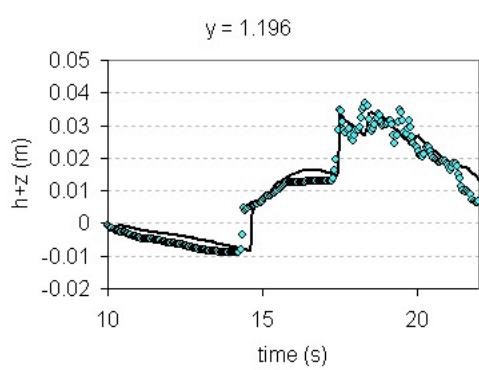
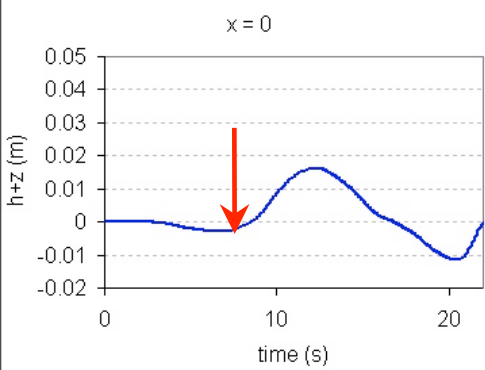
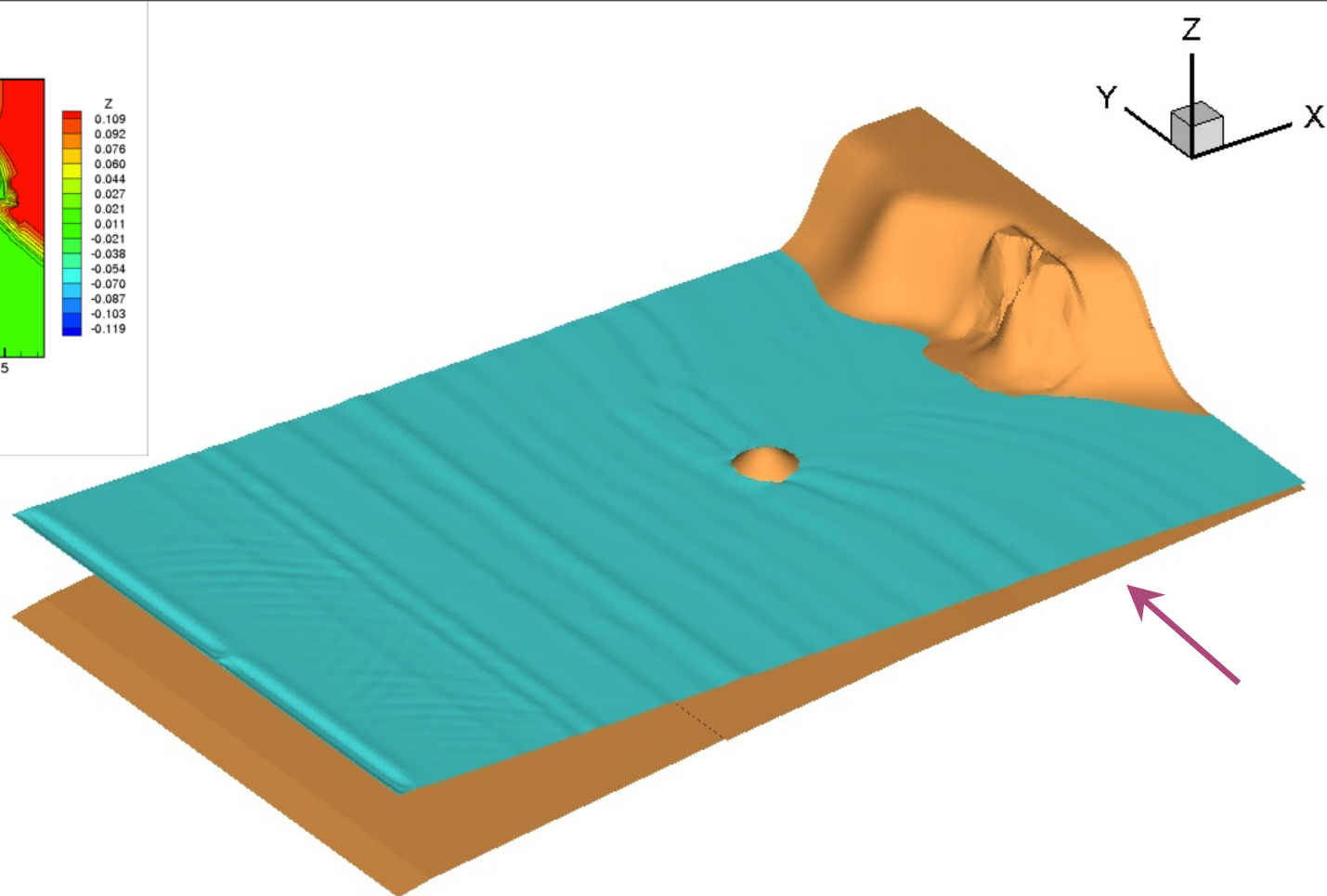
$t = 7 \text{ s}$



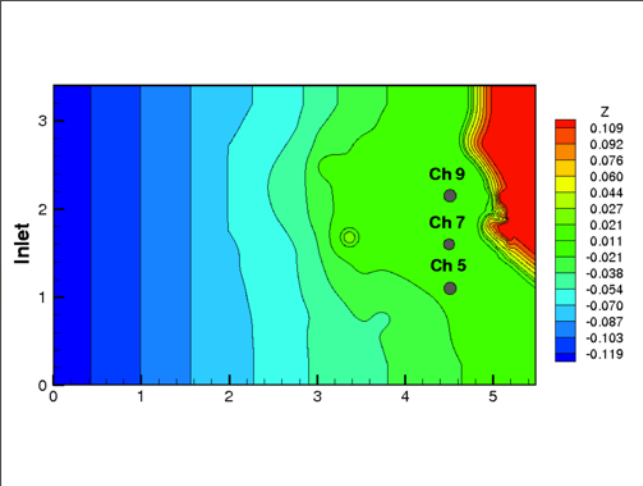
◆ measured — computed



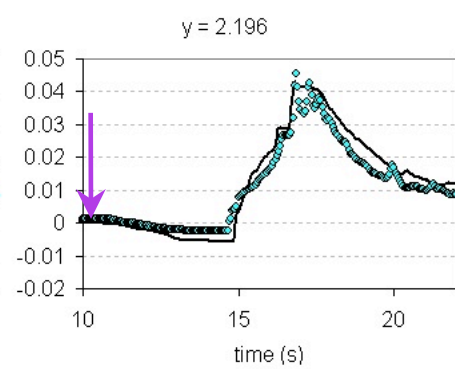
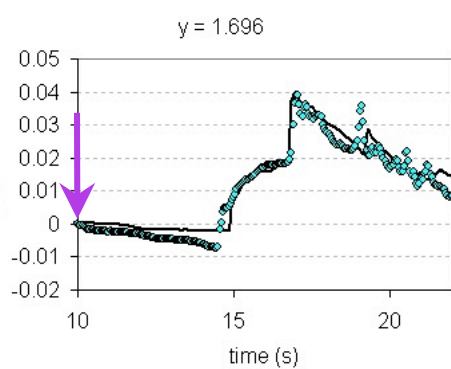
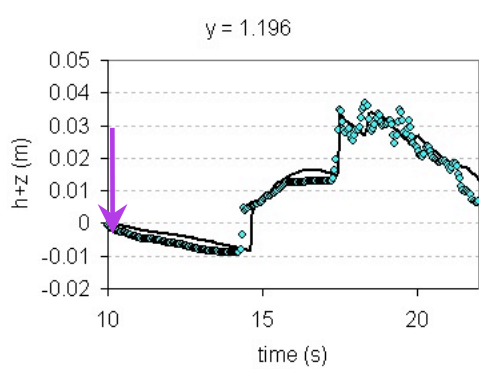
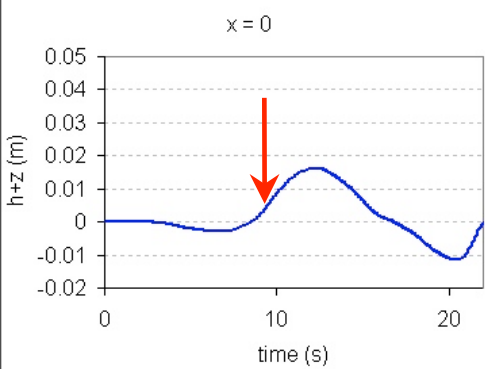
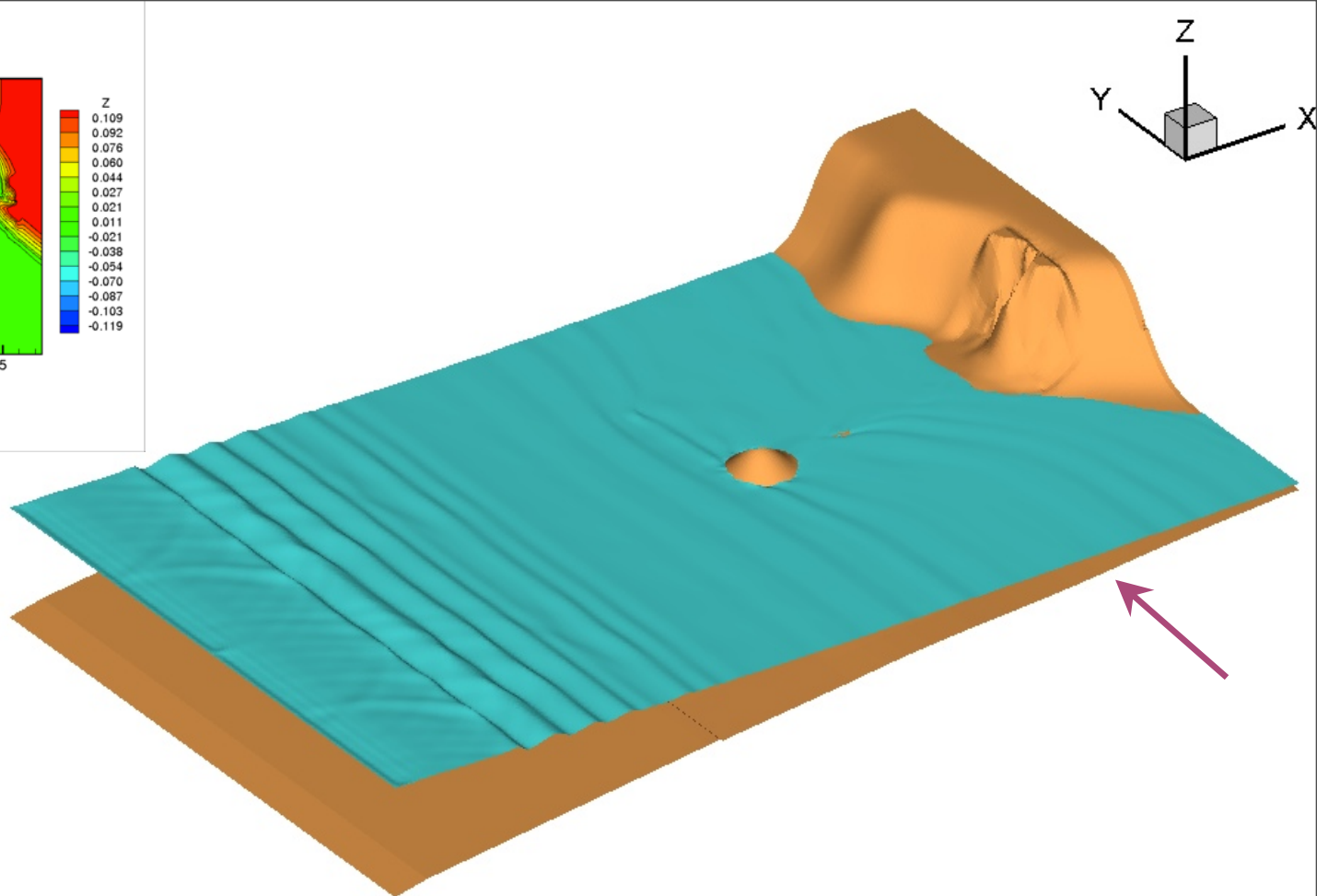
$t = 8 \text{ s}$



◆ measured — computed



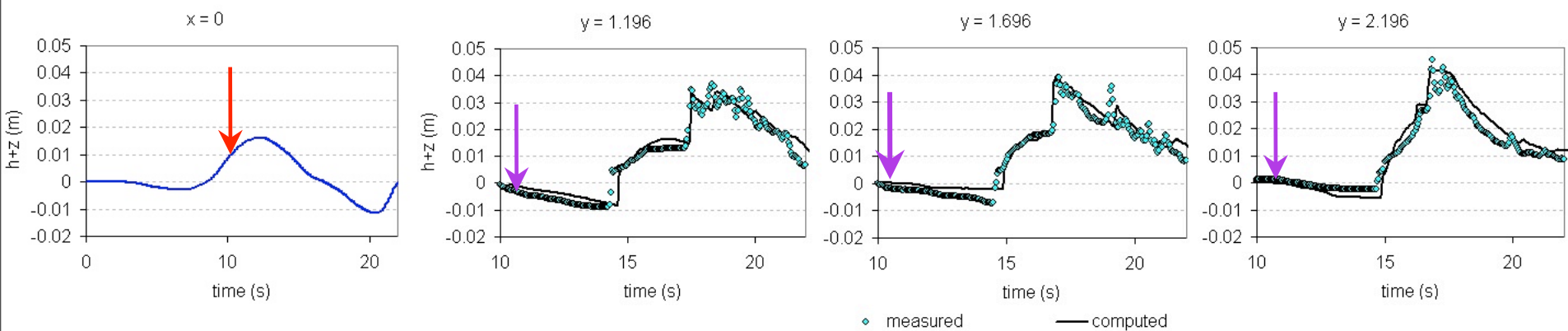
$t = 10 \text{ s}$

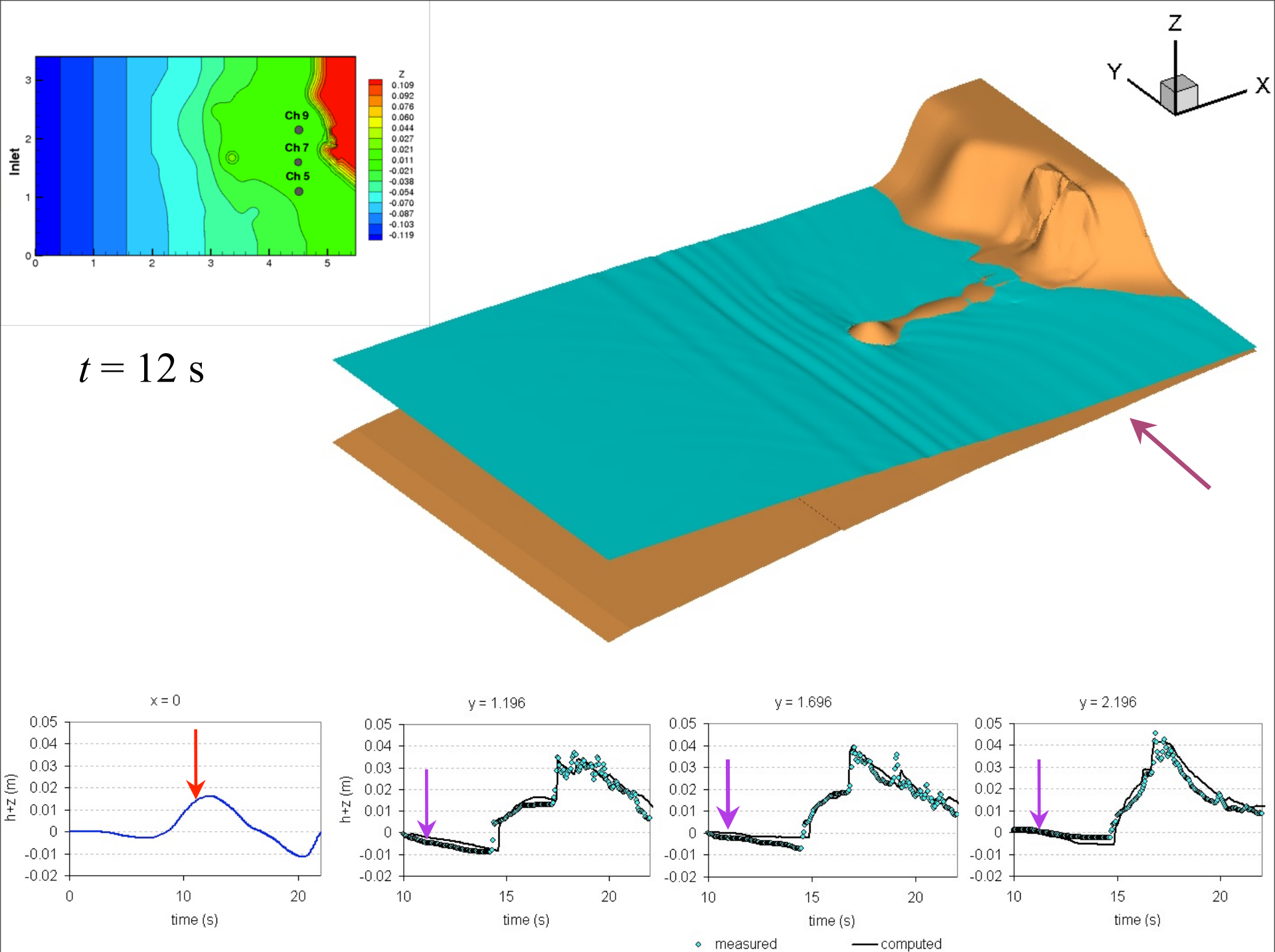


◆ measured — computed



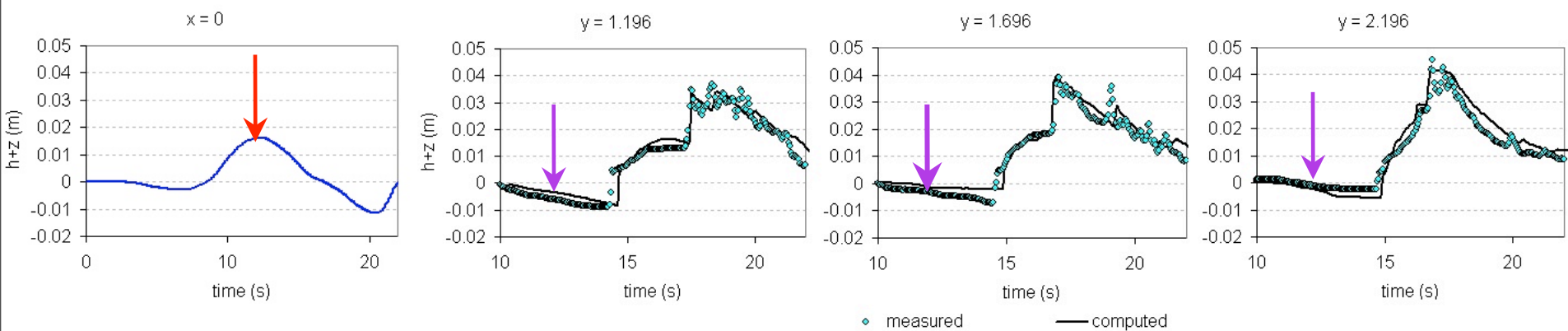
$t = 11 \text{ s}$





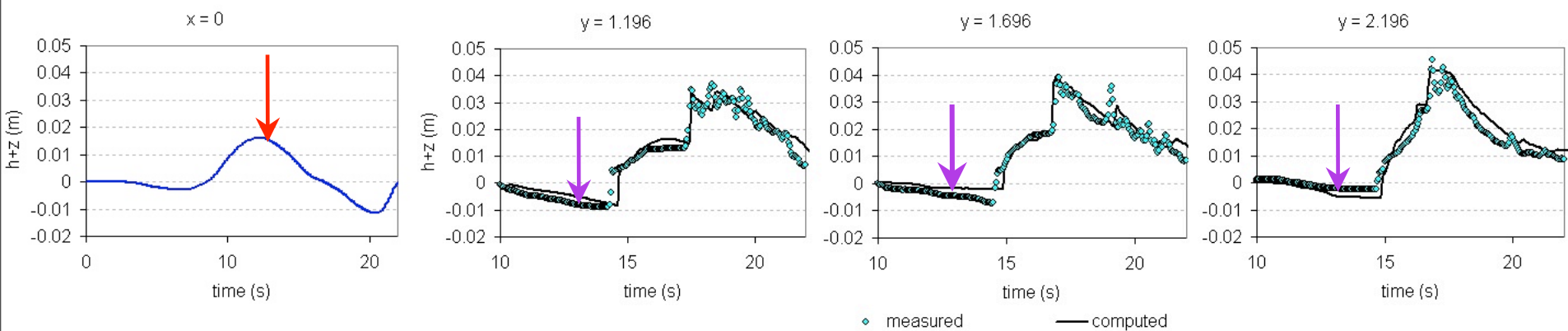


$t = 13 \text{ s}$



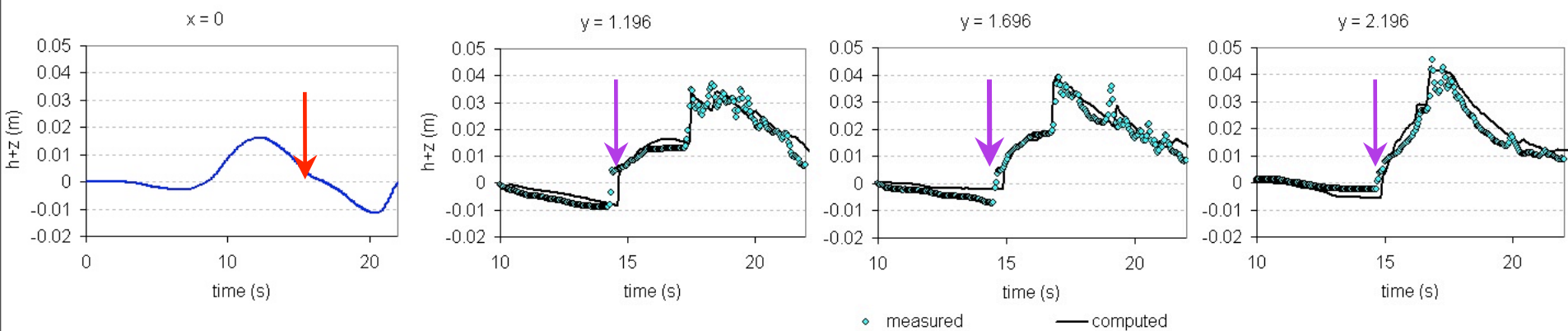


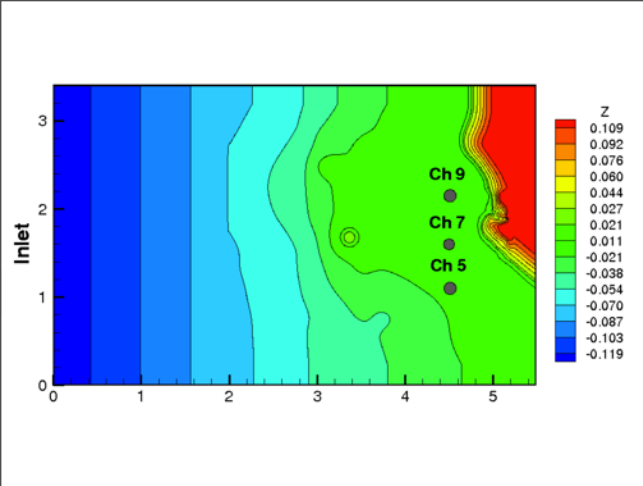
$t = 14 \text{ s}$



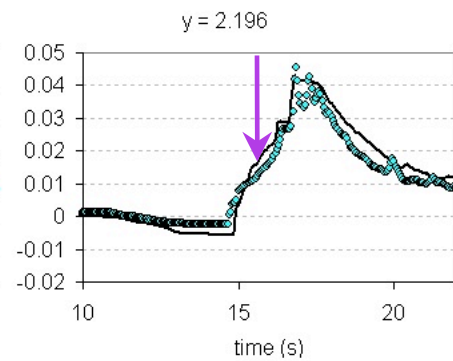
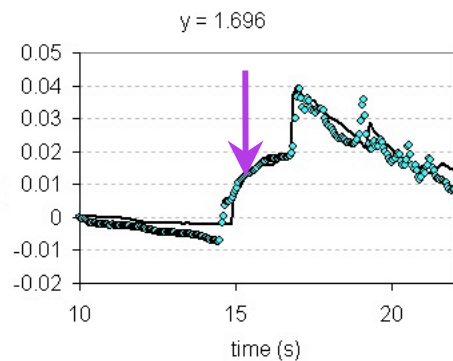
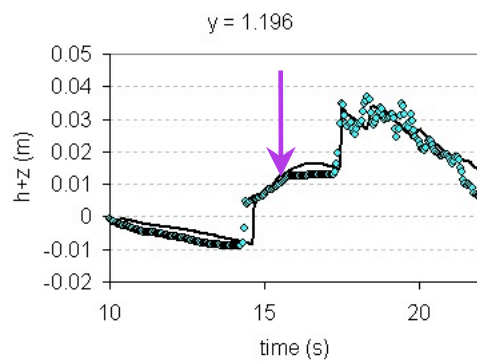
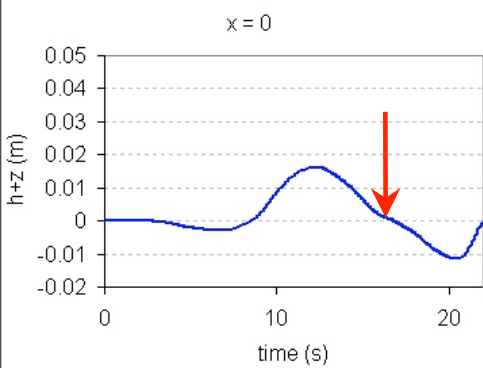
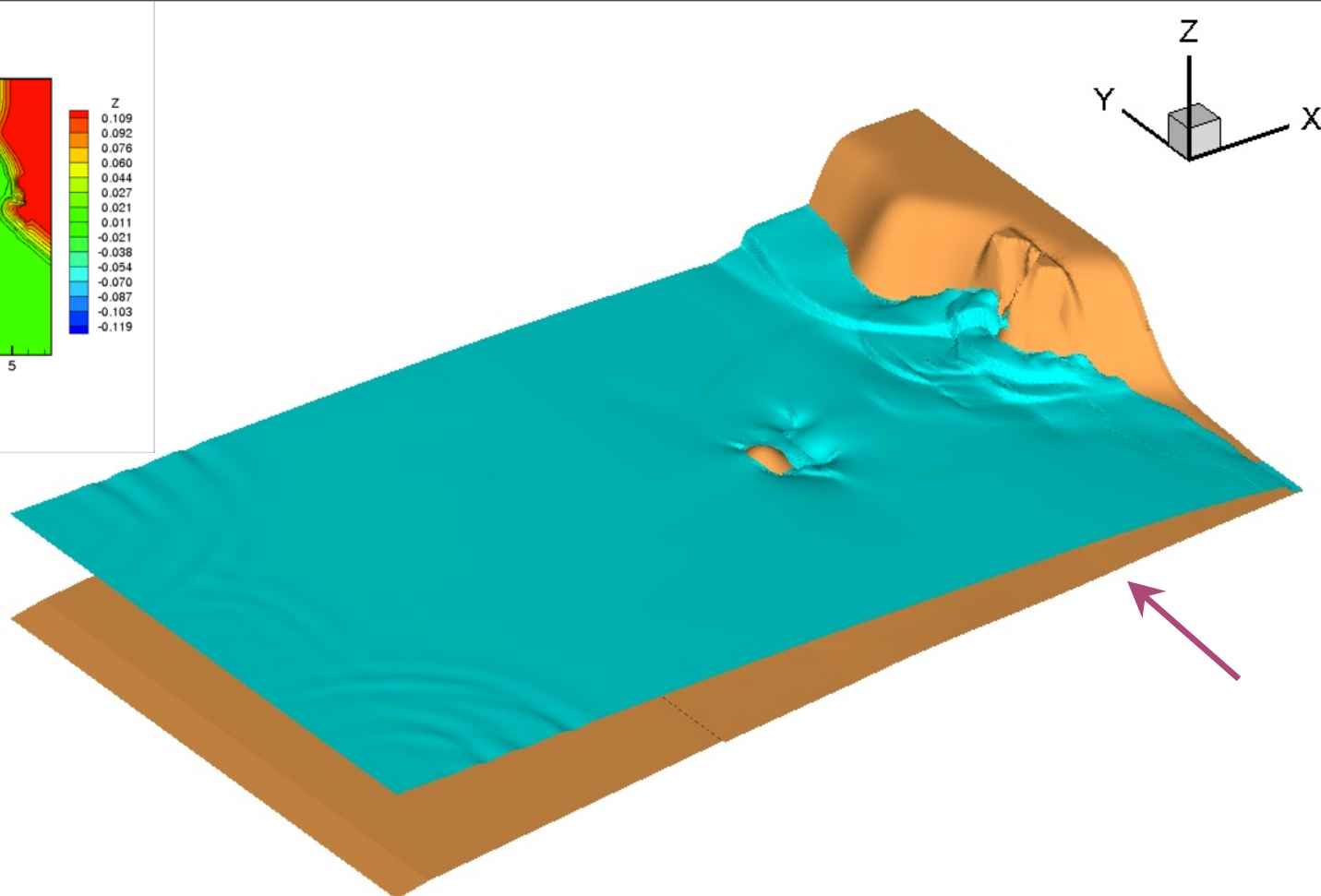


$t = 15 \text{ s}$





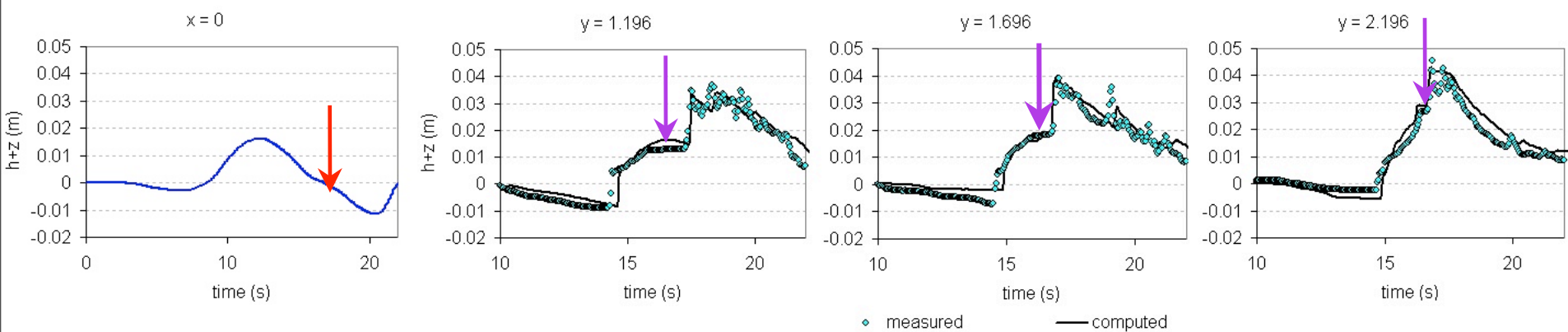
$t = 16 \text{ s}$

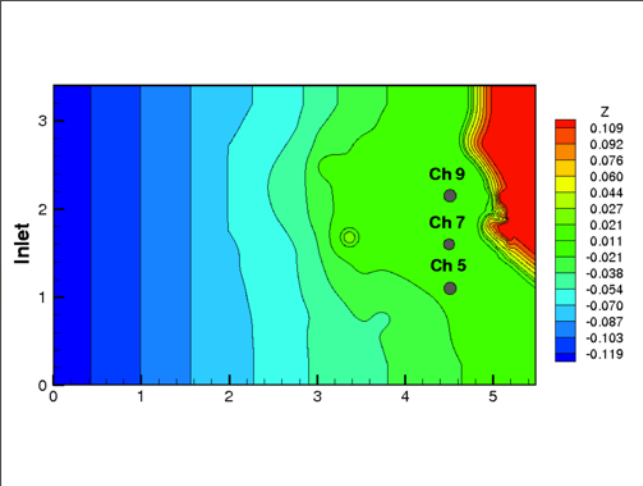


◆ measured — computed

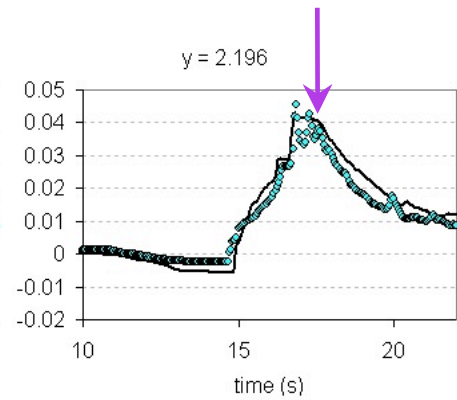
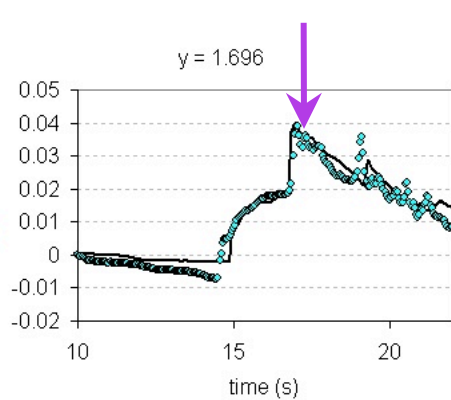
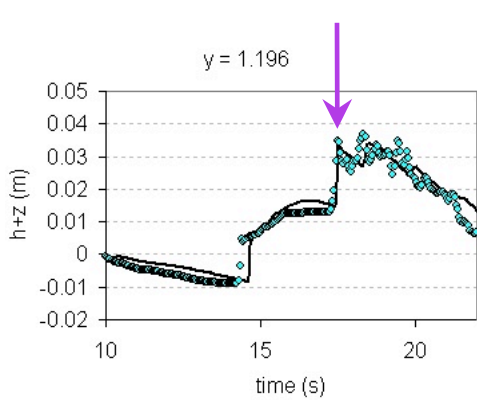
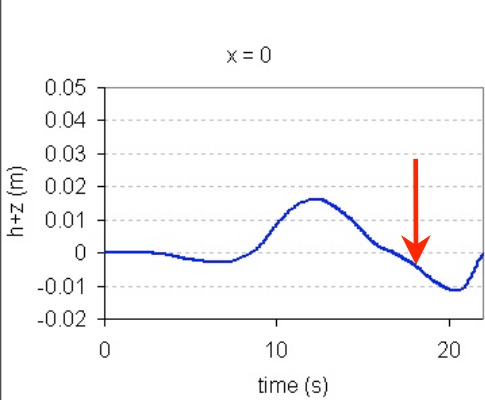
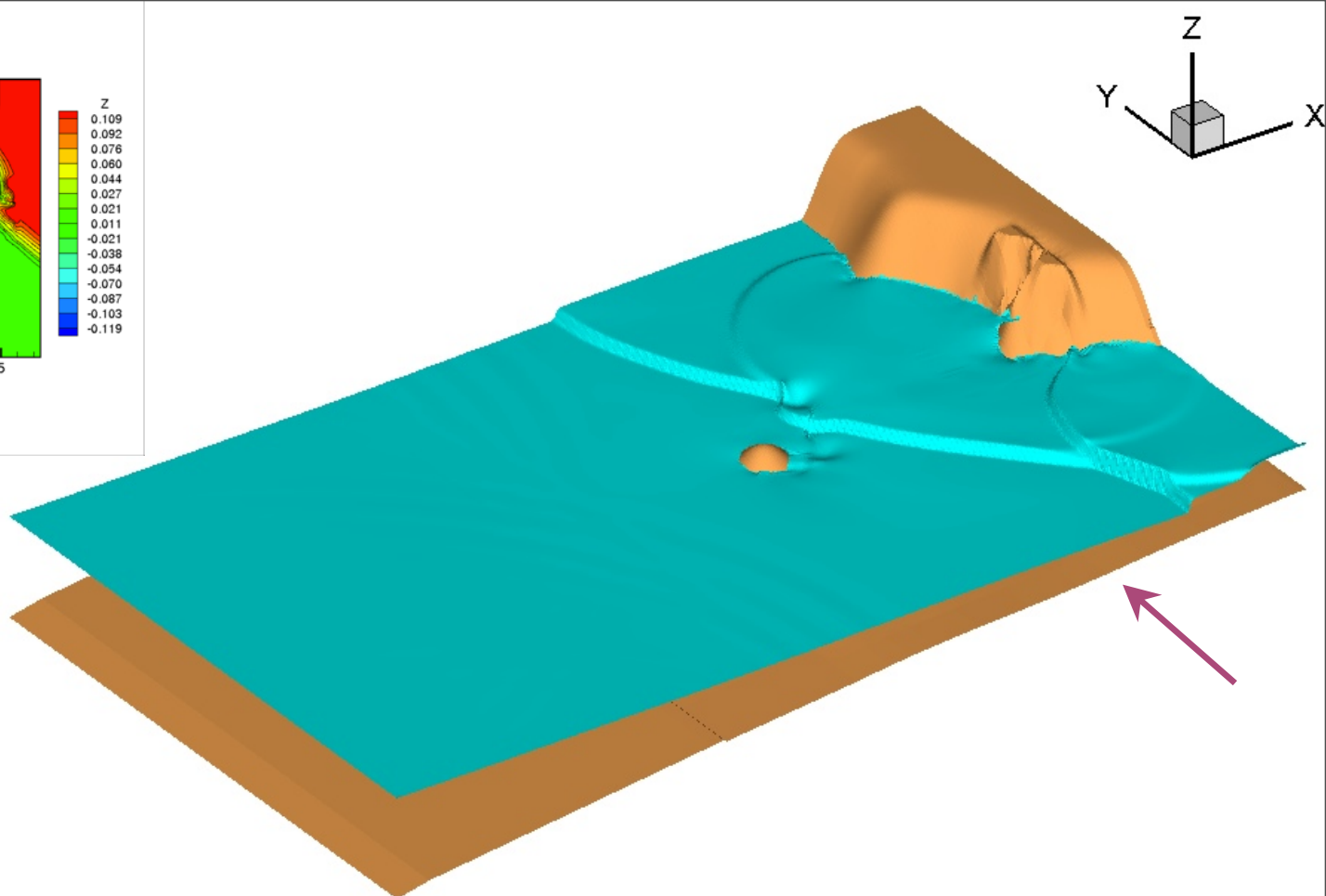


$t = 17 \text{ s}$

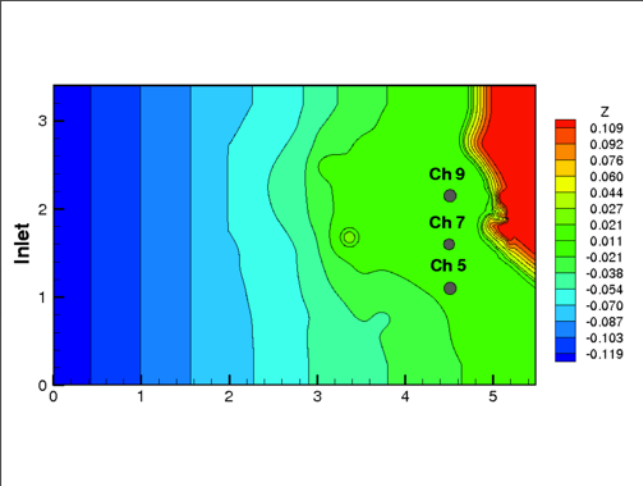




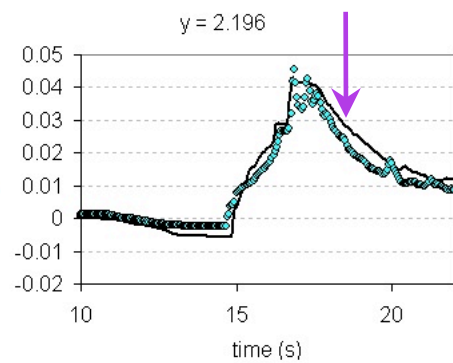
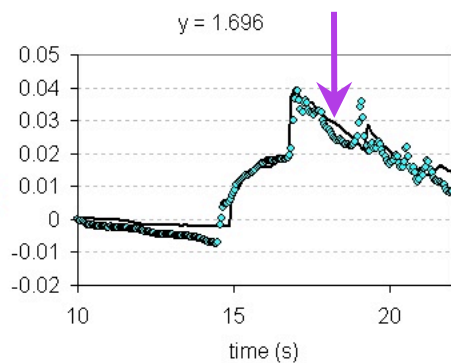
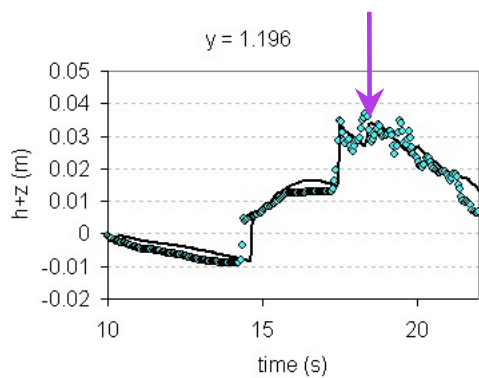
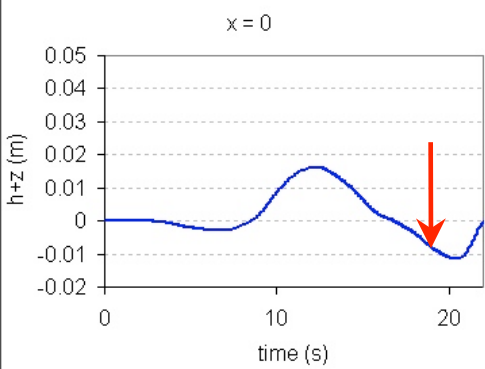
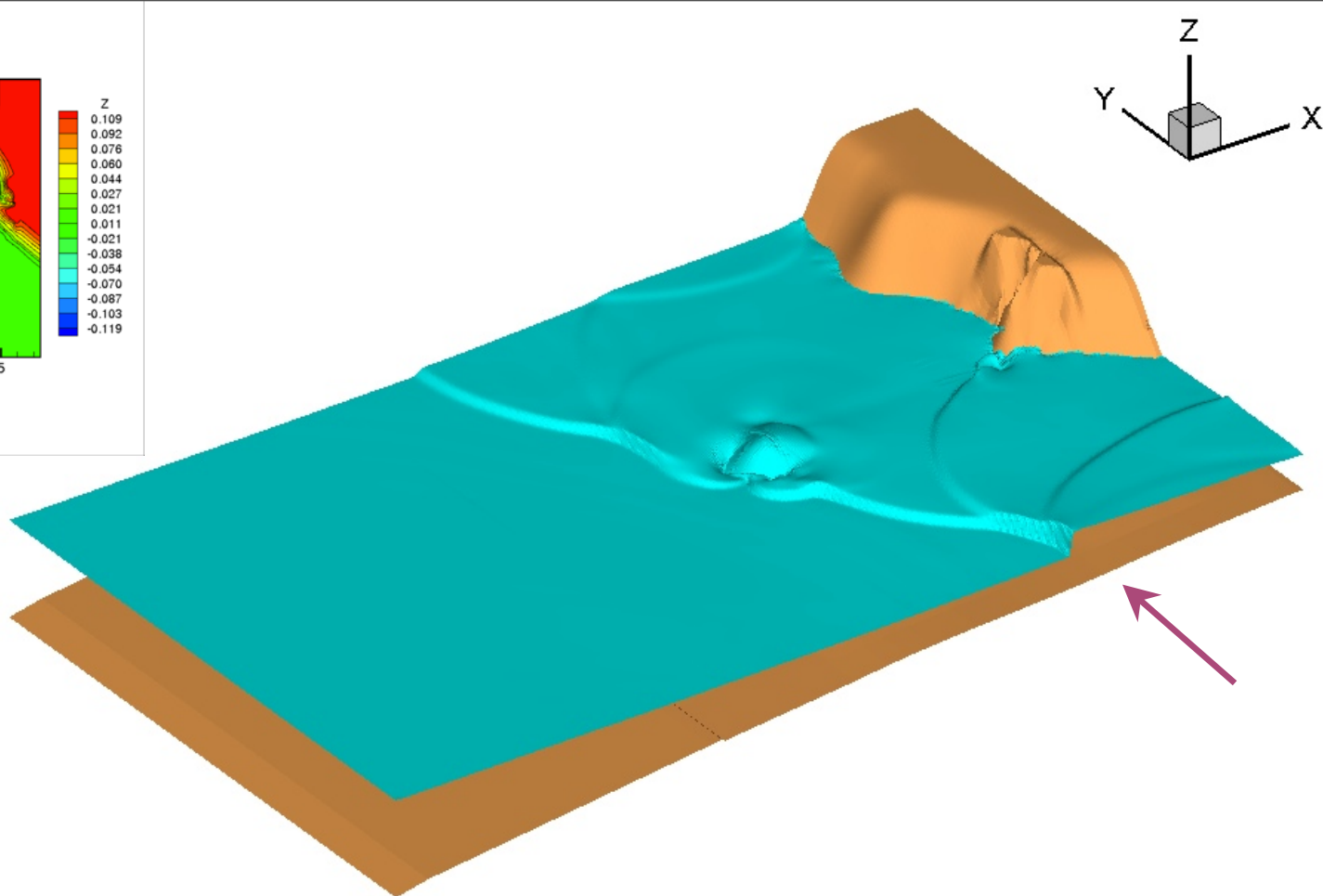
$t = 18 \text{ s}$



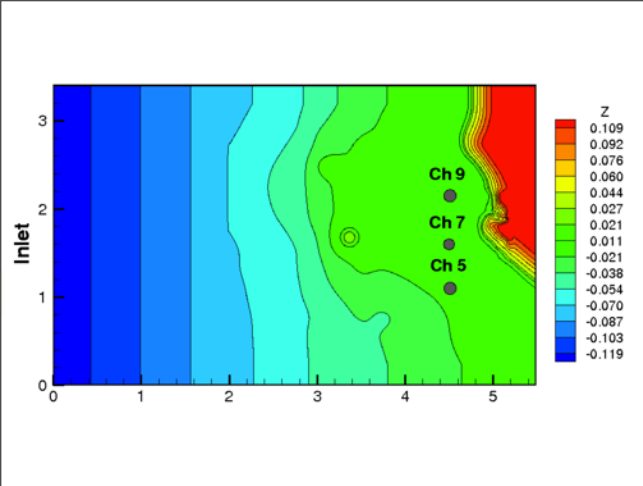
◆ measured — computed



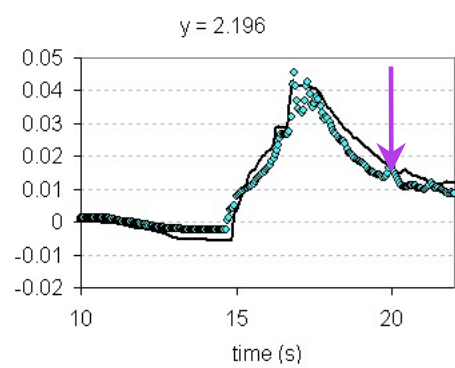
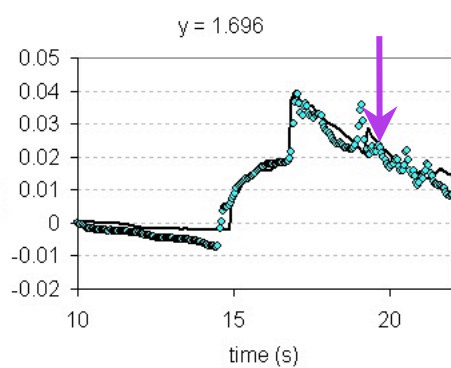
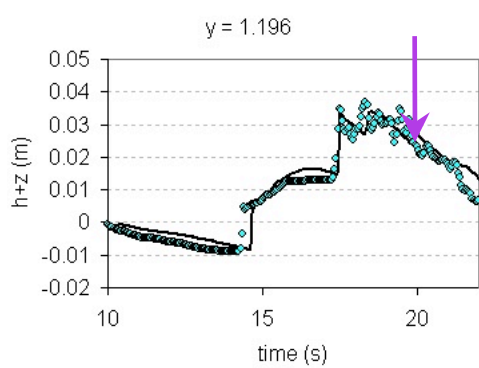
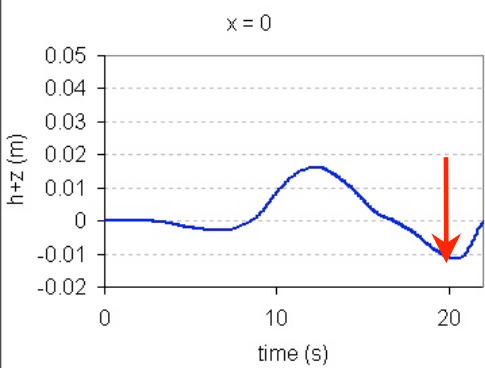
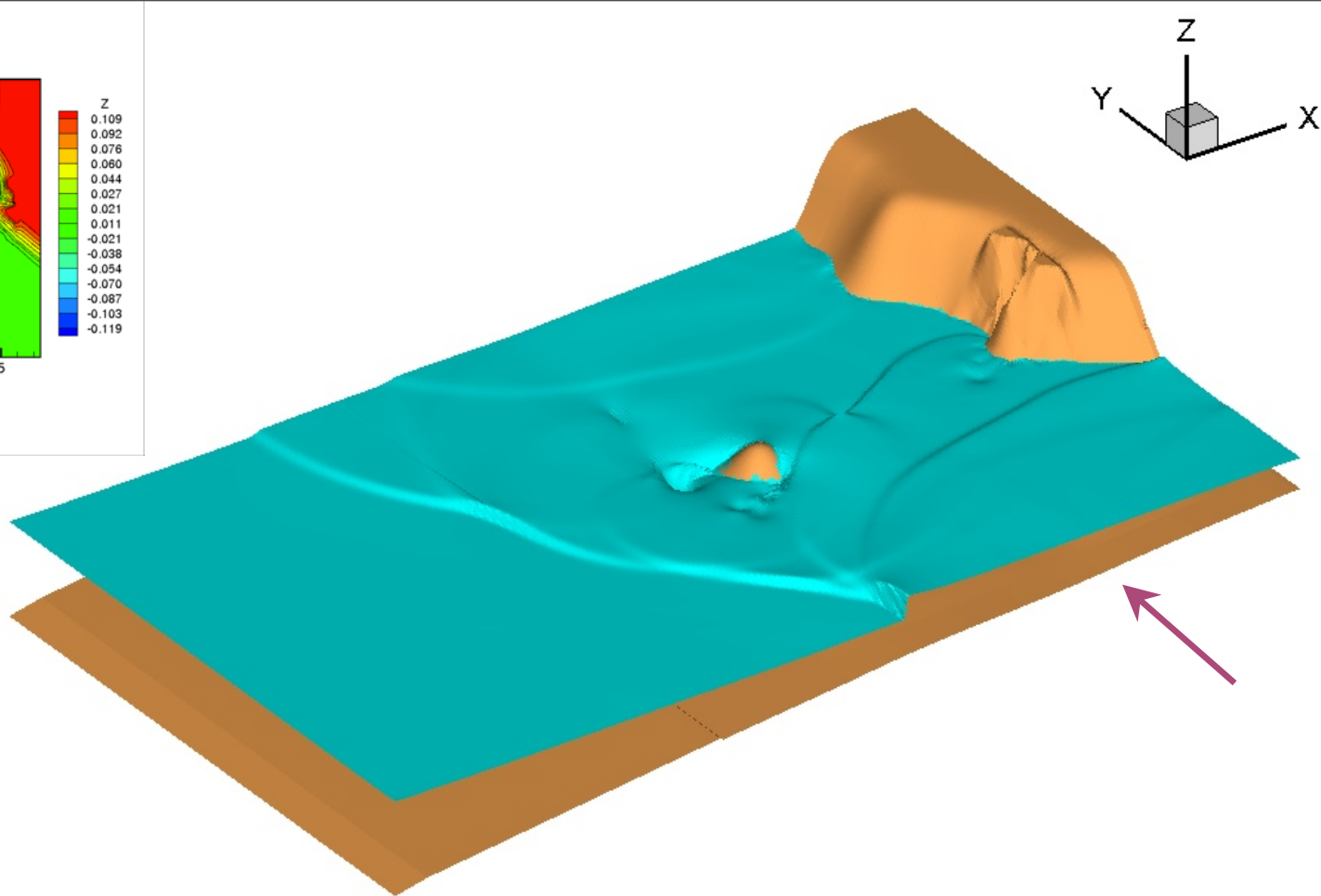
$t = 19 \text{ s}$



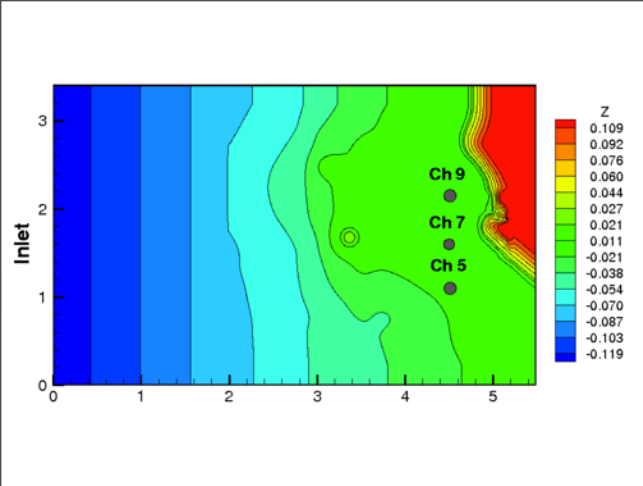
◆ measured
 — computed



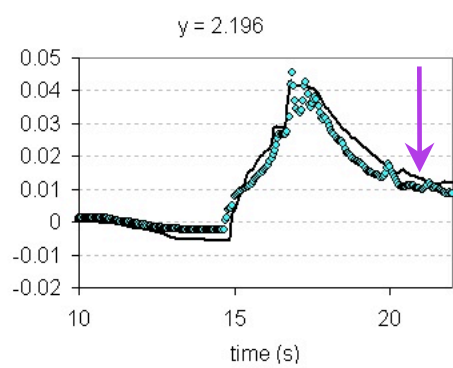
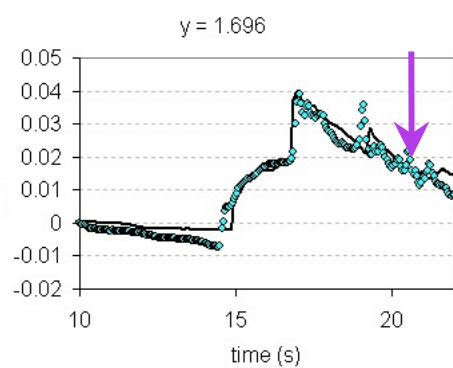
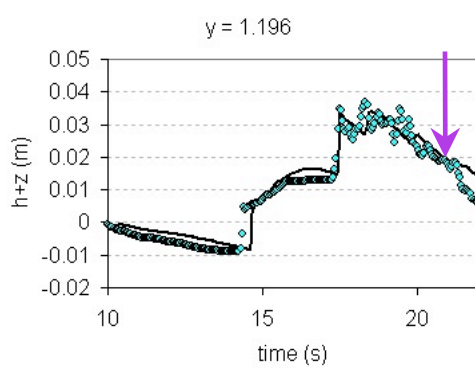
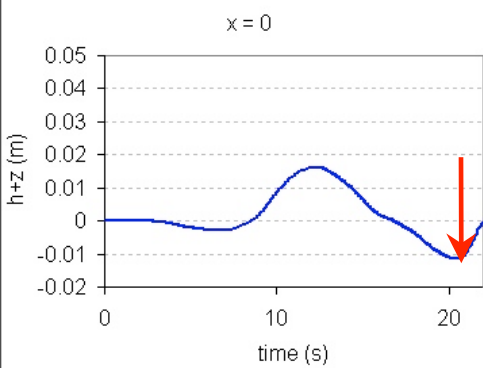
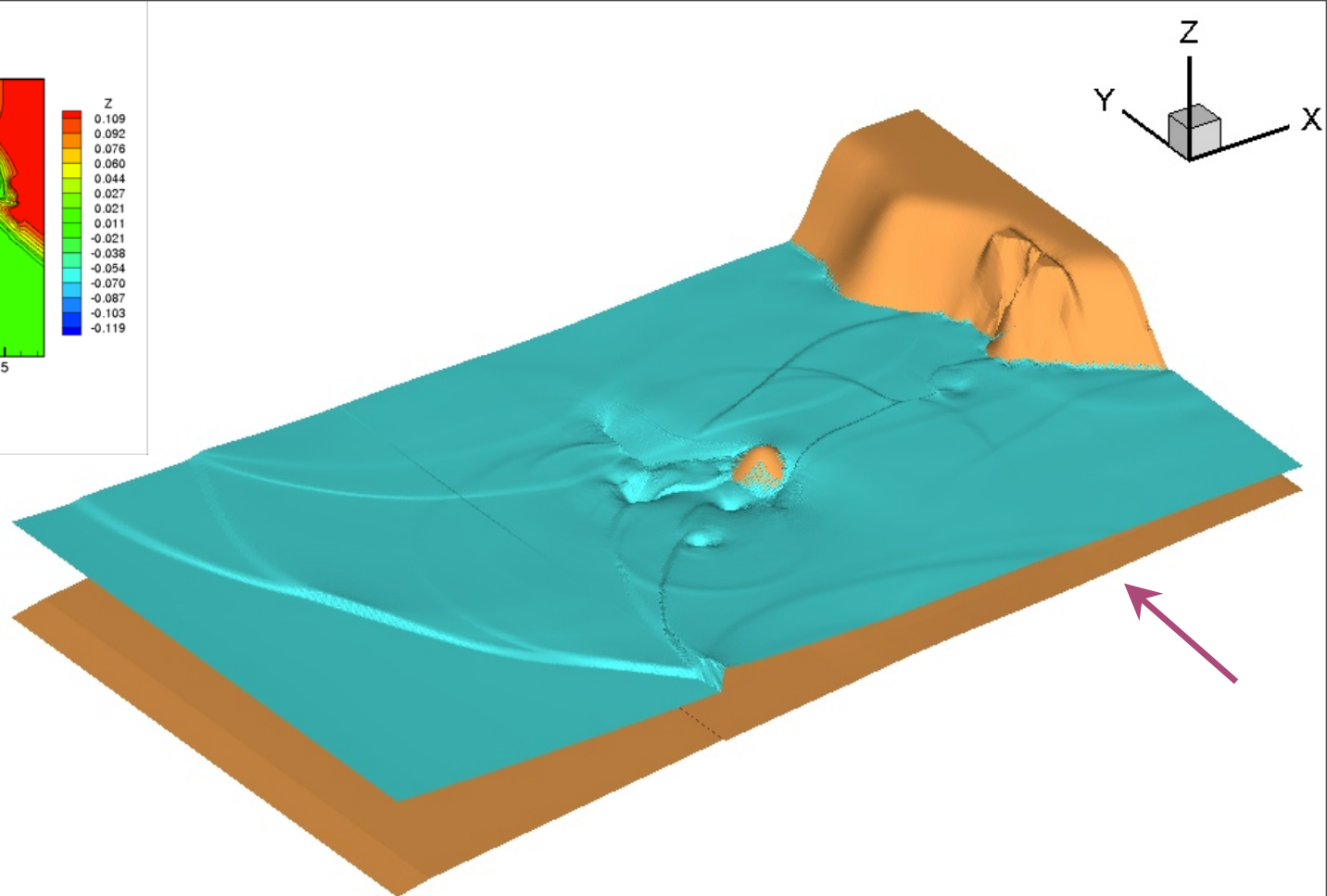
$t = 20 \text{ s}$



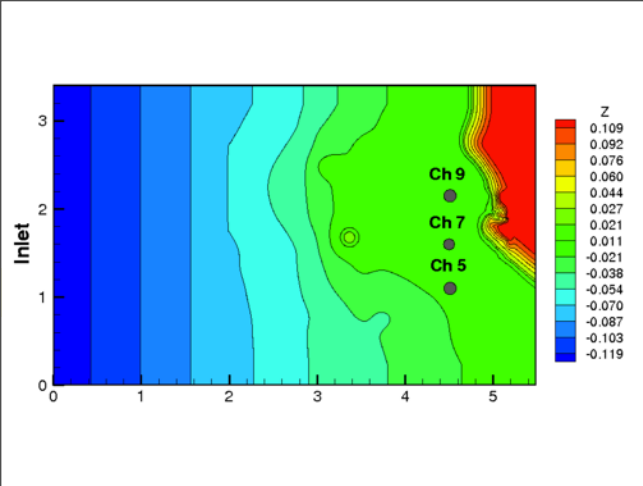
◆ measured — computed



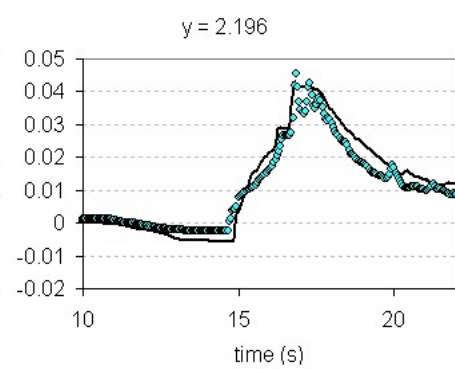
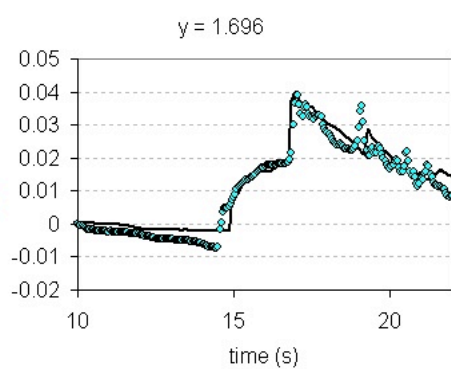
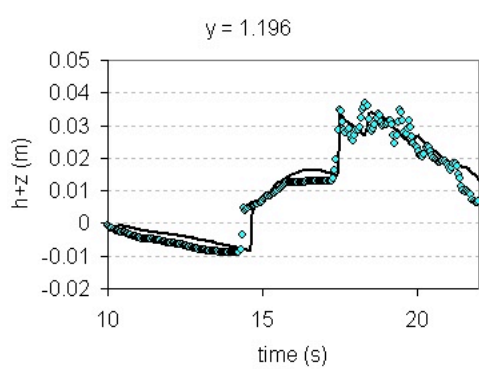
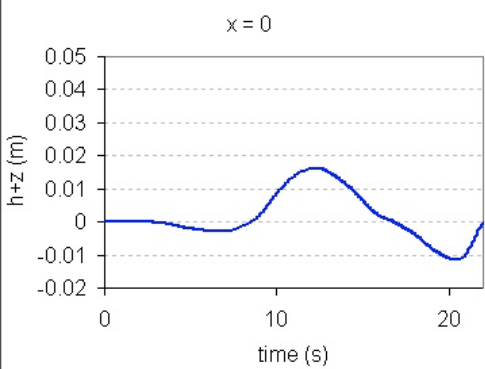
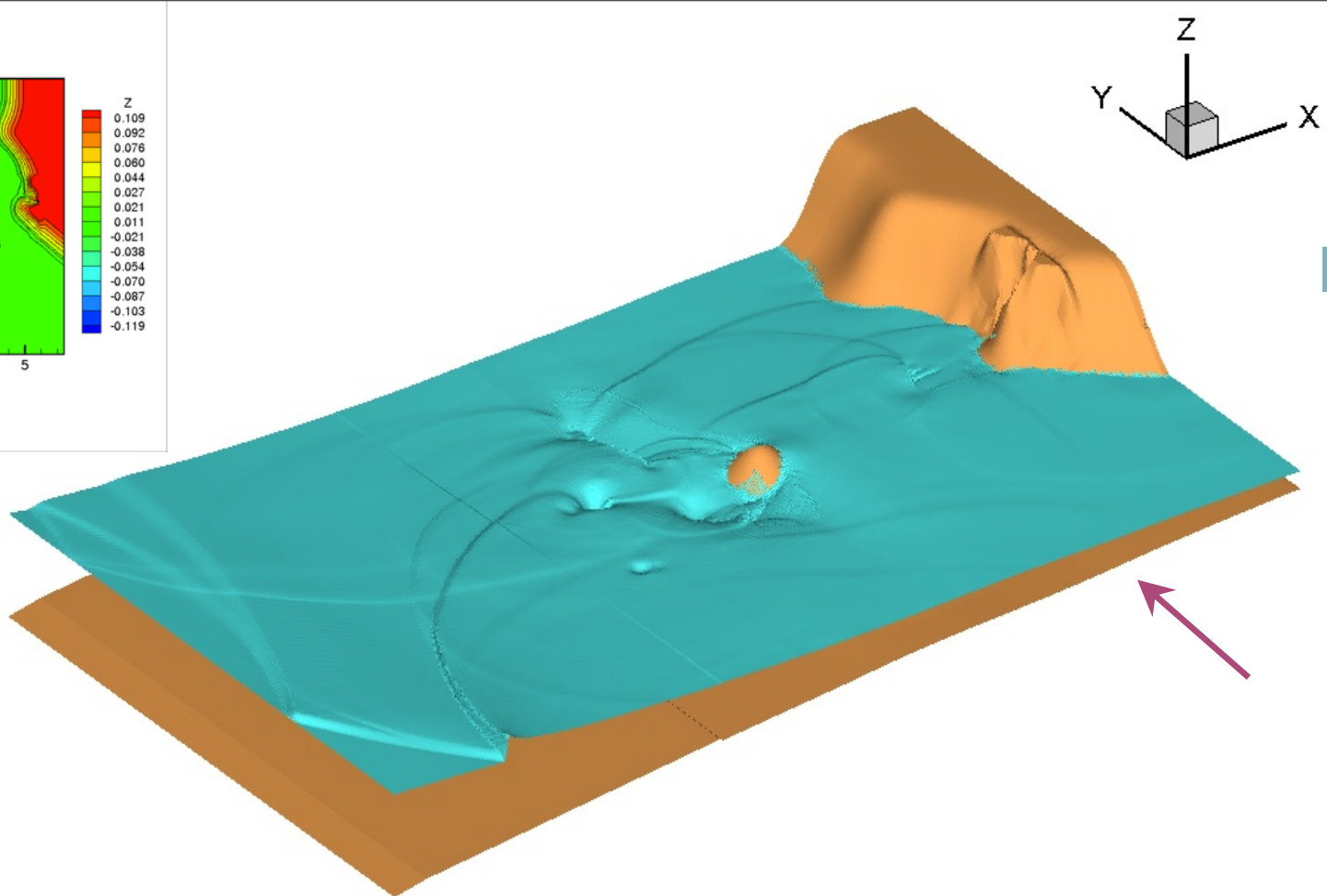
$t = 21 \text{ s}$



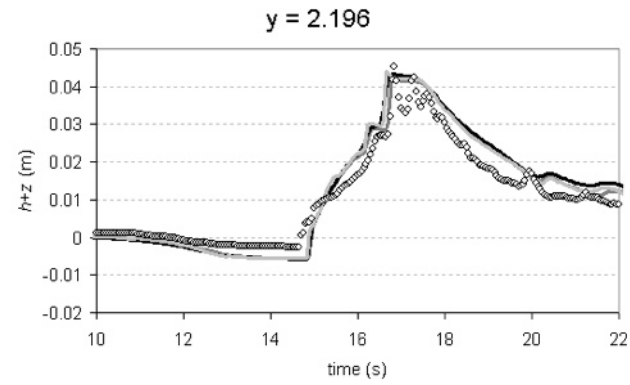
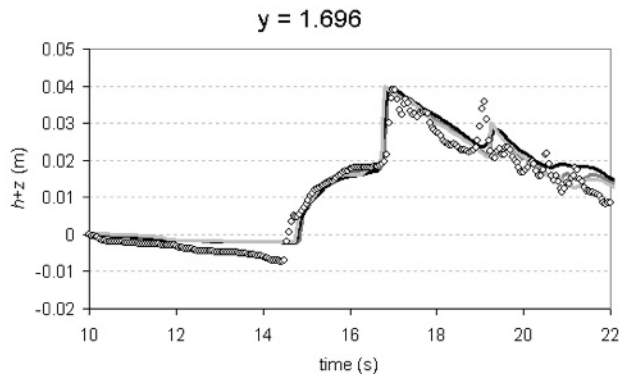
◆ measured — computed



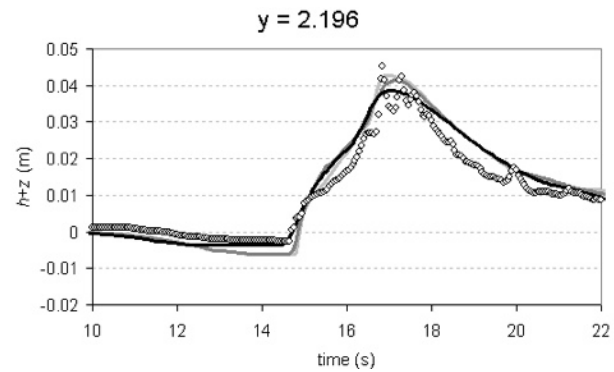
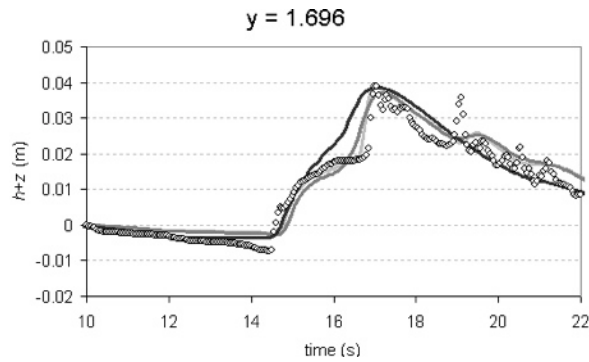
$t = 22 \text{ s}$



◆ measured — computed



— computed M1 — computed M2 — computed M3 ◊ measured



— computed M4 — computed M5 — computed M6 ◊ measured

	M1	M2	M3	M4	M5	M6
length l	0.007	0.014	0.028	0.056	0.112	0.224
cells	762048	190512	47432	11956	2940	750

Obstáculos enfrentados a ondas

(Soares-Frazao et al. 2005)

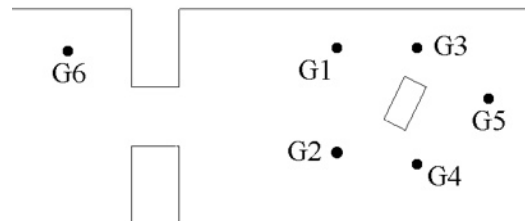
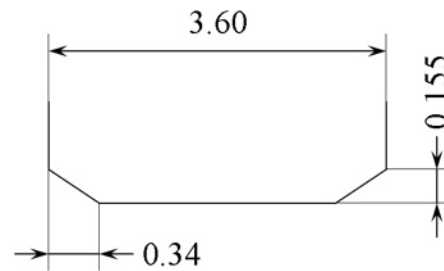
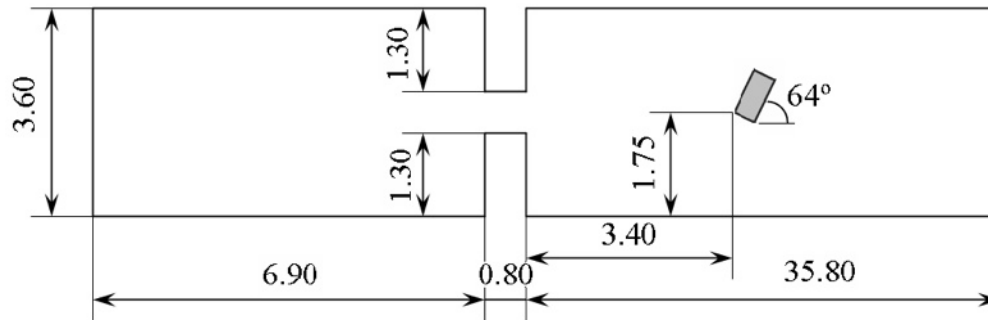


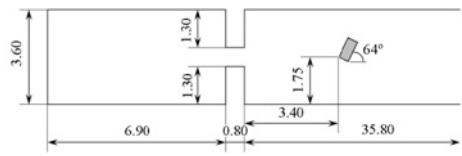
Obstáculos enfrentados a ondas

(Soares-Frazao et al. 2005)



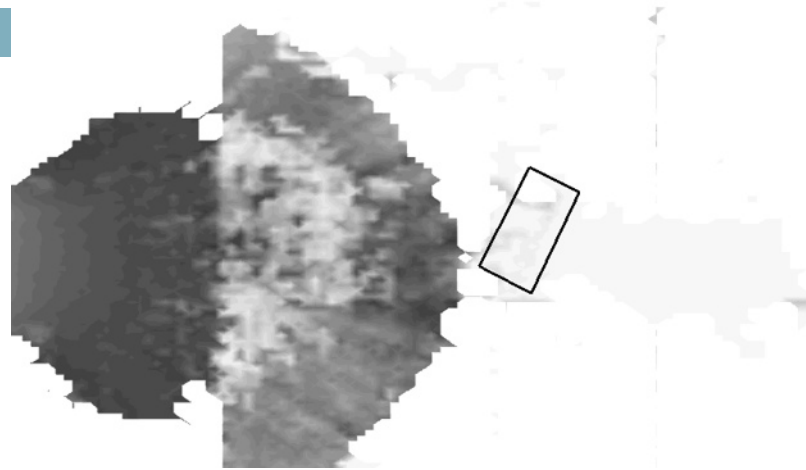
Obstáculos enfrentados a roturas de presas



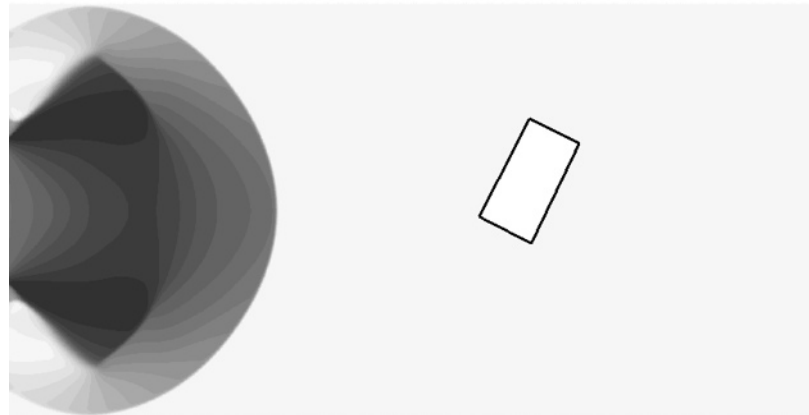


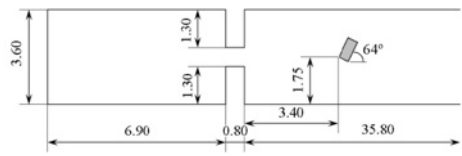
$t = 1s$

Velocidad superficial
medida



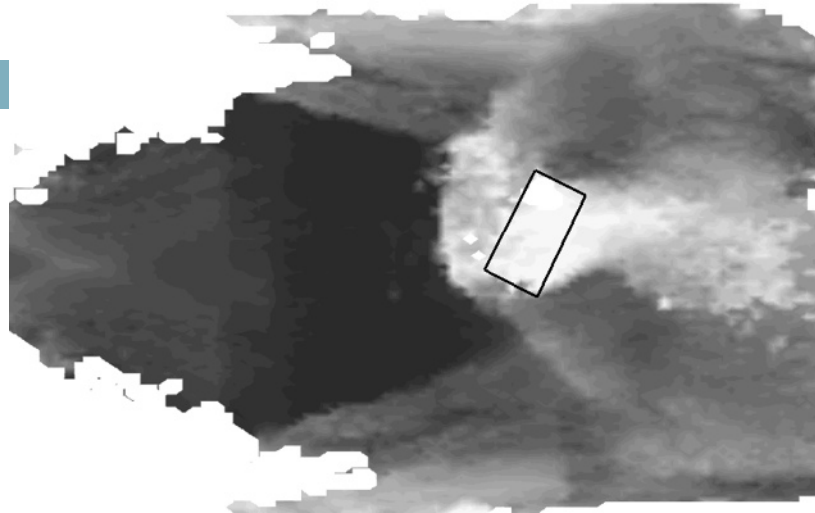
Velocidad (promedio
vertical) calculada



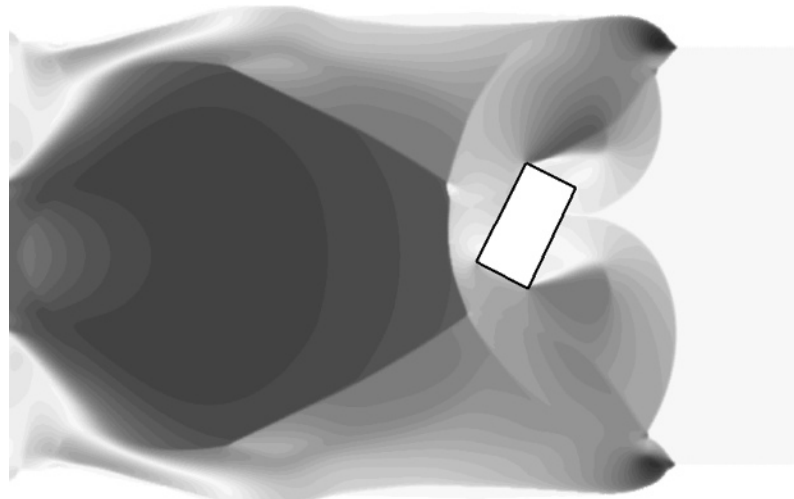


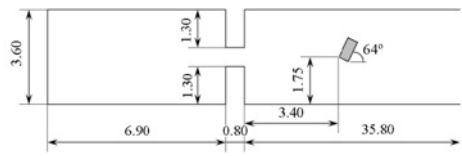
$t = 3s$

Velocidad superficial
medida



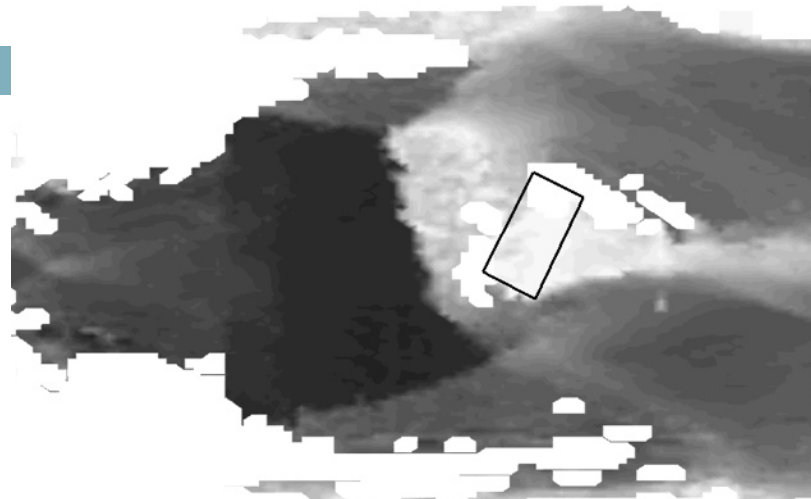
Velocidad (promedio
vertical) calculada



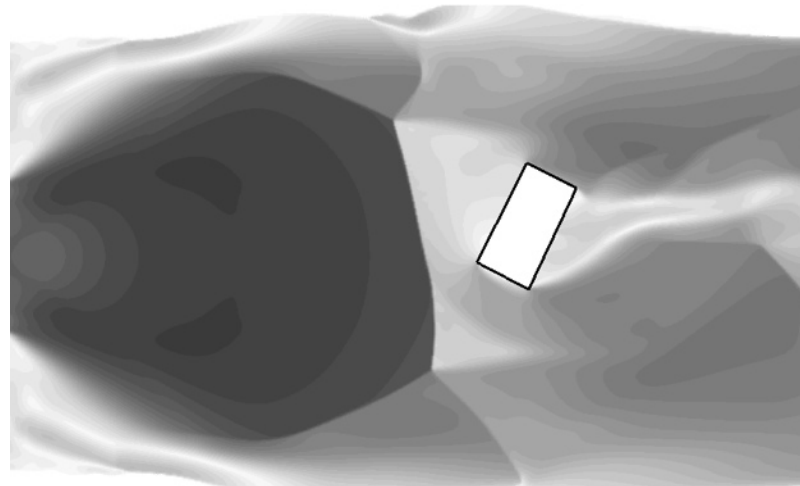


$t = 5s$

Velocidad superficial
medida

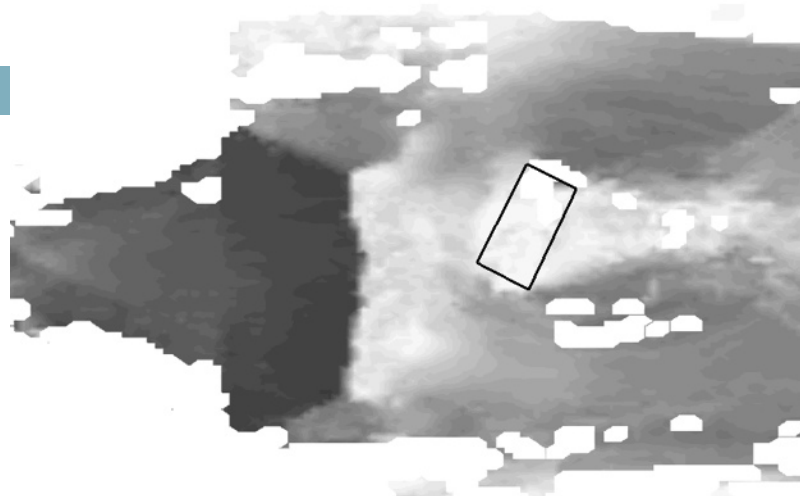


Velocidad (promedio
vertical) calculada

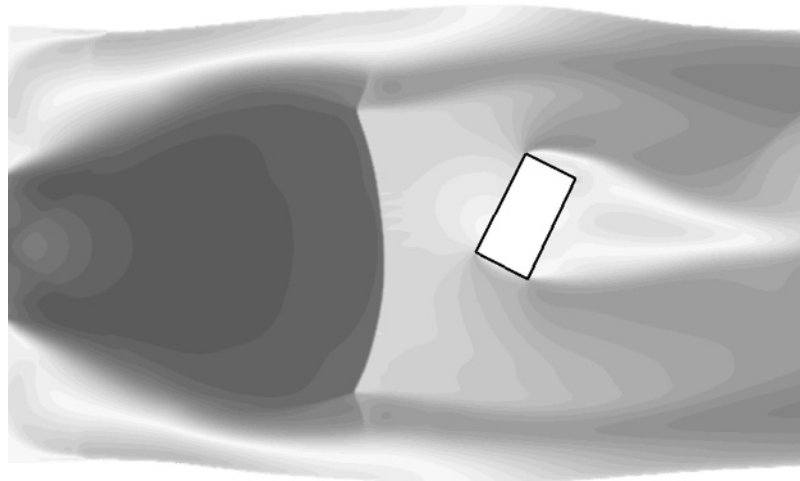


$t = 15s$

Velocidad superficial
medida



Velocidad (promedio
vertical) calculada



Generación de la malla

- Representación discreta
- Precisión
- Coste computacional

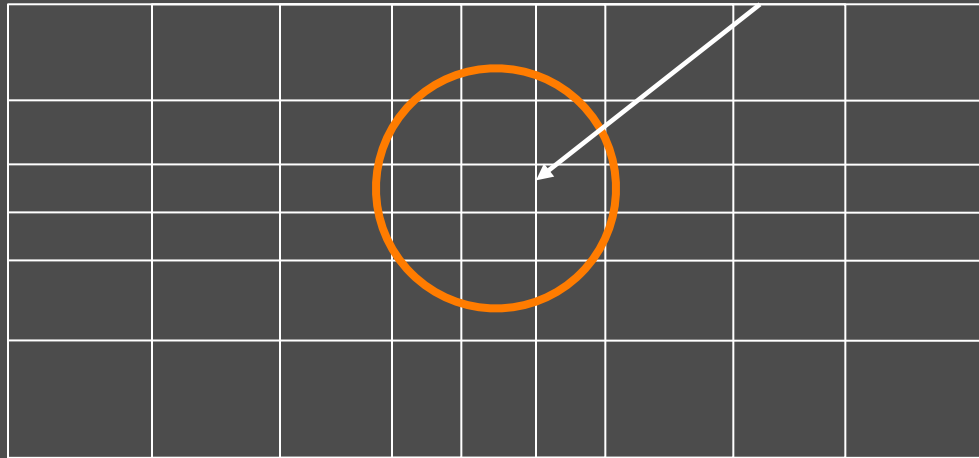
Experiencia computacional

- Celdas grandes para variaciones suaves
- Celdas pequeñas en zonas de fuerte irregularidad.
- Optimización: refinamiento local

DISCRETIZACIÓN DEL TERRENO

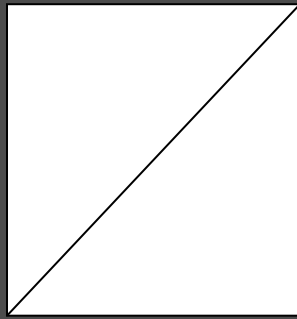
Malla rectangular y refinamiento local

Área de refinamiento

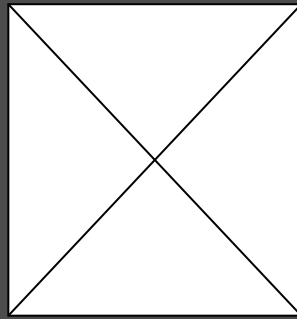


EJEMPLO DISCRETIZACIÓN

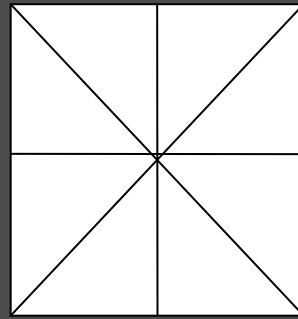
Mallas triangulares de densidad variable



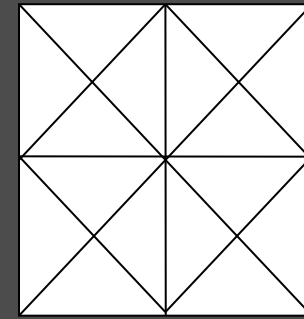
Level 1



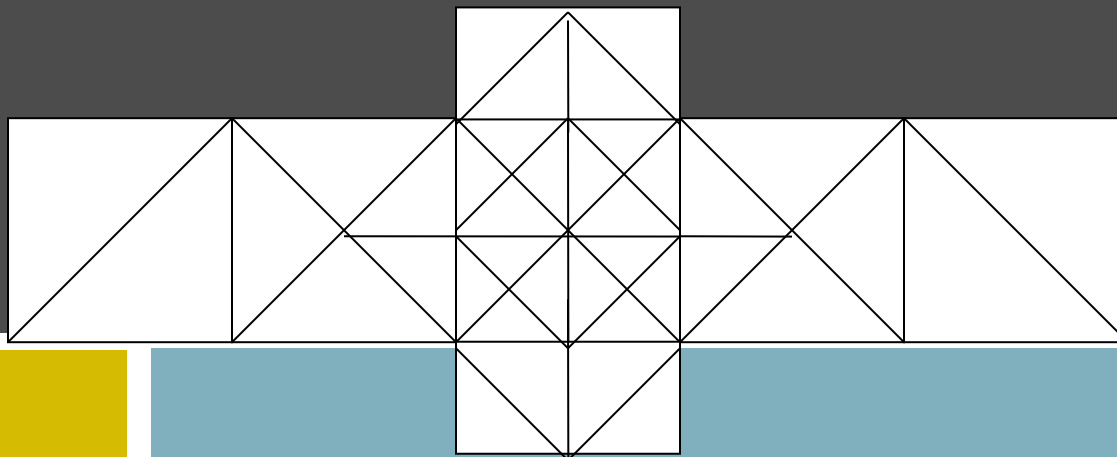
Level 2



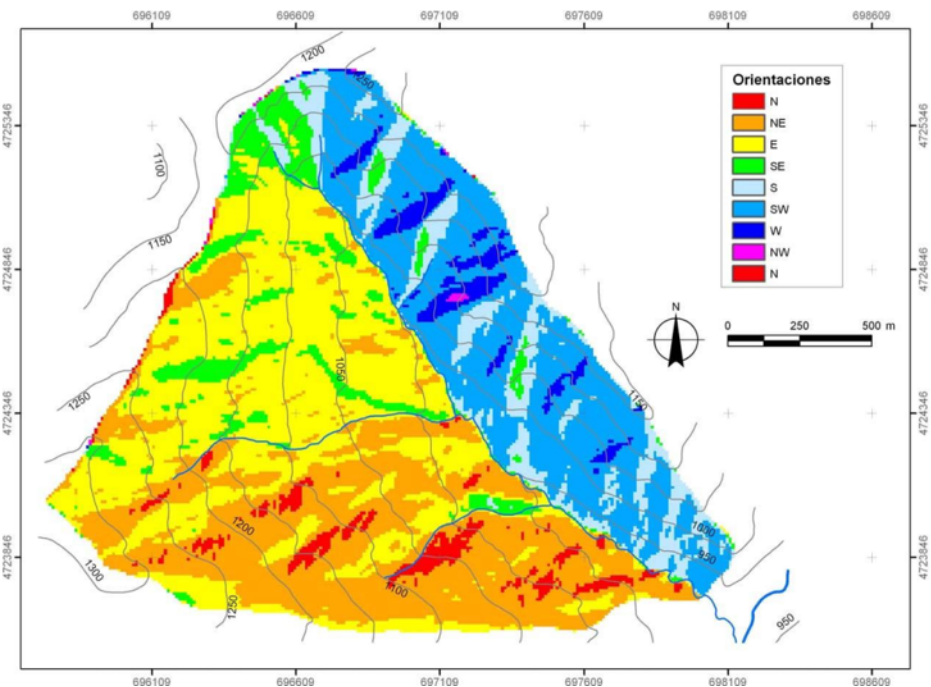
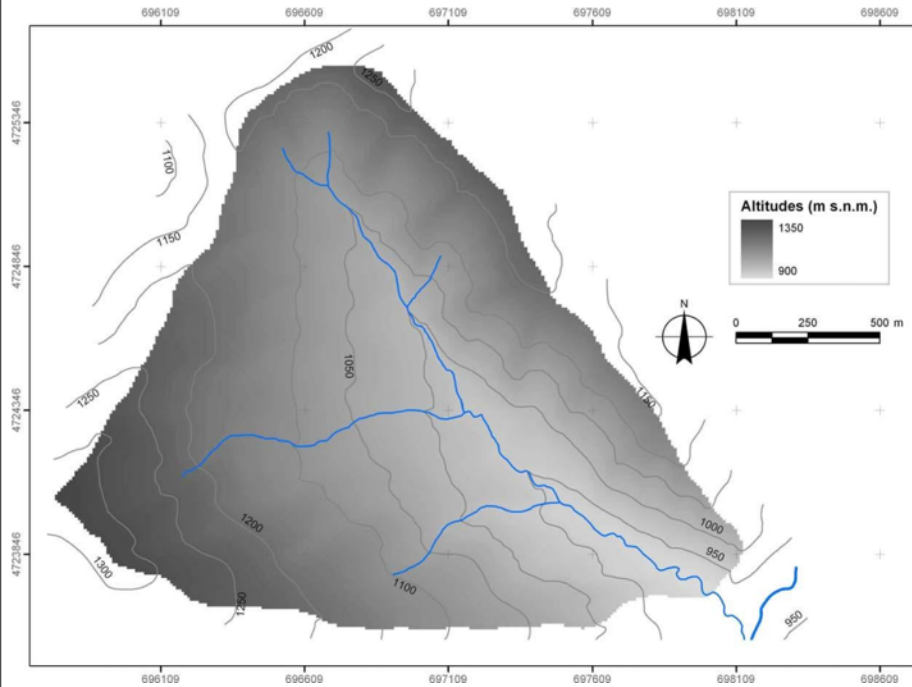
Level 3



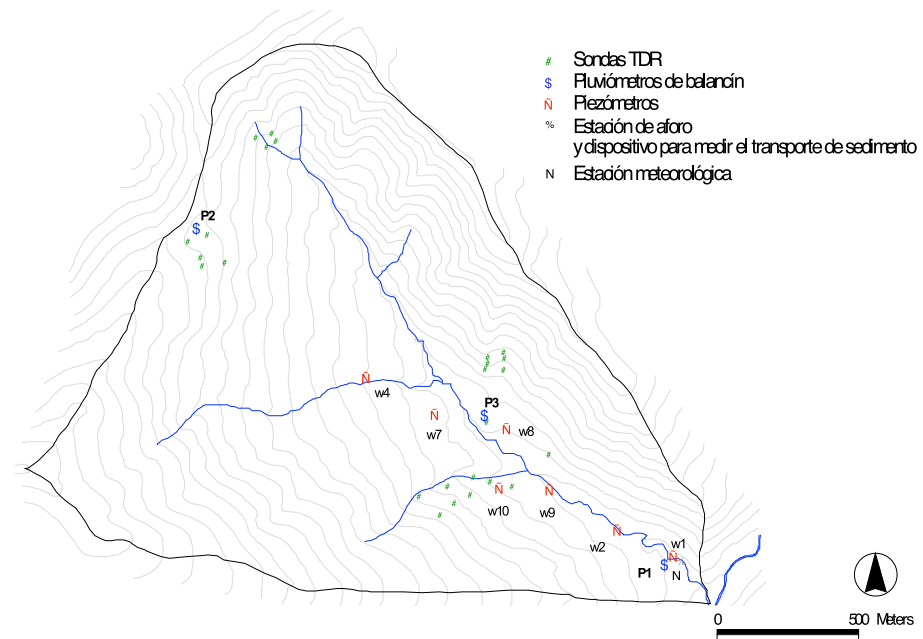
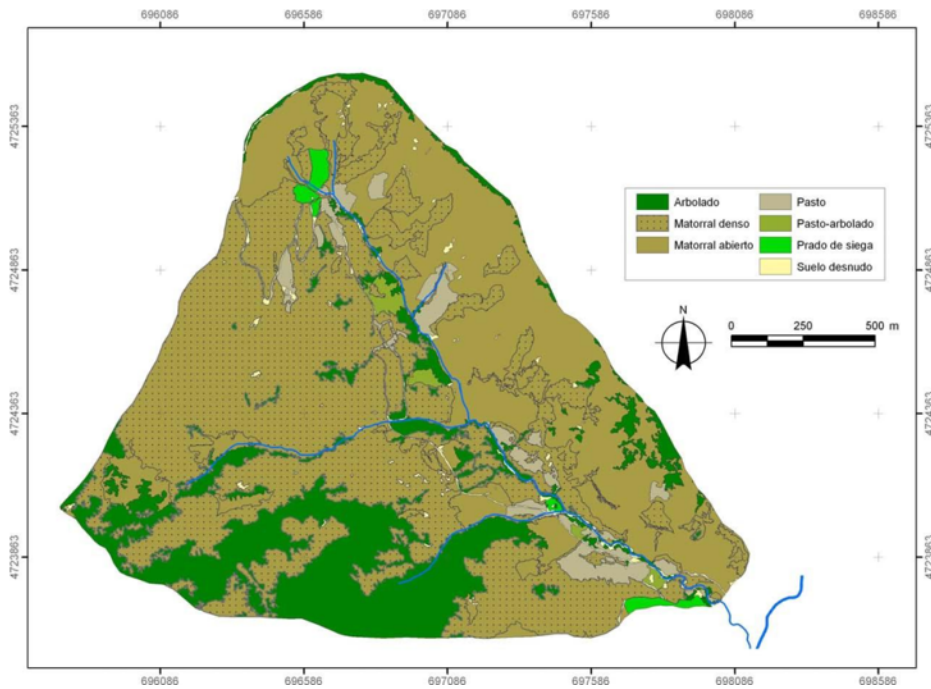
Level 4



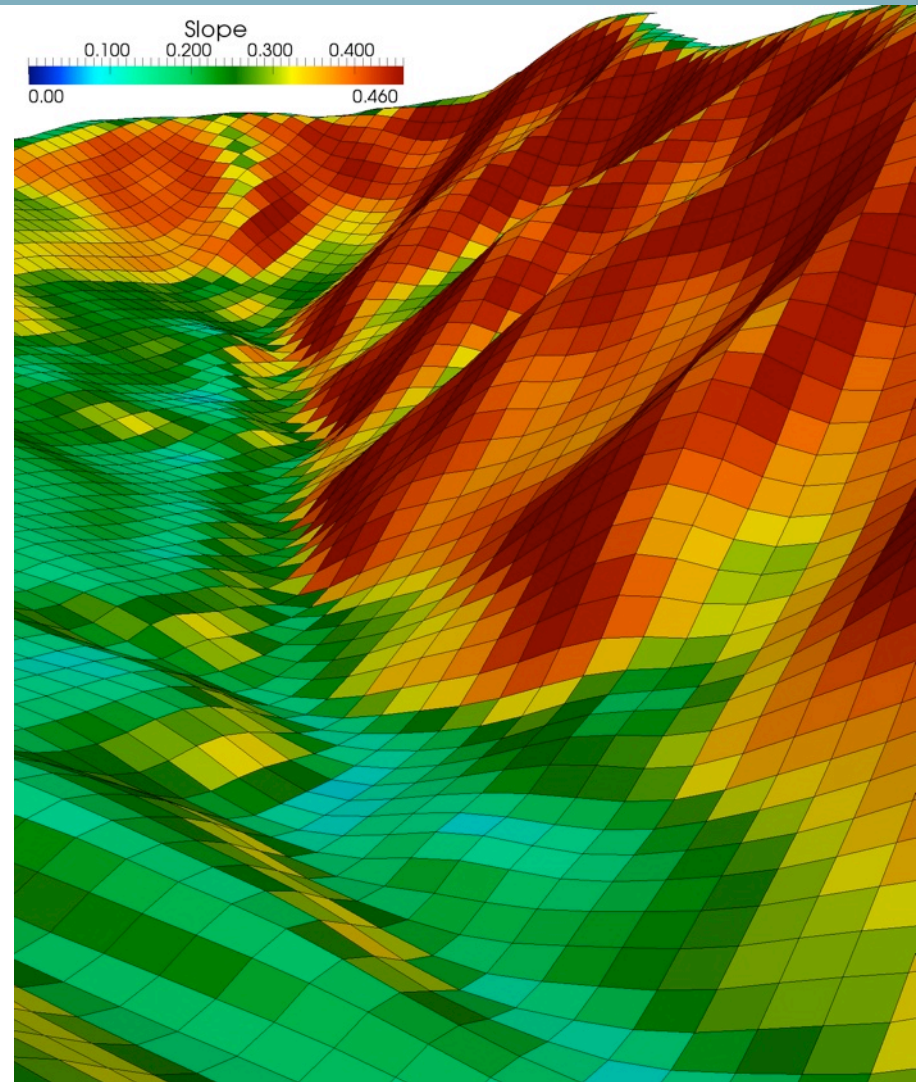
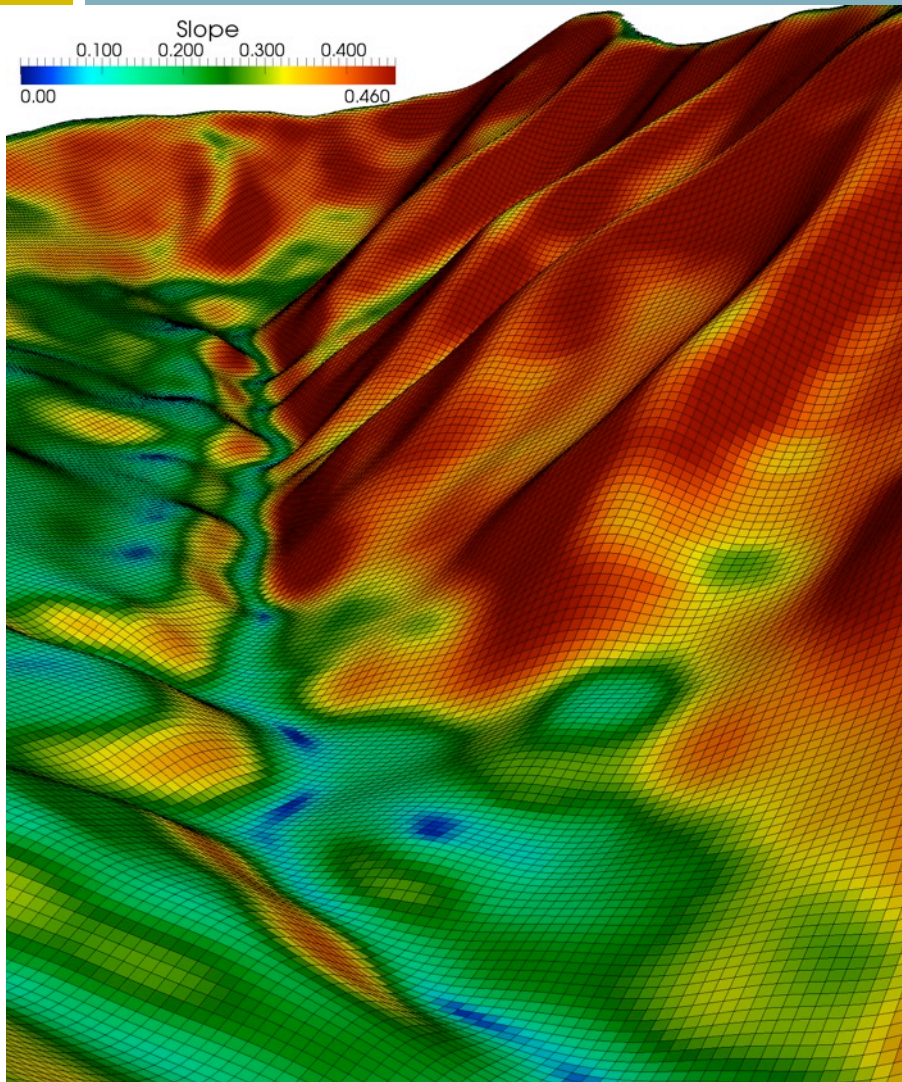
Cuenca de Arnás: terreno



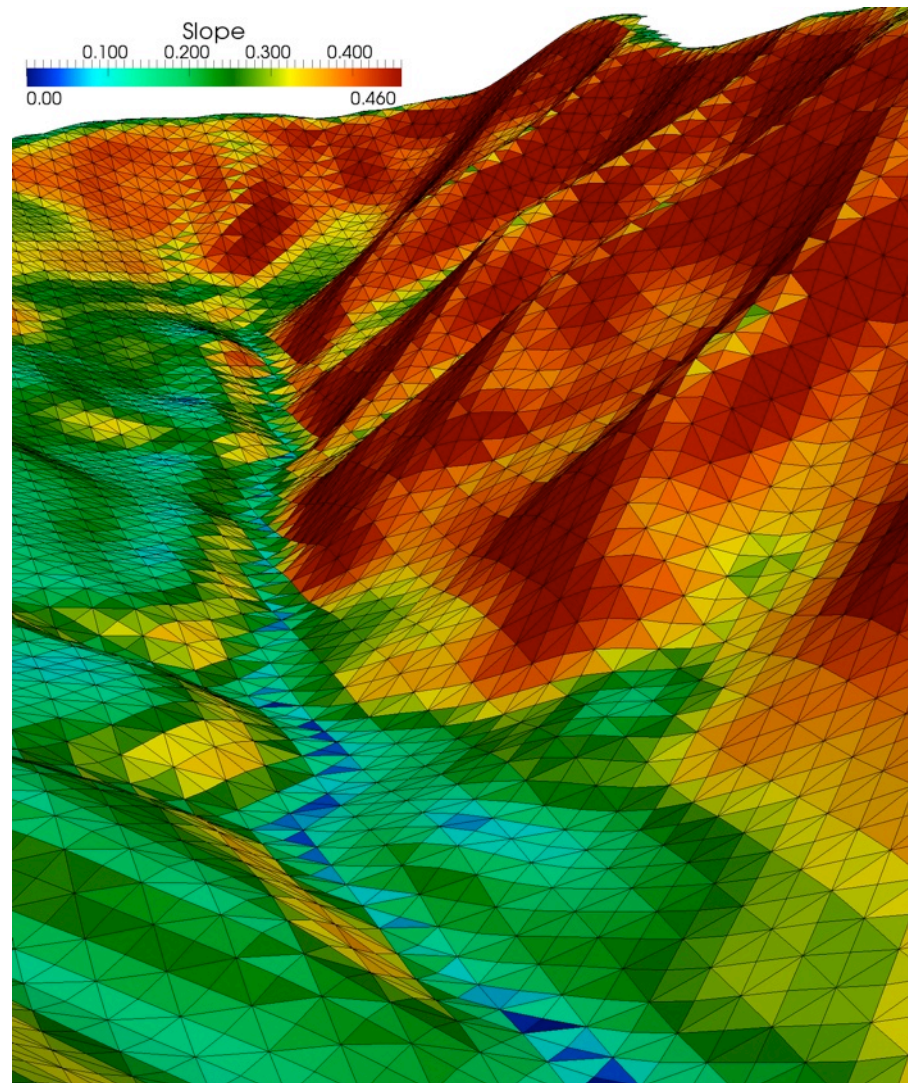
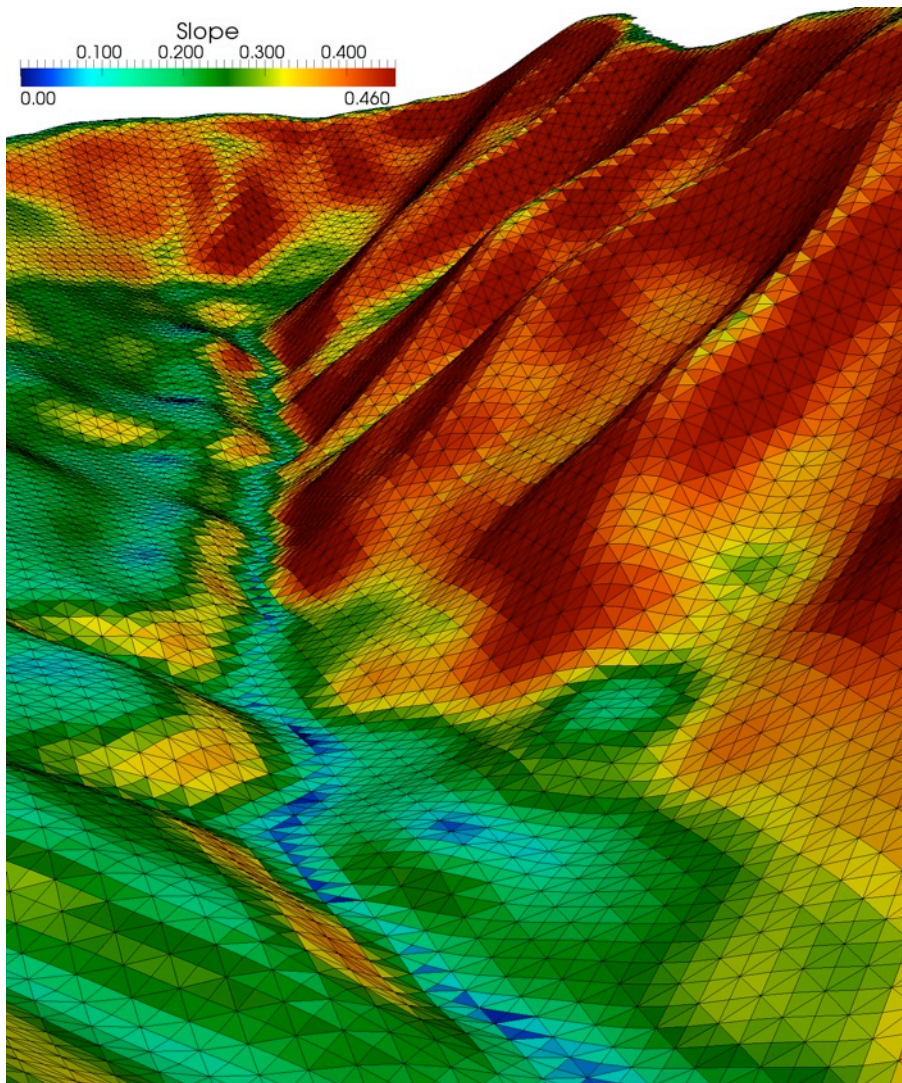
Cuenca de Arnás: vegetación y datos



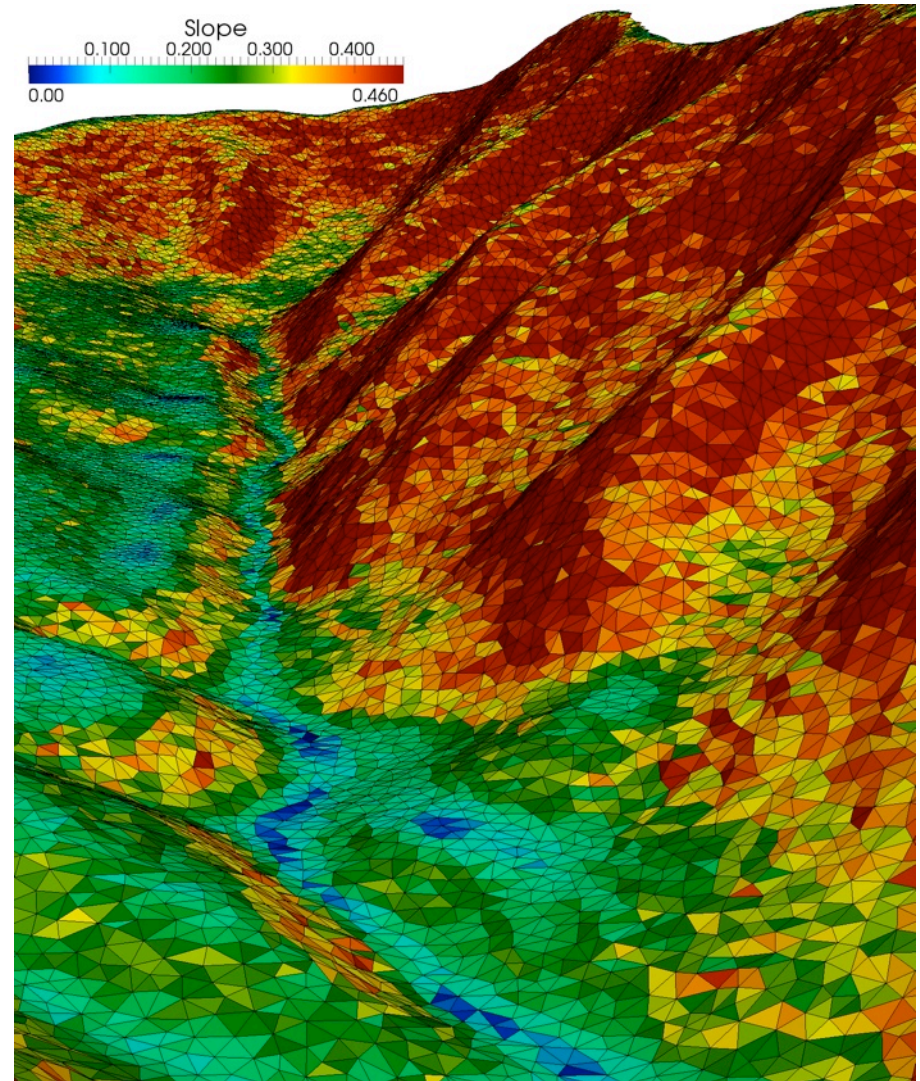
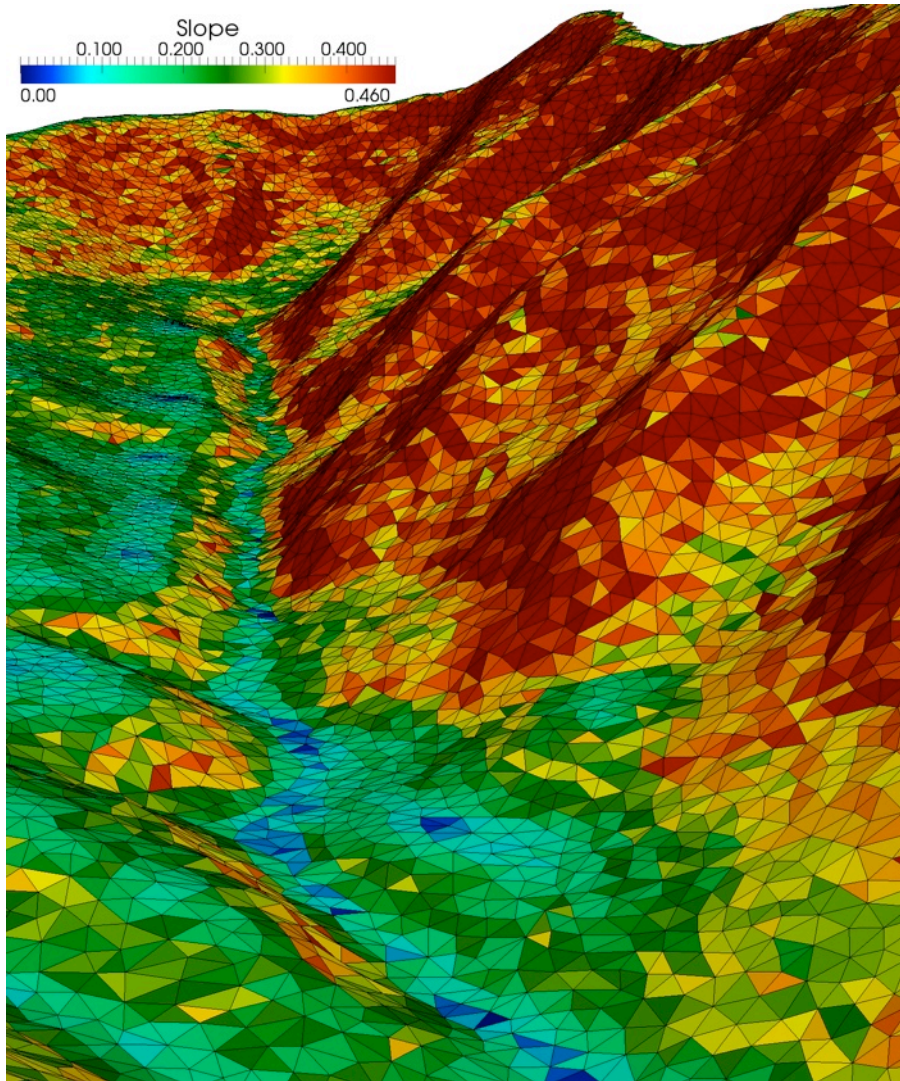
Mallas SS5 y SS20



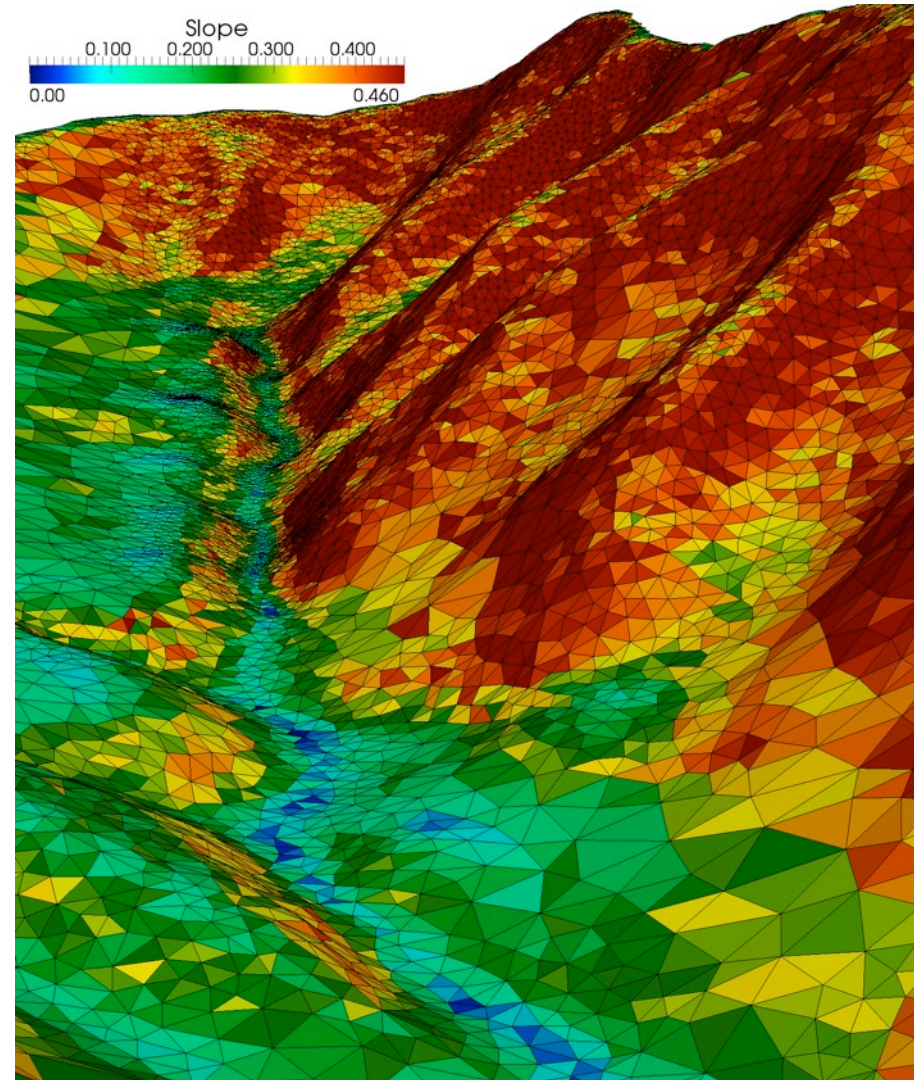
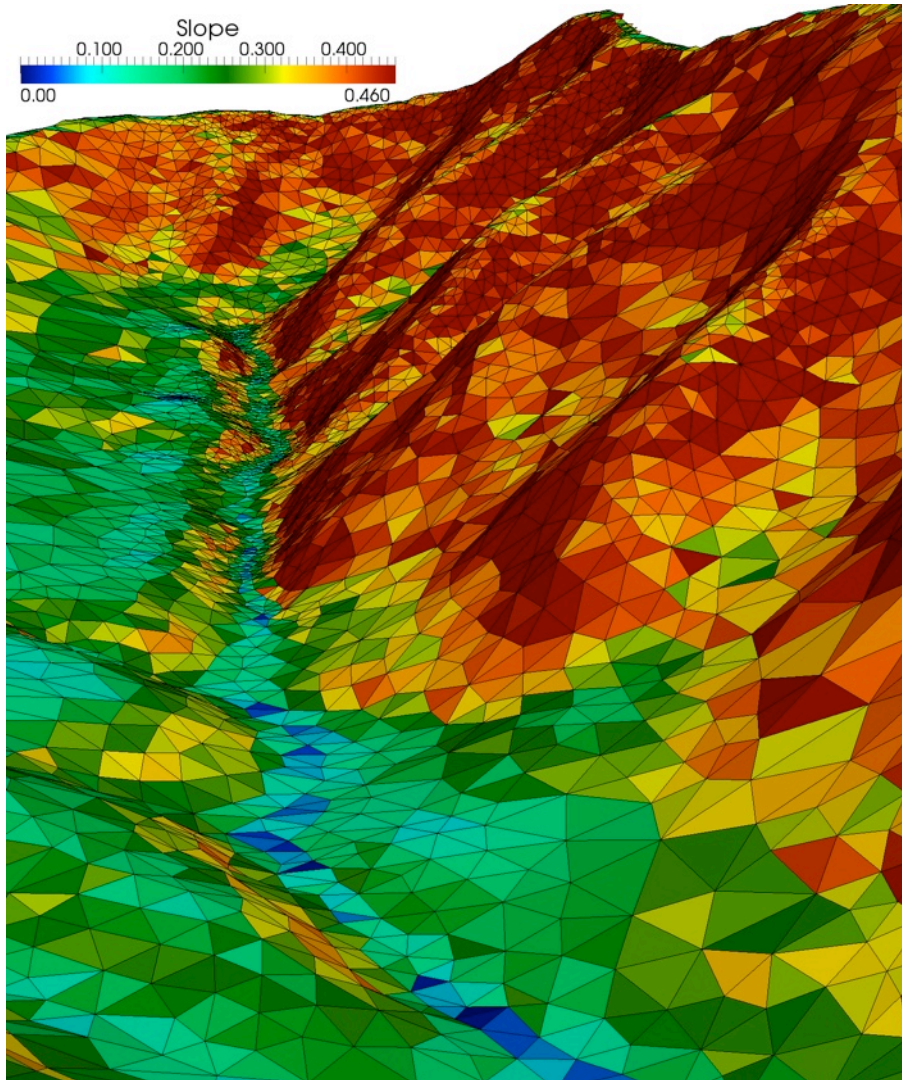
Mallas TS10 y TS15



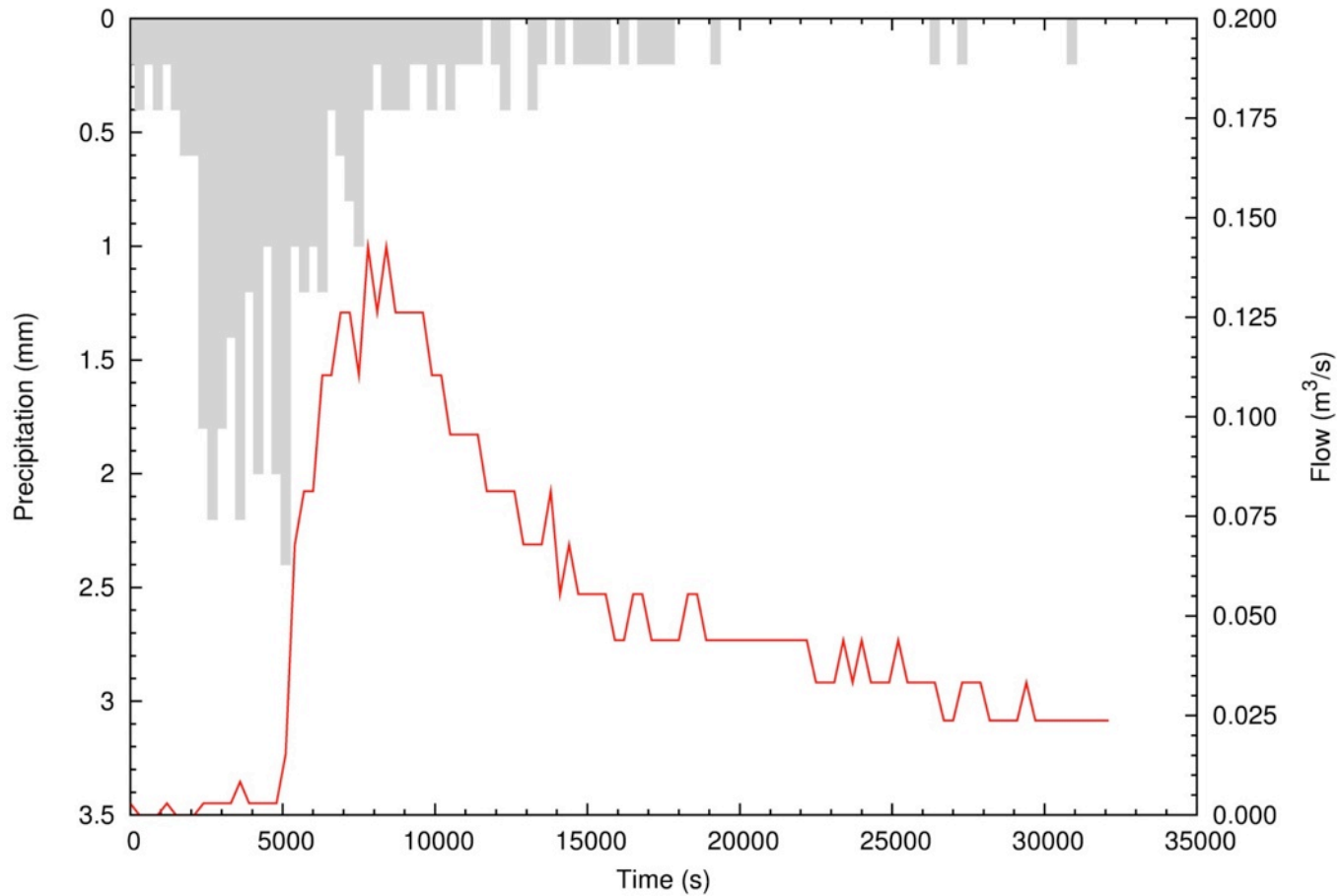
Malas TU10-40 y TU16-60



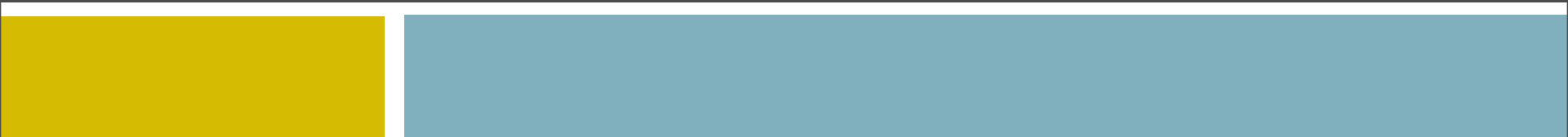
Mallas LMR



Lluvia y respuesta en la cuenca

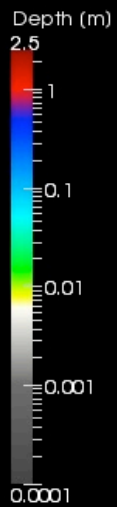
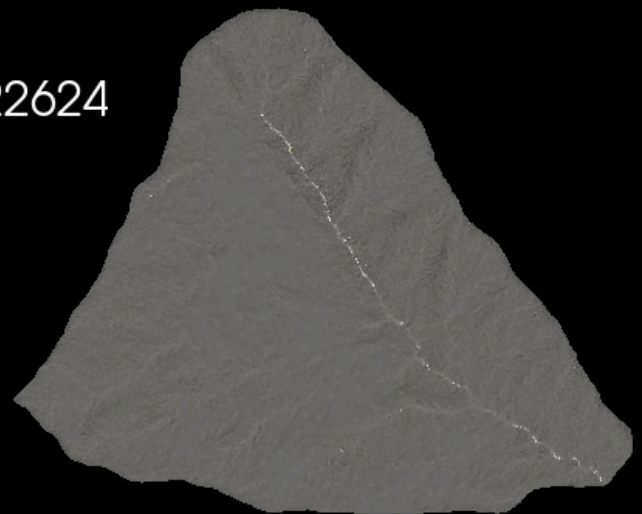
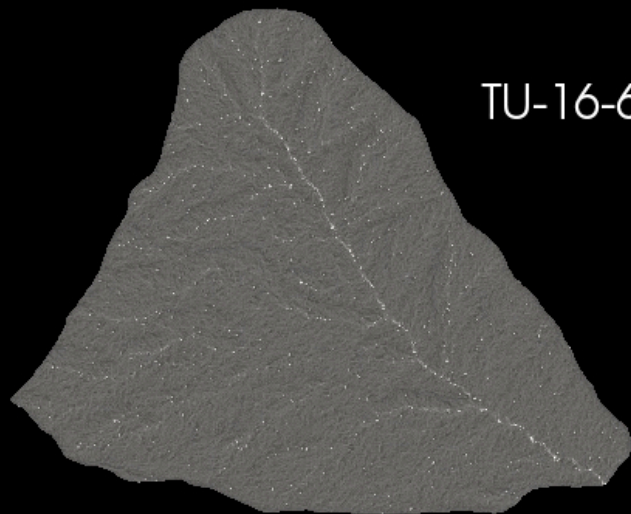
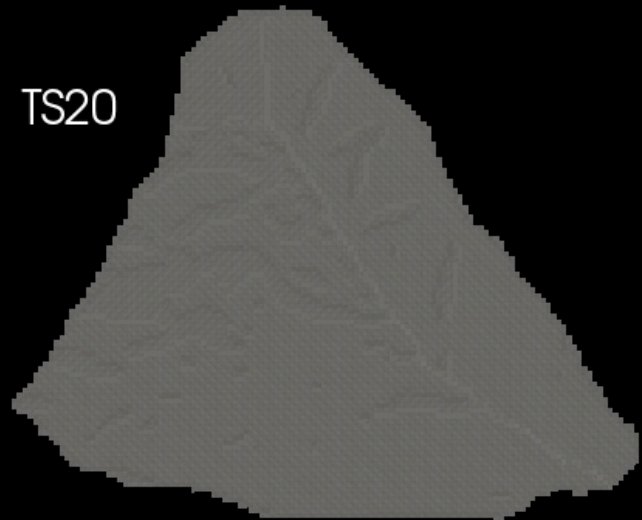


DISCRETIZACION



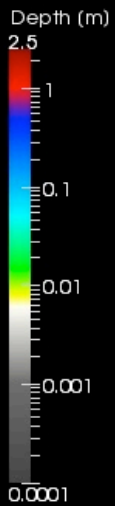
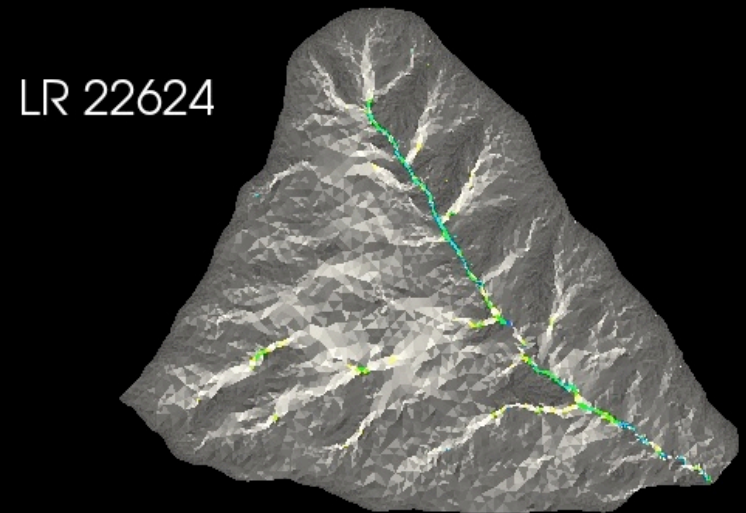
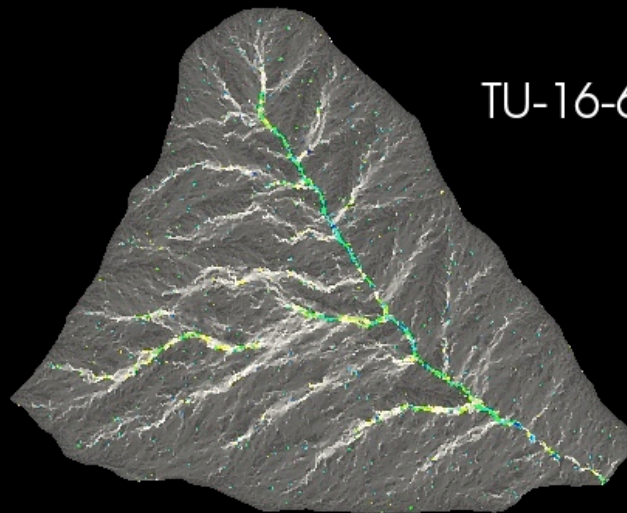
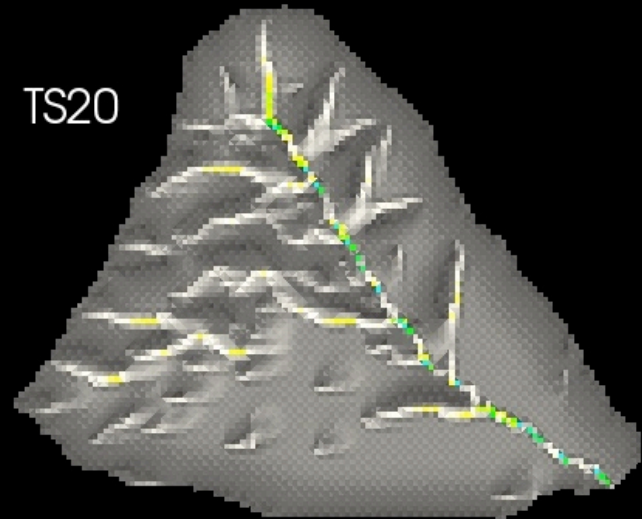
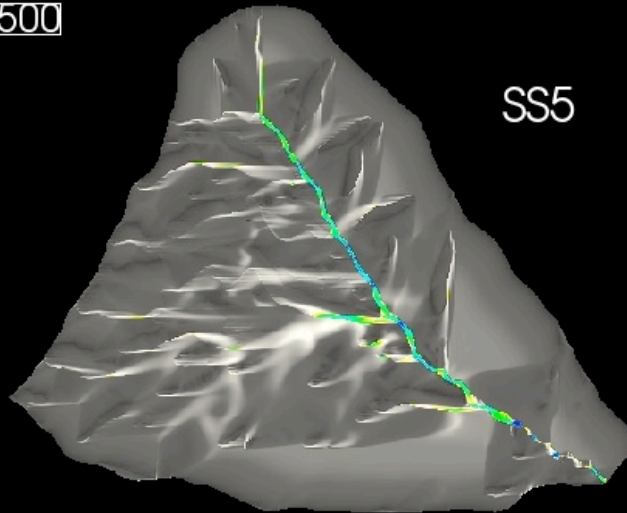
DISCRETIZACION

Time: 500



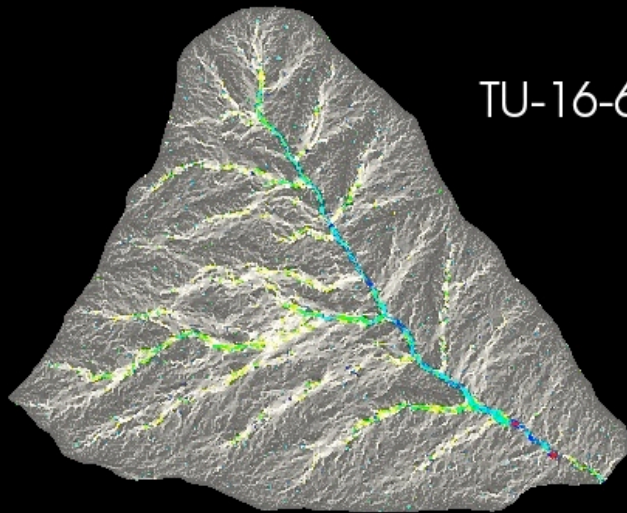
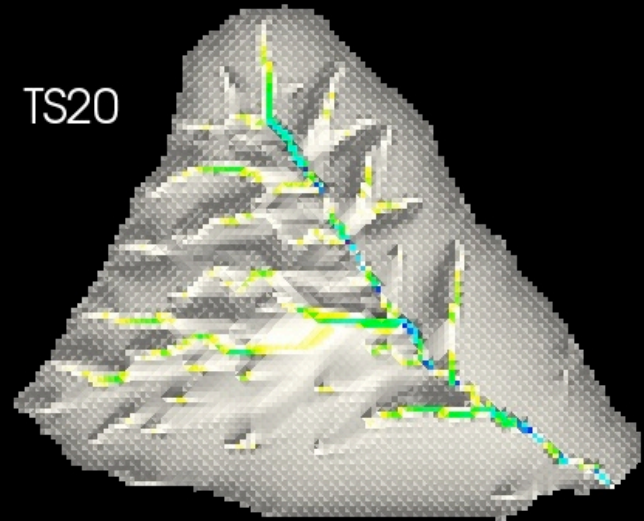
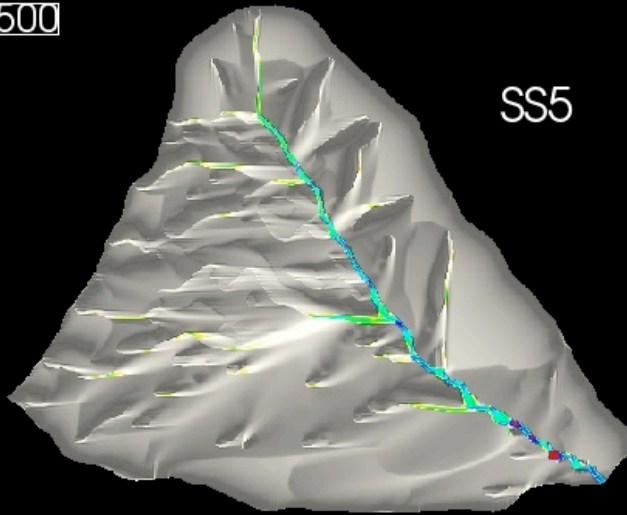
DISCRETIZACION

Time: 1500

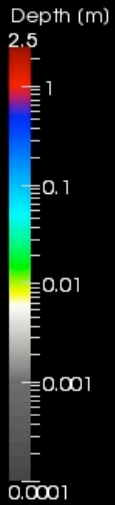
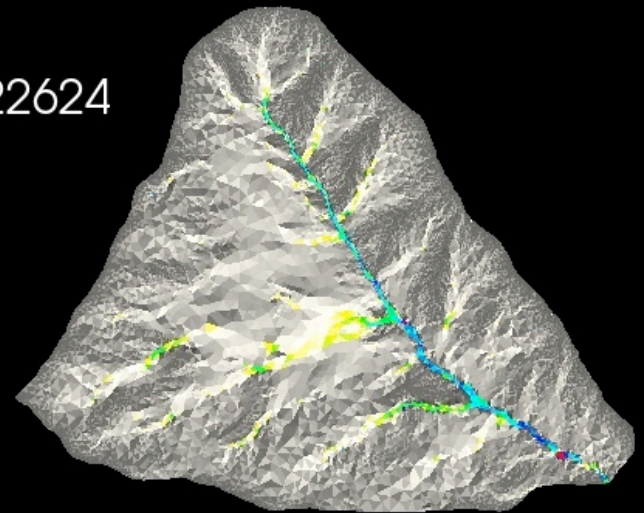


DISCRETIZACION

Time: 2500

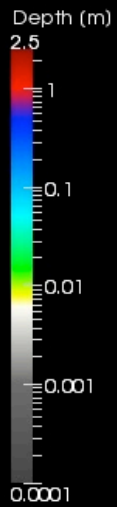
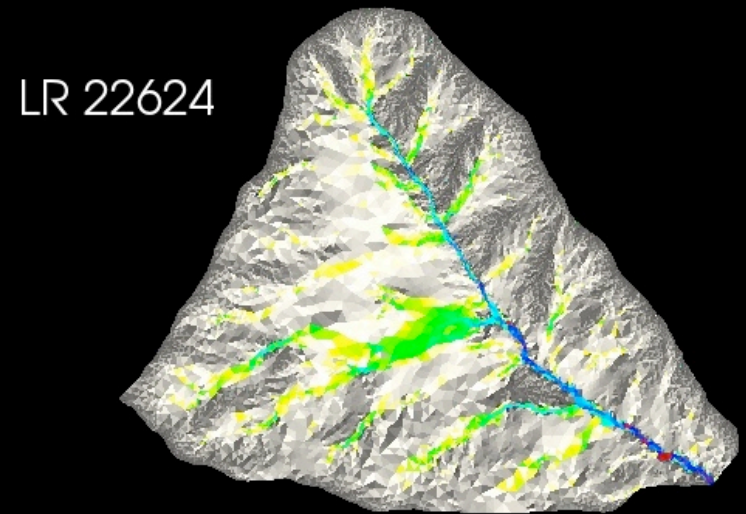
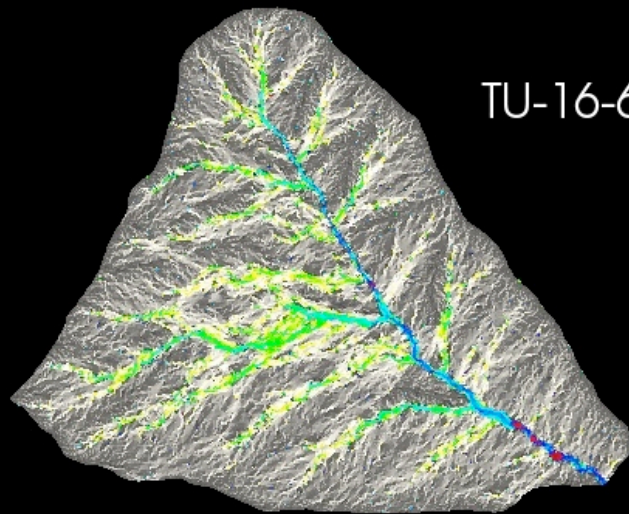
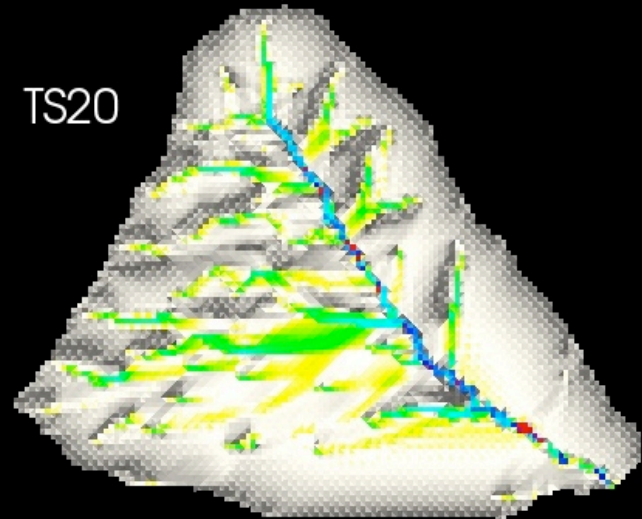
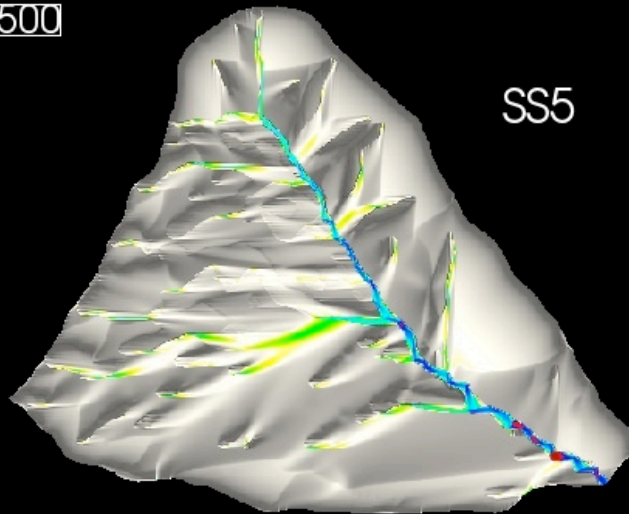


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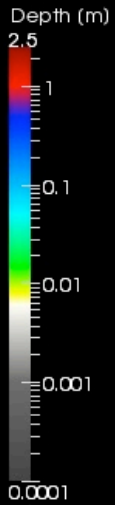
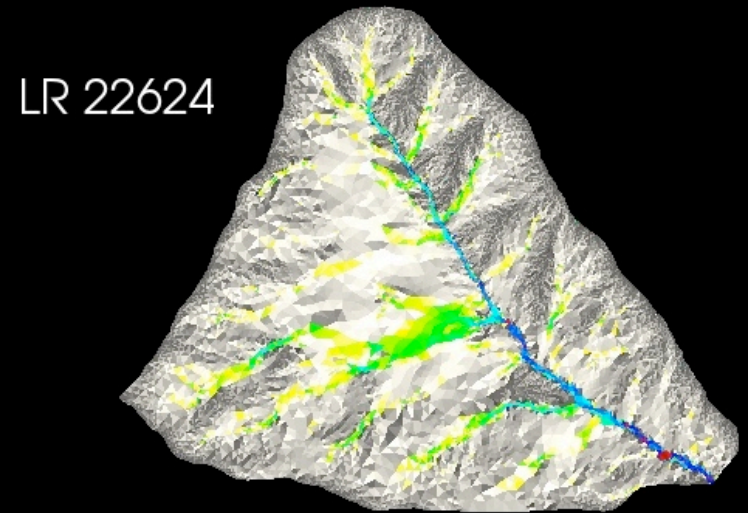
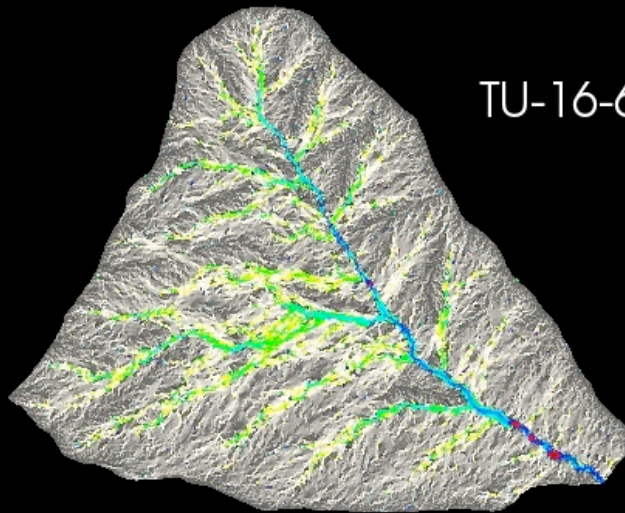
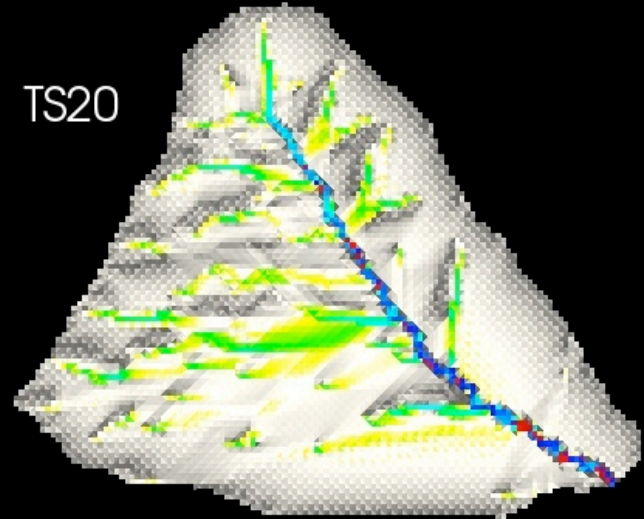
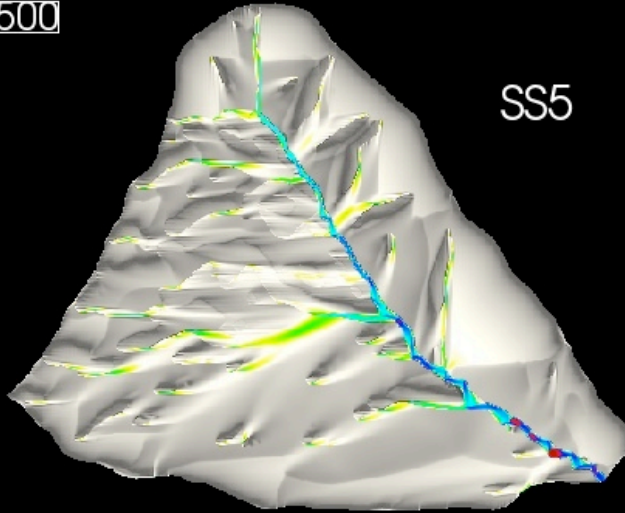
DISCRETIZACION

Time: 3500



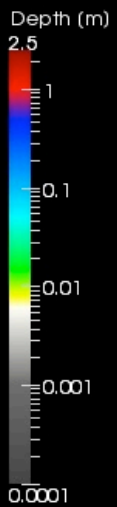
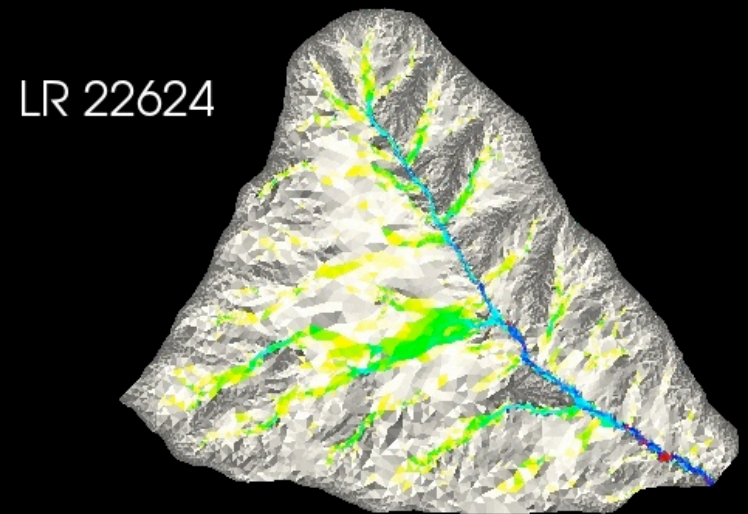
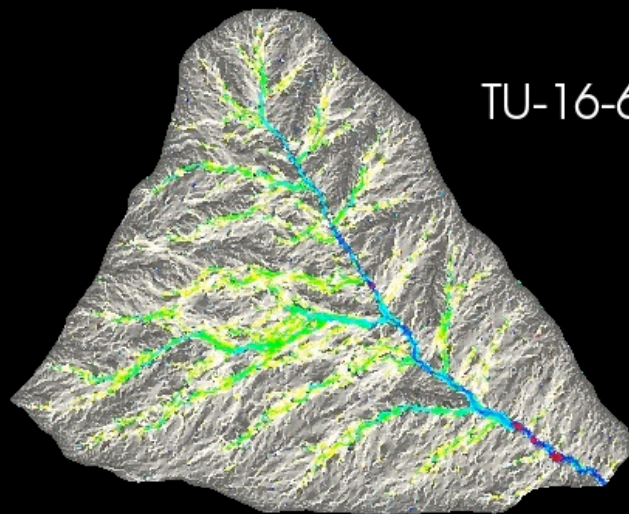
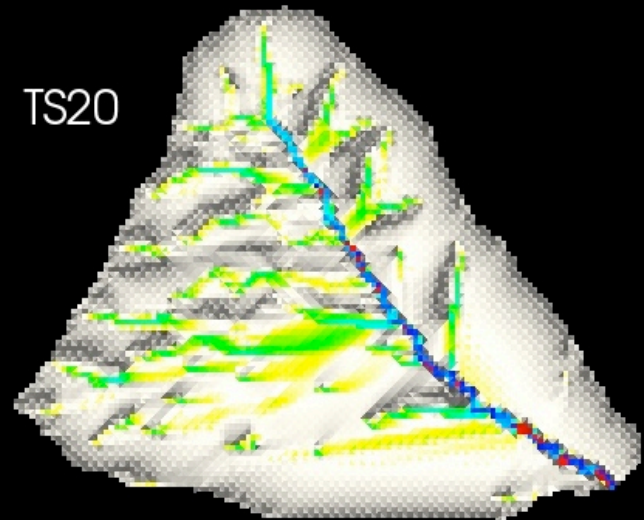
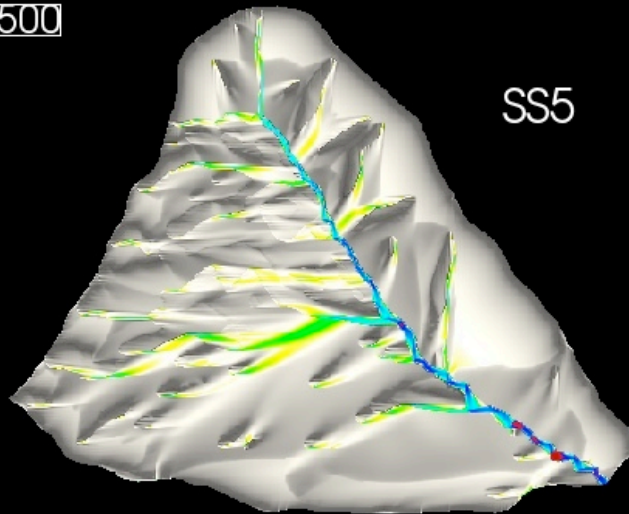
DISCRETIZACION

Time: 4500



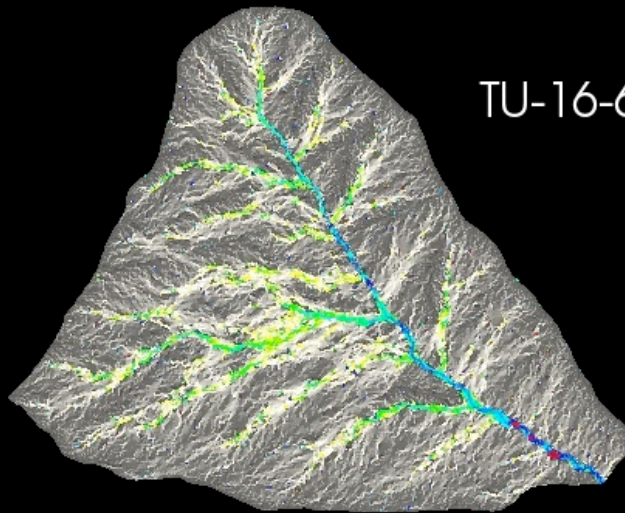
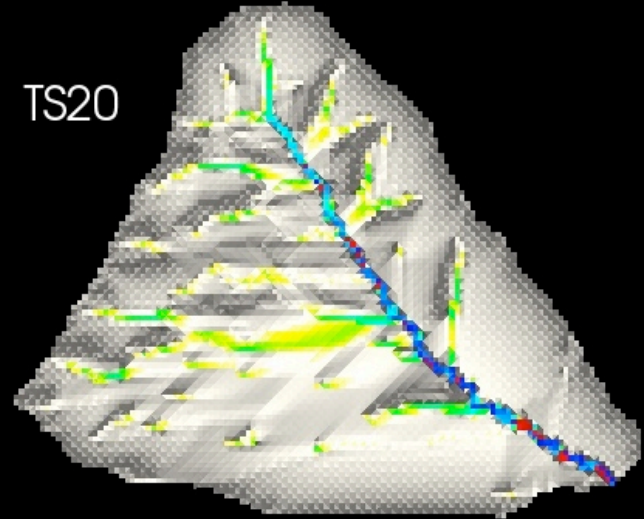
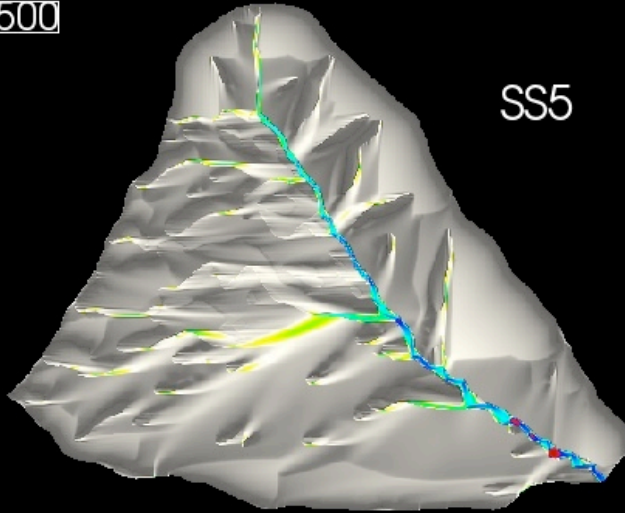
DISCRETIZACION

Time: 5500

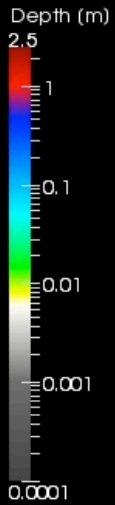
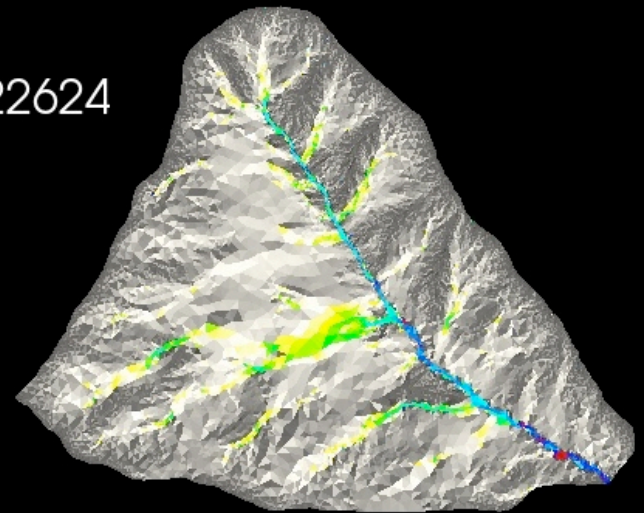


DISCRETIZACION

Time: 6500

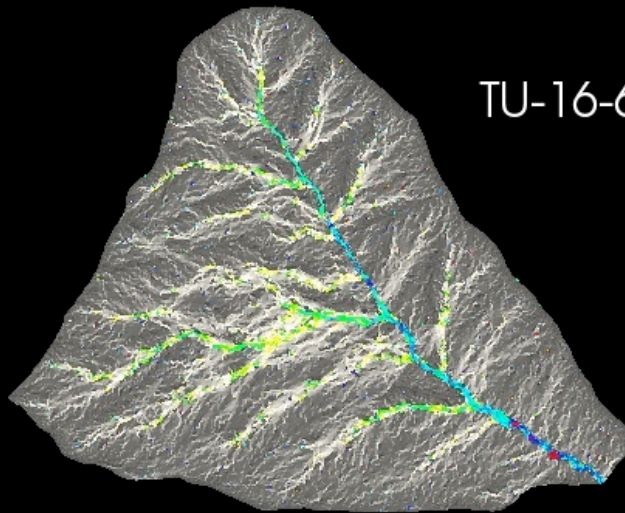
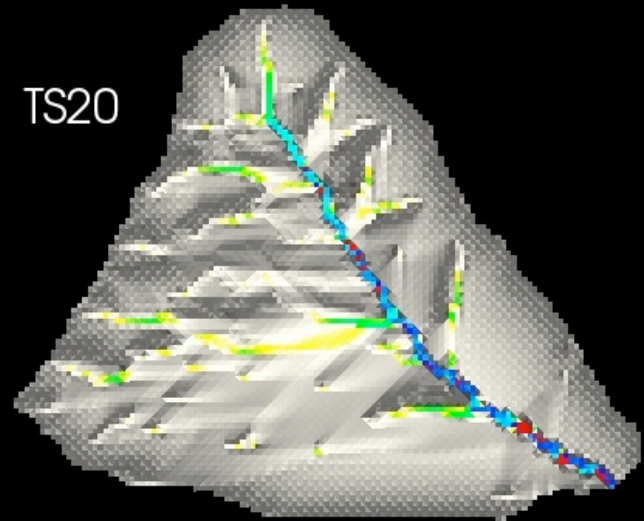
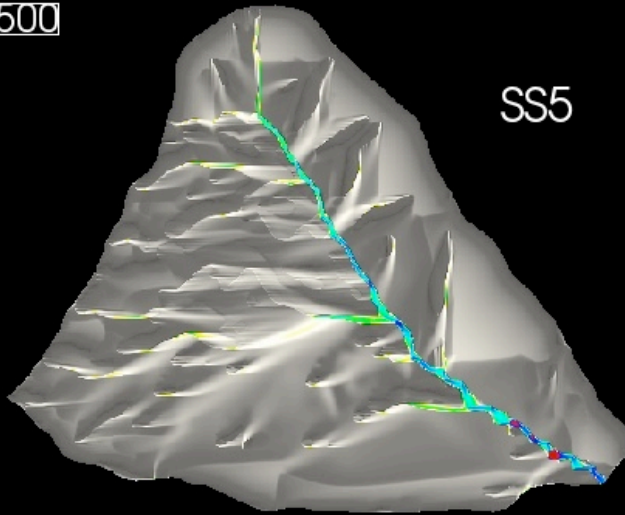


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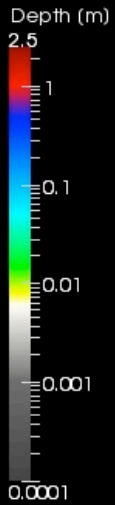
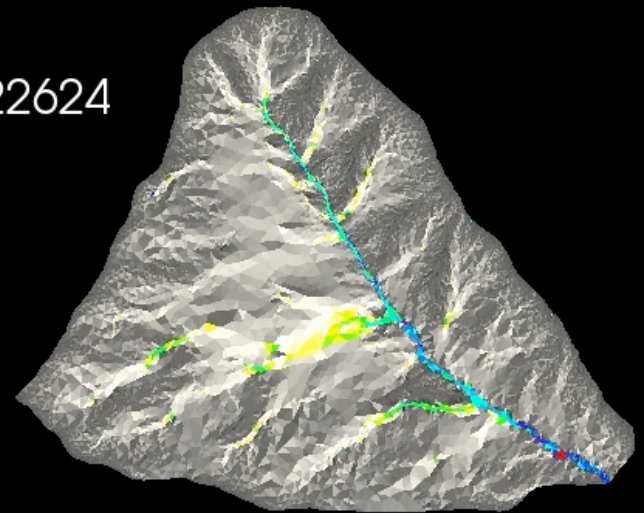


DISCRETIZACION

Time: 7500

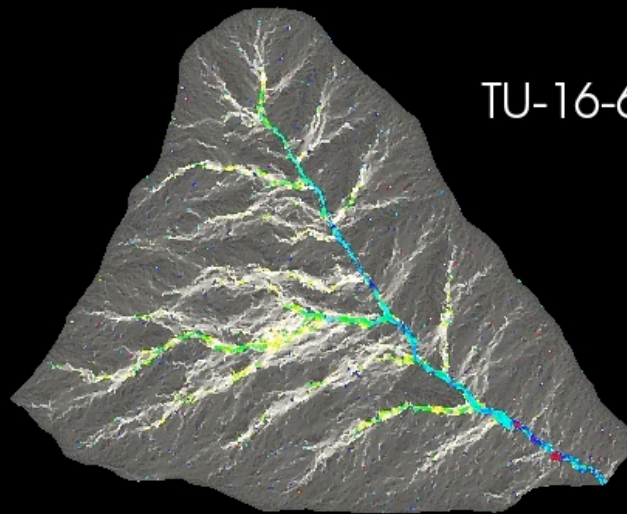
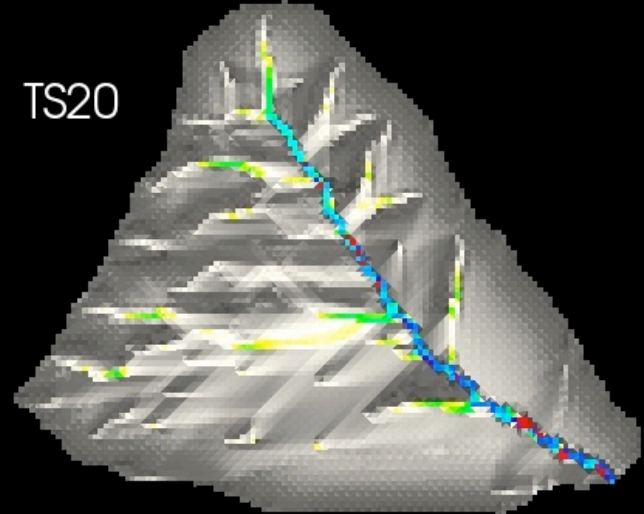
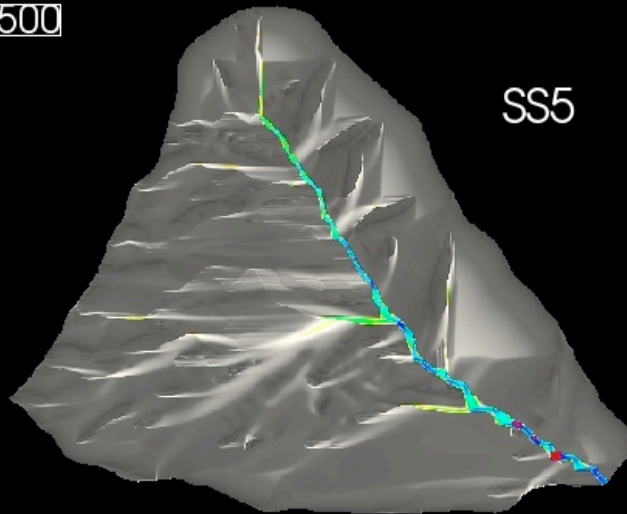


LR 22624

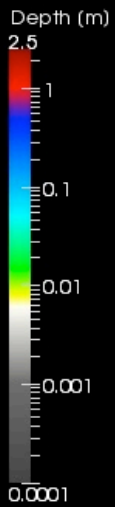
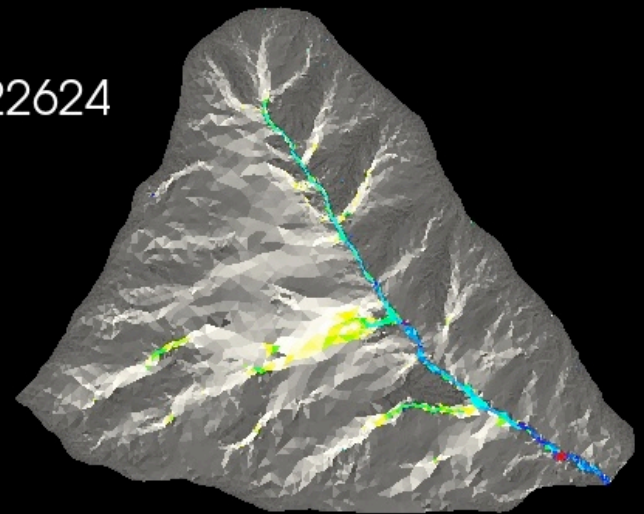


DISCRETIZACION

Time: 8500

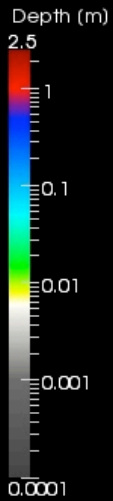
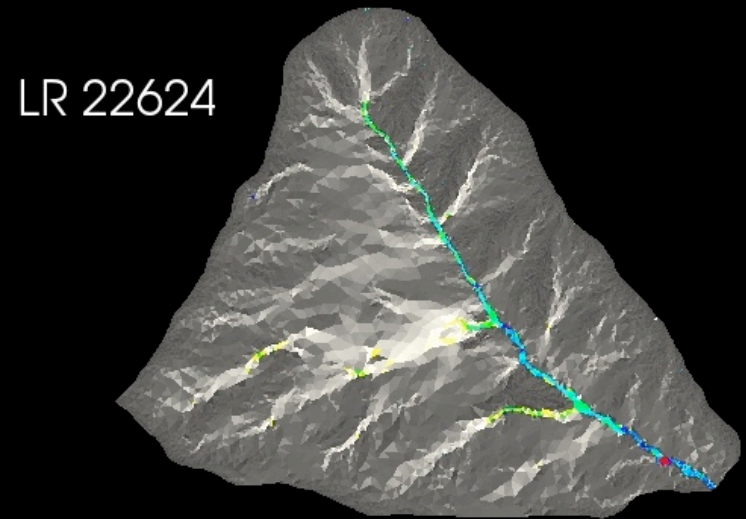
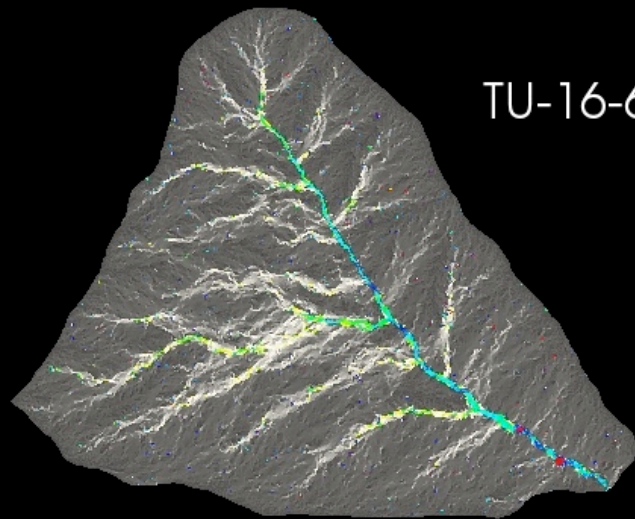
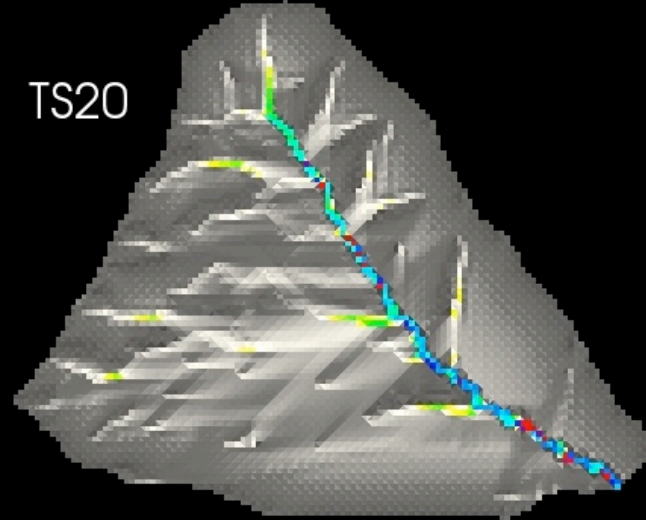
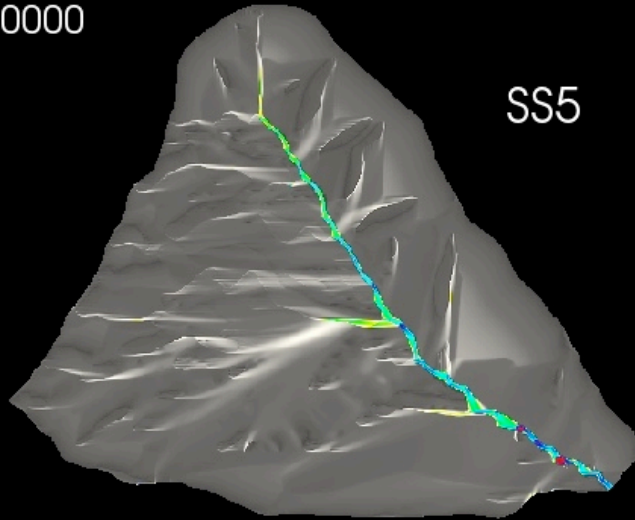


LR 22624



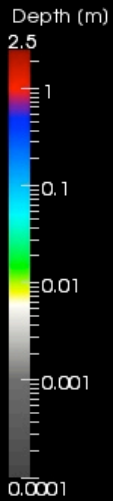
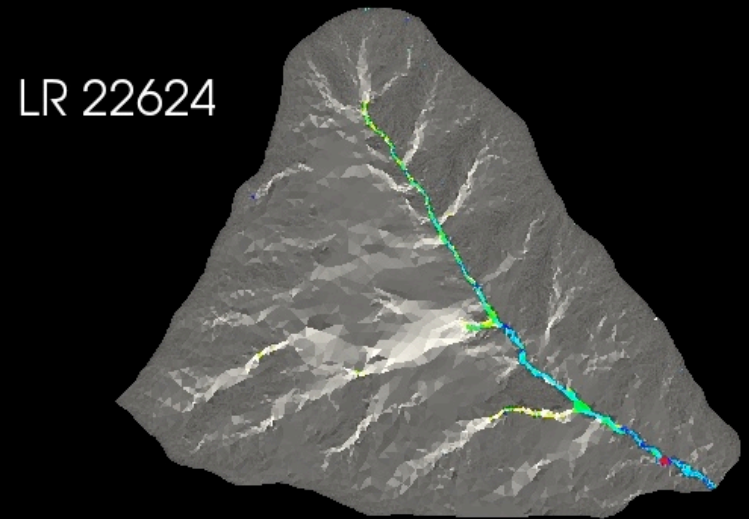
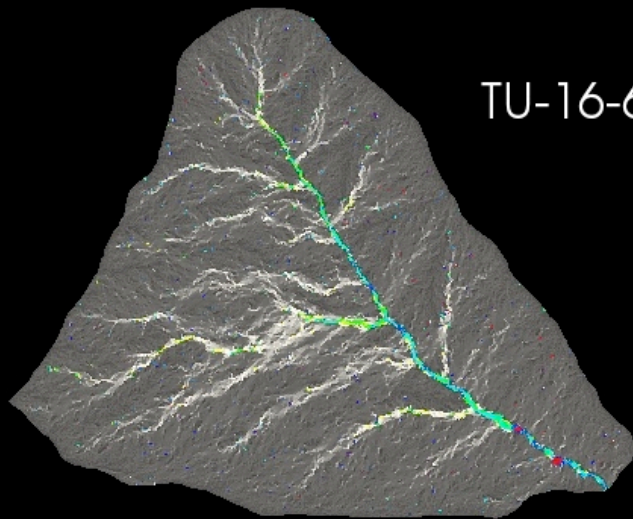
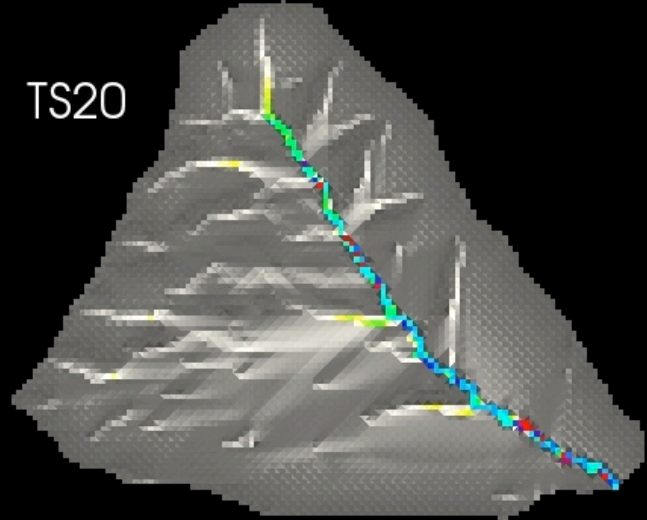
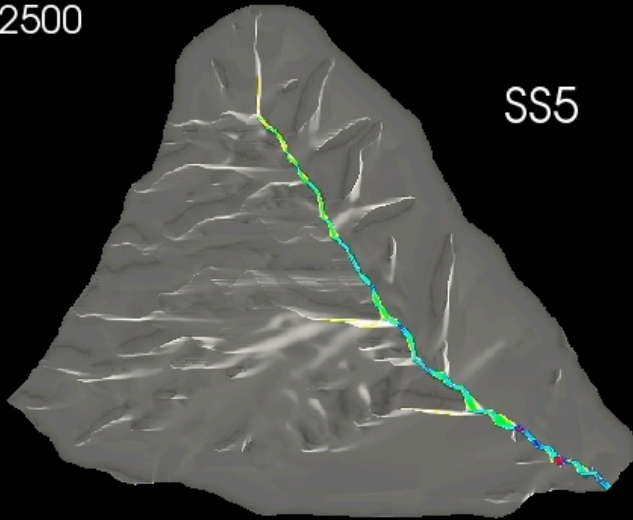
DISCRETIZACION

Time: 10000



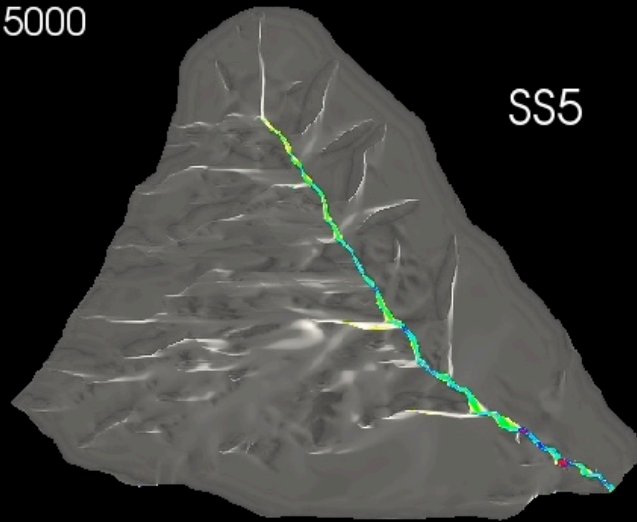
DISCRETIZACION

Time: 12500

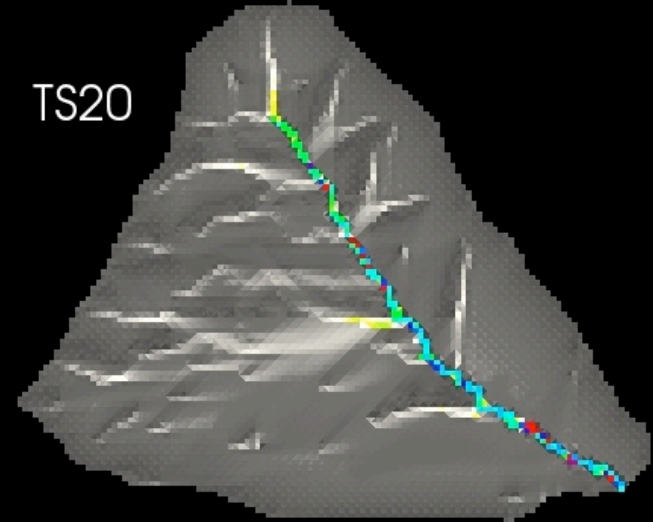


DISCRETIZACION

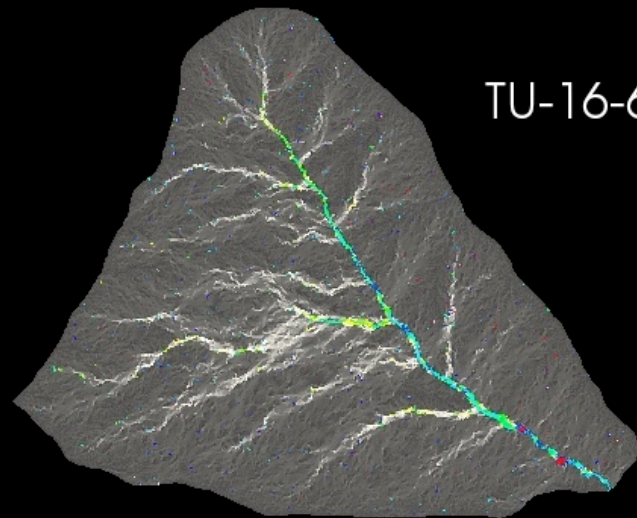
Time: 15000



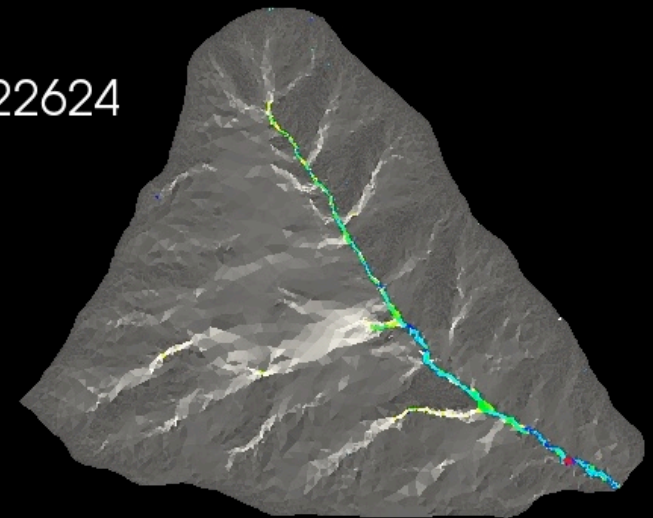
SS5



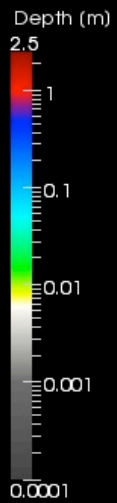
TS20



TU-16-60



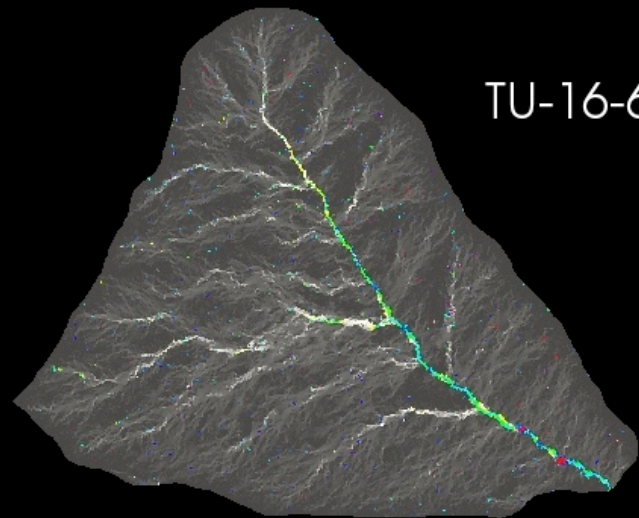
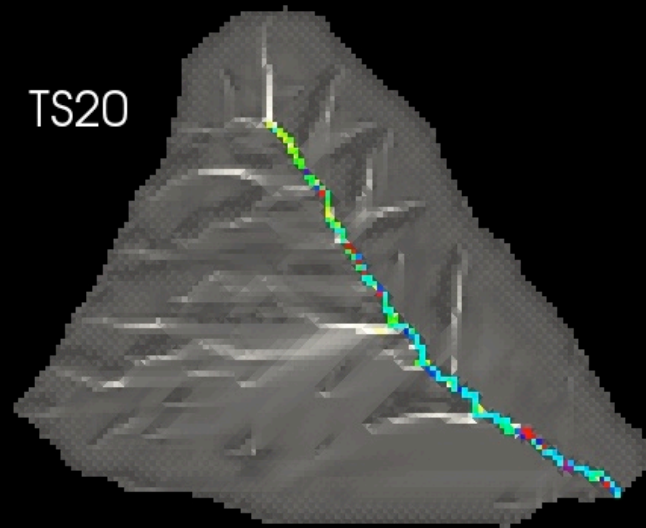
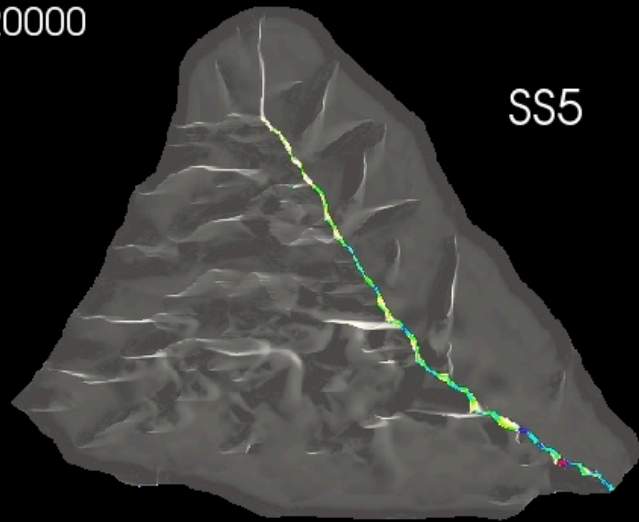
LR 22624



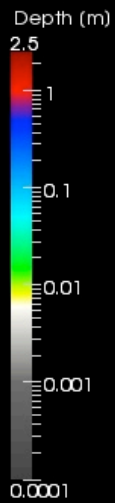
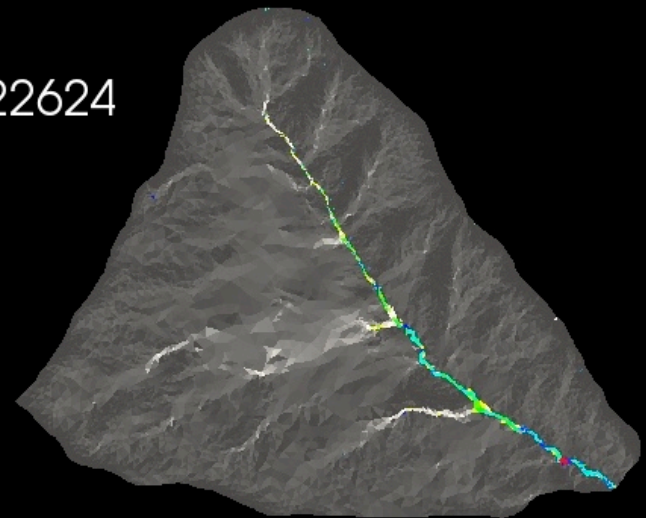
Depth (m)
2.5
1
0.1
0.01
0.001
0.0001

DISCRETIZACION

Time: 20000



LR 22624



DISCRETIZACION

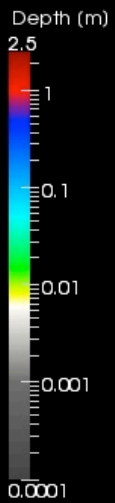
Time: 25000

SS5

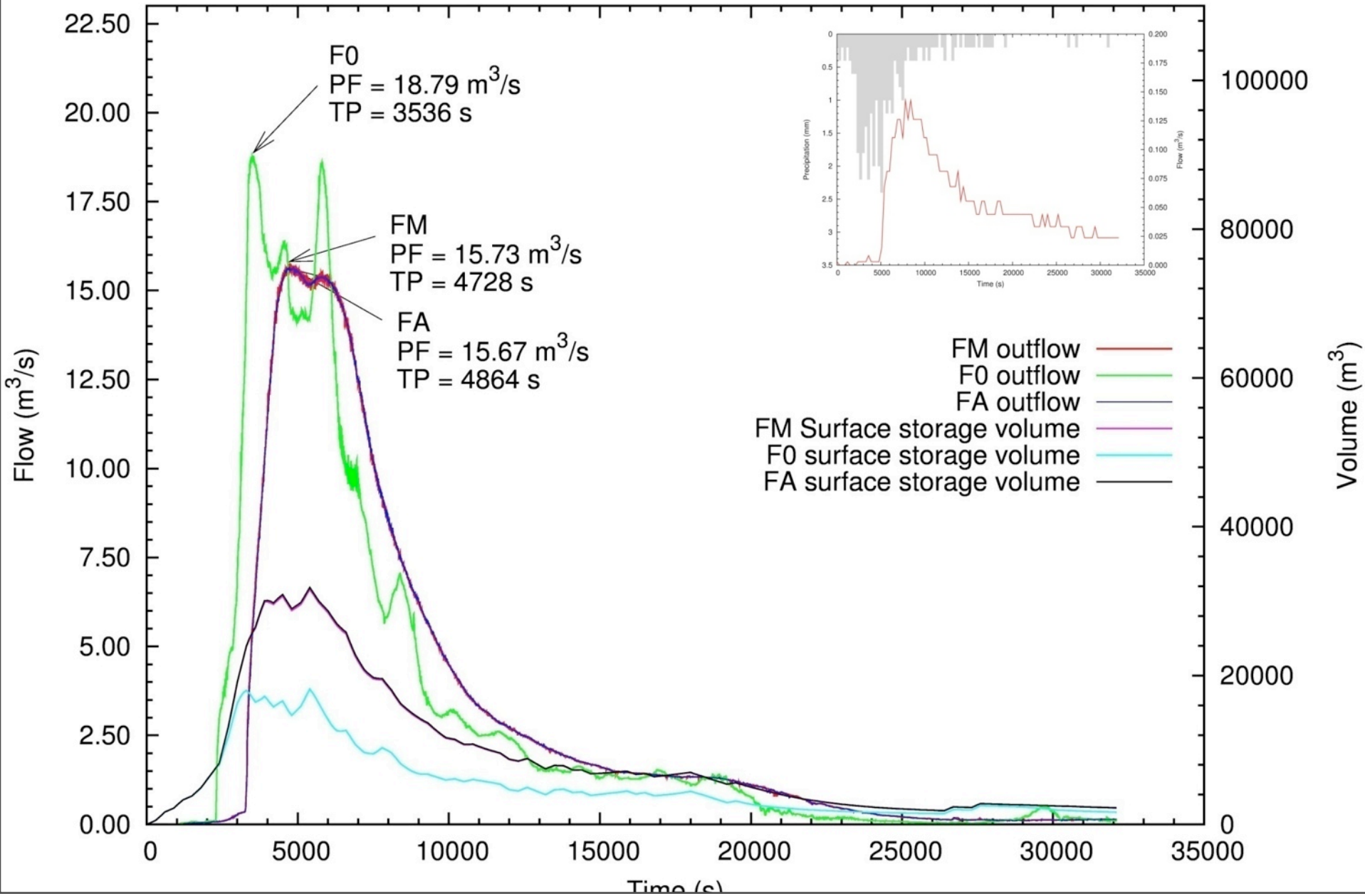
TS20

TU-16-60

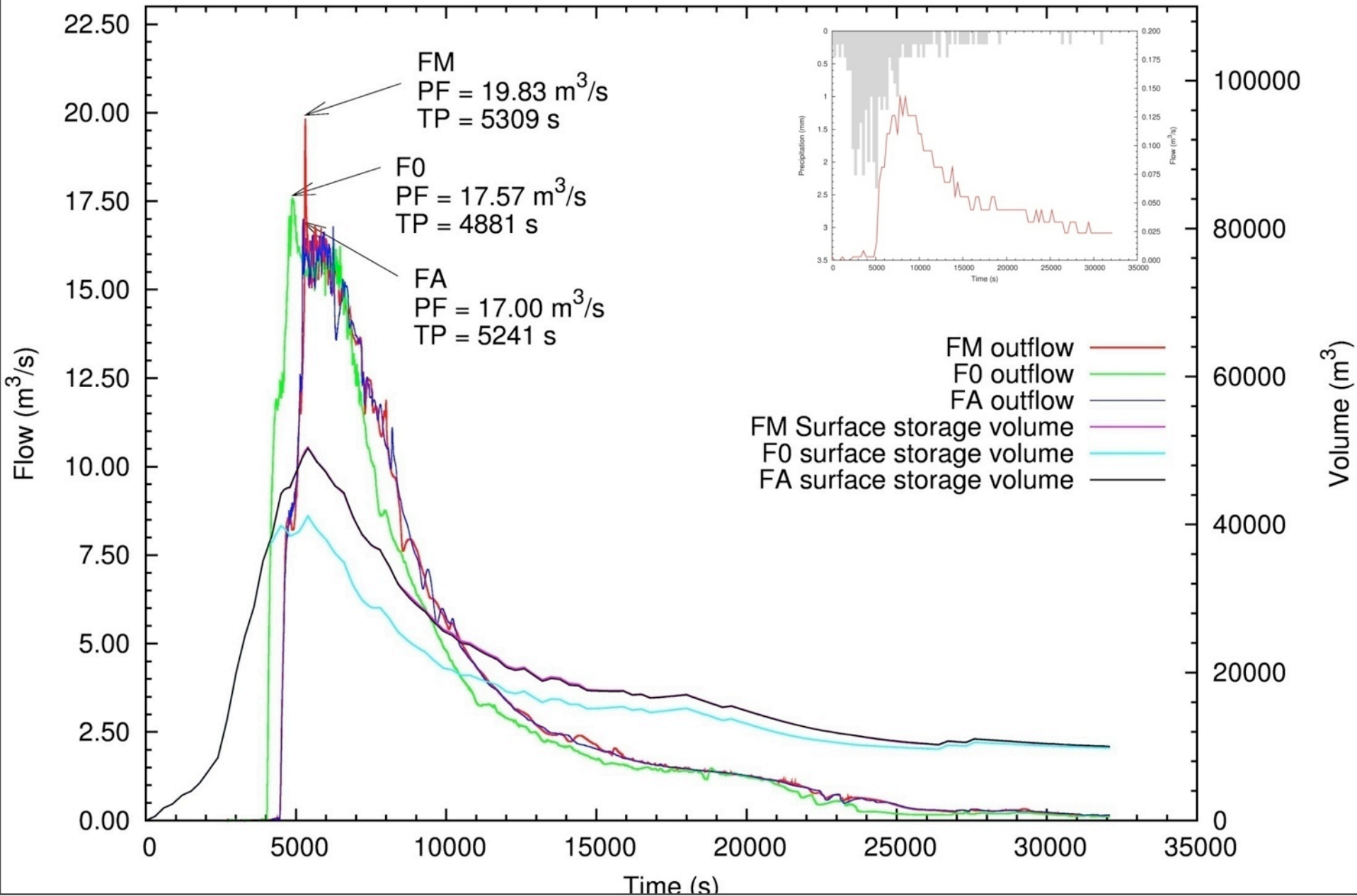
LR 22624



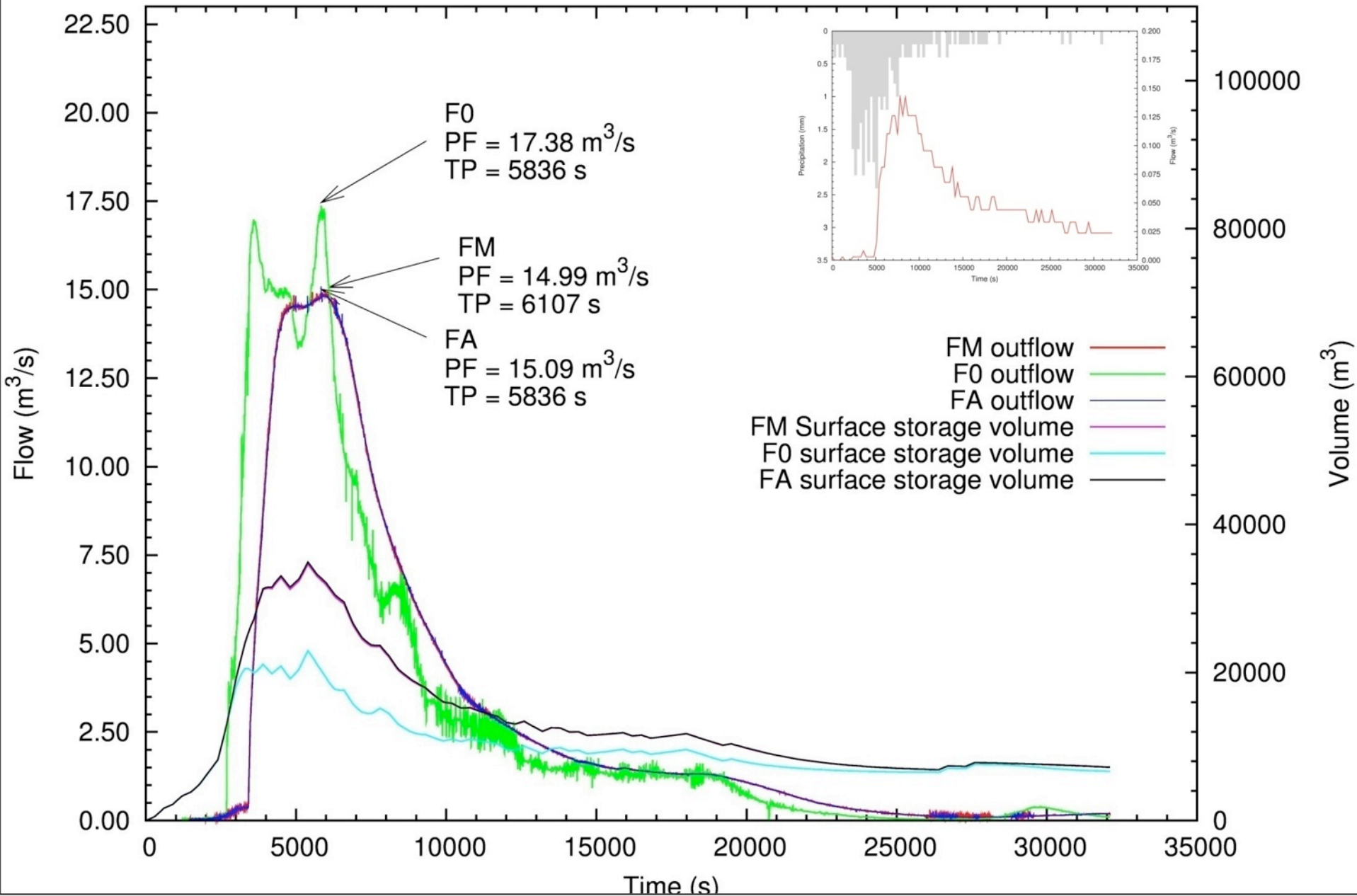
Malla cuadrada estructurada de lado 5m (SS5)



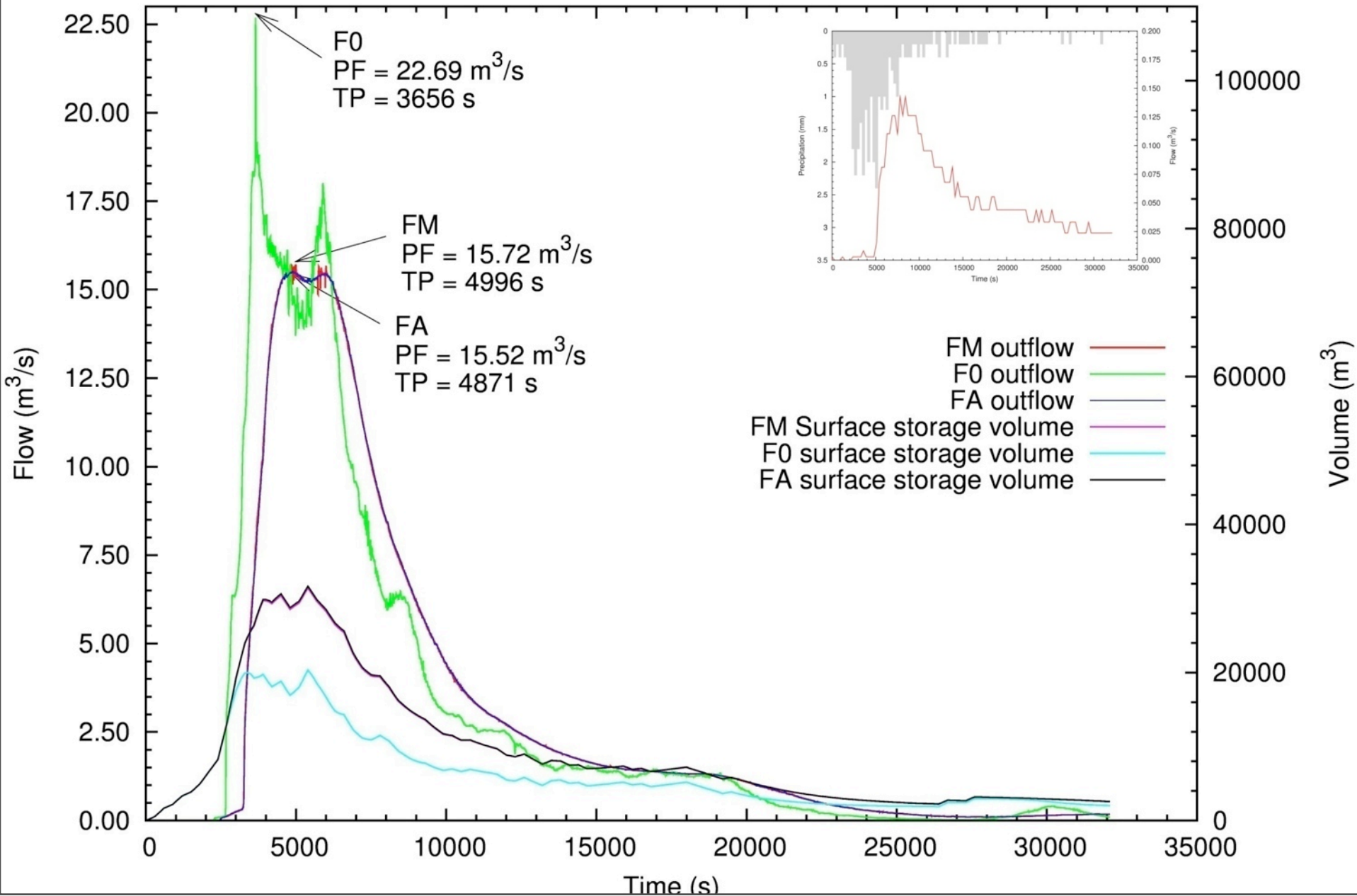
Malla cuadrada estructurada de lado 20m (SS20)



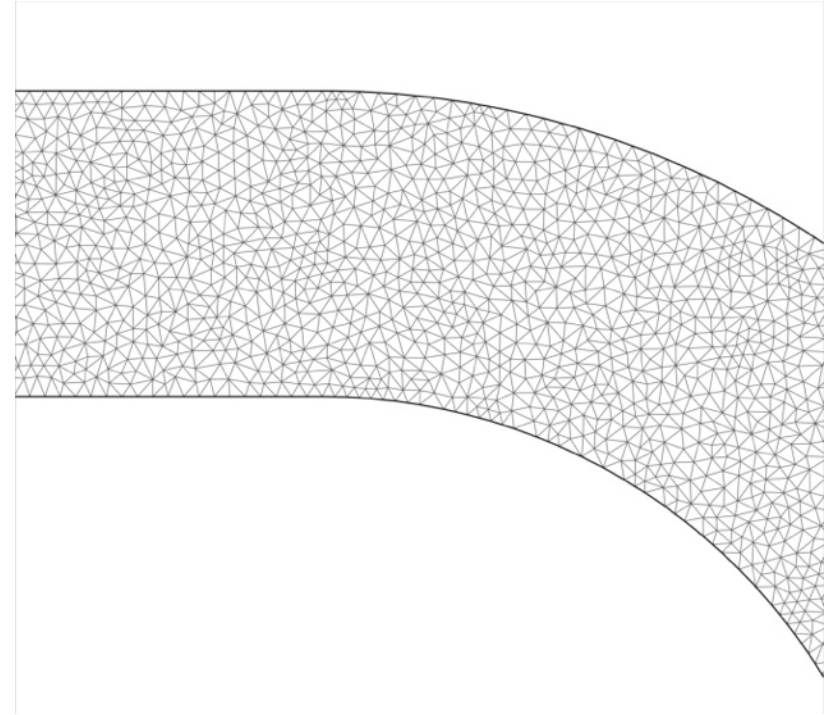
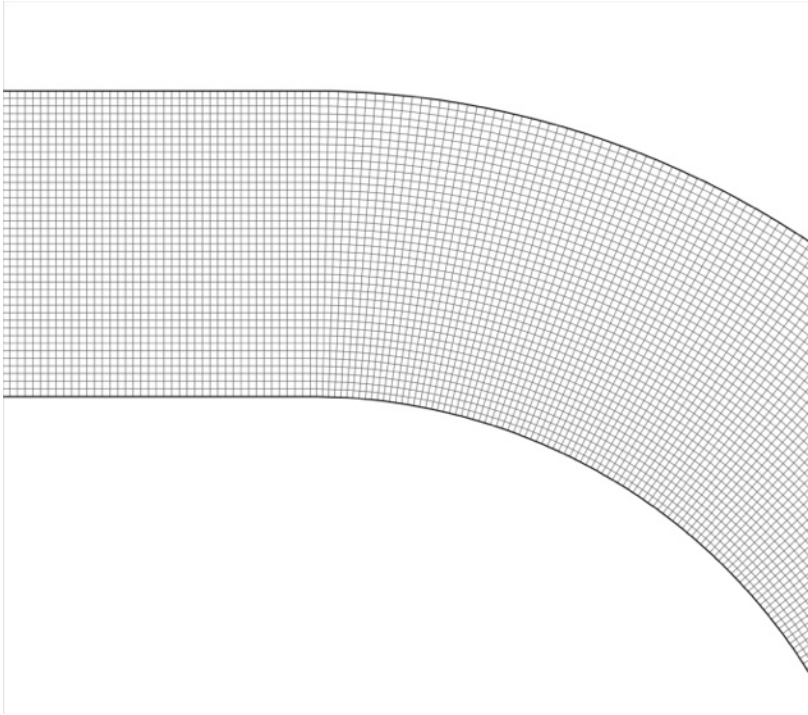
Malla triangular no estructurada (TU1660)



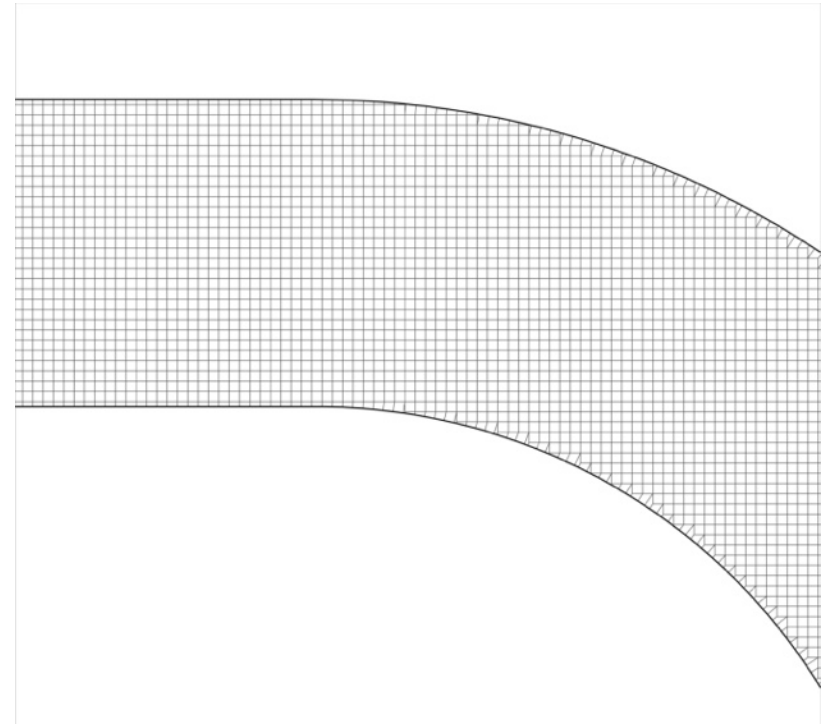
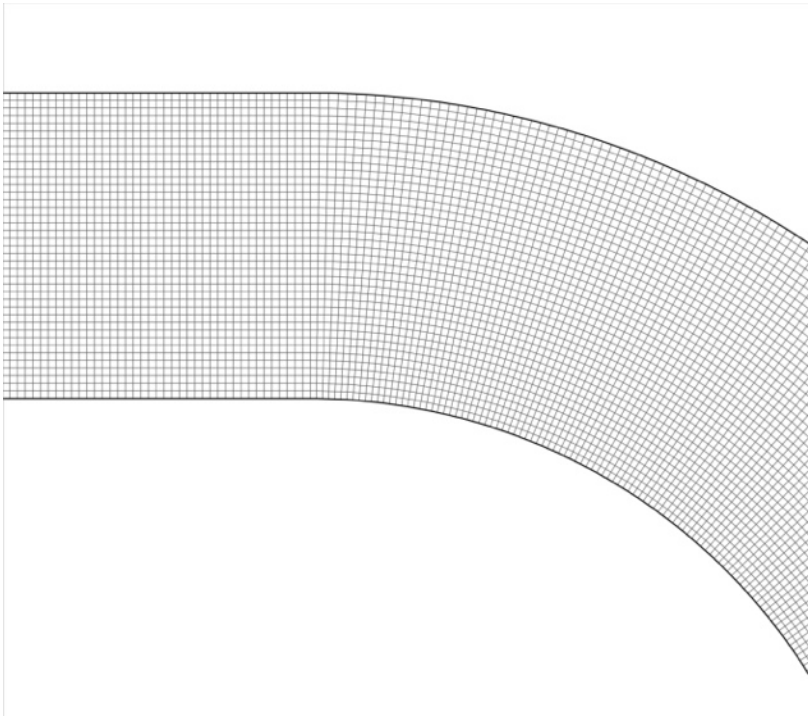
Malla triangular no estructurada adaptada (22624)



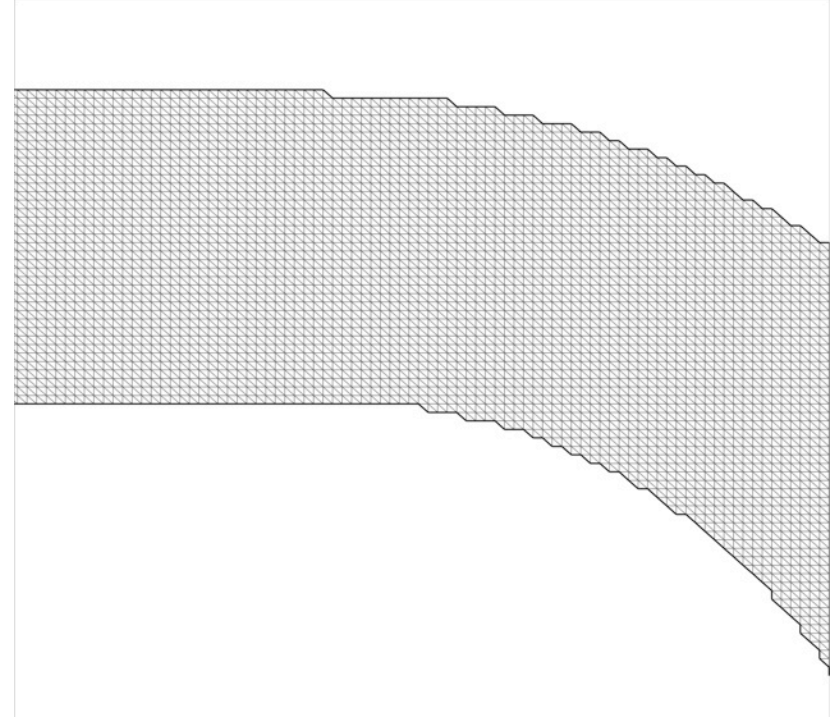
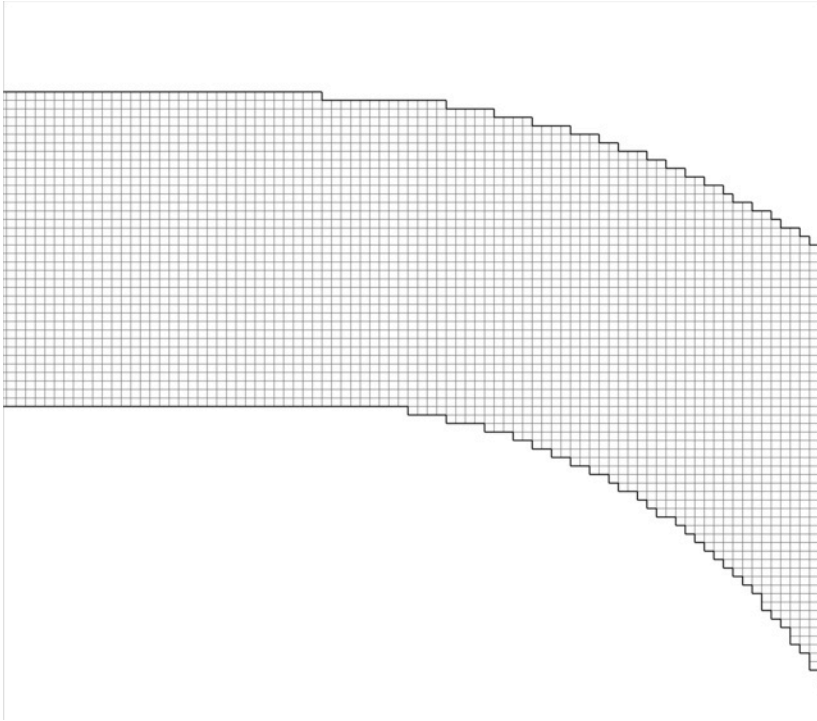
Mallas ajustadas al contorno



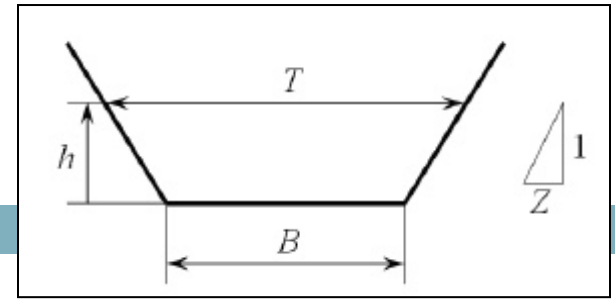
Mallas ajustadas al contorno



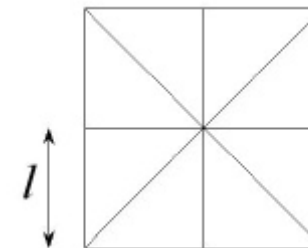
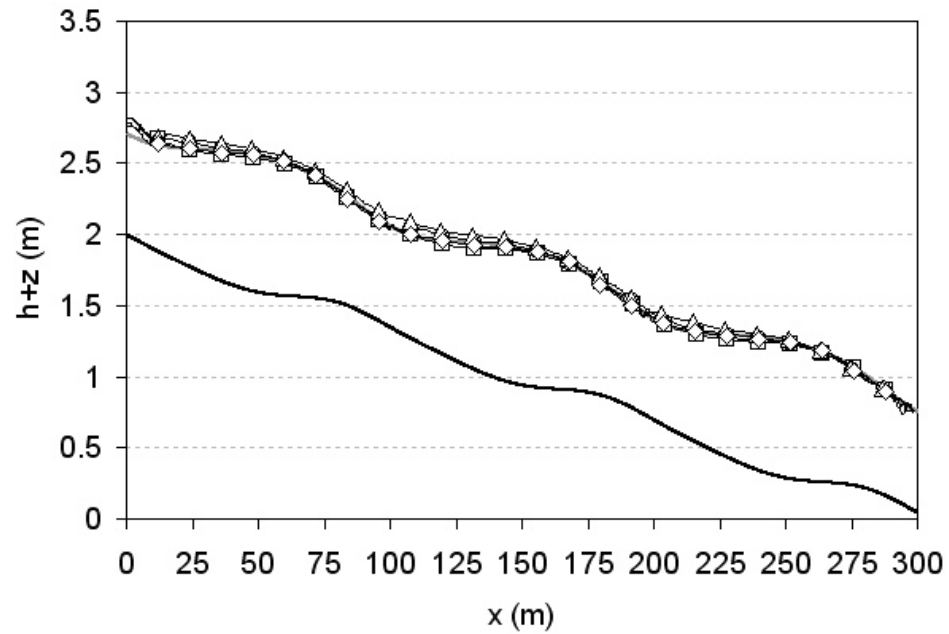
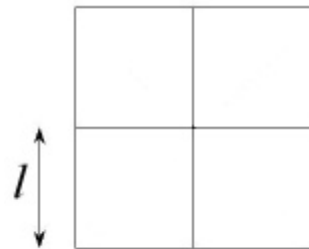
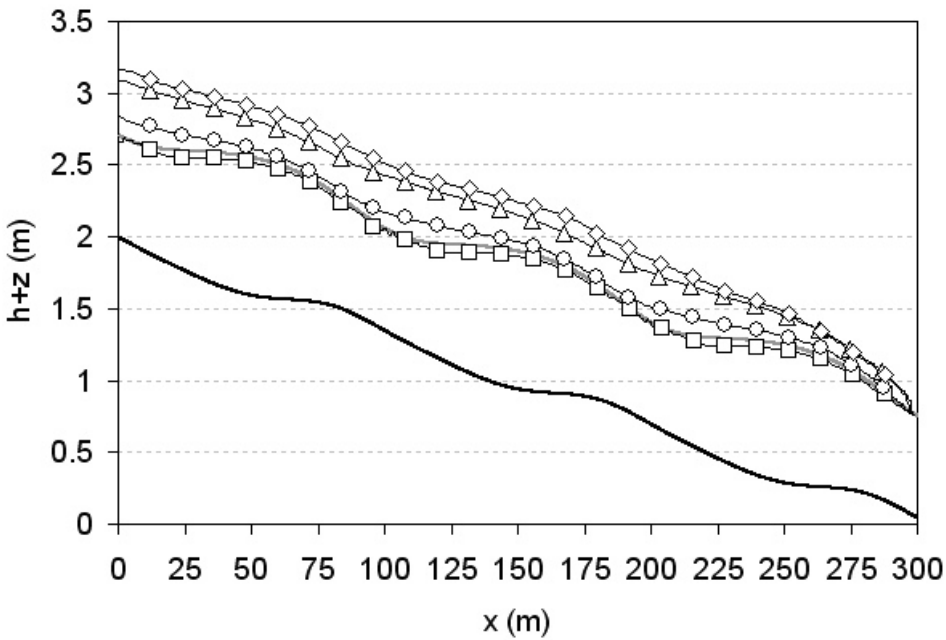
Mallas NO ajustadas al contorno



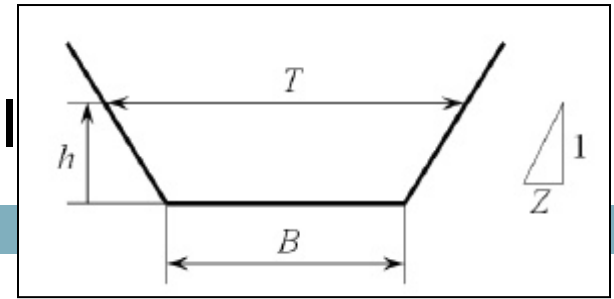
Flujo subcrítico canal trapezoidal



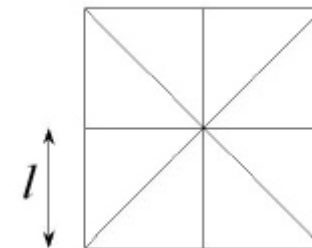
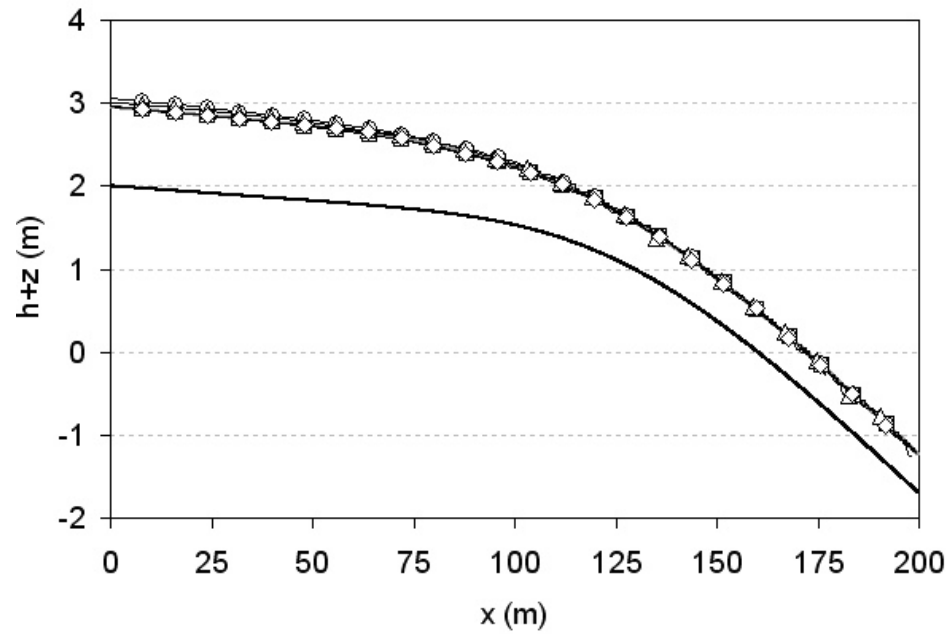
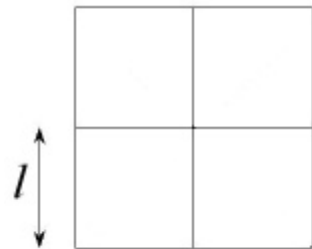
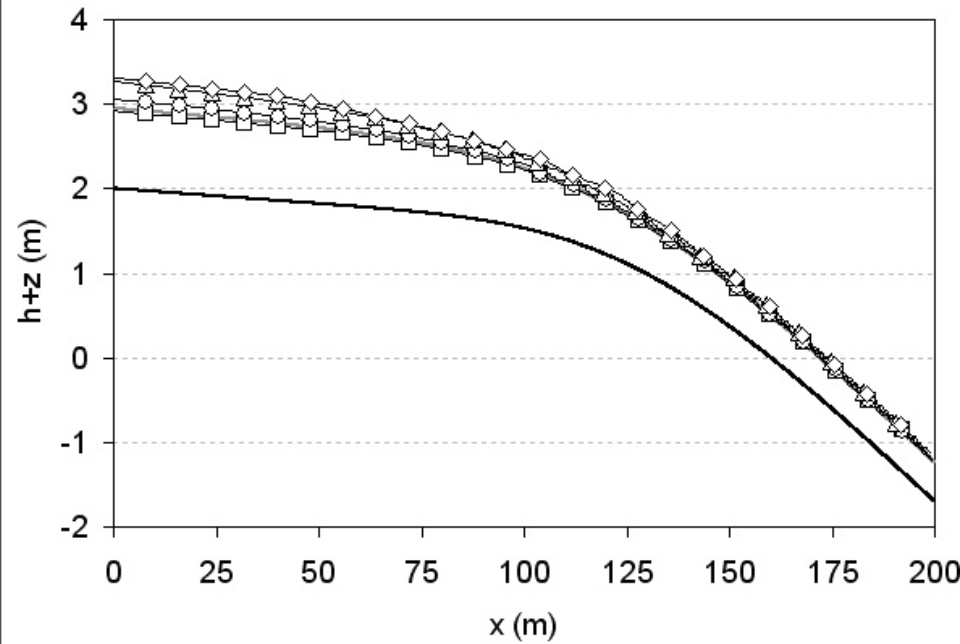
— $h+z$ exact — z □ 0° ○ 15° △ 30° ◇ 45°



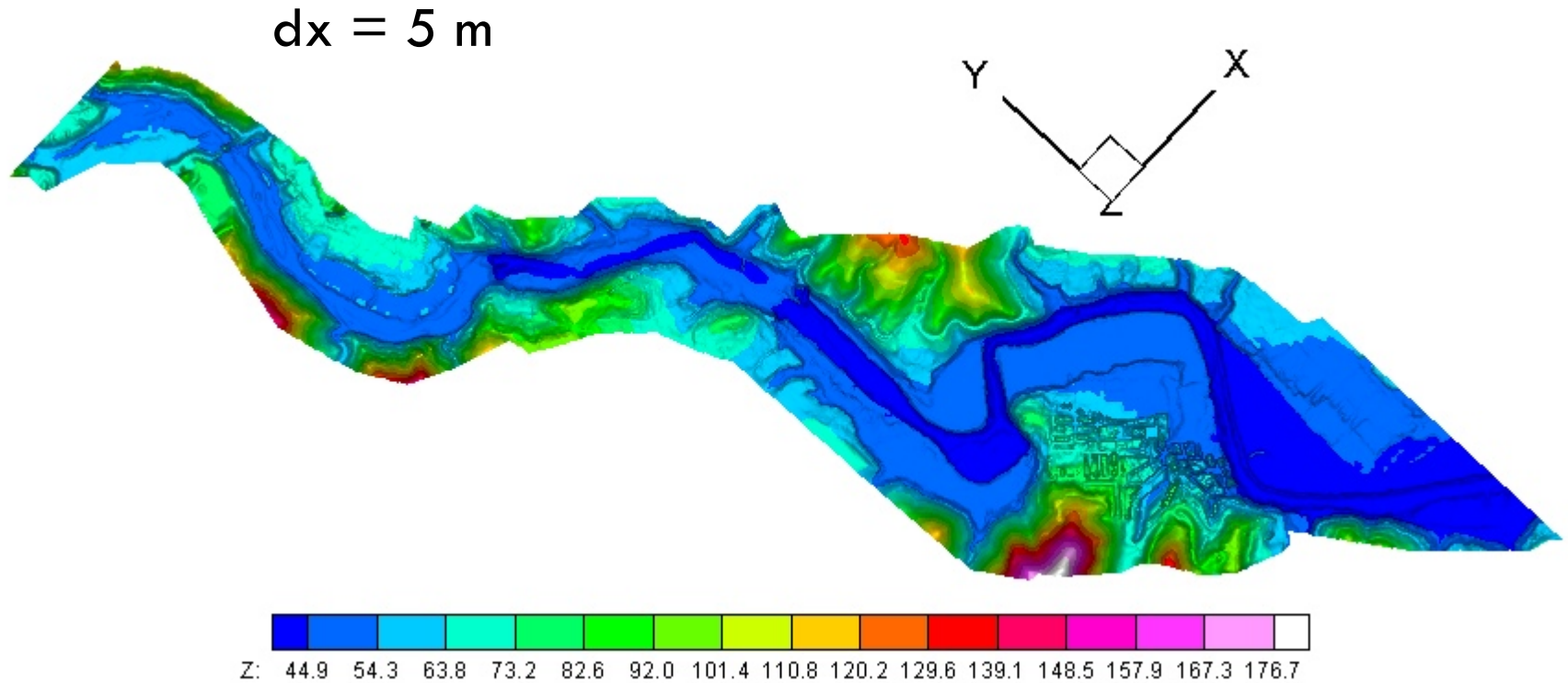
Flujo sub-super crítico canal trapezoidal



— $h+z$ exact — z □ 0° ○ 15° △ 30° ◇ 45°

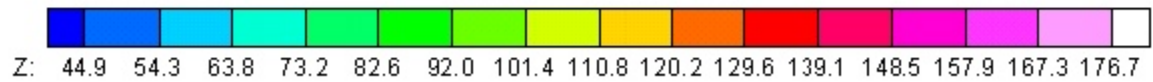
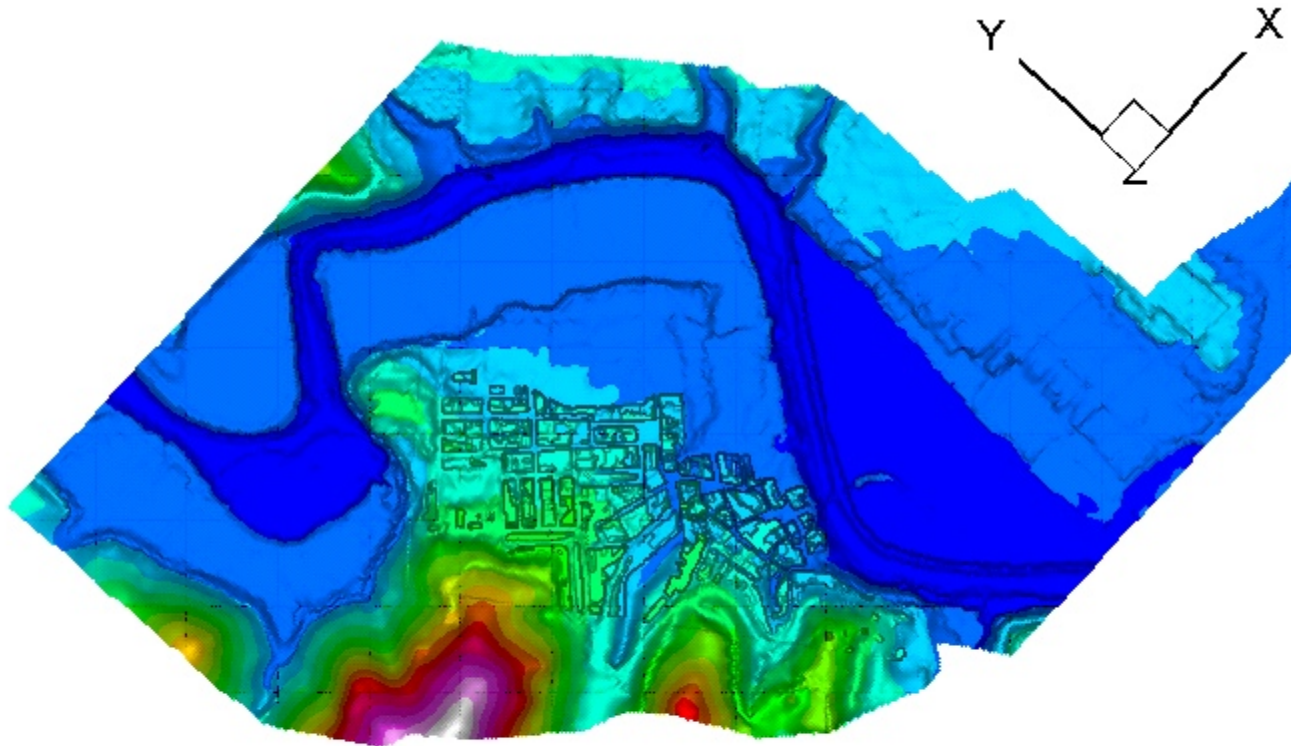


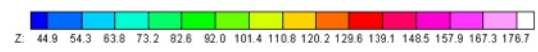
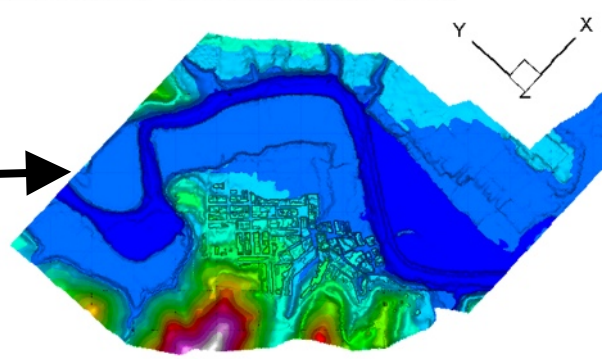
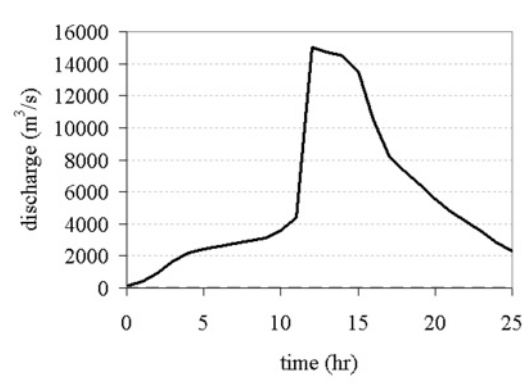
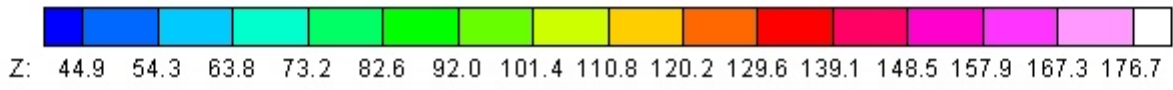
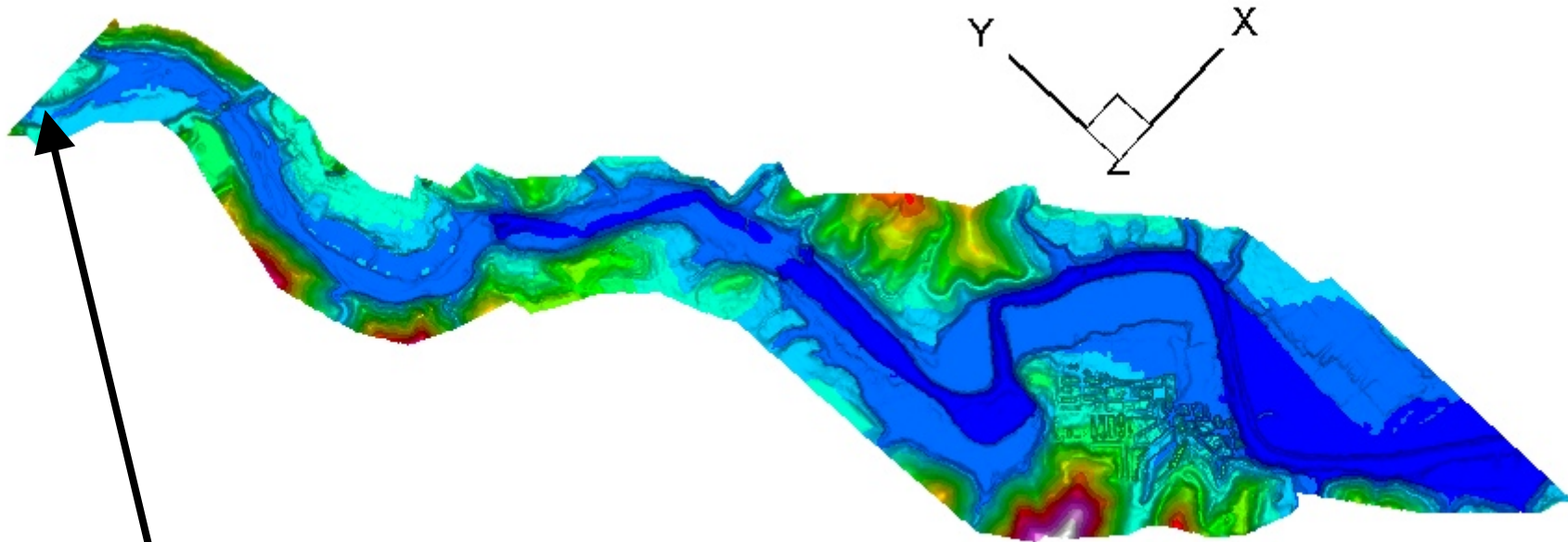
Inundación de Tous-Sumacárcel



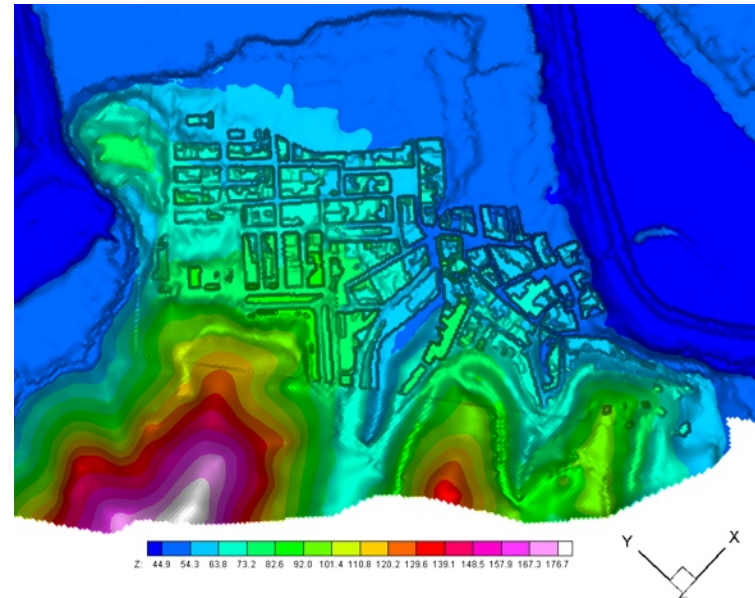
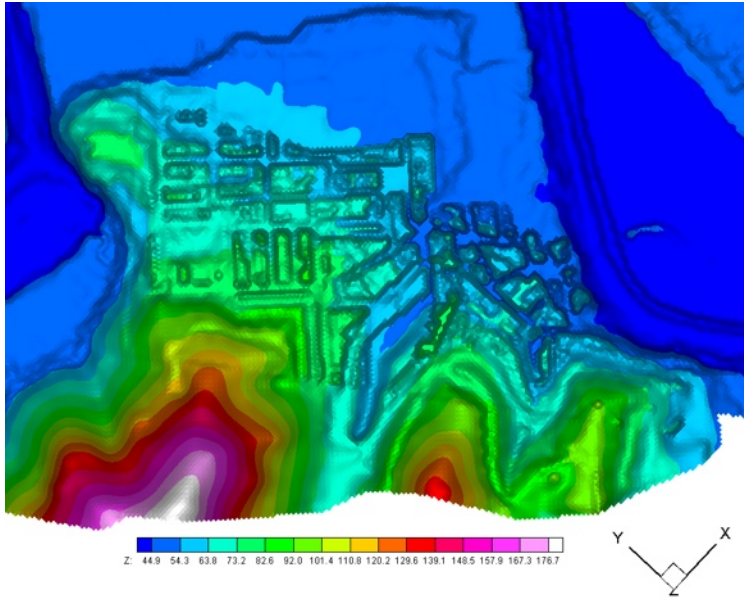
Detalle del pueblo

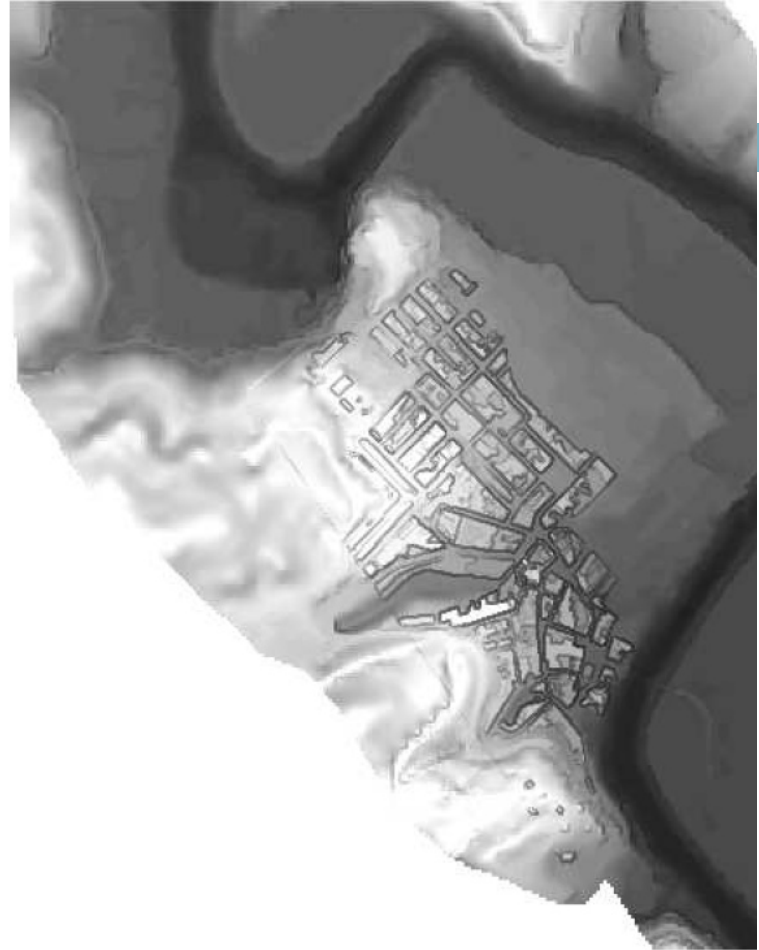
$dx = 2.5$

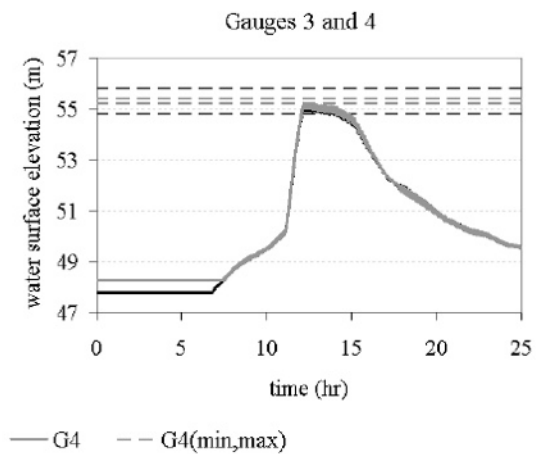
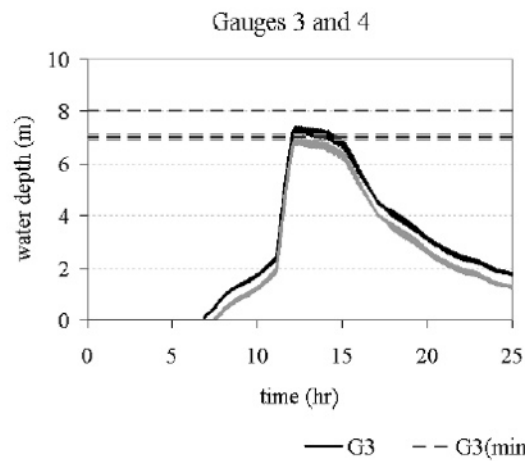
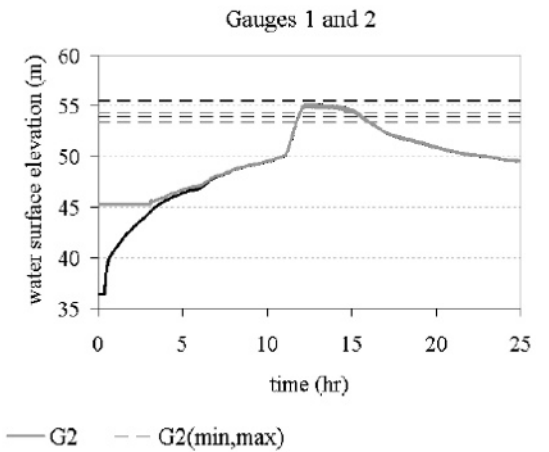
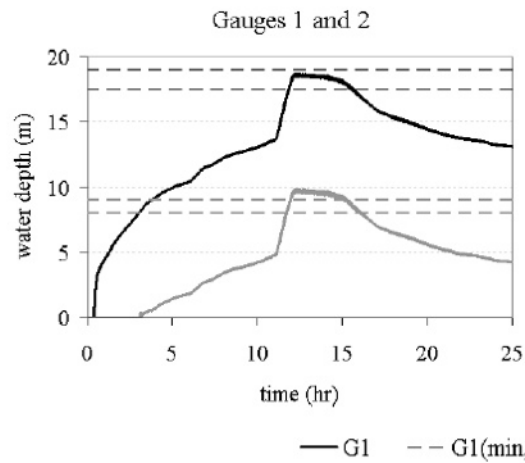
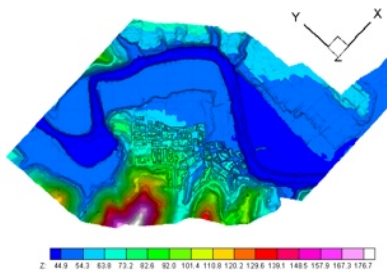


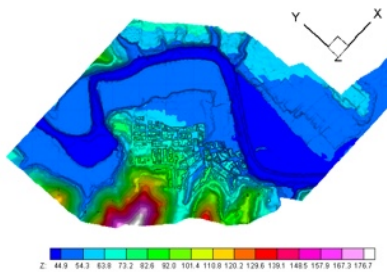


El pueblo con las dos resoluciones de malla

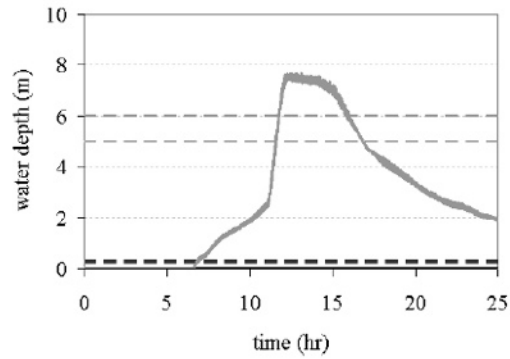




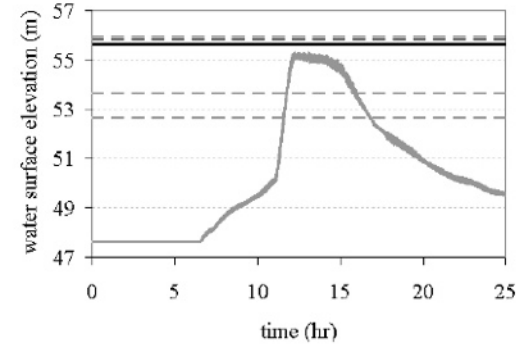




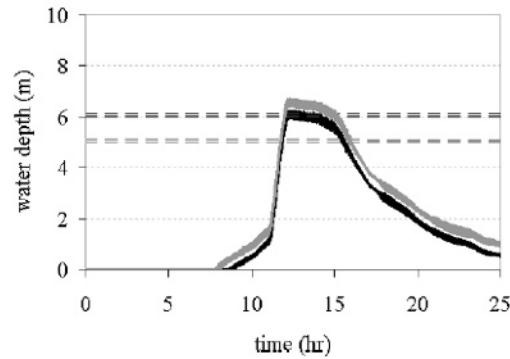
Gauges 5 (dry) and 6



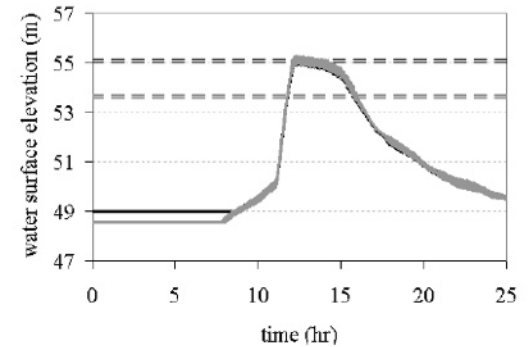
Gauges 5 and 6



Gauges 7 and 8

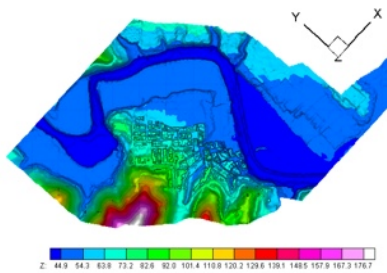


Gauges 7 and 8

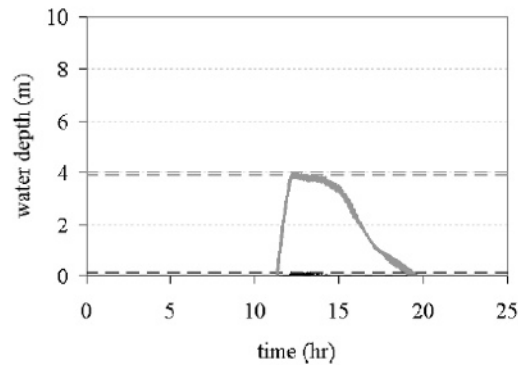


— G5 - - G5(min,max) — G6 - - G6(min,max)

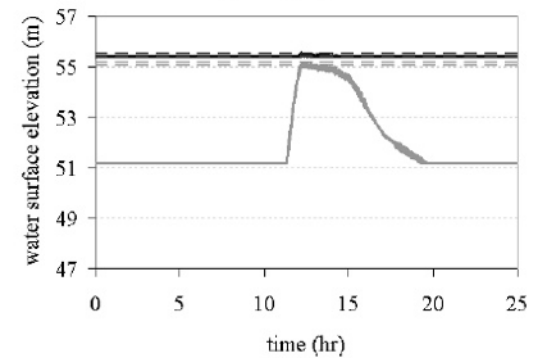
— G7 - - G7(min,max) — G8 - - G8(min,max)



Gauges 9 (dry) and 10

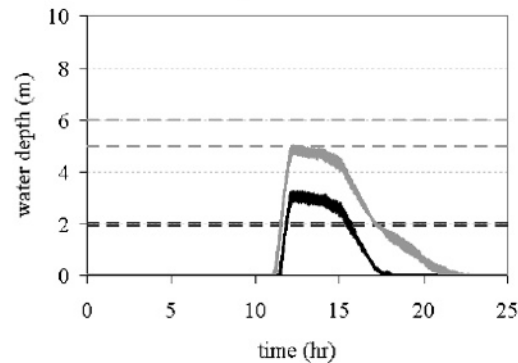


Gauges 9 (dry) and 10

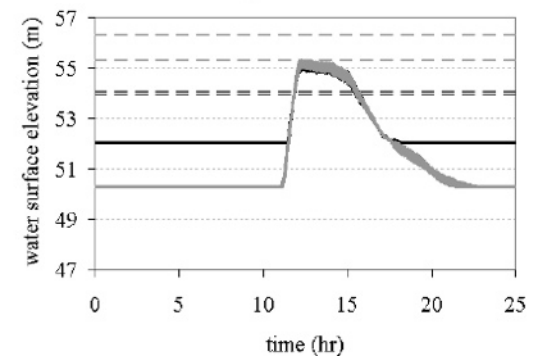


— G9 - - G9(min,max) — G10 - - G10(min,max)

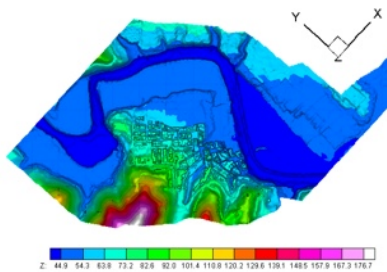
Gauges 11 and 12



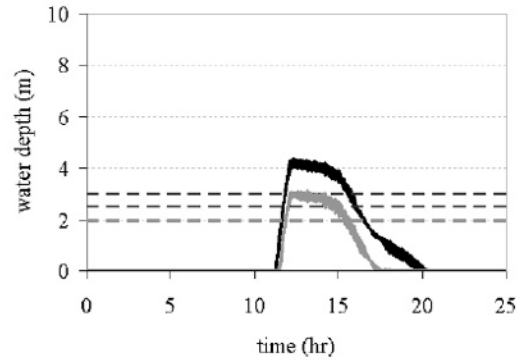
Gauges 11 and 12



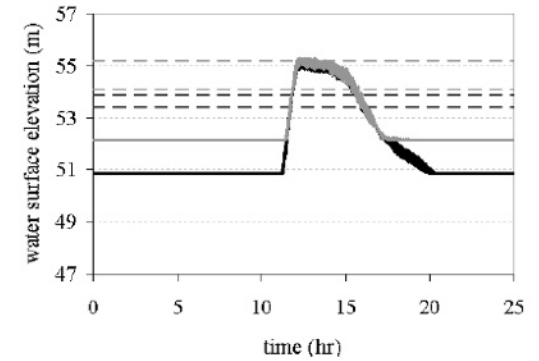
— G11 - - G11(min,max) — G12 - - G12(min,max)



Gauges 13 and 14

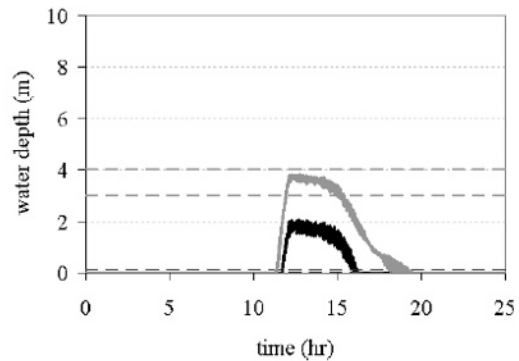


Gauges 13 and 14

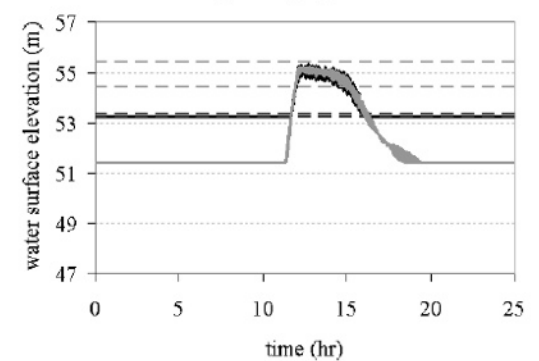


— G13 - - - G13(min,max) — G14 - - - G14(min,max)

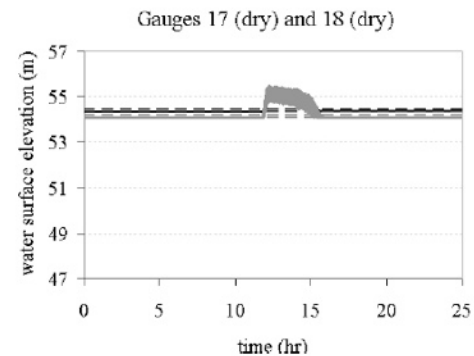
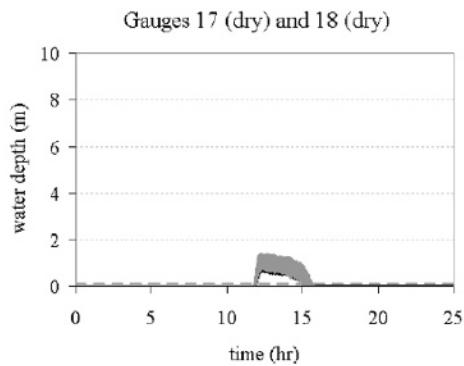
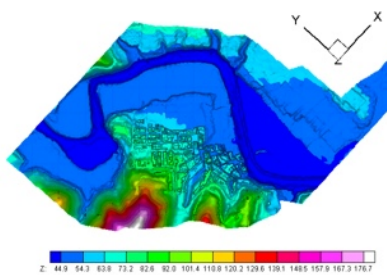
Gauges 15 (dry) and 16



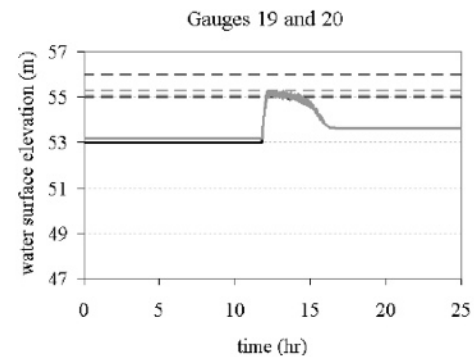
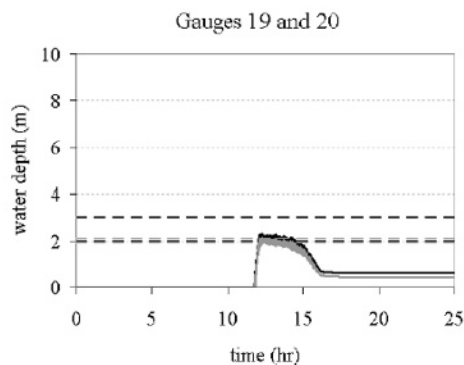
Gauges 15 (dry) and 16



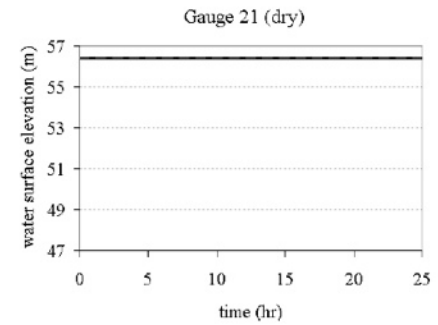
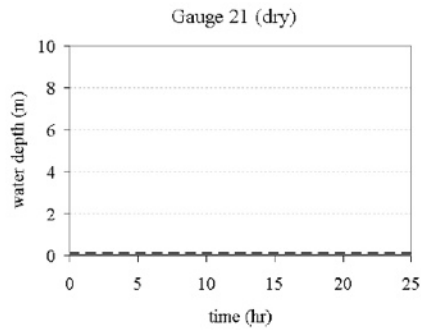
— G15 - - - G15(min,max) — G16 - - - G16(min,max)



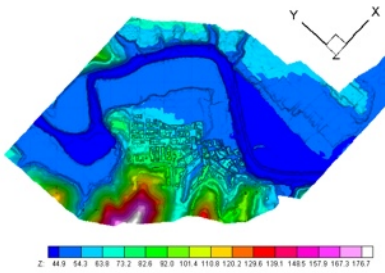
— G17 - - - G17(min,max) — G18 - - - G18(min,max)



— G19 - - - G19(min,max) — G20 - - - G20(min,max)

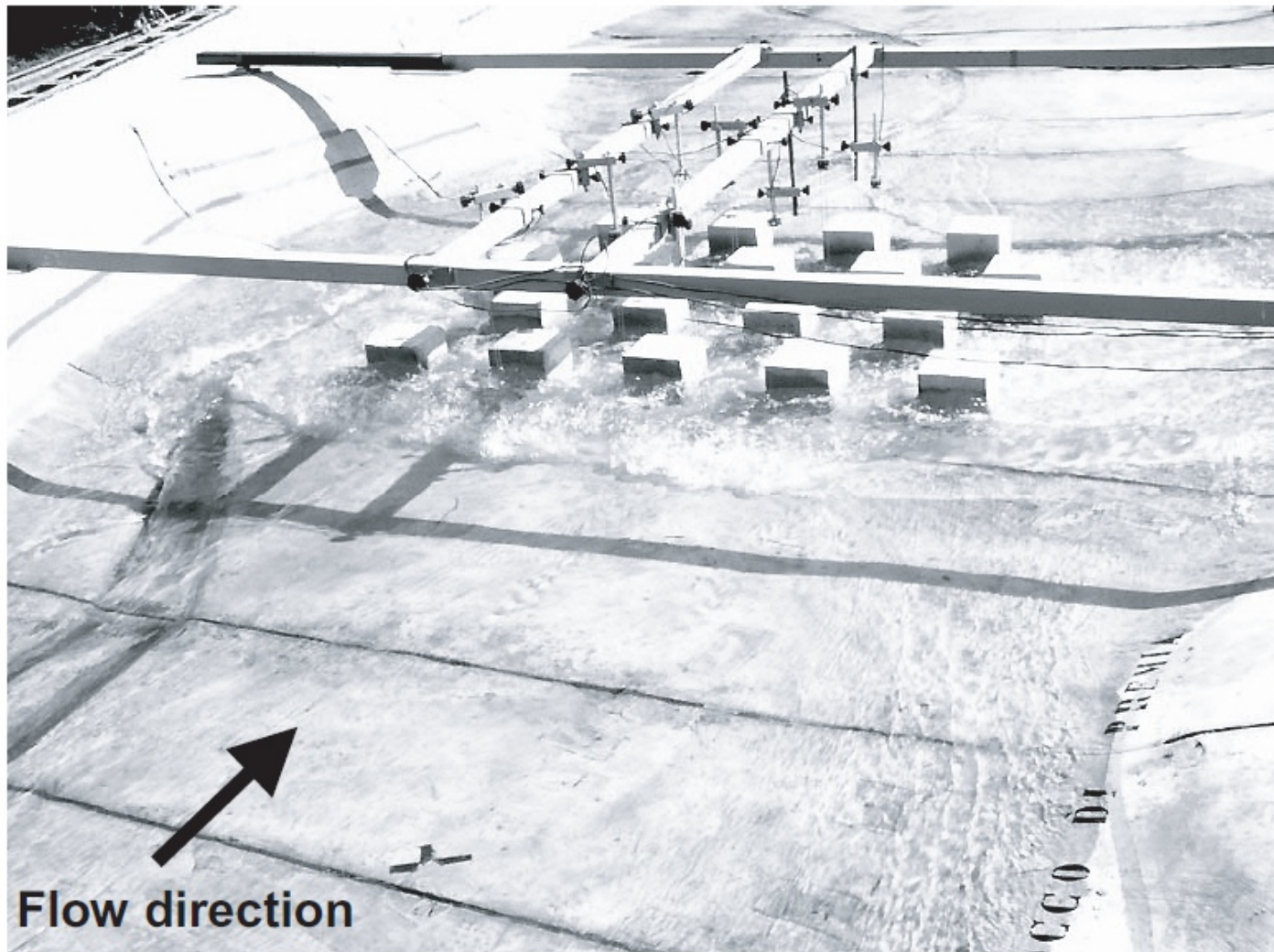


— G21 -- G21(min,max)

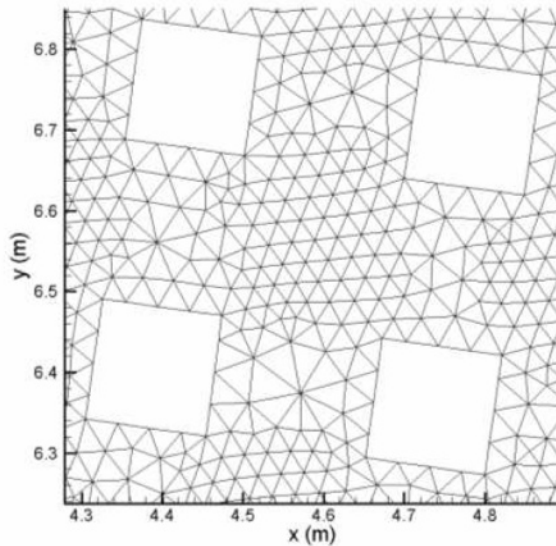
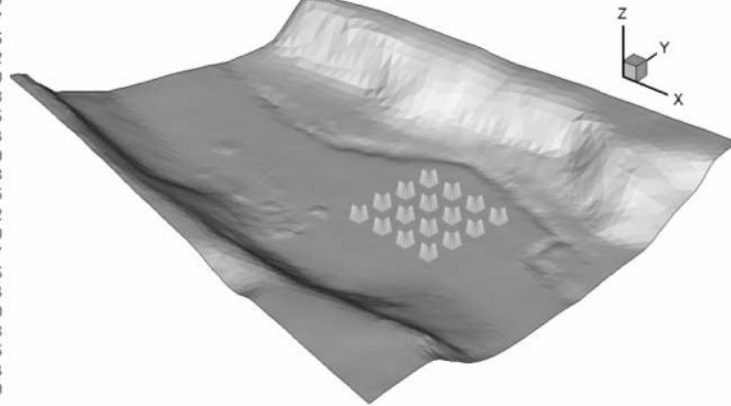
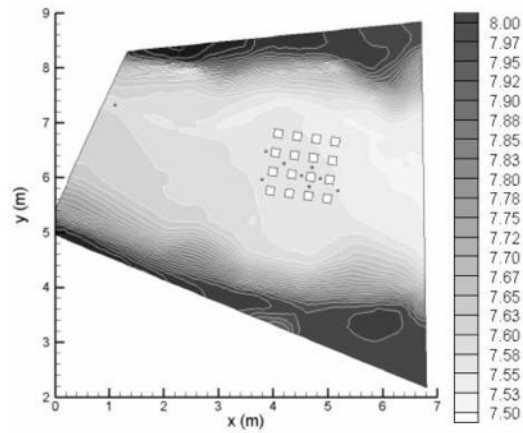


CASO TEST: MODELO A ESCALA

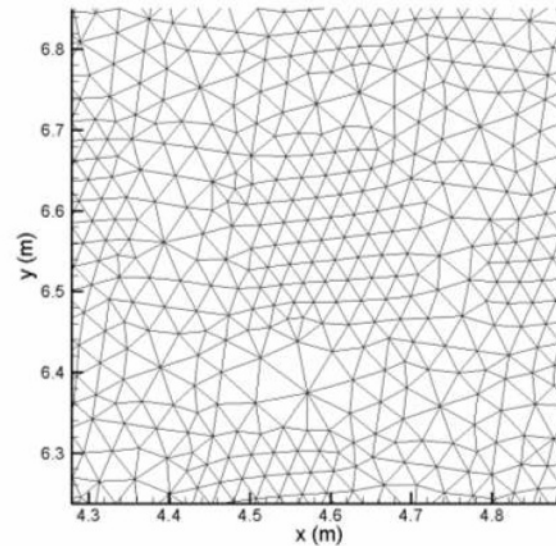
(Soares-Frazaio et al. *Journal of Hydraulic Research* 2007)



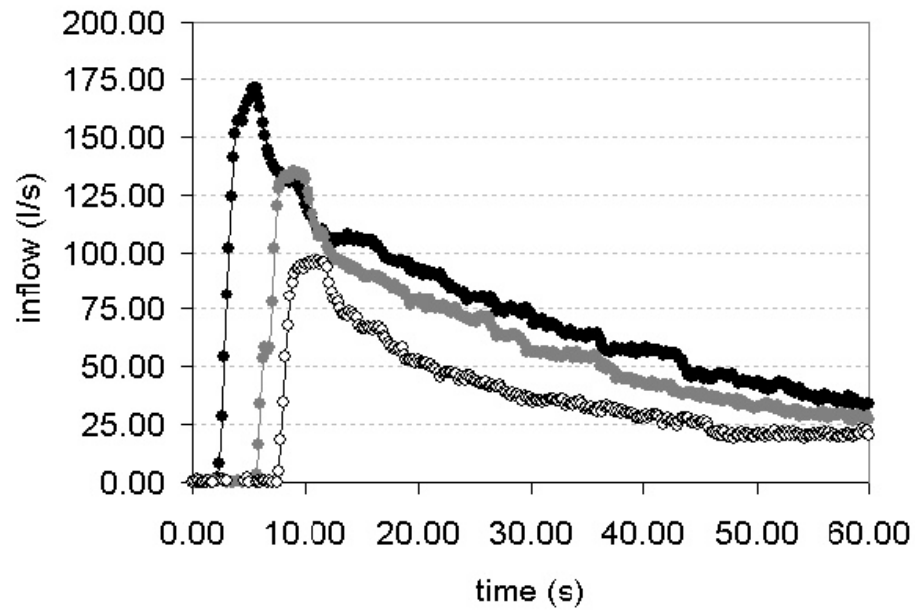
Caso 1: Casas alineadas



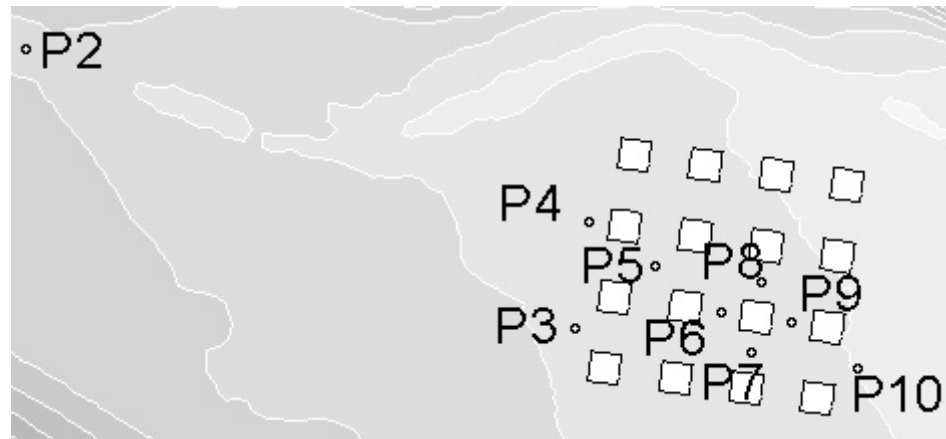
Huecos en la malla

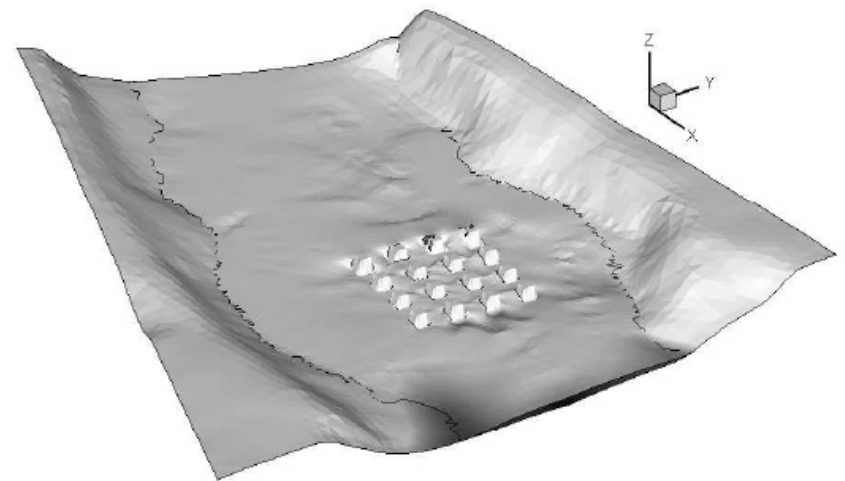
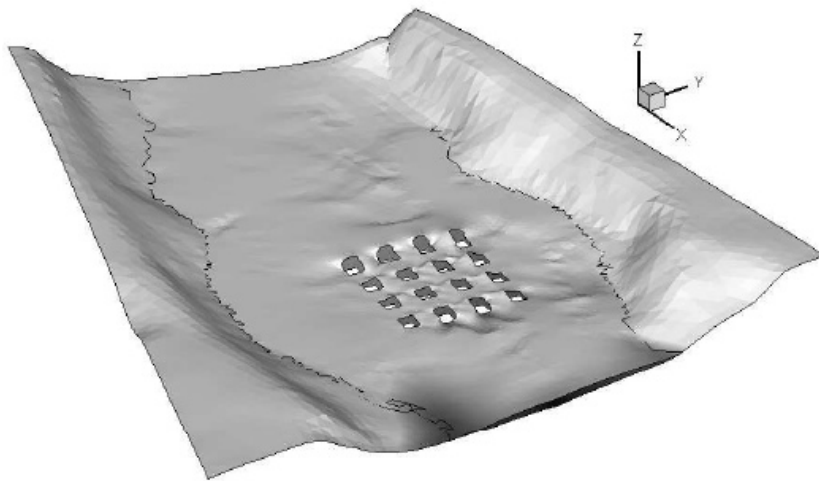
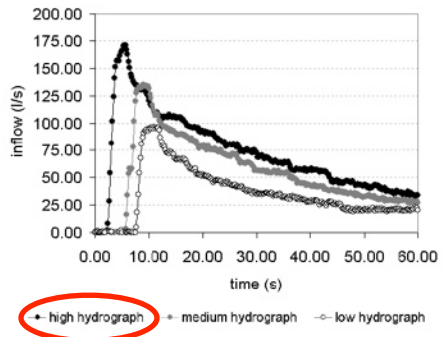


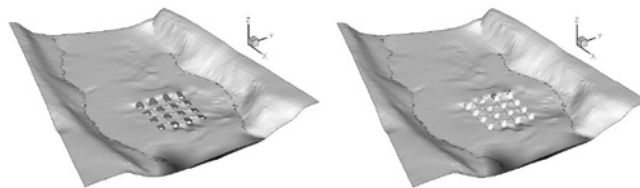
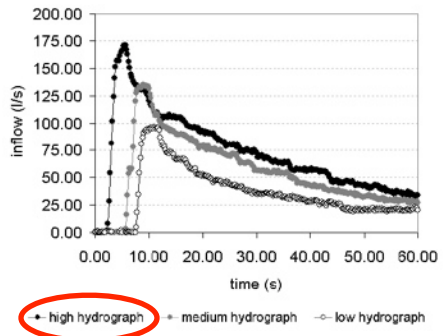
Elevaciones del terreno

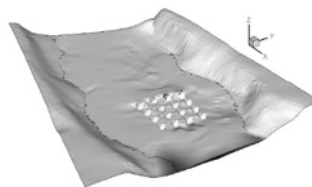
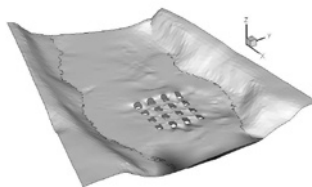
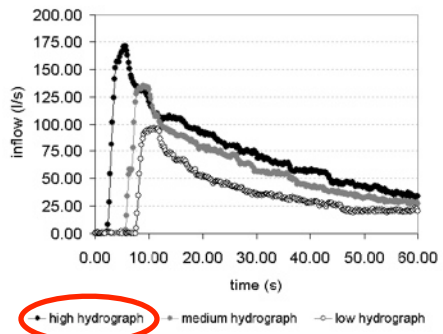


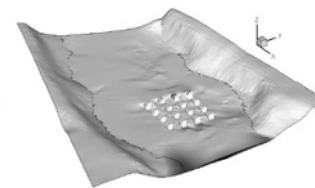
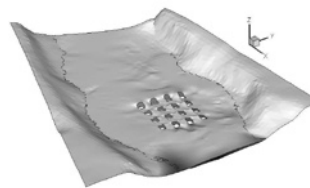
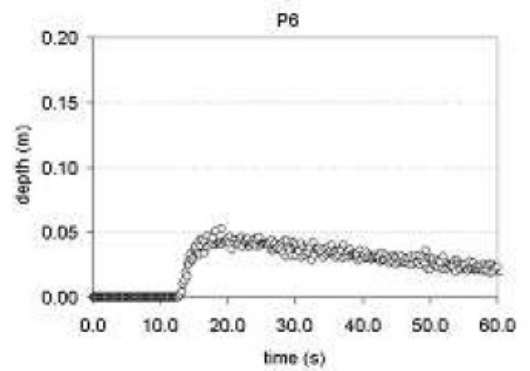
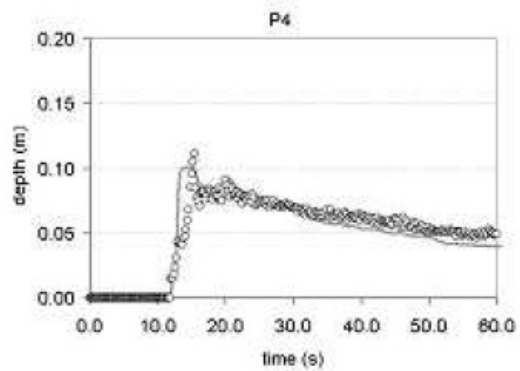
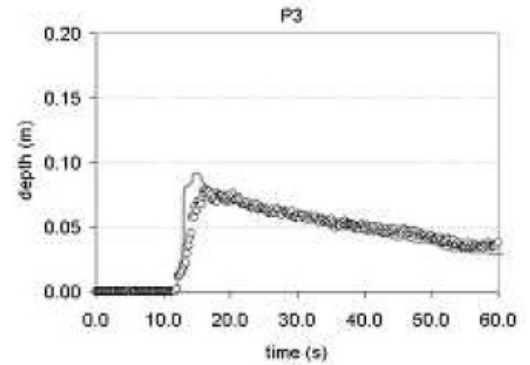
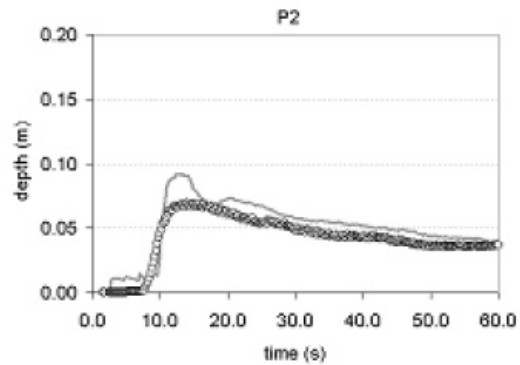
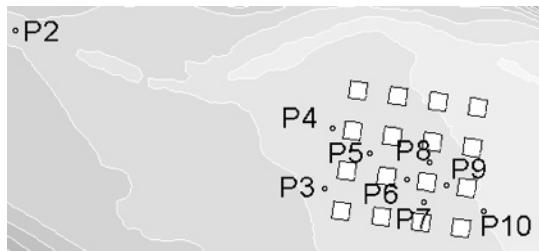
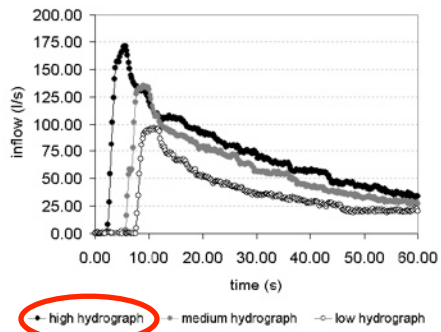
● high hydrograph ● medium hydrograph ○ low hydrograph

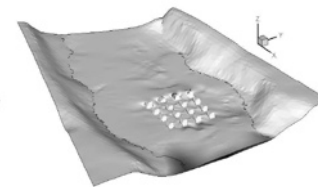
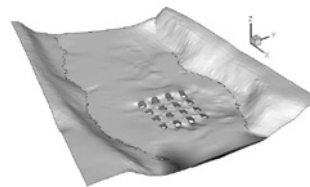
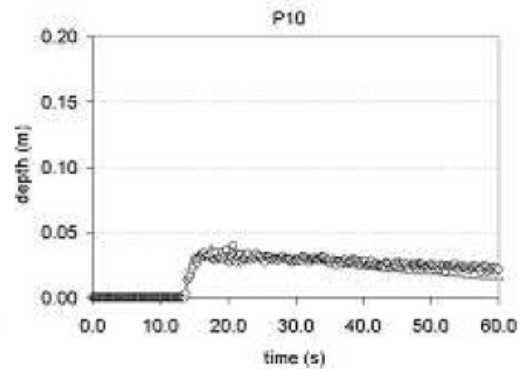
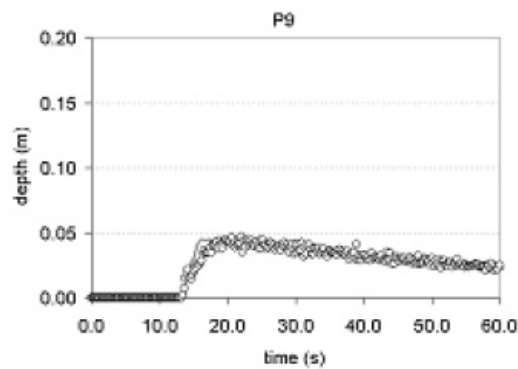
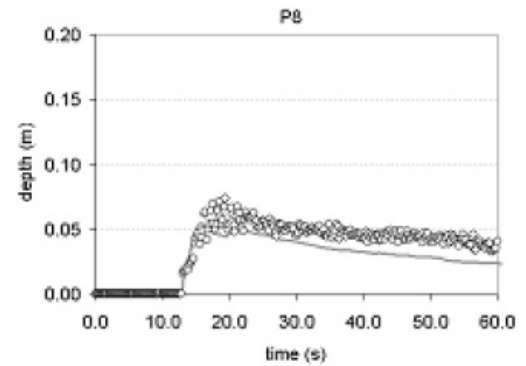
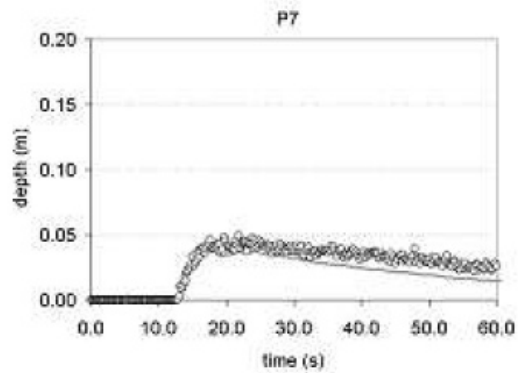
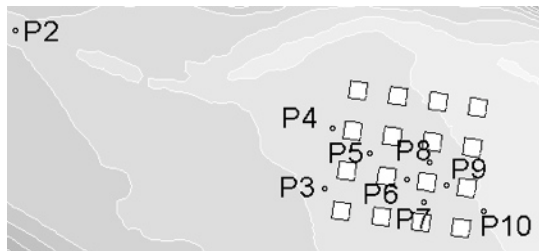
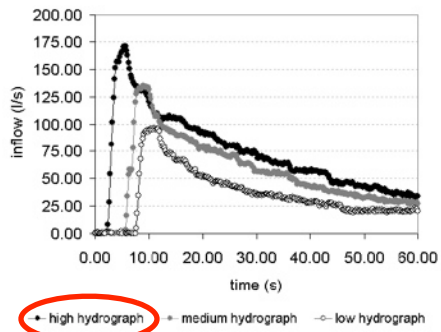












INCORPORACIÓN DEL TRANSPORTE DE UN SOLUTO PASIVO

$$\frac{\partial(h\phi)}{\partial t} + \vec{\nabla}(\mathbf{u} h\phi) = \vec{\nabla}(\mathbf{K} \vec{\nabla}(h\phi))$$

CONSERVACIÓN DE LA SUSTANCIA TRANSPORTADA

Acoplamiento flujo/soluto

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{H}$$

$$\mathbf{U} = (h, hu, hv, h\phi)^T$$

$$\mathbf{F} = (hu, hu^2 + \frac{gh^2}{2}, huv, hu\phi)^T$$

$$\mathbf{G} = (hv, huv, hv^2 + \frac{gh^2}{2}, hv\phi)^T$$

$$\mathbf{H} = (0, gh(S_{0x} - S_{fx}), gh(S_{0y} - S_{fy}), \nabla(\mathbf{K}h\nabla\phi))^T$$

Redefinición del Jacobiano

$$\mathbf{J}_n = \begin{pmatrix} 0 & n_x & n_y & 0 \\ \left(gh - \frac{q_x^2}{h^2}\right)n_x - \frac{q_x q_y}{h^2}n_y & \frac{q_y}{h}n_y + \frac{2q_x}{h}n_x & \frac{q_x}{h}n_y & 0 \\ \left(gh - \frac{q_y^2}{h^2}\right)n_y - \frac{q_x q_y}{h^2}n_x & \frac{q_y}{h}n_x & \frac{q_x}{h}n_x + \frac{2q_y}{h}n_y & 0 \\ -(un_x + vn_y)\phi & \phi n_x & \phi n_y & (un_x + vn_y) \end{pmatrix}$$

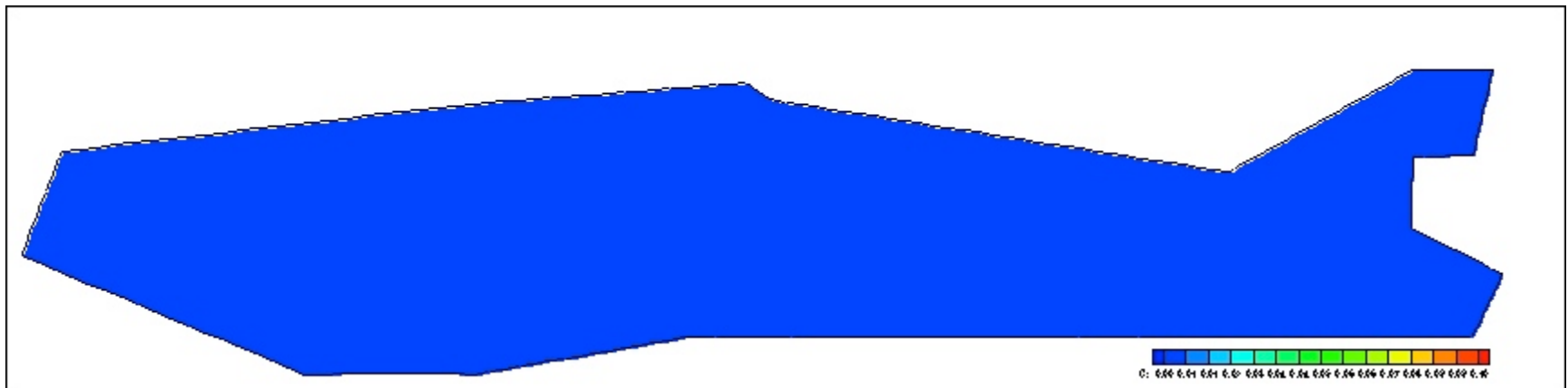
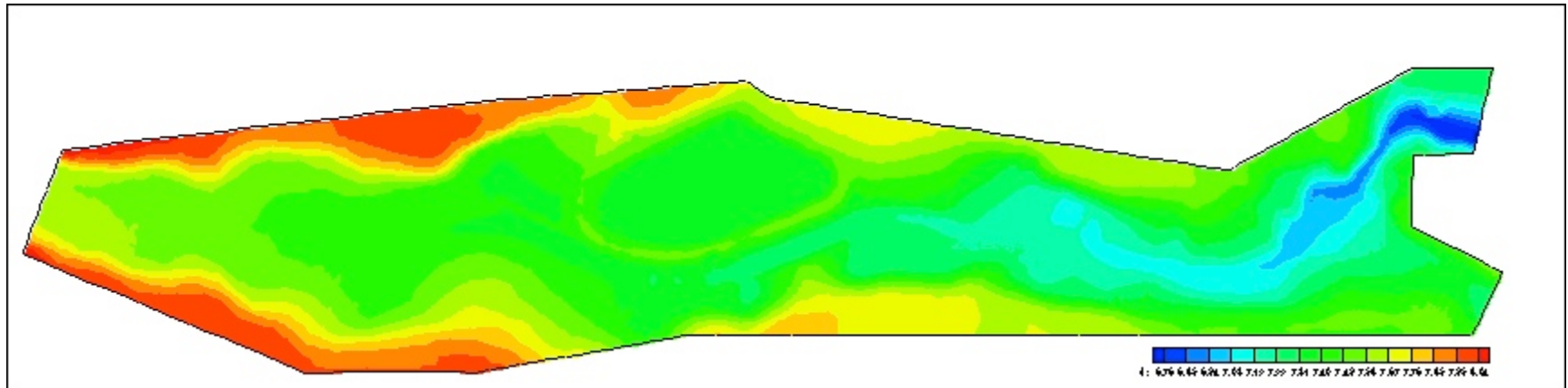
$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} ((\tilde{\lambda}^- \alpha - \beta^-) \tilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$

$$N\lambda = 4$$

Evolución de una mancha de vertido en flujo de inundación

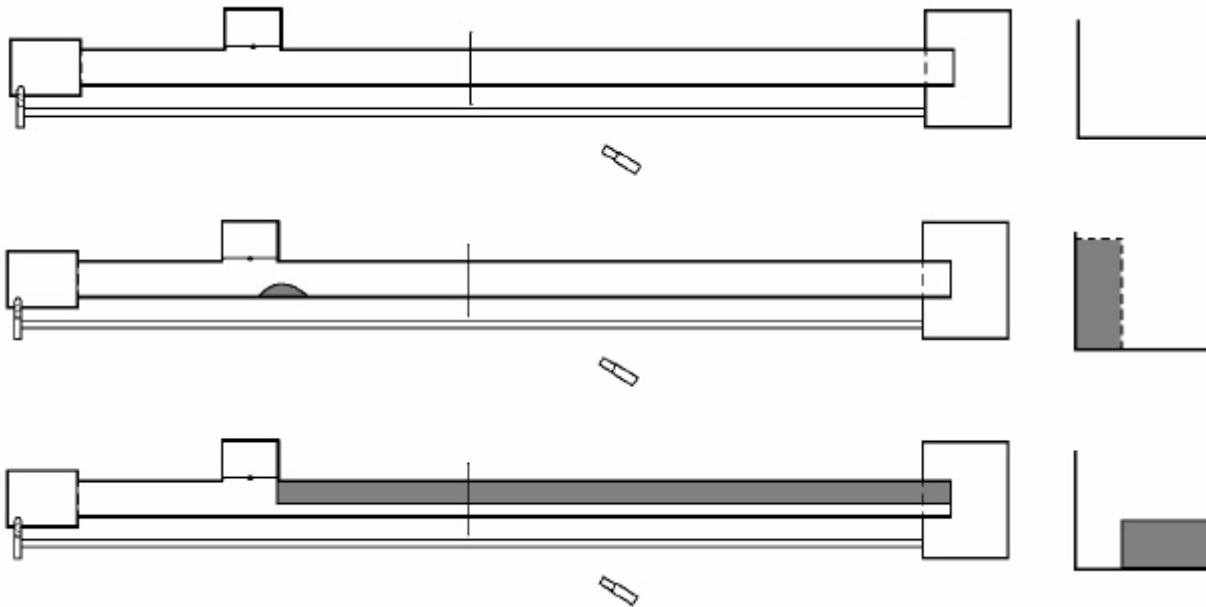
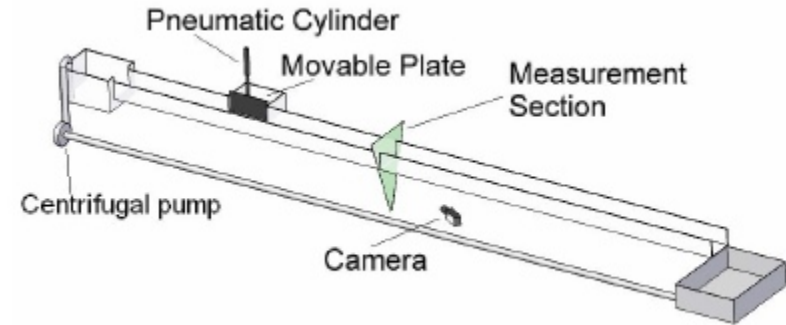


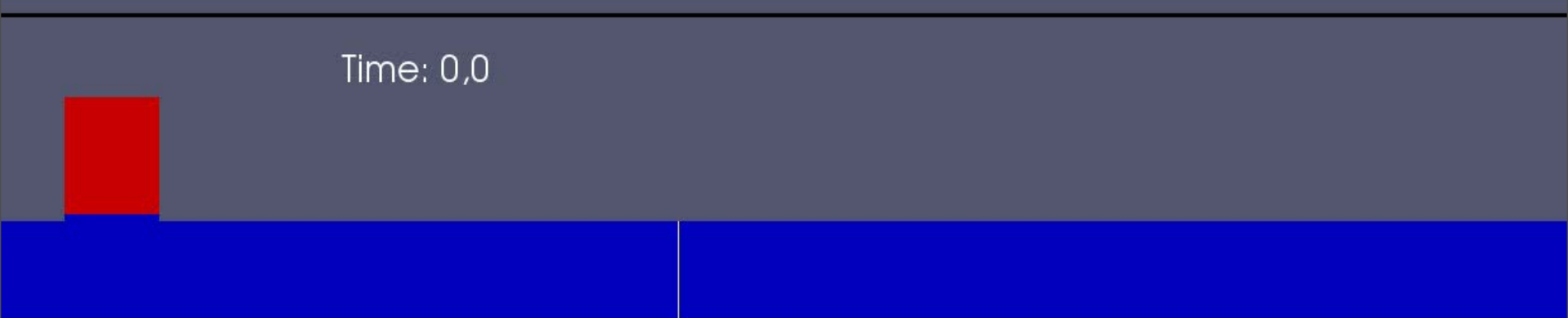
Evolución de una mancha de vertido en flujo de inundación



Trabajo experimental

Canal rectangular
Depósito lateral y compuerta
Laser Ar+. Camara CCD
3 Configuraciones

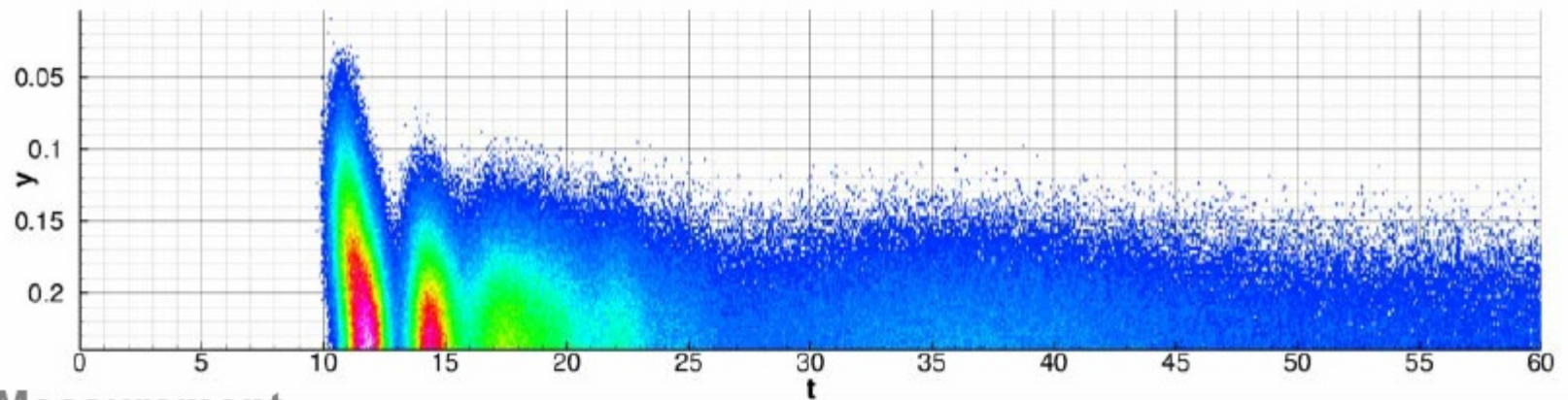




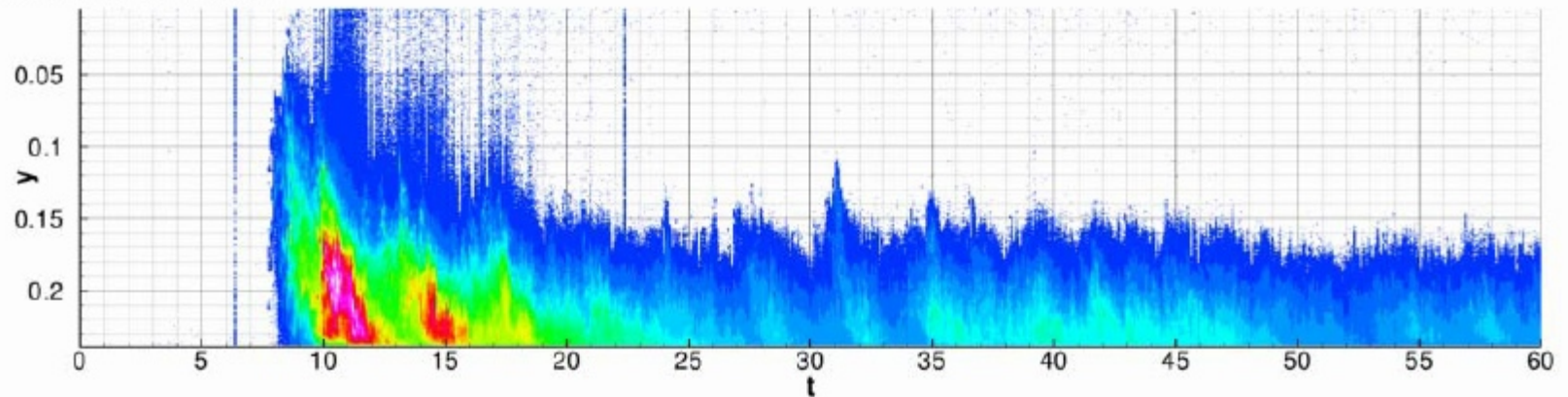
Calibración de la simulación numérica

Difusión numérica/Difusión real

Simulation



Measurement



Incorporación de fondo erosionable



Incorporación del flujo sobre lecho móvil
Modelos de caudal sólido arrastre de fondo
Discretización acoplada/desacoplada
Validación con soluciones exactas
Calibración con datos experimentales
Modelos de carga en suspensión

INCORPORACIÓN DEL LECHO MÓVIL

$$(1 - p) \frac{\partial z}{\partial t} + \frac{\partial q_{s,x}}{\partial x} + \frac{\partial q_{s,y}}{\partial y} = 0$$

Modelo 2D matemático propuesto (Grass)

$$q_{s,x} = A'_g u (u^2 + v^2) \quad q_{s,y} = A'_g v (u^2 + v^2)$$

$$S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}} \quad S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}}$$

Formulación acoplada

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{H}$$

$$\mathbf{U} = \begin{pmatrix} h \\ hu \\ hv \\ z_b \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + \frac{gh^2}{2} \\ huv \\ \xi q_{sx} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + \frac{gh^2}{2} \\ \xi q_{sy} \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} 0 \\ -gh \left(\frac{\partial z_b}{\partial x} + S_{fx} \right) \\ -gh \left(\frac{\partial z_b}{\partial y} + S_{fy} \right) \\ 0 \end{pmatrix}$$

Diferenciabilidad del caudal sólido

¿Existencia de una matriz Jacobiana?

Carácter hiperbólico del sistema ampliado

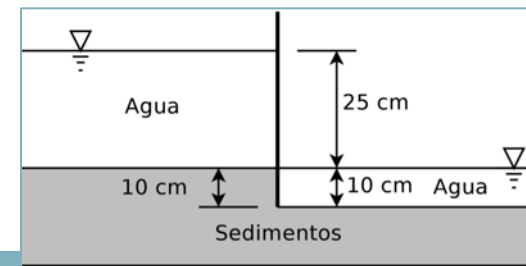
$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{J} \nabla \mathbf{U} = \mathbf{H} \quad \Rightarrow \quad \frac{\partial \mathbf{U}}{\partial t} + \mathbf{J}^* \nabla \mathbf{U} = \mathbf{H}_{friccion}$$

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} ((\tilde{\lambda}^- \alpha - \beta^-) \tilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$

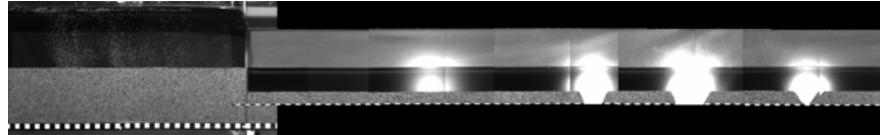
$$N\lambda = 4$$

Validación con datos de laboratorio

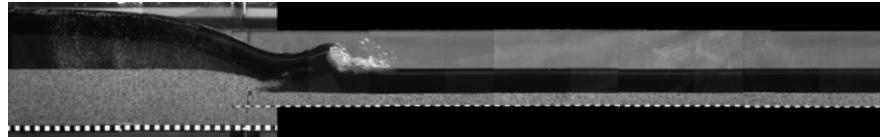
(B. Spinewine, Y. Zech. *Journal of Hydraulic Research* 2007.)



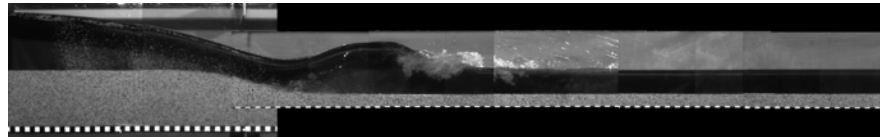
t = 0.000 s



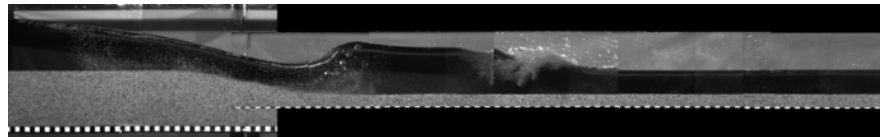
t = 0.250 s



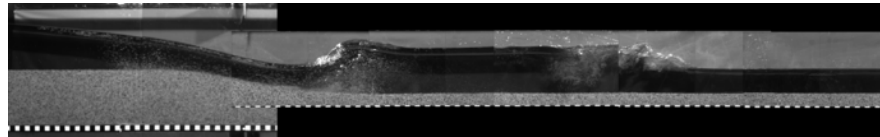
t = 0.500 s



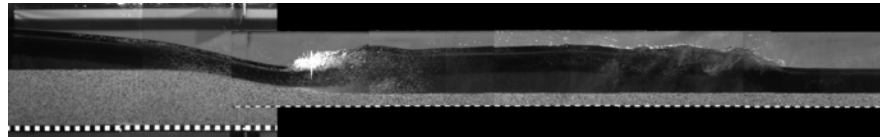
t = 0.750 s



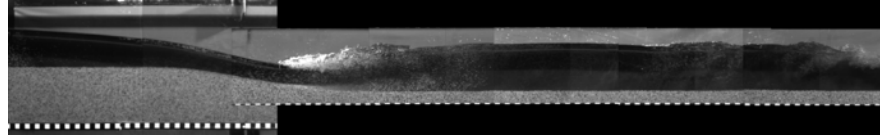
t = 1.000 s

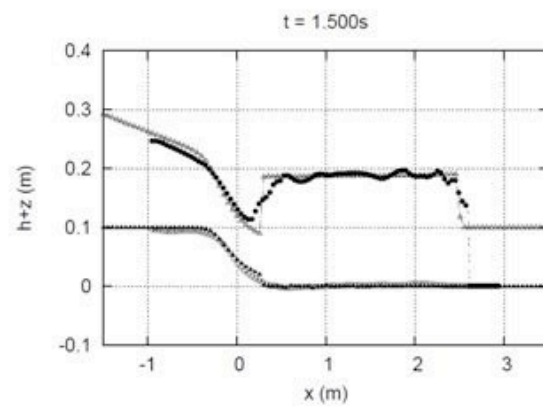
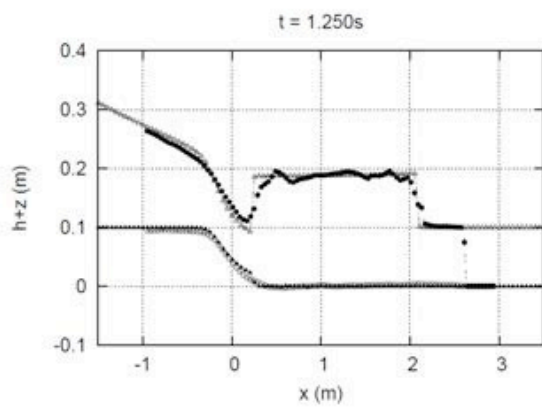
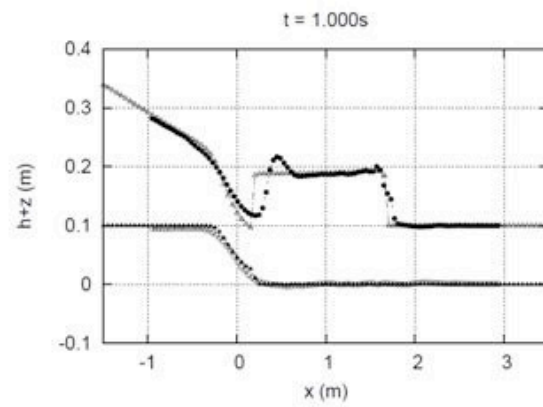
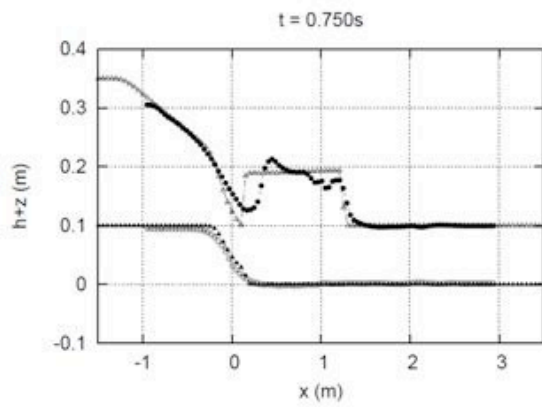
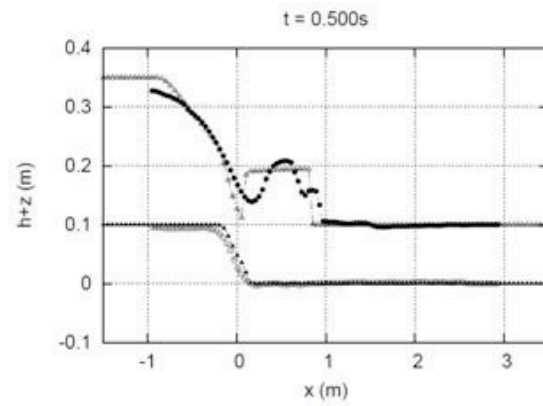
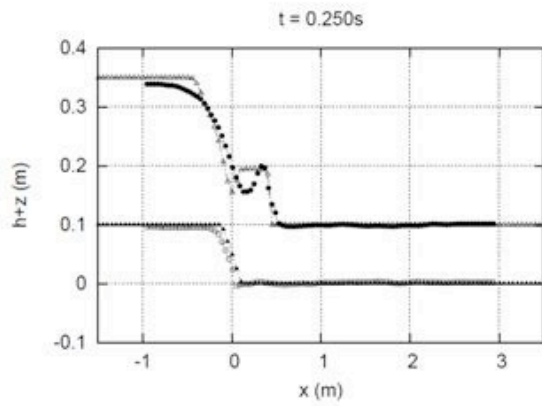


t = 1.250 s

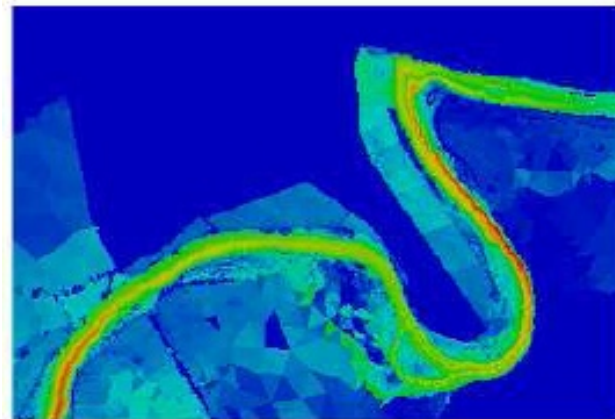
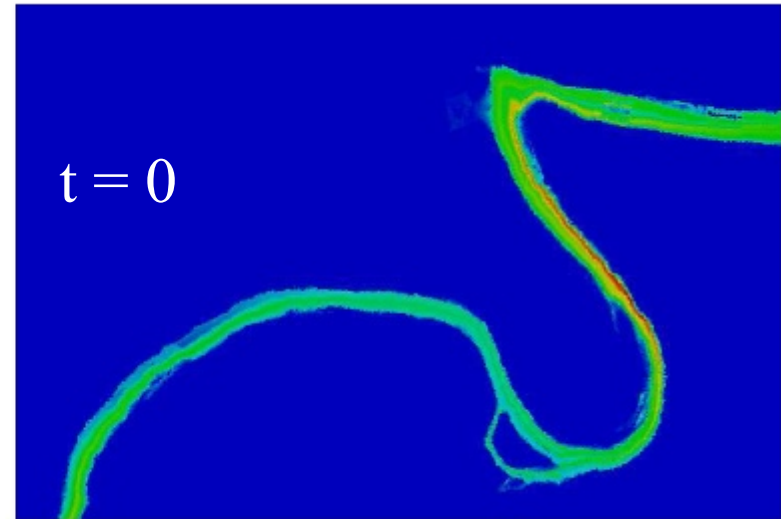
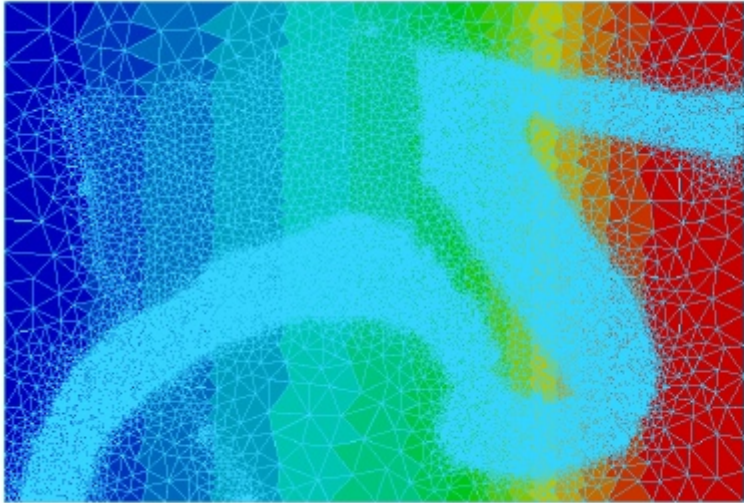


t = 1.500 s





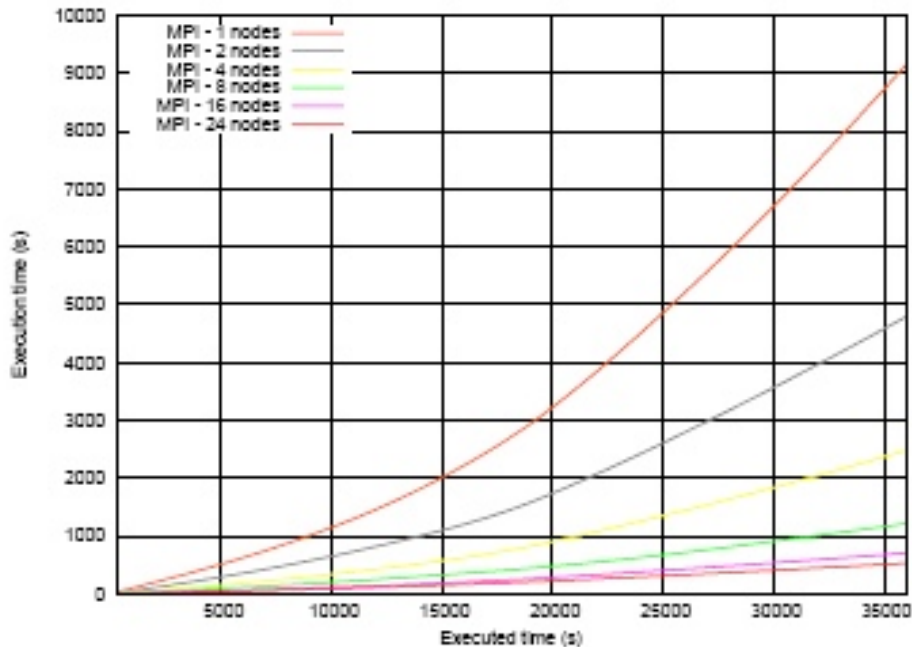
Paralelización en memoria distribuida



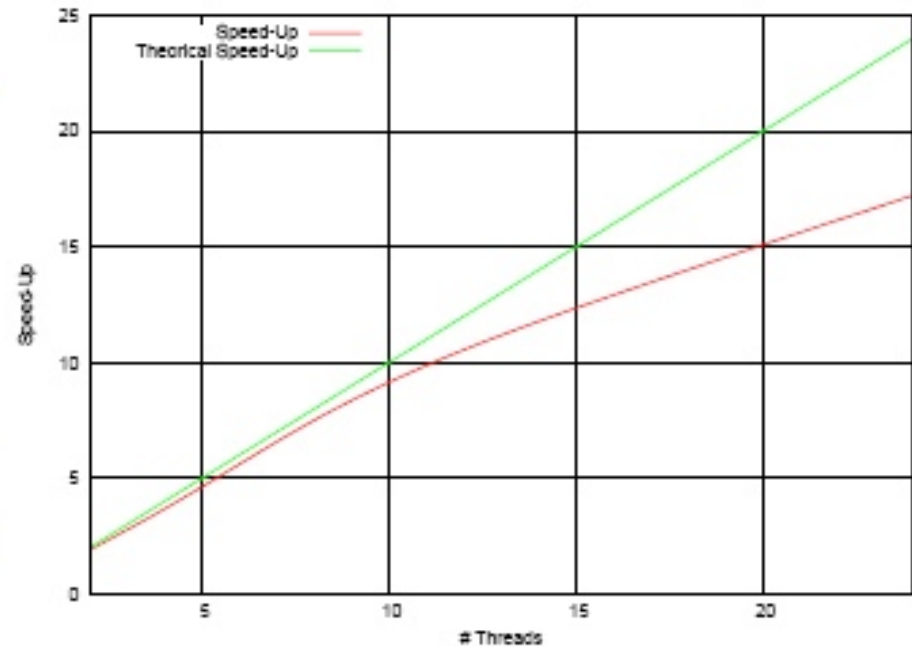
$Q=2500 \text{ m}^3/\text{s}$
 $t = 35000 \text{ s}$

Paralelización en memoria distribuida

Equipo: Cluster con 7 procesadores Intel Core i7 2.8GHz
(4 cores por procesador)



(a) Tiempos de ejecución frente a tiempos ejecutados



(b) Speed-up en 36500 s. de simulación

GRACIAS POR VUESTRA
ATENCIÓN

